

Estimation of Panel Data Models with Two-Sided Censoring*

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Abstract

It is straightforward to construct moment conditions for two-sided censored panel data regression models with strictly exogenous explanatory variables. The contribution of this note is to show that one set of these moment conditions uniquely identify the parameters of the model under a natural full-rank condition. The identification result suggests an estimator that is then applied to evaluate the portfolio allocation effect of a Danish tax reform that increased the after-tax capital income of bonds relative to stocks.

1 Introduction

This paper considers panel data estimation of a regression model with two-sided (“double”) censoring. A leading example of when two-sided censored regression models are useful is when the dependent variable is a fraction. For example, Alan and Leth-Petersen (2006) estimate a portfolio share equation where the portfolio shares are between 0 and 1, with a significant number of observations on either of the limits. Other recent applications in economics of regression models with two-sided censoring include Lafontaine (1993), Petersen and Rajan (1994), Petersen and Rajan (1995), Houston and Ryngaert (1997), Fehr, Kirchler, Weichbold, and Gächter (1998), Huang and

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Hauser (1998), McMillan and Woodruff (1999), de Figueriredo and Tiller (2001), Huang and Hauser (2001), Fenn and Liang (2001), Poterba and Samwick (2002), Nickerson and Silverman (2003), Officer (2004), Charness, Frechette, and Kagel (2004), Andrews, Schank, and Simmons (2005) and Gifford and Bernard (2005).

In a panel data setting, the specific model is

$$y_{it}^* = x_{it}'\beta + \varepsilon_{it}$$

$$y_{it} = \begin{cases} a_1 & \text{if } y_{it}^* < a_1 \\ y_{it}^* & \text{if } a_1 \leq y_{it}^* \leq a_2 \\ a_2 & \text{if } y_{it}^* > a_2 \end{cases} \quad (1)$$

where ε_{it} is stationary conditional on (x_{i1}, \dots, x_{iT}) . This has the usual fixed effects specification with strictly exogenous regressors as a special case, because it is not required that the mean of ε_{it} given (x_{i1}, \dots, x_{iT}) be constant across individuals. When y_{it} is a share, a_1 and a_2 will typically be 0 and 1, respectively.

Honoré (1992) constructed moment conditions for a similar model with one-sided censoring and showed how they can be interpreted as the first-order conditions for a population minimization problem that uniquely identifies the parameter vector, β . It is straightforward to modify the moment conditions in Honoré (1992) so that they apply to the case with two-sided censoring. The contribution of this note is to show that some of those moment conditions can be turned into a minimization problem that identifies β . Since Honoré and Powell (1994) used the moment conditions in Honoré (1992) to construct an estimator for the one-sided censored regression model, it follows immediately how one would modify their estimator to cover the case of two-sided censoring. Since the main contribution of the paper is identification, we will focus on the case where there are two time periods, so $T = 2$. With more than two time periods, one could use all pairs of periods as in Honoré and Kyriazidou (2000).

We also illustrate that one can modify the estimator for a cross-sectional censored regression model with symmetric errors proposed by Powell (1986) to allow for two-sided censoring. This is of less practical relevance, as it is obvious how one would estimate such a model by median regression techniques like the ones in Powell (1984) and Khan and Tamer (2008). Moreover, Powell (1986) already showed how to deal with censoring in one tail and truncation in the other. Zhang and Li (1996) considered estimation of the two-sided censored regression model with independent errors.

In section 5 we apply the estimator developed in section 3 to analyze the portfolio-reshuffling effect of a tax reform that increased the after-tax capital income on bonds relative to stocks in Denmark after 1987.

2 Identification

The key result is Lemma 2 below. The following notation is useful. For $a_1 \leq a_2$, define

$$\text{mami} \{a_1, x, a_2\} = \begin{cases} a_1 & \text{if } x < a_1 \\ x & \text{if } a_1 \leq x \leq a_2 \\ a_2 & \text{if } x > a_2 \end{cases}$$

so (1) can be written as

$$y_{it} = \text{mami} \{a_1, x'_{it}\beta + \varepsilon_{it}, a_2\}.$$

In order to simplify the exposition, we will develop the case when $a_1 = 0$ and $a_2 = 1$. It is then straightforward to adapt the result to the general case.

As mentioned in the introduction, it is straightforward to construct moment conditions that must be satisfied at the true β . Consider an individual, i , in two time periods, t and s . The distribution of $y_{it} - x'_{it}\beta$ will be the same as that of ε_{it} except that the former is censored from below at $-x'_{it}\beta$ and from above at $1 - x'_{it}\beta$. Figure 1 illustrates this. The dotted line depicts the distribution of ε_{it} , while the solid line gives the distribution of $y_{it} - x'_{it}\beta$, which typically has point mass at $-x'_{it}\beta$ and $1 - x'_{it}\beta$ (illustrated by the fatter vertical lines). Since $x'_{it}\beta$ will typically differ from $x'_{is}\beta$, the distributions of $y_{it} - x'_{it}\beta$ and $y_{is} - x'_{is}\beta$ (given (x_{it}, x_{is})) will differ even if $\{\varepsilon_{it}\}$ is stationary. However, it is clear that one could obtain identically distributed “residuals” by artificially censoring $y_{it} - x'_{it}\beta$ and $y_{is} - x'_{is}\beta$ from below at $\max\{-x'_{it}\beta, -x'_{is}\beta\}$ and from above at $\min\{1 - x'_{it}\beta, 1 - x'_{is}\beta\}$. See the dashed lines in Figure 1. One can then form moment conditions from the fact that the difference in these “re-censored” residuals will be orthogonal to functions of (x_{it}, x_{is}) .¹ The contribution here is to show that some of these moment conditions uniquely identify the parameter vector β .

It is useful to define functions $u_1(y_1, \cdot)$ and $u_2(y_2, \cdot)$ over the interval -1 to 1 as follows

$$u_1(y_1, d) = \begin{cases} \max\{y_1 - d, 0\} & \text{for } 1 \geq d \geq 0 \\ \min\{y_1, 1 + d\} & \text{for } 0 \geq d \geq -1 \end{cases}$$

and

$$u_2(y_2, d) = \begin{cases} \min\{y_2, 1 - d\} & \text{for } 1 \geq d \geq 0 \\ \max\{y_2 + d, 0\} & \text{for } 0 \geq d \geq -1 \end{cases}.$$

With these definitions, $u_1(y_{it}, x'_{it}\beta - x'_{is}\beta) - u_2(y_{is}, x'_{it}\beta - x'_{is}\beta)$ will give the difference in the re-censored residuals discussed above. To see this, suppose that $x'_{it}\beta \geq x'_{is}\beta$. Then the difference in

¹Clearly, one can also use the fact that differences in functions of the re-censored residuals will be orthogonal to functions for the explanatory variables. As discussed in Arellano and Honoré (2001), one can also construct moment conditions based on symmetry under the additional assumption that $(\varepsilon_{i1}, \dots, \varepsilon_{iT})$ is exchangeable conditional on (x_{i1}, \dots, x_{iT}) . This is the motivation for the approach in Honoré (1992).

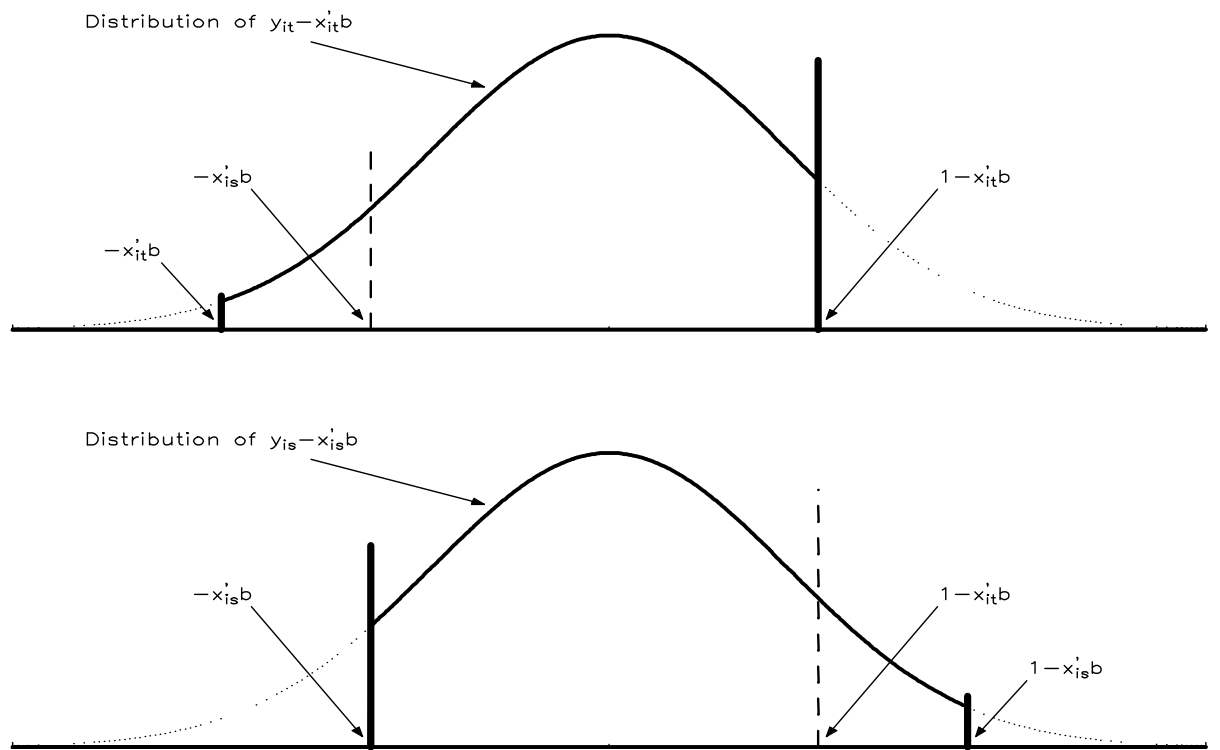


Figure 1: Illustration of Re-Censored Residuals when $x'_{it}\beta > x'_{is}\beta$.

the artificially censored residuals for individual i in periods t and s is

$$\begin{aligned}
& \max \{y_{it} - x'_{it}\beta, -x'_{is}\beta\} - \min \{y_{is} - x'_{is}\beta, 1 - x'_{it}\beta\} \\
&= (\max \{y_{it} - (x'_{it}\beta - x'_{is}\beta), 0\} - x'_{is}\beta) - (\min \{y_{is}, 1 - (x'_{it}\beta - x'_{is}\beta)\} - x'_{is}\beta) \\
&= \max \{y_{it} - (x'_{it}\beta - x'_{is}\beta), 0\} - \min \{y_{is}, 1 - (x'_{it}\beta - x'_{is}\beta)\} \\
&= u_1(y_{it}, x'_{it}\beta - x'_{is}\beta) - u_2(y_{is}, x'_{it}\beta - x'_{is}\beta),
\end{aligned}$$

and when $x'_{it}\beta \leq x'_{is}\beta$

$$\begin{aligned}
& \min \{y_{it} - x'_{it}\beta, 1 - x'_{is}\beta\} - \max \{y_{is} - x'_{is}\beta, -x'_{it}\beta\} \\
&= (\min \{y_{it}, 1 + (x'_{it}\beta - x'_{is}\beta)\} - x'_{it}\beta) - (\max \{y_{is} + (x'_{it}\beta - x'_{is}\beta), 0\} - x'_{it}\beta) \\
&= \min \{y_{it}, 1 + (x'_{it}\beta - x'_{is}\beta)\} - \max \{y_{is} + (x'_{it}\beta - x'_{is}\beta), 0\} \\
&= u_1(y_{it}, x'_{it}\beta - x'_{is}\beta) - u_2(y_{is}, x'_{it}\beta - x'_{is}\beta).
\end{aligned}$$

Also define function $r_1(y_1, \cdot)$ and $r_2(y_2, \cdot)$ over the interval -1 to 1 as

$$r_1(y_1, d) = \begin{cases} d + \frac{1}{2}d^2 + \frac{1}{2}(y_1 - 1)^2 & \text{for } d \leq y_1 - 1 \\ dy_1 & \text{for } y_1 - 1 \leq d \leq 0 \\ dy_1 - \frac{1}{2}d^2 & \text{for } 0 \leq d \leq y_1 \\ \frac{1}{2}y_1^2 & \text{for } y_1 \leq d \end{cases}$$

and

$$r_2(y_2, d) = \begin{cases} -\frac{1}{2}y_2^2 & \text{for } d \leq -y_2 \\ \frac{1}{2}d^2 + dy_2 & \text{for } -y_2 \leq d \leq 0 \\ dy_2 & \text{for } 0 \leq d \leq 1 - y_2 \\ d - \frac{1}{2}d^2 - \frac{1}{2}(y_2 - 1)^2 & \text{for } 1 - y_2 \leq d \end{cases}$$

The functions $r_1(y_1, \cdot)$ and $r_2(y_2, \cdot)$ are constructed so their derivatives are $u_1(y_1, \cdot)$ and $u_2(y_2, \cdot)$, respectively.

Finally, define

$$R(y_1, y_2, d) = r_1(y_1, d) - r_2(y_2, d).$$

Then

Lemma 1

$$\frac{\partial}{\partial d} R(y_1, y_2, d) = u_1(y_1, d) - u_2(y_2, d)$$

PROOF: Trivial. ■

Lemma 2 Suppose

$$y_{i1} = \text{mami}\{0, \delta + \varepsilon_{i1}, 1\}$$

and

$$y_{i2} = \text{mami} \{0, \varepsilon_{i2}, 1\}$$

where ε_{i1} and ε_{i2} are identically distributed random variables with support on the whole real line.

Then

$$\arg \max_{d \in [-1, 1]} E [R(y_{i1}, y_{i2}, d)] = \begin{cases} -1 & \text{if } \delta \leq -1 \\ \delta & \text{if } -1 < \delta < 1 \\ 1 & \text{if } \delta \geq 1 \end{cases}$$

PROOF: See Appendix. ■

The following corollary follows immediately.

Corollary 3 Consider the model

$$y_{it}^* = x'_{it}\beta + \varepsilon_{it}$$

$$y_{it} = \begin{cases} 0 & \text{if } y_{it}^* < 0 \\ y_{it}^* & \text{if } 0 \leq y_{it}^* \leq 1 \\ 1 & \text{if } y_{it}^* > 1 \end{cases}$$

for $t = 1, 2$. If ε_{it} is stationary conditional on (x_{i1}, x_{i2}) with support on the whole real line, then the set of solutions to

$$\max_b E [R(y_{i1}, y_{i2}, \text{mami} \{-1, (x_{i1} - x_{i2})' b, 1\})]$$

is

$$\{b : P(\text{mami} \{-1, (x_{i1} - x_{i2})' b, 1\}) = \text{mami} \{-1, (x_{i1} - x_{i2})' \beta, 1\}) = 1\}$$

The implication of Corollary 3 is that β is identified provided that there is no other b such that $P(\text{mami} \{-1, (x_{i1} - x_{i2})' b, 1\}) = \text{mami} \{-1, (x_{i1} - x_{i2})' \beta, 1\}) = 1$. This is similar to “full rank” conditions for semiparametric censored regression models.

As mentioned, the censoring points are not always 0 and 1. Suppose that

$$y_{it} = \text{mami} \{a_1, x'_{it}\beta + \varepsilon_{it}, a_2\}$$

In that case

$$\frac{y_{it} - a_1}{a_2 - a_1} = \text{mami} \left\{ 0, \frac{x'_{it}}{a_2 - a_1} \beta + \frac{(\varepsilon_{it} - a_1)}{a_2 - a_1}, 1 \right\}$$

and the previous corollary then applies to the function

$$E \left[R \left(\frac{y_{i1} - a_1}{a_2 - a_1}, \frac{y_{i2} - a_1}{a_2 - a_1}, \text{mami} \left\{ -1, \frac{(x_{i1} - x_{i2})'}{a_2 - a_1} b, 1 \right\} \right) \right].$$

The censoring points can differ across observations provided that they are observed for all observations.

3 Estimation

When the censoring points are 0 and 1, the analog estimator based on the idea above is

$$\widehat{\beta} = \arg \min_b \sum_{i=1}^n \sum_{1 \leq s < t \leq T_i} w_{i,t-s} R(y_{is}, y_{it}, \text{mami} \{-1, (x_{is} - x_{it})' b, 1\})$$

where the $w'_{i,t-s}$'s are exogenous weights and T_i is the number of observations for the i 'th individual.² It follows immediately from textbook arguments that $\widehat{\beta}$ is consistent and asymptotically normal under appropriate regularity conditions. The exact structure of the asymptotic covariance matrix depends on the assumptions made on the sampling scheme. Under random sampling (and suitable regularity conditions like the ones in Honoré (1992)),

$$\sqrt{n} (\widehat{\beta} - \beta) \rightsquigarrow N(0, \Gamma^{-1} V \Gamma^{-1})$$

where

$$\begin{aligned} \Gamma = E & \left[\sum_{s < t} w_{i,t-s} 1 \{-1 < (x_{is} - x_{it})' \beta < 1\} \right. \\ & \left(1 \{-1 < (x_{is} - x_{it})' \beta < y_{is} - 1\} - 1 \{0 < (x_{is} - x_{it})' \beta < y_{is}\} \right. \\ & \left. \left. - 1 \{-y_{it} < (x_{is} - x_{it})' \beta < 0\} + 1 \{1 - y_{it} < (x_{is} - x_{it})' \beta < 1\} \right) (x_{is} - x_{it})(x_{is} - x_{it})' \right] \end{aligned}$$

and

$$V = E [v_i v_i']$$

with

$$\begin{aligned} v_i = \frac{1}{T_i} \sum_{s < t} w_{i,t-s} 1 \{-1 < (x_{is} - x_{it})' \beta < 1\} \\ (u_1 (y_{is}, (x_{is} - x_{it})' \beta) - u_2 (y_{it}, (x_{is} - x_{it})' \beta)) (x_{is} - x_{it}) \end{aligned}$$

Following standard arguments, these are consistently estimated by

$$\begin{aligned} \widehat{\Gamma} = \frac{1}{n} \sum_{i=1}^n & \left[\sum_{s < t} w_{i,t-s} 1 \{-1 < (x_{is} - x_{it})' \widehat{\beta} < 1\} \right. \\ & \left(1 \{-1 < (x_{is} - x_{it})' \widehat{\beta} < y_{is} - 1\} - 1 \{0 < (x_{is} - x_{it})' \widehat{\beta} < y_{is}\} \right. \\ & \left. \left. - 1 \{-y_{it} < (x_{is} - x_{it})' \widehat{\beta} < 0\} + 1 \{1 - y_{it} < (x_{is} - x_{it})' \widehat{\beta} < 1\} \right) (x_{is} - x_{it})(x_{is} - x_{it})' \right] \end{aligned}$$

²With unbalanced panels, one might want $w_{i,t-s}$ to depend on T_i . For example, one can think of the usual fixed effects estimator in a linear regression model as minimizing $\sum_{i=1}^n \sum_{1 \leq s < t \leq T_i} \frac{1}{T_i} (y_{is} - y_{it} - (x_{is} - x_{it})' b)^2$

and

$$\widehat{V} = \frac{1}{n} \sum_{i=1}^n \widehat{v}_i \widehat{v}_i'$$

with

$$\widehat{v}_i = \frac{1}{T_i} \sum_{s < t} w_{i,t-s} 1 \left\{ -1 < (x_{is} - x_{it})' \widehat{\beta} < 1 \right\} \\ \left(u_1 \left(y_{is}, (x_{is} - x_{it})' \widehat{\beta} \right) - u_2 \left(y_{it}, (x_{is} - x_{it})' \widehat{\beta} \right) \right) (x_{is} - x_{it})$$

A Gauss program for implementing the estimator can be found at <http://www.princeton.edu/~honore/programs/2side/>.

4 Powell's Model

Powell (1986) studied the censored regression model

$$y_i = \max \{0, x_i' \beta + \varepsilon_i\} \quad (2)$$

with

$$\varepsilon_i | x_i \sim \text{symmetric around } 0.$$

Following the line of argument above, it is easy to show that if (2) is changed to

$$y_i = \text{mami} \{0, \delta + \varepsilon_i, 1\} \quad (3)$$

then

$$\arg \max_{d \in [-1, 1]} E[r(y_i, d)] = \begin{cases} -1 & \text{if } \delta \leq -1 \\ \delta & \text{if } -1 < \delta < 1 \\ 1 & \text{if } \delta \geq 1 \end{cases}$$

where

$$r(y, d) = \begin{cases} -\frac{1}{4}y^2 & \text{if } d < 0 \\ \frac{1}{2}d^2 - \frac{1}{4}y^2 & \text{if } 0 < d < \frac{y}{2} \\ -\frac{1}{2}(y-d)^2 & \text{if } \frac{y}{2} < d < \frac{1+y}{2} \\ \frac{1}{2}(1-d)^2 - \frac{1}{4}(1-y)^2 & \text{if } \frac{1+y}{2} < d < 1 \\ -\frac{1}{4}(1-y)^2 & \text{if } 1 < d \end{cases}$$

Hence,

Corollary 4 *Consider the model*

$$y_i^* = x_i' \beta + \varepsilon_i$$

$$y_i = \begin{cases} 0 & \text{if } y_i^* < 0 \\ y_i^* & \text{if } 0 \leq y_i^* \leq 1 \\ 1 & \text{if } y_i^* > 1 \end{cases}$$

If ε_{it} is symmetrically distributed around 0 conditional on x_i with support on the whole real line, then the set of solutions to

$$\max_b E [r(y_i, \text{mami} \{-1, x_i' b, 1\})]$$

is

$$\{b : P(\text{mami} \{-1, x_i' b, 1\} = \text{mami} \{-1, x_i' \beta, 1\}) = 1\}$$

The analog estimator based on this is

$$\hat{\beta} = \arg \min_b \sum_{i=1}^n r(y_i, \text{mami} \{-1, x_i' b, 1\})$$

As mentioned in the introduction, this is less interesting than the estimator for the panel data version, as it is obvious how one would estimate such a model by median regression techniques like the ones in Powell (1984) and Khan and Tamer (2008). Moreover, Powell (1986) already showed how to deal with censoring in one tail and truncation in the other. Zhang and Li (1996) considered estimation of the two-sided censored regression model with independent errors.

5 Empirical Application

In this section we apply the estimator in Section 3 to analyze the portfolio-reshuffling effect of a tax reform that increased the after-tax capital income on bonds relative to stocks in Denmark in 1987. We use a panel data set constructed from administrative records covering two years before and after the reform to estimate a portfolio share equation for bonds on marginal tax rates of capital income. The analysis presented here follows the literature on taxation and portfolio structure, e.g., Feldstein (1976), Hubbard (1985), King and Leape (1998), Samwick (2000), Poterba and Samwick (2002) and the survey by Poterba (2002). These papers analyze (repeated) cross sections of households.³ Here the analysis is extended by using panel data and controlling for time-invariant correlated heterogeneity, i.e., fixed effects. Controlling for correlated unobserved fixed factors is likely to be important in this context, since the portfolio composition of a household is likely to be influenced by time-invariant factors such as risk aversion and time discounting.

³Bakija (2000) uses the limited panel module of the American Survey of Consumer Finances (SCF) to study portfolio changes around the 1988 tax reform. However, his data set is very small (984 households) and unrepresentative due to the well-known attrition problem in the SCF panel module; see Kennickell and Woodburn (1997). More important in this context, the estimators applied do not exploit the full potential of the panel data in handling unobserved heterogeneity. Ioannides (1992) also employs the 1983-1986 SCF panel module but does not control for unobserved heterogeneity.

In the next subsection we give a brief overview over the tax reform. After this, we introduce the data and present the results.

5.1 The Tax Reform

The tax reform, announced in 1985 and implemented in 1987, broke the link between the marginal tax rates on earned income and capital income. Before the reform, all income was taxed at the same marginal tax rate. With the reform the tax rate on positive capital income for high-income households was decreased from 73 percent to 56 percent. The reform thereby increased the after-tax return on interest-bearing assets and therefore encouraged households to shift their portfolios toward such assets. The reform also changed the tax value of interest deductions from 73 to about 50 percent, and this substantially increased the cost of debt, primarily mortgages, for leveraged high-income households. For such households the reform effectively brought a negative wealth shock, giving them a strong incentive to lower their debt burden.⁴

The exact changes, however, differed across municipalities. The Danish income tax system is built around a proportional local government tax and a progressive tax collected by the central government. While the progressive schedule is the same for everybody in Denmark, the local government tax rates vary across municipalities. A tax ceiling, however, insured that the marginal tax rate could be at the maximum 73 percent. After the reform the tax ceiling on earned income was reduced to 68 percent in the highest bracket⁵ and 56 percent in the middle bracket. Capital income was now taxed at the same rate independently of the level of earned income. The marginal tax rates across tax brackets before and after the reform are summarized in Table 1 (see Appendix 2)

The application of a tax ceiling together with the heterogeneous local government tax rates implies that the reform had differential effects on people living in different municipalities. Figures 2 and 3 illustrate the changes in marginal tax rates due to the reform for a high-tax and a low-tax municipality, respectively.

For a high-income person living in the municipality with the high local government tax, the marginal tax rate on positive net capital income falls by 14.5 percentage points and the marginal tax rate on negative net capital income falls by 20.5 percentage points. For a similar person living in the municipality with the low local government tax rate, the marginal tax rate on positive

⁴Alan and Leth-Petersen (2006) document that the reduced value of the interest deduction led households to liquidate financial assets to lower their mortgage debt. This was possible because pre-payment of mortgage debt is not restricted in Denmark

⁵Approximately 20 percent of the population belong in the top bracket.

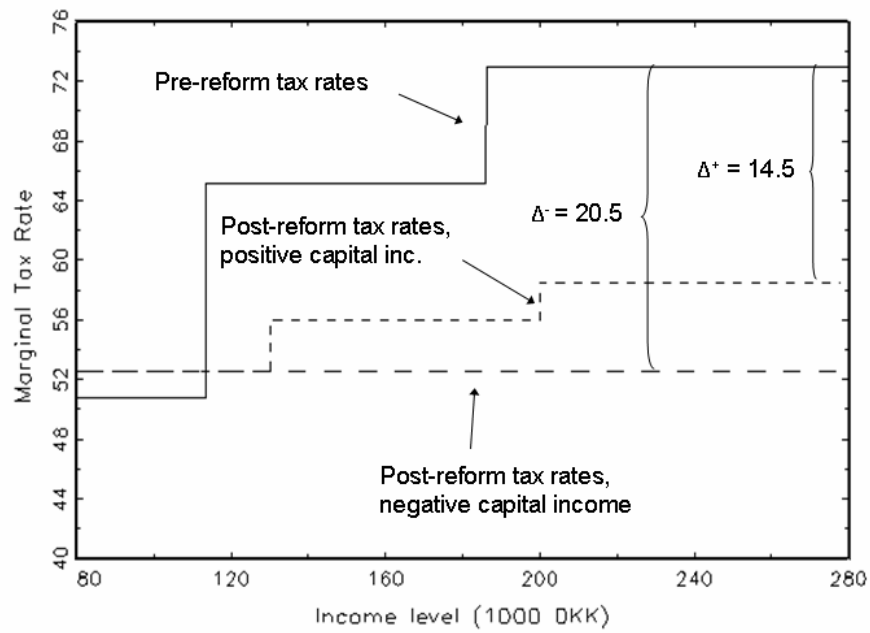


Figure 2: Marginal Tax Rate for High-Tax Municipality.

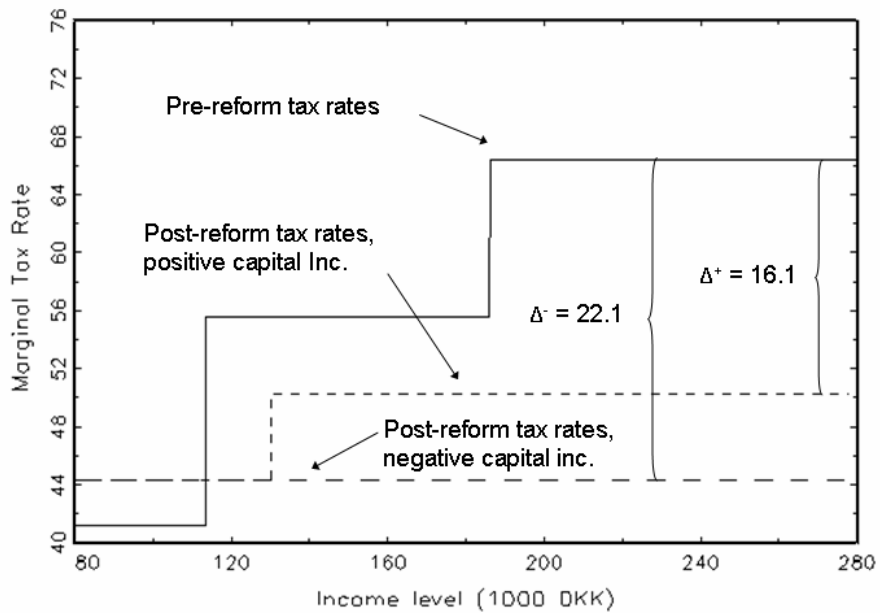


Figure 3: Marginal Tax Rate for Low-Tax Municipality.

net capital income falls by 16.1 percentage points and negative net capital income falls by 22.1 percentage points. It is these differences in changes of marginal tax rates that we will exploit for identifying the effect of changes in marginal tax rates on the portfolio allocation when using the fixed effects estimator.

The marginal tax rates on capital income refer to income received in the form of dividends on stocks and interest payments from interest bearing accounts and bonds. Both before and after the reform, realized capital gains/losses associated with trading assets were generally not taxed. The exemption from this rule is capital gains from corporate stocks held for less than three years. Such capital gains are taxed as earnings. Dividend payments were low relative to interest received from bonds.⁶ This suggests that lowering the marginal tax rate on positive capital income affected bonds and stocks differentially, favoring mainly income from bonds. In the empirical analysis we therefore focus on reshuffling between bonds and stocks.

5.2 Data

The data set is drawn from a random sample of 10 percent of the Danish population observed in the years 1984 to 1988. Information on portfolio allocations, income, wealth and demographics is collected and merged from different public administrative registers for all adult members of the household that the sampled person belongs to. Portfolio and income information is obtained from the income tax register. The portfolio information exists because Denmark had a wealth tax that required all wealth holdings to be reported to the tax authorities. This information allows us to break the wealth of each household into holdings of stocks and bonds. “Stocks” includes all holdings of publicly and privately traded stocks, and “bonds” includes government and corporate bonds. The holdings of stocks and bonds are self-reported through the tax return and then audited by the tax authority.

5.2.1 Sample selection

For our analysis we exclude observations if one of the household members is self-employed, since register data are not likely to contain a good measure of own business wealth and because taxable income is quite volatile for those individuals. Sampled individuals younger than 18 or older than 60 are dropped as are students and individuals living together with his/her parents or living in a common household, i.e., a household with more than one family. To keep the focus on the

⁶The median household in the sample holding stocks received dividends corresponding to 2 percent of the value of the stocks. The median household in the sample holding bonds received interest payments from these corresponding to 10 percent of the value of the bonds.

importance of tax incentives, we include only stable couples, i.e., couples where the partner is the same in 1984 through 1988. On the same grounds we also exclude couples moving in the sample period. For the purpose of the analysis we require that households entering the sample be observed in all years in the period 1984-1988 so that we have a balanced panel.

Our objective is to investigate whether households reshuffle their portfolios in response to a change in tax incentives. As in most industrialized countries many Danish households have fairly undiversified portfolios. Since the decrease in the value of interest deductions generated a large negative wealth shock, clearly, these households are not likely to engage in portfolio reshuffling and hence cannot give us a clean answer regarding portfolio readjustments. We therefore construct a sub-sample of households holding positive amounts of stocks or bonds of at least 5,000 DKK in 1984. We also require households to hold a positive amount of either stocks or bonds throughout the rest of the observation period. This selection is introduced because we want to focus on households with a potential to reshuffle between stocks and bonds. Also renters are deselected because there are few renters with diversified portfolios.⁷ The final subsample includes 8,577 households.⁸

5.3 Results

In this section we investigate if households reshuffled their portfolio of bonds and stocks as a response to the changes in relative after-tax returns on assets brought about by the 1987 tax reform. To do this we employ the estimator presented above to estimate a portfolio share equation where the fraction of bonds in financial wealth, defined as the sum of bonds and stocks, is regressed on the marginal tax rate on positive capital income and some control variables.

The distinguishing feature of our data set is the panel dimension. This facilitates estimating portfolio share equations allowing for correlated unobserved time-invariant heterogeneity. This is important because we believe that unobserved time-constant factors, such as risk aversion and time preferences, are correlated with wealth. High risk aversion may, for example, lead to a higher portfolio share of safe assets, such as bonds, for a given level of wealth.

Before the reform, capital income and earnings were lumped together and taxed according to a progressive tax scheme. This implies that households choose their tax bracket when choosing their portfolio and that the marginal tax rate on capital income is likely to be an endogenous regressor.

⁷For assessing portfolio reshuffling renters could have been included. We have chosen to leave them out of this analysis because there are only a few renters (898) with positive financial wealth of at least 5000 DKK in 1984. Moreover, renters generally do not provide a good comparison group for homeowners, since different preference parameters may govern their behavior.

⁸See Alan and Leth-Petersen (2006) for a more detailed analysis.

We address this by calculating the marginal tax rate on capital income based on the household's income in 1984, the year before the reform was announced, but using current year rules. In this way the individual level tax bracket is allocated based on information that was predetermined relative to the portfolio response to the reform.

We regress portfolio shares on the marginal tax rate on positive capital income, the log of total financial assets, i.e., assets held in stocks and bonds, and a set of year dummies. Tax rate changes vary across municipalities, but most of the change in tax rates is common across municipalities. Year dummies control for the effect of this common part, thereby also removing the major part of the wealth effect brought about by the reform. Effectively, by introducing year dummies, the coefficients on marginal tax rates are identified by differences in changes of marginal tax rates. Year dummies may also pick up common effects relating to fluctuations in assets. Financial assets control for any remaining wealth effect that might be present.⁹

Table 2 of Appendix 2 presents the parameter estimates from estimating random effects tobit and fixed effects censored regression models for the portfolio share of bonds in financial wealth. The estimated parameter on the marginal tax rate is negative.¹⁰ If year dummies and financial assets pick up the wealth effect related to the reform, in particular the effect of the reduction in the value of interest deduction that led households to liquidate financial assets, then this is exactly what economic theory predicts. Households should substitute from stocks toward bonds, whose relative after-tax return increased, and this is what the results indicate. Considering the corresponding random effects estimates, we can see that the parameter estimates on financial assets and on year dummies are quite different, and the test of parameter equality between the random effects and fixed effects specifications rejects.

⁹An alternative identification strategy could be based on comparing the behavior of households in different tax brackets. Households in the lowest tax bracket faced only very small changes in marginal tax rates on capital income, and households in the middle tax bracket faced different changes in marginal tax rates than households in the highest tax bracket. In our case this is not a natural approach to follow. High- and low-income people are different in terms of wealth levels and portfolio composition and possibly different with respect to preference parameters such as the discount rate and the level of risk aversion. Households in lower tax brackets therefore do not represent a natural control group for high-income households.

¹⁰As explained in Honoré (2008), the parameter estimates for both the random effects and the fixed effects models can be converted to marginal effects by multiplying them by the fraction of observations that are not censored.

6 Concluding Remarks

It is straightforward to construct moment conditions for linear panel data models with two-sided censoring. The contribution of this note is to show that one set of these moment conditions uniquely identify the parameters of the model under a natural full-rank condition. The main limitation is that the approach requires strictly exogenous explanatory variables. The methodological point of the empirical application is to show that the method is feasible in practice. The empirical finding is that the conventional random effects model is rejected and that some of the estimates are quite sensitive to whether one specifies a random effects or fixed effects model.

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7 Appendix 1: Proof of Lemma 2

For $1 \geq d \geq 0$

$$\begin{aligned}
 E[u_{1i}(y_{i1}, d) - u_{2i}(y_{i2}, d)] &= E[\max\{y_{i1} - d, 0\} - \min\{y_{i2}, 1 - d\}] \\
 &= E[\max\{\text{mami}\{0, \delta + \varepsilon_{i1}, 1\} - d, 0\} - \min\{\text{mami}\{0, \varepsilon_{i2}, 1\}, 1 - d\}] \\
 &= E[\max\{\text{mami}\{0 - d, \delta - d + \varepsilon_{i1}, 1 - d\}, 0\} - \text{mami}\{0, \varepsilon_{i2}, 1 - d\}] \\
 &= E[\text{mami}\{0, \delta - d + \varepsilon_{i1}, 1 - d\} - \text{mami}\{0, \varepsilon_{i2}, 1 - d\}]
 \end{aligned}$$

If ε_{i1} (and ε_{i2}) have full support, then this is negative for $d > \delta$ and positive for $d < \delta$.

For $-1 \leq d \leq 0$

$$\begin{aligned}
 E[u_{1i}(y_{i1}, d) - u_{2i}(y_{i2}, d)] &= E[\min\{y_{i1}, 1 + d\} - \max\{y_{i2} + d, 0\}] \\
 &= E[\min\{\text{mami}\{0, \delta + \varepsilon_{i1}, 1\}, 1 + d\} - \max\{\text{mami}\{0, \varepsilon_{i2}, 1\} + d, 0\}] \\
 &= E[\text{mami}\{0, \delta + \varepsilon_{i1}, 1 + d\} - \max\{\text{mami}\{d, \varepsilon_{i2} + d, 1 + d\}, 0\}] \\
 &= E[\text{mami}\{0, \delta + \varepsilon_{i1}, 1 + d\} - \text{mami}\{0, \varepsilon_{i2} + d, 1 + d\}]
 \end{aligned}$$

If ε_{i1} (and ε_{i2}) have full support, then this is negative for $d > \delta$ and positive for $d < \delta$.

Since

$$E[R'(y_{i1}, y_{i2}, d)] = E[u_1(y_{i1}, d) - u_2(y_{i2}, d)]$$

the argument above shows that

$$\arg \max_{d \in [-1, 1]} E[R(y_{i1}, y_{i2}, d)] = \begin{cases} -1 & \text{if } \delta \leq -1 \\ \delta & \text{if } -1 < \delta < 1 \\ 1 & \text{if } \delta \geq 1 \end{cases} \quad \blacksquare$$

8 Appendix 2: Tables

Table 1: Marginal tax rates before and after implementation of the 1987 tax reform

Before Reform		After Reform			
Tax bracket	Earnings + Cap inc	Tax bracket	Earnings	inc < 0	inc. > 0 ⁽¹⁾
0-113	$M + 19.75$	0 – 130	$M + 22.00$	$M + 22.00$	$M + 22.00$
113-186	$M + 34.15$	130 – 200	$M + 28.00$	$M + 22.00$	$M + 28.00$
186-	$M + 44.95$	200–	$M + 40.00$	$M + 22.00$	$M + 28.00$
Tax ceiling	73	Tax ceiling	68.00/56.00 ⁽²⁾		56.00

Note: M is the local government tax rate. Threshold values for the tax brackets are given in 1000 DKK. Thresholds are adjusted yearly. Threshold values used in the table are for 1986 (before the reform) and 1987 (after the reform). The marginal tax rates refer to personal income (as opposed to household income).

(1) The tax brackets for positive net capital income refer to the sum of earnings and positive net capital income.

After the reform positive capital income is taxed progressively up to the first threshold, 130,000 DKK. For a married couple the progression threshold is 260,000 based on the sum of their joint positive net capital income and earnings.

(2) The 68 percent tax ceiling applies only to the top bracket.

Table 2.: Random and Fixed Effects Censored Regression Estimates of the Portfolio Share of Bonds in Financial Wealth.

	Fixed Effects	Random Effects
MTR capital income	-0.130 (0.071)	-0.206 (0.045)
Ln(Financial Assets)	0.177 (0.008)	0.085 (0.003)
D85	-0.214 (0.006)	-0.157 (0.006)
D86	-0.314 (0.009)	-0.248 (0.006)
D87	-0.318 (0.013)	-0.285 (0.008)
D88	-0.383 (0.013)	-0.331 (0.008)
Constant	—	0.047 (0.046)
# households/observations	8,577 / 42,885	
# left/right censored obs	7,529 / 15,655	
Test of Parameter Equality (d.f.)	167 (6)	

Standard errors in parenthesis