Homework 9.

1. Symbolize, taking the domain of discourse to be persons, and using *only* the following vocabulary:

$Pxy \equiv x$ is a parent of y	$Mx \equiv x$ is male	$Ixy \equiv x$ is identical to y
$Lxy \equiv x$ loves y	$Txy \equiv x$ is taller than y	
a = Alice	b = Bob	

   (a) Everyone has two grandfathers.

   (b) There is a person who has no first cousins.

   (c) Everyone except Alice loves Bob.

   (d) Alice is Bob's only daughter.

   (e) Alice is the tallest of Bob's children.

2. For each of the following sentences, find an interpretation with domain $\{1, 2, 3, 4\}$ and nonempty extension of "Fxy" that makes the sentence true, and another such interpretation that makes the sentence false.

   (a) $(x)(y)(Fxy \rightarrow (\exists z)(Fxz \,\&\, Fyz))$

   (b) $(x)((y)(Fyx \rightarrow Fxy) \rightarrow (y)(Fxy \rightarrow Fyx))$

   (c) $(\exists x)(\exists y)(Fxy \,\&\, Fyx) \,\&\, (x)(y)((\exists z)(Fxz \,\&\, Fzy) \rightarrow Fxy)$

3. For each of the following pairs of sentences, give an interpretation that shows that the first sentence does not imply the second.

   (a) $(\exists x)(y) - Fxy \,\&\, (\exists x)(y)Fxy$ $\qquad$ $(x)((\exists y)Fxy \rightarrow (y)Fxy)$

   (b) $(x)(-Lx \rightarrow (\exists y)(Ly \,\&\, Ayx))$ $\qquad$ $(x)(-Lx \rightarrow (y)(Ly \rightarrow Ayx))$

   (c) $(x)(\exists y)(Gxy \,\&\, -Gyx)$ $\qquad$ $(x)((\exists y)Gxy \rightarrow (\exists y)Gyx)$

4. Construct a proof of **one** of the following two arguments. (You may use any of the inference rules.)

   (a) (1) $(x)(((\exists y)Gxy \vee (\exists z)Gzx) \rightarrow Gxx)$
   $// \ (x)(y)(Gxy \rightarrow (Gxx \,\&\, Gyy))$

   (b) (1) $(x)(y)(Gxy \rightarrow (Gxx \,\&\, Gyy))$
   $// \ (x)(((\exists y)Gxy \vee (\exists z)Gzx) \rightarrow Gxx)$