Homework 8 Key

A. For each of the following sentences, give an interpretation with domain \{1, 2, 3, 4\} and nonempty extension of “Rxy” that makes the sentence true, and another such interpretation that makes the sentence false.

1. \((\exists x)(y)(Ryx \rightarrow Ryy)\)
   To make true: Ext(Rxy) = \{<1,1>\}
   To make false: Ext(Rxy) = \{<1,2>, <2,3>, <3,4>, <4,1>\}

2. \((x)(y)(Rxy \rightarrow (\exists z)(Rxz \& Ryz))\)
   To make true: Ext(Rxy) = \{<1,1>\}
   To make false: Ext(Rxy) = \{<1,2>\}

3. \((x)[(y)(Ryx \rightarrow Rxy) \rightarrow (y)(Rxy \rightarrow Ryx)]\)
   To make true: Ext(Rxy) = \{<1,1>\}
   To make false: Ext(Rxy) = \{<1,2>\}

4. \((\exists x)(\exists y)(Rxy \& Ryx) \& (x)(y)[(\exists z)(Rxz \& Rzy) \rightarrow Rxy]\)
   To make true: Ext(Rxy) = \{<1,1>\}
   To make false: Ext(Rxy) = \{<1,2>\}
B. For each of the following pairs of sentences, give an interpretation that shows that the first sentence does not imply the second.

1. \((x)(\exists y)(Rxy \lor Ryx)\) \hspace{1cm} \((x)(\exists y)Rxy \lor (x)(\exists y)Ryx\)

   Domain = \{1,2\} \hspace{1cm} \text{Ext}(Rxy) = \{<1,2>\}

   \[
   \begin{array}{c}
   1 \\
   \hline
   2
   \end{array}
   \]

2. \((\exists x)(y) \sim Rxy \& (\exists x)(y)Rxy\) \hspace{1cm} \((x)[(\exists y)Rxy \to (y)Rxy]\)

   Domain = \{1,2,3\} \hspace{1cm} \text{Ext}(Rxy) = \{<1,1>, <1,2>, <1,3>, <2,3>\}

   \[
   \begin{array}{c}
   1 \\
   \hline
   2 \\
   \hline
   3
   \end{array}
   \]

3. \((y)[(\exists z)Ryz \to (\exists z)Rzy]\) \hspace{1cm} \((y)[(z)Ryz \to (z)Rzy]\)

   Domain = \{1,2\} \hspace{1cm} \text{Ext}(Rxy) = \{<1,1>, <1,2>\}

   \[
   \begin{array}{c}
   1 \\
   \hline
   2
   \end{array}
   \]

4. \((x)(\exists y)(Rxy \& \sim Ryx)\) \hspace{1cm} \((x)[(\exists y)Rxy \to (\exists y)Ryx]\)

   Domain = \{1,2,3,4\} \hspace{1cm} \text{Ext}(Rxy) = \{<1,2>, <2,3>, <3,4>, <4,2>\}

   \[
   \begin{array}{c}
   1 \\
   \hline
   2 \\
   \hline
   3 \\
   \hline
   4
   \end{array}
   \]