LIQUIDITY RISK AND CONTAGION

Rodrigo Cifuentes  
Banco Central de Chile  

Gianluigi Ferrucci  
Bank of England  

Hyun Song Shin  
London School of Economics

Abstract  
This paper explores liquidity risk in a system of interconnected financial institutions when these institutions are subject to regulatory solvency constraints and mark their assets to market. When the market’s demand for illiquid assets is less than perfectly elastic, sales by distressed institutions depress the market prices of such assets. Marking to market of the asset book can induce a further round of endogenously generated sales of assets, depressing prices further and inducing further sales. Contagious failures can result from small shocks. We investigate the theoretical basis for contagious failures and quantify them through simulation exercises. Liquidity requirements on institutions can be as effective as capital requirements in forestalling contagious failures. (JEL: G21, G28)

1. Introduction

Prudential regulations are designed to enhance the resilience to shocks of financial systems by requiring institutions to maintain prudent levels of liquidity and capital under a broad range of market conditions. However, at times of market turbulence the remedial actions prescribed by these regulations may have perverse effects on systemic stability. Forced sales of assets may feed back on market volatility and produce a downward spiral in asset prices, which in turn may affect adversely other financial institutions. This paper investigates these issues. In particular, it looks at the consequences mark-to-market accounting of firms’ balance sheets when there are externally imposed regulatory solvency requirements.

Acknowledgments: We thank Rafael Repullo, Martin Summer, Philipp Hartmann, Anjan Thakor, Around Boot, Andy Haldane, Vicky Saporta, and Charles Goodhart for their comments on earlier versions of this paper. The opinions expressed in this paper are those of the authors, and do not necessarily reflect those of the Central Bank of Chile, or the Bank of England.

E-mail addresses: Cifuentes: rcifuent@bcentral.cl; Ferrucci: gianluigi.ferrucci@bankofengland.co.uk; Shin: h.s.shin@lse.ac.uk
We examine two channels of contagion—direct balance sheet interlinkages between financial institutions and contagion via changes in asset prices. The former has been studied extensively, but the latter has received only scant attention. Our aim is to redress the balance. Changes in asset prices may interact with externally imposed solvency requirements or the internal risk controls of financial institutions to generate amplified endogenous responses that are disproportionately large relative to any initial shock. A shock that reduces the market value of a firm’s assets will elicit the disposal of assets. If the market’s ability to absorb these sales in less than perfect, such disposals will result in a short-run change in market prices. When assets are marked to market at the new prices, the externally imposed solvency constraints, or the internally imposed risk controls may dictate further disposals. In turn, such disposals will have a further impact on market prices. In this way, the combination of mark-to-market accounting and solvency constraints have the potential to induce an endogenous response that far outweighs the initial shock.1

To be sure, we model only the ex post stability effects of capital requirements and marking to market for given portfolio choices. We do not model the ex ante effects explicitly, but the optimal level of liquid assets held by individual financial institutions may still be suboptimal from the system point of view of minimizing systemic risk. This is because individual institutions do not internalize the externalities of the effects of their actions on other firms. Our full results can be obtained from our longer paper (Cifuentes, Ferrucci, and Shin 2004). Here, we report some of the highlights from this work.

Regulators are familiar with the potentially destabilizing effect of solvency constraints in distressed markets. In the U.K., for instance, the usual resilience test applied to life insurance companies in which the firm has to demonstrate solvency in the face of a further 25% market decline was suspended for several weeks. Also, following the decline in European stock markets in the summer of 2002, the Financial Services Authority diluted the resilience test so as to preempt the destabilizing forced sales of stocks by the major market players.2 The LTCM crisis of 1998 can also be seen as an instance where credit interconnections and asset prices acted in concert as the main channel propagating widespread market distress (see BIS 1999, Furfine 1999, and Morris and Shin 1999).

There has been a substantial body of work that has examined balance sheet interlinkages as a possible source of contagious failures of financial

---

1. Our paper is related to the recent literature on banking and financial crises that has emphasized the limited capacity of the financial markets to absorb sales of assets (see Allen and Gale (2002, 2004, 2005), Gorton and Huang (2004), and Schnabel and Shin (2002)). The inefficient liquidation of long assets in Diamond and Rajan (2000) has an analogous effect.

institutions. The main focus of these papers is on finding estimates of inter-bank credit exposures. Once this is determined, systemic robustness is assessed by simulating the effects on the system of the failure of one bank at a time. Solvency is assessed mostly on fixed prices that do not change through time. Invariably, a consistent finding is that contagion is never significant in practice, even in the presence of large shocks. In the absence of price effects, this is hardly surprising, as interbank loans and deposits represent only a limited fraction of banks’ balance sheets.

Our work suggests that systemic risk in these networks may be larger than thought, even in the presence of collateralization. The reason is that the risk that materializes is not a credit risk but the interaction of credit risk and market risk, exacerbated by counterparty risk. For financial firms that hold mainly marketable assets—such as insurance companies, hedge funds or investment banks—the assumption of fixed prices would be highly unrealistic. Even for commercial banks, whose assets are mainly loans, the collateral assets that back these loans would be marked to market. The mechanism identified in our paper would thus be applicable to banks, with this modification.

Prudential regulation (in the form of minimum capital requirements) when combined with mark-to-market rules can sometimes generate undesirable spillover effects. Marking to market enhances transparency but it may introduce a potential channel of contagion and may become an important source of systemic risk. Liquidity requirements can mitigate contagion, and can play a similar role to capital buffers in curtailing systemic failure.

The paper is organized as follow. Section 2 illustrates the framework. Section 3 presents the results of simulations of a hypothetical banking system. Section 4 concludes.

2. Framework

There are $n$ interlinked financial institutions (we call them “banks”). The liability of bank $i$ to bank $j$ is denoted by $L_{ij}$. The total liability of bank $i$ is then

3. Most papers calibrate the models using actual cross-exposures in real banking systems (or an approximation of them) and simulate the effects of a shock to the system resulting from the failure of one or more institutions. Sheldon and Maurer (1998) and Müller (2004) study the Swiss banking system. Upper and Worms (2001) consider the German system. Furfine (1999) analyses interlinkages in the U.S. Federal Funds market. Wells (2002) focuses on the U.K. banks. Elsinger, Lehar, and Summer (2002) consider an application to the Austrian banking system, and provide a stochastic extension of the framework (using the concept of value at risk). Cifuentes (2002) uses the same framework to analyze the link between banking concentration and systemic risk. See also Eichberger and Summer (2005).

4. It seems intuitive to conjecture that when players are faced with illiquid markets, they would try to insure against liquidity black holes by holding more liquid assets. The argument in Jackson, Perraudin, and Saporta (2002) should apply also to liquidity—i.e., that market discipline would induce banks to hold more liquid assets. That said, each individual bank will have no incentive to internalize any network externalities, and the level of liquidity may not be optimal.
Cifuentes, Ferrucci & Shin  Liquidity Risk and Contagion  559

the sum

$$\bar{x}_i = \sum_j L_{ij}$$

Denote by $x_i$ the market value of bank $i$’s interbank liabilities. This can be different from the notional value because the debtor may be unable to fulfill these liabilities in full. Interbank claims are of equal seniority, so that if the market value falls short of the national liability, then the bank’s payments are proportional to the notional liability. Let $\pi_{ij} = L_{ij}/\bar{x}_i$. Then, the payment by $i$ to $j$ is given by

$$x_i \pi_{ij},$$

while the total payment received by bank $i$ from all other banks is

$$\sum_j x_j \pi_{ji}.$$

Banks $i$’s endowment of the illiquid asset is given by $e_i$. The price of the illiquid asset is denoted by $p$. In addition, bank $i$ has holding of the liquid asset given by $c_i$. Thus, the net worth or equity value of bank $i$ is

$$pe_i + c_i + \sum_j x_j \pi_{ji} - x_i.$$

Limited liability of the bank implies that its equity value is non-negative. Priority of debt over equity implies that equity value is strictly positive only when $x_i = \bar{x}_i$ (i.e., bank $i$’s payment is equal to its notional obligation). Thus, the vector of payments $x = (x_1, x_2, \ldots, x_n)$ is such that for each $i$,

$$x_i = \min \left\{ \bar{x}_i, w_i(p) + \sum_j x_j \pi_{ji} \right\},$$

where $w_i(p) = pe_i + c_i$ is the marked-to-market value of the liquid and illiquid assets of bank $i$. More succinctly, we can write (2) in vector form as

$$x = \bar{x} \wedge (w(p) + \Pi^T x),$$

where $w(p) = (w_1(p), \ldots, w_n(p))$, $\Pi^T$ is the transpose of the exposure matrix, and $\wedge$ is the pointwise minimum operator. Thus, a clearing vector $x$ that satisfies (3) is a fixed point of the mapping

$$H(x) \equiv \bar{x} \wedge (w(p) + \Pi^T x).$$

$H(\cdot)$ is an increasing function on the lattice $\mathbb{R}^n_+$ (with infimum defined by the operator $\wedge$), and where $H(0) \geq 0$ and $H(\bar{x}) \leq \bar{x}$. Hence, by Tarski’s fixed point
Theorem, there is at least one fixed point of $H(.)$, and hence at least one clearing vector $x$. Eisenberg and Noe (2001) have proved that under mild regularity conditions, there is a unique fixed point of such a function. Thus for any fixed $p$ the net worth of each bank is determined fully. Then, by appealing to the result of Eisenberg and Noe (2001), we have:

**Lemma 1.** Suppose the banking system is connected, and that at price $p$, there is at least one bank that has positive equity value. Then, there is a unique clearing vector $x$ such that

$$x = \bar{x} \wedge (w(p) + \Pi^T x)$$

Write $x(p)$ as the unique clearing vector when the price of the illiquid asset is given by $p$. Then each payment $x_{ij}$ is determined by the pro rata rule (1). This allows us to write each $x_{ij}$ as a function of $p$.

Assets held by the bank attract a regulatory minimum capital ratio, which stipulates that the ratio of the bank’s equity value to the mark to market value of its assets must be above some prespecified ratio $r^*$. When a bank finds itself violating this constraint, it sells assets so as to reduce the size of its balance sheet. Denote by $t_i$ the units of the liquid asset sold by bank $i$, and denote by $s_i$ the units of the illiquid asset sold by bank $i$. The liquid asset has constant price of 1. The illiquid asset has price $p$, which is determined in equilibrium.

The capital adequacy constraint puts a lower bound on the capital asset ratio of the bank. The constraint is given by

$$\frac{pe_i + c_i + \sum_j x_j \pi_{ji} - x_i}{p(e_i - s_i) + (c_i - t_i) + \sum_j x_j \pi_{ji}} \geq r^*$$

(4)

The numerator is the equity value of the bank where the interbank claims and liabilities are calculated in terms of the realized payments. The denominator is the marked-to-market value of its assets after the sale of $s_i$ units of the illiquid asset and sale $t_i$ of the liquid asset. The underlying assumption is that the assets are sold for cash, and that cash does not attract a capital requirement. Thus, if the bank sells $s_i$ units of the illiquid asset, then it has $ps_i$ in cash (assuming for simplicity that it starts with zero cash), and holds $p(e_i - s_i)$ worth of the illiquid asset. Hence, we have the sum of these (given by $pe_i$) on the numerator, while we have only the mark to market value of the illiquid asset (given by $p(e_i - s_i)$) on the denominator. The same holds for the liquid asset. Thus, by selling its assets

---

5. A sufficient condition for the existence of a unique fixed point is that, first, the system is connected in the sense that the banks cannot be partitioned into two or more unconnected subsystems, and that there is at least one bank that has positive equity value in the system.

6. The other possibility is to raise fresh capital, but we do not consider this.
for cash, the bank can reduce the size of its balance sheet and hence reduce the
denominator, making the capital asset ratio larger.

We make two assumptions. First, the bank cannot short-sell the assets. Thus,
\( s_i \in [0, e_i] \) and \( t_i \in [0, c_i] \). Second, the bank sells all its liquid assets before it
starts selling its illiquid assets. Thus, \( s_i > 0 \) only if \( t_i = c_i \). Any value maximizing
bank will follow this rule, and hence this assumption is not a strong one. An
equilibrium is the triple \((x, s, p)\) consisting of a vector of payments \( x \), vector of
sales of illiquid asset \( s \), and the price \( p \) of the illiquid asset such that:

1. For all banks \( i \), \( x_i = \min \left\{ x_i, pe_i + c_i + \sum_j x_j \pi_{ji} \right\} \)
2. For all banks \( i \), \( s_i \) is the smallest sale that ensures that the capital adequacy
   condition is satisfied. If there is no value of \( s_i \in [0, e_i] \) for which the capital
   adequacy condition is satisfied, then \( s_i = e_i \).
3. There is a downward sloping inverse demand function \( d^{-1}(.) \) such that \( p =
   d^{-1} \left( \sum_i s_i \right) \).

The first clause reiterates the limited liability of equity holders, and the priority
and equal seniority of the debt holders. The second clause says that either the
bank is liquidated altogether, or its sales of illiquid assets (possibly zero) reduces
its assets sufficiently to comply with the capital adequacy ratio. Finally, the third
clause states that the price of the illiquid asset is determined by the intersection
of a downward sloping demand curve and the vertical supply curve given by
aggregate sales.

By rearranging the capital adequacy condition (4) together with the condition
that \( s_i \) is positive only if \( t_i = c_i \), we can write the sale \( s_i \) as a function of \( p \), where
\( s_i = 0 \) if the capital adequacy condition can be met by sales of the liquid asset or
from no sales of assets, but otherwise is given by

\[
 s_i = \min \left\{ e_i, \frac{x_i - (1 - r_*) \left( \sum_j x_j \pi_{ji} + pe_i \right) - c_i}{r_* p} \right\}
\]

The interbank payments \( x_{ij} \) are all functions of \( p \). Thus, \( s_i \) itself is a function
of \( p \), and we write \( s_i(p) \) the sales by bank \( i \) as a function of the price \( p \). Let
\( s(p) = \sum_i s_i(p) \) be the aggregate sale of the illiquid asset given price \( p \). Since
each \( s_i(.) \) is decreasing in \( p \), the aggregate sale function \( s(p) \) is decreasing in \( p \).

The inverse demand curve for the illiquid asset is assumed to be

\[
p = e^{-\alpha \left( \sum_i s_i \right)}
\]  

(5)

where \( \alpha > 0 \) is a positive constant. The maximum price is \( p = 1 \), which occurs
when sales are zero. An equilibrium price of the illiquid asset is a price \( p \) for
which \( s(p) = d(p) \). From equation (7), we have at least one equilibrium price,
given by \( p = 1 \). This is the status quo price where the banking system has not
suffered any adverse shock. However, an equilibrium price lower than 1 is possible
Figure 1. Amplification of shock through asset sales.

provided that the \( s(p) \) curve lies above the \( d(p) \) curve for some ranges of price (see Figure 1).

The price adjustment process can be depicted as a step adjustment process in the arc below the \( s(p) \) curve, but above the \( d(p) \) curve. The process starts with a downward shock to the price of the illiquid asset. At the lower price \( p_0 \), the forced sales of the banks puts quantity \( s(p_0) \) on the market. However, this pushes the price further down to \( p_1 = d^{-1}(s(p_0)) \). This elicits further sales, implying total supply of \( s(p_1) \). Given this increased supply, the price falls further to \( p_2 = d^{-1}(s(p_1)) \), and so on. The price falls until we get to the nearest intersection point where the \( d(p) \) curve and \( s(p) \) curve cross. Equivalently, we may define the function \( \Phi_1: [p, 1] \rightarrow [p, 1] \) as

\[
\Phi_1(p) = d^{-1}(s(p))
\]

and an equilibrium price is a fixed point of the mapping \( \Phi(.) \). The function \( \Phi(.) \) has the following interpretation. For any given price \( p \), the value \( \Phi(p) \) is the market-clearing price of the illiquid asset that results when the price of the illiquid asset on the banks’ balance sheets are evaluated at price \( p \). Thus, when \( \Phi(p) < p \), we have the precondition for a downward spiral in the illiquid asset’s price. The price that results from the sales is lower than the price at which the balance sheets are evaluated. We can summarize our results as follows.

**Proposition 2.** If \( \Phi(p) \geq p \) for all \( p \), there is a unique equilibrium in which \( p = 1 \). In this case, the value of the banking system declines only by the size of the initial shock.
Proposition 3. If \( \Phi(p) < p \) for some values of \( p \), then there is an equilibrium in which \( p \) is strictly below 1, and in which there are sales of the illiquid asset. In this case, the banking system will reach this equilibrium by the step adjustment process provided that the initial shock is big enough.

The first proposition is immediate. When the \( \Phi(p) \) curve lies above the 45-degree line, there is no endogenous fall in the asset value of the banking system. The second proposition follows from the continuity of the \( \Phi(.) \) mapping, which inherits its continuity from the continuity of \( d(p) \) and \( s(p) \). In this case, there is an amplification effect that arises from the endogenous responses generated by the forced sales.

3. Simulations

We conducted simulations on a hypothetical banking system consisting of ten ex ante identical banks with the following balance sheets.

<table>
<thead>
<tr>
<th>Liquid and Illiquid assets</th>
<th>Equity</th>
<th>Interbank assets</th>
<th>Deposits</th>
<th>Interbank liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>7</td>
<td>30</td>
<td>63</td>
<td>30</td>
</tr>
<tr>
<td>Total assets</td>
<td>100</td>
<td>Net worth and liabilities</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Given the space constraints in our current paper, we cannot describe the simulations in full. We refer the reader to the longer version of our paper (Cifuentes, Ferrucci, and Shin (2004) for the full description. Our algorithm mimics the step-wise adjustment mechanism depicted in Figure 1. We inject a shock in the initial stage arising from the failure of one of the ten banks. We then track the sale of assets and fall in asset price. We do this across possible banking structures. We
Figure 3. Number of defaults given initial liquidity ratio and number of counterparties.

vary the number of counterparties that a particular bank has, and the results refer to the average across all possible configurations that have the stated number of interconnections.

Figure 2 shows the cumulative sales of illiquid assets in successive iterations of the simulation. The initial sale of assets arises from the shock. However, the subsequent sales are the endogenously generated response of the system. As can be seen, the effects can be very large. The case with two interconnections among banks is especially interesting. There, the initial shock is followed by a lull—the "eye of the storm". However, the subsequent response is quite dramatic.

The total number of bank failures depends on the size of the initial shock and the elasticity of the residual demand curve. It also depends on the network structure and the liquidity of the banking system as a whole. Figure 3 depicts the number of bank failures as a function of the liquidity ratio (the ratio of liquid assets to total liquid and illiquid assets) measured on the horizontal axis, and the network structure (the number of interconnections) on the vertical axis.

The Figure 3 shows that there is a threshold liquidity level beyond which no systemic contagion via asset prices occurs. Additionally, for this combination of shock and price elasticity there is a nonmonotonic relationship between the number of interlinkages and the liquidity threshold. Contagion is small either when there are no interconnections, or when every bank is connected to every other bank (so that shocks are diffused evenly). Contagion is worst when there are moderately many interconnections.

4. Conclusions

Under certain circumstances, prudential regulations can have perverse effects on the stability of a financial system. One message that emerges from our analysis is that liquidity buffers play a role similar to capital buffers. In some circumstances,

---

7. In the case of a single link in the interbank market, there is evidence of contagion to at least one other bank in the system, for any level of the liquidity ratio. However, this is due to direct credit exposure, and not to asset price contagion.
liquidity requirements may be more effective than capital buffers in forestalling systemic effects. When the residual demand curve is extremely inelastic (such as during periods of major financial distress when risk appetite is very low), even a large capital buffer may be insufficient to prevent contagion, since the price impact of sales into a falling market would be very high. Liquidity is a public good. Liquidity requirements can internalize some of the externalities that are generated by the price impact of selling into a falling market.

References


This article has been cited by:
