object cannot be asserted by means of a sentence"; *translation:* "A sentence in which a property-word of a certain kind occurs is not a sentence." Sentence 13 means: "The higher facts cannot be expressed by means of sentences"; *translation:* "The higher sentences are not sentences."

Let it be once more called to mind that the distinction between the formal and the material modes of speech does not refer to genuine object-sentences and therefore not to the sentences of the empirical sciences, or to sentences of this kind which occur in the discussions of the logic of science (or of philosophy). (See the three columns, on p. 286.) It is here a question of the sentences of the proper logic of science. According to the ordinary use of language it is customary to formulate these partly in the form of logical sentences and partly in the form of object-sentences. Our investigations have shown that the supposititious object-sentences of the logic of science are pseudo-object-sentences, or sentences which apparently speak about objects, like the real object-sentences, but which in reality are speaking about the designations of these objects. This implies that all the sentences of the logic of science are logical sentences; that is to say, sentences about language and linguistic expressions. And our investigations have further shown that all these sentences can be formulated in such a way as to refer not to sense and meaning but to the syntactical form of the sentences and other expressions—they can all be translated into the formal mode of speech, or, in other words, into syntactical sentences. *The logic of science is the syntax of the language of science.*

B. THE LOGIC OF SCIENCE AS SYNTAX

§ 82. The Physical Language

The logical analysis of physics—as a part of the logic of science—is the syntax of the physical language. All the so-called epistemological problems concerning physics (in so far as it is not a question of metaphysical pseudo-problems) are in part empirical questions, the majority of which belong to psychology, and in part logical questions which belong to syntax. A more exact exposition of the logical analysis of physics as the syntax of the physical language must be left for a special investigation. Here we shall only offer a few suggestions towards it.

The logical analysis of physics will have, in the first place, to formulate *rules of formation* for sentences and other kinds of expressions of the physical language (see § 40). The most important expressions which occur as arguments are the point-expressions (designations of a spatio-temporal point, consisting of four real-number expressions, namely, three space-co-ordinates and one time-co-ordinate) and the domain-expressions (designations of a limited space-time domain). The physical coefficients of states are represented by descriptive functors. The descriptive functors and predicates can be divided into those having point-expressions and those having domain-expressions as arguments.

The sentences can be classified according to their degree of generality. We will here only discuss the two extreme kinds of sentences and, for the sake of simplicity, only those in which all the interior arguments are point- or domain-expressions: the *concrete sentences* contain no unrestricted variables; the *laws* contain no constants as interior arguments.

Either L-rules alone, or L-rules and P-rules, can be laid down as *transformation rules* of the physical language. If P-rules are desired, they will generally be stated in the form of P-primitive sentences. In the first place, certain most general laws will be formulated as P-primitive sentences; we will call these *primitive laws*. In addition, descriptive synthetic sentences of another form—even concrete ones—may be stated as P-primitive sentences. In the majority of cases, the primitive laws will have the form of a universal sentence of implication or of equivalence. The primitive laws and the other valid laws can be either deterministic or *laws of probability*; the latter can be formulated, for instance, with the help of a probability implication. Since the *concept of probability* is a very significant one for physics, particularly in view of the latest developments, the logical analysis of physics will have thoroughly to investigate the syntax of the sentences of probability; and it may be found possible to establish a connection with the concept of range in the general syntax.

We cannot go more fully into the concept of probability here. See the lectures and discussions of the Prague Congress (*Erkenntnis* 1, 1930); further bibliographical references are given in *Erkenntnis*

II, 189 f., 1931; there are also investigations, as yet unpublished, by Reichenbach, Hempel, and Popper.* On the probability implication, see Reichenbach [*Wahrscheinlichkeitslogik*].

Syntactical rules will have to be stated concerning the forms which the *protocol-sentences*, by means of which the results of observation are expressed, may take. [On the other hand, it is not the task of syntax to determine which sentences of the established protocol form are to be actually laid down as protocol-sentences, for 'true' and 'false' are not syntactical terms; the statement of the protocol-sentences is the affair of the physicist who is observing and making protocols.]

A sentence of physics, whether it is a P-primitive sentence, some other valid sentence, or an indeterminate assumption (that is, a premiss whose consequences are in course of investigation), will be *tested* by deducing consequences on the basis of the transformation rules of the language, until finally sentences of the form of protocol-sentences are reached. These will then be compared with the protocol-sentences which have actually been stated and either confirmed or refuted by them. If a sentence which is an L-consequence of certain P-primitive sentences contradicts a sentence which has been stated as a protocol-sentence, then some change must be made in the system. For instance, the P-rules can be altered in such a way that those particular primitive sentences are no longer valid; or the protocol-sentence can be taken as being non-valid; or again the L-rules which have been used in the deduction can also be changed. There are no established rules for the kind of change which must be made.

Further, it is not possible to lay down any set rules as to how new primitive laws are to be established on the basis of actually stated protocol-sentences. One sometimes speaks in this connection of the method of so-called *induction*. Now this designation may be retained so long as it is clearly seen that it is not a matter of a regular method but only one of a practical procedure which can be investigated solely in relation to expedience and fruitfulness. That there can be no rules of induction is shown by the fact that the L-content of a law, by reason of its unrestricted universality, always goes beyond the L-content of every finite class of protocol-

* (*Note*, 1935.) These works have meantime appeared; see Bibliography.

sentences. On the other hand, exact rules for deduction can be laid down, namely, the L-rules of the physical language. Thus the laws have the character of *hypotheses* in relation to the protocol-sentences; sentences of the form of protocol-sentences may be L-consequences of the laws, but a law cannot be an L-consequence of any finite synthetic class of protocol-sentences. The laws are not inferred from protocol-sentences, but are selected and laid down on the grounds of the existing protocol-sentences, which are always being re-examined with the help of the ever-emerging new protocol-sentences. Not only laws, however, but also concrete sentences are formulated as hypotheses, that is to say, as P-primitive sentences—such as a sentence about an unobserved process by which certain observed processes can be explained. There is in the strict sense no refutation (falsification) of an hypothesis; for even when it proves to be L-incompatible with certain protocol-sentences, there always exists the possibility of maintaining the hypothesis and renouncing acknowledgment of the protocol-sentences. Still less is there in the strict sense a complete confirmation (verification) of an hypothesis. When an increasing number of L-consequences of the hypothesis agree with the already acknowledged protocol-sentences, then the hypothesis is increasingly confirmed; there is accordingly only a gradually increasing, but never a final, confirmation. Further, it is, in general, impossible to test even a single hypothetical sentence. In the case of a single sentence of this kind, there are in general no suitable L-consequences of the form of protocol-sentences; hence for the deduction of sentences having the form of protocol-sentences the remaining hypotheses must also be used. Thus *the test applies, at bottom, not to a single hypothesis but to the whole system of physics as a system of hypotheses* (Duhem, Poincaré).

No rule of the physical language is definitive; all rules are laid down with the reservation that they may be altered as soon as it seems expedient to do so. This applies not only to the P-rules but also to the L-rules, including those of mathematics. In this respect, there are only differences in degree; certain rules are more difficult to renounce than others. [If, however, we assume that every new protocol-sentence which appears within a language is synthetic, there is this difference between an L-valid, and therefore analytic, sentence $\mathfrak{S}_1$ and a P-valid sentence $\mathfrak{S}_2$, namely, that such a new protocol-sentence—independently of whether it is acknowledged as valid or not—can be, at most, incompatible with $\mathfrak{S}_2$ but never with $\mathfrak{S}_1$. In spite of this, it may come about that, under the inducement of new protocol-sentences, we alter the language to such an extent that $\mathfrak{S}_1$ is no longer analytic.]

If a new P-*primitive sentence* $\mathfrak{S}_1$ is stated, but without sufficient transformation rules by which, from $\mathfrak{S}_1$ in conjunction with the other P-primitive sentences, sentences of the form of protocol-sentences could be deduced, then in principle $\mathfrak{S}_1$ *cannot be tested*, and is therefore useless from the scientific point of view. If, however, sentences of the form of protocol-sentences are deducible from $\mathfrak{S}_1$ in conjunction with the remainder of the P-primitive sentences, but only such as are deducible from the remaining P-primitive sentences alone, then $\mathfrak{S}_1$ as a primitive sentence is unproductive, and scientifically superfluous.

A *new descriptive symbol which is to be introduced* need not be reducible by means of a chain of definitions to symbols which occur in protocol-sentences. A symbol of this kind may also be introduced as a *primitive symbol* by means of new P-primitive sentences. If these primitive sentences are testable, i.e. if sentences of the form of protocol-sentences are deducible from them, then thereby the primitive symbols are reduced to symbols of the protocol-sentences.

Example: Let protocol-sentences be the observation sentences of the usual form. The electric field vector of classical physics is not definable by means of the symbols which occur in such protocol-sentences; it is introduced as a primitive symbol by the Maxwell equations which are formulated as P-primitive sentences. There is no sentence equipollent to such an equation, which contains only symbols of the protocol-sentences, although, of course, sentences of protocol form can be deduced from the Maxwell equations in conjunction with the other primitive sentences of classical physics; in this way, the Maxwell theory is empirically tested. *Counter-example.* The concept of "entelechy", employed by the neo-vitalists, must be rejected as a pseudo-concept. It is, however, not a sufficient justification for this rejection to point out that no definition of that concept is given by means of which it could be reduced to the terms of the observation sentences; for the same thing is also true of a number of abstract physical concepts. The decisive point is rather the fact that no laws which can be empirically tested are laid down for that concept.

The *explanation* of a single known physical process, the *deduction* of an unknown process in the past or in the present, from one that is known, and the *prediction* of a future event, are all operations of the same logical character. In all three cases it is, namely, a matter of deducing the concrete sentence which describes the process from valid laws and other concrete sentences. To explain a law (in the material mode of speech: a universal fact) means to deduce it from more general laws.

The construction of the physical system is not effected in accordance with fixed rules, but by means of conventions. These conventions, namely, the rules of formation, the L-rules, and the P-rules (hypotheses), are, however, not arbitrary. The choice of them is influenced, in the first place, by certain practical methodological considerations (for instance, whether they make for simplicity, expedience, and fruitfulness in certain tasks). This is the case for all conventions, including, for example, definitions. But in addition the hypotheses can and must be tested by experience, that is to say, by the protocol-sentences—both those that are already stated and the new ones that are constantly being added. Every hypothesis must be compatible with the total system of hypotheses to which the already recognized protocol-sentences also belong. That hypotheses, in spite of their subordination to empirical control by means of the protocol-sentences, nevertheless contain a conventional element is due to the fact that the system of hypotheses is never univocally determined by empirical material, however rich it may be.

Let us make brief mention of two theses held by us, upon which, however, the above view regarding the physical language does not depend. The thesis of *physicalism* maintains that the physical language is a universal language of science—that is to say, that every language of any sub-domain of science can be equipollently translated into the physical language. From this it follows that science is a unitary system within which there are no fundamentally diverse object-domains, and consequently no gulf, for example, between natural and psychological sciences. This is the thesis of the *unity of science*. We will not examine these theses in greater detail here. It is easy to see that both are theses of the syntax of the language of science.

On the view of the physical language here discussed and on the theses of physicalism and of the unity of science, see Neurath [*Physicalism*], [*Physikalismus*], [*Soziol. Phys.*], [*Protokollsätze*], [*Psychol.*]; Carnap [*Phys. Sprache*], [*Psychol.*], [*Protokollsätze*]. In the discussions of the Vienna Circle, Neurath has been conspicuous for his early—often initiatory—and especially radical adoption of new theses. For this reason, although many of his formulations are not unobjectionable, he has had a very stimulating and fruitful influence upon its investigations; for instance, in his demand for a unified language which should not only include the domains of science but also the protocol-sentences and the sentences about sentences; in his emphasis on the fact that all rules of the physical language depend upon conventional decisions, and that none of its sentences—not even the protocol-sentences—can ever be definitive; and, finally, in his rejection of so-called pre-linguistic elucidations and of the metaphysics of Wittgenstein. It was Neurath who suggested the designations "Physicalism" and "Unity of science".—One of the most important problems of the logical analysis of physics is that of the form of the protocol-sentences and of the operation of testing (problem of verification); on this point, see also Popper.

On the view here expounded the domain of the scientific sentences is not so restricted as on the one formerly held by the Vienna Circle. It was originally maintained that every sentence, in order to be significant, must be *completely verifiable* (Wittgenstein; Waismann [*Wahrscheinlichkeit*] p. 229; and Schlick [*Kausalität*] p. 150); every sentence therefore must be a molecular sentence formed of concrete sentences (the so-called elementary sentences) (Wittgenstein [*Tractatus*] pp. 102, 118; Carnap [*Aufbau*]). On this view there was no place for the *laws of nature* amongst the sentences of the language. Either these laws had to be deprived of their unrestricted universality and be interpreted merely as report-sentences, or they were left their unrestricted universality, and regarded not as proper sentences of the object-language, but merely as directions for the construction of sentences (Ramsey [*Foundations*] pp. 237 ff.; Schlick [*Kausalität*] pp. 150 f., with references to Wittgenstein), and hence as a kind of syntactical rules. In accordance with the principle of tolerance, we will not say that a construction of the physical language corresponding to this earlier view is inadmissible; it is equally possible, however, to construct the language in such a way that the unrestrictedly universal laws are admitted as proper sentences. The important difference between laws and concrete sentences is not obliterated in this second form of language, but remains in force. It is taken into account in the fact that definitions are framed for both kinds of sentences, and their various syntactical properties are investigated. The choice between the two forms of language is to be made on the grounds of expedience. The second form, in which the laws are treated as equally privileged proper sentences of the object-language, is, as it appears, much simpler and better adapted to the ordinary use of language in the actual sciences than the first form. A detailed criticism of the view according to which laws are not sentences is given by Popper.

The view here presented allows great freedom in the introduction of new primitive concepts and new primitive sentences in the language of physics or of science in general; yet at the same time it retains the *possibility of differentiating pseudo-concepts and pseudo-sentences* from real scientific concepts and sentences, *and thus of eliminating the former.* [This elimination, however, is not so simple as it appeared to be on the basis of the earlier position of the Vienna Circle, which was in essentials that of Wittgenstein. On that view it was a question of "*the* language" in an absolute sense; it was thought possible to reject both concepts and sentences if they did not fit into *the* language.] A newly stated P-primitive sentence is shown to be a pseudo-sentence if either no sufficient rules of formation are given by means of which it can be seen to be a sentence or no sufficient rules of transformation by means of which it can, as previously indicated, be submitted to an empirical test. The rules need not be explicitly given; they may also be tacitly laid down, provided only that they are exhibited in the use of language. A newly stated descriptive term is shown to be a pseudo-concept if it is neither reduced to previous terms by means of a definition, nor introduced by means of P-primitive sentences that can be tested (see the example and counter-example on p. 319).

Like the individual sentences of the logic of science previously discussed, this presentation of a conception of the logic of science is intended only as an example. Its truth is not here in question. The example is only for the purpose of making it clear that the logical analysis of physics is the syntax of the physical language, and of further stimulating the formulation, within the domain of syntax, of views, questions, and investigations concerning the logic of science (in the ordinary mode of expression: epistemology) and thus making the subject more precise and more fruitful.

§ 83. The so-called Foundations of the Sciences

Much has been said in recent times about the problems of the so-called philosophical or logical foundations of the individual sciences, by which are understood (in our method of designation) certain problems of the logic of science in relation to the domains of the sciences. Taking the most important examples, we shall show briefly that these problems are questions of the syntax of the language of science.

The chief *problems of the foundations of physics* have already been spoken of in the previous section, and, earlier, in Examples 49 to 53 (on p. 307). We have seen that the problem of the structure of time and space is concerned with the syntax of the space and time co-ordinates. The problem of causality is concerned with the syntactical form of laws; and in particular the controversy regarding determinism with a certain property of completeness of the system of physical laws. The problem of empirical foundation (problem of verification) is an inquiry into the form of the protocol-sentences and the consequence-relations between the physical sentences—especially the laws—and the protocol-sentences. The question of the logical foundations of physical measurement is the question of the syntactical form of quantitative physical sentences (containing functors) and of the relations of derivation between these sentences and the non-quantitative sentences (containing predicates; for instance, sentences about pointer-coincidences). Further, such questions as those concerning the relation between macro- and micro-magnitudes or between macro- and micro-laws are to be formulated as syntactical questions; the elucidation of the concept of genidentity also belongs to syntax.

The problems of the foundations of biology refer mainly to the connection between biology and the physics of the inorganic, or, more exactly, to the possibility of translating the biological language S_1 into that sub-language S_2 of the physical language which contains the necessary terms for the purpose of describing the inorganic processes and the necessary laws for the explanation of these processes; in other words: to the relations between S_1 and S_2 on the basis of the total language S_3 which contains both as sub-languages. There are, most importantly, two questions which must be distinguished: (1) Can the *concepts* of biology be reduced to those of the physics of the inorganic? In syntactical form: Is every descriptive primitive symbol of S_1 synonymous in S_3 with a symbol which is definable in S_2? If this is the case, then there is in relation to S_3 an equipollent translation of the L-sub-language of S_1 into that of S_2. (2) Can the *laws* of biology be reduced to those of the physics of the inorganic? In syntactical form: is every primitive law of S_1 equipollent in S_3 to a law which is valid in S_2? If so, then there is, in relation to S_3, an equipollent translation of S_1 (as a P-language) into S_2. This second question constitutes the scientific core of the problem of *vitalism*, which is, however, often entangled with extra-scientific pseudo-problems.

The *problems of the foundations of psychology* contain analogues to those of biology just mentioned. (1) Can the *concepts* of psychology be reduced to those of physics in the narrower sense? (2) Can the *laws* of psychology be reduced to those of physics in the narrower sense? (Physicalism answers the first question in the affirmative, but leaves the second open.) The so-called *psycho-physical problem* is usually formulated as a question concerning the relation of two object-domains: the domain of the psychical processes and the domain of the parallel physical processes in the central nervous system. But this formulation in the material mode of speech leads into a morass of pseudo-problems (for instance: "Are the parallel processes merely functionally correlated, or are they connected by a causal relation? Or is it the same process seen from two different sides?"). With the use of the formal mode of speech it becomes clear that we are here concerned only with the relation between two sub-languages, namely, the psychological and the physical language; the question is whether two parallel sentences are always, or only in certain cases, equipollent with one another, and, if so, whether they are L- or P-equipollent. This important problem can only be grappled with at all if it is formulated correctly, namely, as a syntactical problem—whether in the manner indicated or in some other. In the controversy regarding *behaviorism* there are two different kinds of question to be distinguished. The empirical questions which are answered by the behavioristic investigators on the basis of their observations do not belong here; they are object-questions of a special science. On the other hand, the fundamental question of behaviorism, which is sometimes designated as a methodological or an epistemological problem, is a problem of the logic of science. It is often formulated in the material mode of speech as a pseudo-object-question (e.g. "Do mental processes exist?", "Is psychology concerned only with physical behaviour?", and so on). If, however, instead of being formulated in this way it is formulated in the formal mode, it will be seen that here again the question is one of the reducibility of the psychological concepts; the fundamental thesis of behaviorism is thus closely allied to that of physicalism.

The *problems of the foundations of sociology* (in the widest sense, including the science of history) are for the most part analogous to those of biology and psychology.

§ 84. The Problem of the Foundation of Mathematics

What should a logical foundation of mathematics achieve? On this question there are various views; the fundamental antithesis between them is particularly clearly brought out in two doctrines, *logicism*, which was founded by Frege (1884), and *formalism*, represented by Frege's opponents. (The designations 'logicism' and 'formalism' only appeared later.) Frege's opponents maintained that the logical foundation of mathematics is effected by the construction of a formal system, a calculus, a system of axioms, which makes possible the proof of the formulae of classical mathematics; in this the meaning of the symbols is not to be taken into consideration, the symbols are, so to speak, implicitly defined by the primitive sentences of the calculus; the question as to what numbers actually are—which goes beyond the domain of the calculus—must be rejected. Formalism today represents a view which is in essentials the same, but which has been improved upon in several important points, notably by Hilbert. According to this view, mathematics and logic are constructed together in a common calculus; the question of freedom from contradiction is made the centre of the investigations; the formal treatment (the so-called metamathematics) is carried out more strictly than before. As opposed to the formalist standpoint, Frege maintained that the logical foundation of mathematics has the task, not only of setting up a calculus, but also, and pre-eminently, of giving an account of the meaning of mathematical symbols and sentences. He tried to perform this task by reducing the symbols of mathematics to the symbols of logic by means of definitions, and proving the sentences of mathematics by means of the primitive sentences of logic with the help of the logical rules of inference ([*Grundgesetze*]). Later Russell and Whitehead, also representing the standpoint of logicism, carried out in an improved form the construction of mathematics on the basis of logic ([*Princ. Math.*]). We will not go into certain difficulties with which a structure of this kind is faced (see Carnap [*Logizismus*]), for we are here not so much concerned with the question whether mathematics can be derived from logic or must be constructed simultaneously with it, as with the question whether the construction is to be of a purely formal nature, or whether the meaning of the symbols must be determined. The apparently complete antithesis of the opposing views on this point can, however, be overcome. The formalist view is right in holding that the construction of the system can be effected purely formally, that is to say, without reference to the meaning of the symbols; that it is sufficient to lay down rules of transformation, from which the validity of certain sentences and the consequence relations between certain sentences follow; and that it is not necessary either to ask or to answer any questions of a material nature which go beyond the formal structure. But the task which is thus outlined is certainly not fulfilled by the construction of a logico-mathematical calculus alone. For this calculus does not contain all the sentences which contain mathematical symbols and which are relevant for science, namely those sentences which are concerned with the *application of mathematics*, i.e. synthetic descriptive sentences with mathematical symbols. For instance, the sentence "In this room there are now two people present" cannot be derived from the sentence "Charles and Peter are in this room now and no one else" with the help of the logico-mathematical calculus alone, as it is usually constructed by the formalists; but it can be derived with the help of the logicist system, namely on the basis of Frege's definition of '2'. A logical foundation of mathematics is only given when a system is built up which enables derivations of this kind to be made. The system must contain general rules of formation concerning the occurrence of the mathematical symbols in synthetic descriptive sentences also, together with consequence-rules for such sentences. Only in this way is the application of mathematics, i.e. calculation with numbers of empirical objects and with measures of empirical magnitudes, rendered possible and systematized. *A structure of this kind fulfils, simultaneously, the demands of both formalism and logicism.* For, on the one hand, the procedure is a purely formal one, and on the other, the meaning of the mathematical symbols is established and thereby the application of mathematics in actual science is made possible, namely, by *the inclusion of the mathematical calculus in the total language.* The logicist requirement only appears to be in contradiction with the formalist one; this apparent antithesis arises as a result of the ordinary formulation in the material mode of speech, namely, "an interpretation for mathematics must be given in order that it may be applied to reality". By translation into the formal mode of speech this relation is reversed: the interpretation of mathematics is effected by means of the rules of application. The *requirement of logicism* is then formulated in this way: *the task of the logical foundation of mathematics is not fulfilled by a metamathematics (that is, by a syntax of mathematics) alone, but only by a syntax of the total language, which contains both logico-mathematical and synthetic sentences.*

Whether, in the construction of a system of the kind described, only logical symbols in the narrower sense are to be included amongst the primitive symbols (as by both Frege and Russell) or also mathematical symbols (as by Hilbert), and whether only logical primitive sentences in the narrower sense are to be taken as L-primitive sentences, or also mathematical sentences, is not a question of philosophical significance, but only one of technical expedience. In the construction of Languages I and II we have followed Hilbert and selected the second method. Incidentally, the question is not even accurately formulated; we have in the general syntax made a formal distinction between logical and descriptive symbols, but a precise classification of the logical symbols in our sense into logical symbols in the narrower sense and mathematical symbols has so far not been given by anyone.

The logical analysis of geometry has shown that it is necessary to distinguish clearly between mathematical and physical geometry. The sentences belonging to the two domains, although they often have the same wording in the ordinary use of language, have a very different logical character. *Mathematical geometry* is a part of pure mathematics, whether it is constructed as an axiomatic system or in the form of analytical geometry. The questions of the foundation of mathematical geometry thus belong to the syntax of the geometrical axiom-systems, or to the syntax of the systems of co-ordinates respectively. *Physical geometry*, on the other hand, is a part of physics; it arises from a system of mathematical geometry by means of the construction of the so-called correlative definitions (see § 25). In the case of the problems of the foundation of physical geometry, the question is one of the syntax of the geometrical system as a sub-language of the physical language. The principal theses, for example, of the empiricist view of geometry: "The theorems of mathematical geometry are analytic", "The theorems of physical geometry are synthetic but P-valid", are obviously syntactical sentences.

§ 85. Syntactical Sentences in the Literature of the Special Sciences

In all scientific discussions, object-questions and questions of the logic of science, i.e. syntactical questions, are bound up with one another. Even in treatises which have not a so-called epistemological problem or problem of foundation as their subject, but are concerned with specialized scientific questions, a considerable, perhaps even a preponderant, number of the sentences are syntactical. They speak, for instance, *about* certain definitions, about the sentences of the domain which have been hitherto accepted, about the statements or derivations of an opponent, about the compatibility or incompatibility of different assumptions, and so on.

It is easy to realize that a *mathematical* treatise is predominantly metamathematical, that is to say, that it contains, in addition to proper mathematical sentences (for instance: "Every even number is the sum of two prime numbers"), syntactical sentences (of such forms as: "From...it follows that...", "By substitution we get...", "We will transform the expression...", and the like). The same thing is equally true, however, of treatises of *empirical science*. We will illustrate this by an example from physics. In the following table the first column contains the initial sentences (abbreviated) of Einstein's *Zur Elektrodynamik bewegter Körper* (1905). The reformulation in the second column is merely for the purpose of making clear the character of the sentences. In the third column, the character of the individual sentences or descriptions is stated, and it is shown that the majority of these are syntactical.

Sentences from the original	*Paraphrase*	*Kinds of sentence*
		(p.s. = pure-syntactical. d.s. = descriptive-syntactical.)
That Maxwell's electro-dynamics ...	In the laws which are consequences of the Maxwell equations	p.s. description of sentences.
lead to asymmetries in their application to bodies in motion	certain asymmetries are shown	p.s. sentence about laws
which do not appear to appertain to the phenomena	which do not occur in the appertaining protocol-sentences.	and about protocol-sentences.
is well known.	Contemporary physicists know that	Historical d.s. sentence.
For example, if one thinks of ... reciprocal causation	*Example:* the reciprocal causation-sentences	p.s. description of sentences.
Here the observable phenomenon is dependent only upon the relative motion of conductor and magnet,	The protocol-sentences are dependent only upon such and such sentences of the system.	p.s. sentence.
while, according to the usual view, the case in which the one body is in motion must be strictly separated from the case in which the other is in motion.	In the ordinary form of the system the two concrete sentences '...' and '...' are not equipollent to each other.	p.s. sentence (with descriptions of two sentences).
If, namely, the magnet moves ..., then an electric field ... is the result,	If a magnet moves ..., then an electric field ... results.	Object-sentence (physical law).
which produces an electric current.	If an electric field ... arises, a current ... results.	As before.
But if the magnet does not move ... then no field ... results,	(Analogous.)	As before.
but on the other hand an electro-motive power results in the conductor	(Analogous.)	As before.

Sentences from the original	*Paraphrase*	*Kinds of sentence*
which, however, ... causes ... electric currents.	(Analogous.)	As before.
Examples of a similar kind,	A 1. Sentences similar to the previous ones.	(Loose) p.s. description of sentences.
like the unsuccessful attempts to prove a motion of the earth relative to the "light medium",	A 2. Such and such protocol-sentences occurring in the history of physics. By means of these protocol-sentences such and such an hypothesis is refuted.	Historical d.s. description of sentences. p.s. sentence
lead to the supposition that	The sentences A suggest the tentative construction of a physical system S for which the sentences B are true (that is to say, S is a system of hypotheses which is confirmed by the sentences A).	p.s. sentence.
... in electro-dynamics no properties of the observable phenomena ... correspond to the concept of absolute rest,	B 1. There is no term in the appertaining protocol-sentences (of the system S) corresponding to the term 'absolute rest' in the sentences of electro-dynamics.	p.s. sentence.
but rather that ... the same electro-dynamic ... laws are valid for all co-ordinate systems	B 2. The ... laws (of the system S) have the same form in relation to all co-ordinate systems.	p.s. sentence (about certain transformations).
We will take this supposition		
(whose content will be called in what follows the "Principle of Relativity")	B 2 shall be called the "Principle of Relativity".	p.s. definition.
as an hypothesis.	B 2 is stated as a hypothetical P-rule.	p.s. convention (definition of 'P-valid in S').

§ 86. The Logic of Science is Syntax

We have attempted to show by a brief examination of the problems of the logical analysis of physics and of the so-called problems of foundation of the different domains—which also belong to the logic of science—that these are, at bottom, syntactical, although the ordinary formulation of the problems often disguises their character. Metaphysical philosophy tries to go beyond the empirical scientific questions of a domain of science and to ask questions concerning the nature of the objects of the domain. These questions we hold to be pseudo-questions. The non-metaphysical logic of science, also, takes a different point of view from that of empirical science, not, however, because it assumes any metaphysical transcendency, but because it makes the language-forms themselves the objects of a new investigation. On this view, it is only possible, in any domain of science, to speak either *in* or *about* the sentences of this domain, and thus only object-sentences and syntactical sentences can be stated.

The fact that we differentiate these two kinds of sentences does not mean that the two investigations must always be kept separate. In the actual practice of scientific research, on the contrary, the two points of view and the two kinds of sentences are linked with one another. We have seen from the example of a treatise on physics that investigations in the domains of the special sciences contain many syntactical sentences. But it is also true, conversely, that researches in the logic of science always contain numerous object-sentences; these sentences are in part object-sentences of the domain to which logical analysis is being applied, and in part sentences concerning the psychological, sociological, and historical circumstances under which work is being done in that field. So although we can divide the concepts into logical and descriptive concepts, and the sentences of simpler form into sentences of the logic of science (that is to say, syntactical sentences) and object-sentences, on the other hand no strict classification of the investigations themselves and the treatises in which they are set forth is possible. Treatises in the domain of biology, for instance, contain in part biological, and in part syntactical, sentences; there are only differences of degree, according to which of the two sorts of question predominates; and on this basis one may, in practice, distinguish between specially biological treatises and treatises of the logic of science. He who wishes to investigate the questions of the logic of science must, therefore, renounce the proud claims of a philosophy that sits enthroned above the special sciences, and must realize that he is working in exactly the same field as the scientific specialist, only with a somewhat different emphasis: his attention is directed more to the logical, formal, syntactical connections. Our thesis that the logic of science is syntax must therefore not be misunderstood to mean that the task of the logic of science could be carried out independently of empirical science and without regard to its empirical results. The syntactical investigation of a system which is already given is indeed a purely mathematical task. But the language of science is not given to us in a syntactically established form; whoever desires to investigate it must accordingly take into consideration the language which is used in practice in the special sciences, and only lay down rules on the basis of this. In principle, certainly, a proposed new syntactical formulation of any particular point of the language of science is a convention, i.e. a matter of free choice. But such a convention can only be useful and productive in practice if it has regard to the available empirical findings of scientific investigation. [For instance, in physics the choice between deterministic laws and laws of probability, or between Euclidean and non-Euclidean geometry, although not univocally determined by empirical material, is yet made in consideration of this material.] All work in the logic of science, all philosophical work, is bound to be unproductive if it is not done in close co-operation with the special sciences.

Perhaps we may say that the researches of non-metaphysical philosophy, and especially those of the logic of science of the last decades, have all, at bottom, been syntactical researches, although unconsciously. This essential character of such investigations must now also be recognized in theory and systematically observed in practice. Only then will it be possible to replace traditional philosophy by a strict scientific discipline, namely, that of the logic of science as the syntax of the language of science. The step from the morass of subjectivist philosophical problems on to the firm ground of exact syntactical problems must be taken. Then only shall we have as our subject-matter exact terms and theses that can be clearly apprehended. Then only will there be any possibility of fruitful co-operative work on the part of the various investigators working on the same problems—work fruitful for the individual questions of the logic of science, for the scientific domain which is being investigated, and for science as a whole. In this book we have only created a first working-tool in the form of syntactical terms. The use of this instrument for dealing with the numerous and urgent contemporary problems of the logic of science, and the improvement of it which will follow from its use, demands the co-operation of many minds.