1 Competing Risk

1.1 Setup

Consider the competing risks model

$$(T, I) = (\min\{T_1, T_2\}, \arg\min\{T_1, T_2\})$$

with a binary explanatory variable X. Moreover assume that a discretized version of T is observed. The joint distribution of the data is then characterized by $P(T = t_k, I = i|\, X = x)$. There is a one–to–one relationship between $P(T = t_k, I = i|\, X = x)$ and the discrete–time hazard

$$P(T = t_k, I = i|\, X = x, T \geq t_k)$$

Honoré and Lleras–Muney (2005) consider the linear programming problem for given a and b

$$f(a, b) = \max_{\{v_i\},\{p(\cdot,\cdot)\}} \sum -v_i \tag{1}$$

subject to

$$v_k + \sum_{\substack{t_k < s_1 < t_{k+1} \\ s_2 > s_1}} p(s_1, s_2) = P(T = t_k, I = 1|\, X = 0) \qquad k = 1, \ldots M, \tag{2}$$

$$v_{M+k} + \sum_{\substack{t_k < s_2 < t_{k+1} \\ s_1 > s_2}} p(s_1, s_2) = P(T = t_k, I = 0|\, X = 0\ ) \qquad k = 1, \ldots M, \tag{3}$$

$$v_{2M+k} + \sum_{\substack{t_k < as_1 < t_{k+1} \\ bs_2 > as_1}} p(s_1, s_2) = P(T = t_k, I = 1|\, X = 1\ ) \qquad k = 1, \ldots M, \tag{4}$$

$$v_{3M+k} + \sum_{\substack{t_k < bs_2 < t_{k+1} \\ as_1 > bs_2}} p(s_1, s_2) = P(T = t_k, I = 0|\, X = 1\ ) \qquad k = 1, \ldots M, \tag{5}$$

$$v_{4M+1} + \sum_{s_1, s_2} p(s_1, s_2) = 1, \qquad p(s_1, s_2) \geq 0 \qquad \text{for all } (s_1, s_2), \tag{6}$$

$$v_i \geq 0 \qquad k = 1, \ldots 4M + 1 \tag{7}$$

1.2 *grids_fcts*

The gauss routine

$$grids\_fcts(h1, h2, startt, maxt, thmin1, thmax1, dth1, thmin2, thmax2, dth2)$$

calculates the function value for the linear programming problem (1) in Honoré and Lleras–Muney (2005) over a grid of points.

Input

The inputs $h1$ and $h2$ are matrices, $startt$ and $maxt$ are positive integers, and $thmin1$, $thmax1$, $dth1$, $thmin2$, $thmax2$ and $dth2$ are reals such that $thmin1 < thmax1$ and $thmin2 < thmax2$.

$h1$ and $h2$ both have two columns. $h1\left[startt+1 : maxt, i\right]$ and $h2\left[startt+1 : maxt, i\right]$ refer to the discrete time hazards, $P\left(T = t_k, I = i \mid X = x, T \geq t_k\right)$, for $x = 0$ and $x = 1$, respectively. We assume that the durations are censored after $maxt$ periods.

The points on the grid are constructed so they cover the set $[thmin1, thmax1] \times [thmin2, thmax2]$. In terms of the notation of Honoré and Lleras–Muney (2005), the program searches for values of a in $[thmin1, thmax1]$ and b in $[thmin2, thmax2]$. The distance between points in the grid is less than $dth1$ for the first element and less than $dth2$ for the second element.

Output

The output of the routine is a matrix with three columns. The first column contains the function value for a grid point. The second and third columns contain the coordinates for the grid point, i.e., the values if (a, b).

2 Existence of solution to linear constraints.

Let A be a matrix and let b be a vector. Consider the equations

$$Ax = b.$$

a solution with $x_i \geq 0$ for all i exists if and only if 0 equals

$$\max_{\{x_i\},\{v_i\}} -\sum v_i$$

subject to

$$\begin{aligned} Ax - b &= v \\ v_i &\geq 0 \end{aligned}$$

The gauss routine

$$lp\_max\_value(A, b, nmet)$$

calculates

$$\max_{\{x_i\},\{v_i\}} -\sum v_i$$

subject to

$$\begin{aligned} Ax - b &= v, \\ v_i &\geq 0. \end{aligned}$$

If $nmet = 1$ then Gauss's linear programming routine are used. If $nmet = 2$ then the program uses a linear programming routine based on Press et al.