

Practice Proofs

1 Proofs that use only Stage 1 rules

1. Associativity of $\wedge$

$$P \wedge (Q \wedge R) \dashv\vdash (P \wedge Q) \wedge R$$

2. Commutativity of $\wedge$

$$P \wedge Q \vdash Q \wedge P$$

3. Idempotence of $\wedge$

$$P \dashv\vdash P \wedge P$$

2 Proofs that use CP but not $\vee$ -Elimination or RAA

1. Weakening the Consequent

$$P \rightarrow (Q \wedge R) \vdash P \rightarrow Q$$

$$P \rightarrow Q \vdash P \rightarrow (Q \vee R)$$

2. Strengthening the Antecedent

$$(P \vee Q) \rightarrow R \vdash P \rightarrow R$$

$$P \rightarrow R \vdash (P \wedge Q) \rightarrow R$$

3. Transitivity

$$P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$$

4. Contrapositive

$$P \rightarrow Q \vdash \neg Q \rightarrow \neg P$$

5. Prefixing

$$P \rightarrow Q \vdash (R \rightarrow P) \rightarrow (R \rightarrow Q)$$

6. Suffixing

$$P \rightarrow Q \vdash (Q \rightarrow R) \rightarrow (P \rightarrow R)$$

7. Contraction

$$P \rightarrow (P \rightarrow Q) \dashv\vdash P \rightarrow Q$$

8. Permutation

$$P \rightarrow (Q \rightarrow R) \vdash Q \rightarrow (P \rightarrow R)$$

9. Fusion

$$P \rightarrow (Q \rightarrow R) \dashv\vdash (P \wedge Q) \rightarrow R$$

10. Positive Paradox

$$P \vdash Q \rightarrow P$$

11. [No name]

$$\neg(P \wedge Q) \rightarrow \neg(R \vee S) \vdash R \rightarrow Q$$

12. [No name]

$$(P \wedge Q) \rightarrow R, P \rightarrow Q \vdash P \rightarrow R$$

13. [No name]

$$P \vdash Q \rightarrow (P \wedge Q)$$

14. [No name]

$$P \rightarrow (Q \wedge R) \dashv\vdash (P \rightarrow Q) \wedge (P \rightarrow R)$$

15. [No name]

$$P \vdash (P \rightarrow Q) \rightarrow Q$$

16. [No name]

$$P \rightarrow (Q \rightarrow R) \vdash (P \rightarrow Q) \rightarrow (P \rightarrow R)$$

17. [No name]

$$\vdash P \rightarrow P$$

3 Proofs that use $\vee E$ but not RAA

1. Distribution

$$P \wedge (Q \vee R) \dashv\vdash (P \wedge Q) \vee (P \wedge R)$$

$$P \vee (Q \wedge R) \dashv\vdash (P \vee Q) \wedge (P \vee R)$$

2. [No name]

$$P \rightarrow Q, R \rightarrow S \vdash (P \vee R) \rightarrow (Q \vee S)$$

3. Commutativity of $\vee$

$$P \vee Q \vdash Q \vee P$$

4. Associativity of $\vee$

$$P \vee (Q \vee R) \dashv\vdash (P \vee Q) \vee R$$

5. Idempotence of $\vee$

$$P \dashv\vdash P \vee P$$

4 Proofs that use RAA

1. Ex Falso Quodlibet

$$P, \neg P \vdash Q$$

2. [No name]

$$P \rightarrow \neg P \dashv\vdash \neg P$$

3. [No name]

$$\neg P \rightarrow P \dashv\vdash P$$

4. Negative Paradox

$$\neg P \vdash P \rightarrow Q$$

5. Chain Order

$$\vdash (P \rightarrow Q) \vee (Q \rightarrow P)$$

6. [No name]

$$\vdash (P \rightarrow Q) \vee (Q \rightarrow R)$$

7. Material Implication

$$P \rightarrow Q \dashv\vdash \neg P \vee Q$$

$$\neg(P \rightarrow Q) \dashv\vdash P \wedge \neg Q$$

8. De Morgan's Laws

$$\neg(P \vee Q) \dashv\vdash \neg P \wedge \neg Q$$

$$\neg(P \wedge Q) \dashv\vdash \neg P \vee \neg Q$$

9. Modus Tollendo Ponens (Disjunctive Syllogism)

$$P \vee Q, \neg P \vdash Q$$

10. Modus Ponendo Tollens

$$\neg(P \wedge Q), P \vdash \neg Q$$

11. [No name]

$$(P \rightarrow Q) \rightarrow Q \vdash P \vee Q$$

12. Peirce's Law

$$\vdash ((P \rightarrow Q) \rightarrow P) \rightarrow P$$

13. Excluded Middle

$$\vdash P \vee \neg P$$

14. Law of Non-Contradiction

$$\vdash \neg(P \wedge \neg P)$$

15. [No name]

$$(P \rightarrow Q) \vee (P \rightarrow R) \dashv\vdash P \rightarrow (Q \vee R)$$

16. [No name]

$$(P \wedge Q) \rightarrow R \dashv\vdash P \rightarrow (\neg Q \vee R)$$

17. [No name]

$$(P \wedge Q) \rightarrow R \dashv\vdash P \rightarrow \neg(Q \wedge \neg R)$$

18. Tautologies absorbed by Conjunctions

$$P \wedge (Q \vee \neg Q) \dashv\vdash P$$

19. Inconsistencies absorbed by Disjunctions

$$P \vee (Q \wedge \neg Q) \dashv\vdash P$$

20. Biconditional

$$P \leftrightarrow Q \dashv\vdash (P \wedge Q) \vee (\neg P \wedge \neg Q)$$

$$\neg(P \leftrightarrow Q) \dashv\vdash (\neg P \leftrightarrow Q)$$

$$\neg(P \leftrightarrow Q) \dashv\vdash (P \vee Q) \wedge (\neg P \vee \neg Q)$$

$$\vdash (P \leftrightarrow Q) \vee (P \leftrightarrow R) \vee (Q \leftrightarrow R) \quad \text{Warning: Do not try this at home.}$$

Updated: March 7, 2012