PHI 340: Final Exam Study Guide

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The Final Exam will be cumulative, but with large emphasis on those topics covered after the midterm exam.

I. A reminder of what we covered before the midterm exam:

- Basics of set theory. Definitions, axioms, how to prove things.
  - Cartesian products; relations
  - Functions
  - Inductively defined sets

  *There will be no questions on set theory for its own sake.*

- Basics of languages and logics.
  - What are the components of a language?
  - What are the valuations of classical propositional logic (CPL)?
  - What is the definition of \( \mathcal{X} \models A \)?
  - What is a tautology?
  - What is a base for a valuation? Do all valuations have bases?
  - When is one language an extension of another?
  - What is a logic?
  - Define: soundness, completeness, argument completeness, strong completeness, compactness.
  - What is the difference between \( \models \) and \( \vdash \)?
  - Calculus of deductive systems. e.g. be able to show that: \( Cn(Cn(\mathcal{X})) = Cn(\mathcal{X}) \); if \( \mathcal{X} \subseteq \mathcal{Y} \) then \( Cn(\mathcal{X}) \subseteq Cn(\mathcal{Y}) \).

  *There will be no questions about extensions of languages.*

- Modal logics: K, D, T, S4, B, S5, NN, NN''.
  - Models and restrictions on the accessibility relation \( \mathcal{R} \).
  - What is the relationship between “models” and “valuations”?
– Tableau methods for the normal modal logics (K, D, T, S4, B, S5).
– Characteristic sentences for the normal modal logics.
– Paradoxes of material implication.
– What are the valuations of NN?
– The rule of necessitation:
  \[ \text{If } A_1, \ldots, A_n \models B \text{ then } \Box A_1, \ldots, \Box A_n \models \Box B. \]
  For which modal logics is this true?
– Natural deduction for S5. (Optional: You will have a choice between natural deduction problems and problems that don’t use a system of natural deduction.)
– Do the valuations of S5, S4, etc. have bases?

II. The main emphases of the final exam:

**Intuitionistic Logic**

- You should be able to decide whether or not an argument is valid in intuitionistic logic. If it is valid, you should either be able to give a Lemmon style proof, or you should be able to produce a tableaux with all branches dead. If it is invalid, you should be able to give a Kripke (possible worlds) model demonstrating its invalidity.
- You should be able to sketch the argument that intuitionistic logic does not permit a finite-valued semantics.
- You should understand how intuitionistic logic can be translated into S4.

**Relevance Logic**

- FDE tableaux or the FDE algorithm for deciding validity. (You need to be proficient at one of these two methods.)
- You should be able to prove that in FDE there are no tautologies or contradictions. (In general, you should be proficient at these sorts of arguments using induction on the construction of sentences.)
• You should be able to give counterexamples using Sugihara’s semantics.

• You should be able to give natural deduction proofs of arguments for the languages tAC, R, and RM. (Recall that “taC” is the logic that assumes $t = $, associativity, and commutativity of the semicolon.) e.g. you should be able to prove “$\vdash P \lor \neg P$” in the logic R.

Of course, you should understand all of this stuff so well that you can answer integrative questions that ask you to compare and contrast the various logics. You should also be able to answer conceptual questions about what motivations there are for the various logics, especially what conceptual problems of classical propositional logic they can and cannot solve.