Contagious Adverse Selection*

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Abstract

We illustrate the corrosive effect of even small amounts of adverse selection in an asset market and show how it can lead to the total breakdown of trade. The problem is the failure of “market confidence”, defined as approximate common knowledge of an upper bound on expected losses. Small probability events can unravel market confidence. We discuss the role of contagious adverse selection and the problem of “toxic assets” in the recent financial crisis.

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1 Introduction

Some participants in financial markets have either private information or better expertise in evaluating new financial instruments and markets. This gives rise to adverse selection. Financial markets may nonetheless operate smoothly when there are sufficient gains from trade among traders without private information or expertise. In this case, expert traders are able to extract some rent from their expertise but there are enough gains from trade to fund these rents and yet allow mutually beneficial trade.

However, as illustrated vividly by the recent financial crisis, financial markets appear to exhibit fragility where shocks to the economy can suddenly lead to abrupt breakdowns in markets. One element of such market breakdowns to be explored in our paper is the amplifying effect of strategic complementarities in participation decisions and the inefficient coordination among ordinary uninformed traders’ participation decisions. If I am expecting other traders to run, then I expect that my trade will be with an expert trader who will take advantage of me. As an ordinary uninformed trader, my incentive to participate in the market is greater when other ordinary uninformed traders participate more.

An important insight from game theory is that in coordination problems, what matters is not so much what each agent knows about the returns to alternative actions, but rather what is common knowledge (or approximate common knowledge) between them. It is their shared understanding that matters. This insight fits well with much commentary on financial markets suggesting that “market confidence” is critical. We can give an interpretation of “market confidence” as embodying such shared understanding. It is not enough that each market participant believes that the fundamentals are sound. Market confidence requires that the fundamental soundness is commonly understood among market participants.

Our purpose in this paper is to develop a link between adverse selection in trading environments with game theoretic insights about coordination and common knowledge. We will shortly describe how our approach relates to the (large) literature on trade under adverse selection. Our main contribution is to highlight the importance of small departures from common knowledge, and what kinds of departures matter for the breakdown of trade.

We consider the following classic trading problem. It is common knowledge among two groups of traders - potential buyers and potential sellers - that an asset is worth $2c$ more to potential buyers than to potential sellers at
every state of the world, where $c > 0$ is a known constant. Thus it is common knowledge that the ex post gains from trade are $2c$. Ex ante welfare is thus equal to $2c$ times the probability of trade. From a welfare perspective, the traders should always trade with each other.

However, suppose that a trader attaches a small probability $\delta$ to the possibility that his partner knows something that creates a large benefit $M$ to the partner at his expense. The “loss ratio” is the ratio of the expected losses of an uninformed agent and his known gains from a split the difference trade. In the benchmark case where there is common knowledge about the expected loss ratio, trade takes place if the loss ratio is less than one.

However, the more intriguing case is when there is incomplete information about the loss ratio. In other words, each agent is unsure exactly what his partner’s perception of expected losses are. In such circumstances, the adverse selection can be shown to have a much more corrosive effect where the fear of asymmetric information reverberates throughout the information structure and gets amplified in the process. It is possible that even when the ex ante probability of adverse selection is small, there can be a catastrophic breakdown in trade. Essentially, the incomplete information leads to an unraveling result in a coordination game among differentially informed traders. Each uninformed trader would like to trade if the trading partner is also an uninformed trader. Otherwise, the trading partner is likely to be an informed party who will take advantage of the uninformed trader.

Drawing on the insights from the earlier literature on common knowledge, we can characterize the threshold condition for the sustainability of trade. At the core of our construction is the self-referential nature of “market confidence”. Loosely, market confidence rests on approximate common knowledge of mutually beneficial trade. The exact threshold depends on the loss ratio faced by the traders. The higher is the loss ratio, the more rigorous must be the notion of common knowledge that will sustain trade. Stated more precisely, a trader has market confidence if he expects the proportion of regular traders who (themselves) have market confidence exceeds his loss ratio. Notice how this definition is self-referential and thus implicitly incorporates a notion of approximate common knowledge. We show that only traders with market confidence participate in the market. To the extent that approximate common knowledge can be sensitive to the interaction of the payoff fundamentals with the information structure, it is possible that even small changes in the underlying parameters of the problem can lead to abrupt breakdowns of market confidence, and hence of trade. Exploring the
subtleties of how market confidence depends on the parameters gives us a
great deal of insight into the underlying economics of trade under adverse
selection. In the penultimate section of our paper, we draw on the insights
from our framework to revisit the breakdown in the market for securities
based on subprime mortgages in the United States.

The outline of our paper is as follows. Before presenting our formal frame-
work, we begin with a brief review of the literature on trade with adverse
selection with an emphasis on how our results can be related to the insights
gained from the existing literature. The formal framework is then presented
in several stages. We start by stating the fundamentals of our trading envi-
ronment and posit a trading institution where trade takes place if and only if
both traders say 'yes' to a proposed trade at a price that splits the difference.
We then introduce adverse selection, and introduce the idea that the severity
of the adverse selection can be a subject of incomplete information among
the traders.

The core of our paper is the characterization of market confidence in
terms of approximate common belief and the demonstration that our notion
of market confidence is the right one when considering the occurrence of
trade in equilibrium. Having introduced our key concepts in the initially
stark setting, we follow up by showing that the insights from the simplified
setting can be embedded in more general settings, and that the intuitions
from the common knowledge literature can help our understanding in these
more general settings. As we have flagged already, we conclude with a brief
discussion of how our results can help to shed light on aspects of the subprime
crisis. Before we embark on the main body of our paper, we begin with a
brief survey of how our discussion links with the existing literature on trade
with adverse selection.

1.1 Related Literature

In Akerlof (1970) and the classical adverse selection models that followed,
there is market unravelling with equally informed traders on both sides of the
market. We are concerned with situations where, on both sides of the market,
some traders are informed and some are not informed. This then translates
into a coordination problem among uninformed traders. This coordination
problem among uninformed traders plays an explicit or implicit role in a wide
variety of finance models.

Bhattacharya and Spiegel (1991) consider a competitive model of a mar-
ket for a risky asset when there are gains from trade from risk sharing among a pool of uninformed traders, but there is a single informed trader. The price does not fully reveal the informed trader’s information because of idiosyncratic motives for trade. Bhattacharya and Spiegel (1991) identify conditions under which there is a market breakdown (i.e., no trade). While the analysis is competitive rather than explicitly strategic, there are strategic complementarities in the sense that lower participation of uninformed traders in the market reduces the incentive of other uninformed traders to participate. This framework has been used to address questions such as when new securities can shut down markets (Bhattacharya, Reny and Spiegel (1995)) and when public disclosure rules can mitigate the adverse selection problems (Spiegel and Subrahmanyam (2000)). Pagano (1989) and Dow (2004) highlight the coordination problem among uninformed traders and draw implications for designing financial institutions. In these models, the coordination problem is among identical uninformed traders, so there is no role for considering what is or is not common knowledge among them. A distinctive feature of our analysis is that we examine the coordination among traders who, while all uninformed, have different beliefs about the amount of adverse selection they face.

A large literature in game theory examines the importance of what is or is not common knowledge or approximate common knowledge in coordination games. Rubinstein (1989), Monderer and Samet (1989) and Carlsson and van Damme (2003) are key early contributions. The insight from this literature is that coordination - i.e., taking an action which is only optimal if others do so - requires approximate common knowledge, in a sense to be made precise below. The methodological contribution of this paper is providing a tractable framework in which adverse selection generates a coordination problem among uninformed traders and then showing how the relevant common knowledge requirement for coordination can be expressed in terms of the underlying adverse selection problem.

We consider an environment where it is common knowledge that an object is worth more to buyers than to sellers, but there is a lack of common knowledge about a common value component. This environment is a classic one in the mechanism design literature, with Myerson (1985) being an early reference on optimal mechanisms in this setting. We analyze what happens in simple but realistic trading mechanisms but arbitrary beliefs and higher order beliefs of traders. Dang (2008, 2009) also examines what happens in this environment in simple trading mechanisms (ultimatum offer bargaining
and double auction, respectively) with a simple information structure but endogenous information acquisition that gives rise to an endogenous lemons problem.

A number of papers have examined channels by which adverse selection may cause a market freeze. Adverse selection interacts with funding constraints in Kiyotaki and Moore (1997), and Kurlat (2009) shows how learning can exacerbate endogenous adverse selection in such settings. In Bolton, Santos and Scheinkman (2009a, 2009b), decisions of firms whether to sell early or late in response to a liquidity shock create coordination problems because of endogenous adverse selection. Kirabaeva (2010) introduces adverse selection into a Diamond and Dybvig (1983) banking model.

The recent papers of Pagano and Volpin (2008) and Dang, Gorton and Holmstrom (2009) emphasize the role of information in preventing trade. These papers highlight the payoff structure of debt as a financial claim that minimizes the information sensitivity of asset returns. In such a setting, an aggregate shock may have a disproportionate impact because it increases the information sensitivity of asset returns and thus triggers adverse selection. Our own paper shares with these earlier papers the perspective that the arrival of information (not lack of "transparency") may trigger market collapse. However, our focus is on modelling the commonality of information in an abstract trading model, while Dang, Gorton and Holmstrom (2009) endogenize the choice of financial instrument and have endogenous information acquisition in a simpler informational environment.

The notion of market confidence as approximate common knowledge ties in with a broader set of arguments on the importance of institutions that ensure common understanding, and common knowledge of the important fundamentals. We argued elsewhere (Morris and Shin (2007a)) that there are important tradeoffs between providing accuracy (individually correct beliefs) and commonality (approximate common knowledge of beliefs) for many problems in economics. Holmstrom (2009) has argued that this tradeoff is particularly important in thinking about the regulatory reforms on transparency, and the coarse nature of credit ratings. The coarse nature of the credit ratings may have a rationale in terms of promoting common understanding, at the expense of a finer grid for the fundamentals. In Morris and Shin (2007a), we argue that coarse accounting standards can be seen as an institution that could potentially provide the commonality.

Our objective in this paper is to highlight the importance of commonality of information, and how the lack of such commonality can lead to sub-optimal
outcomes. In order to emphasize our key theme, the model is deliberately stark, but we believe that our results shed much light on the broader problem of common knowledge and trade, as well as providing a spur for a more systematic investigation of the role of adverse selection in exacerbating the current financial crisis.

2 The Setting

There are \( N \) potential buyers and \( N \) potential sellers of an asset. Sellers each own one unit of the asset with private value \( v - c \), while buyers’ valuations are \( v + c \). It would be efficient for all sellers to transfer their object to the buyers, who each have unit demand, say at a price of \( v \) which splits the gains from trade.

However, there is uncertainty about the common value component \( v \): its expected value is \( \bar{v} \) and it is equal to its expectation in what we’ll refer to as the normal state with probability \( 1 - 2\delta \). However, with probability \( \delta \), the asset is a "peach" and \( v = \bar{v} + M \) and, with probability \( \delta \), the asset is a "lemon" with \( v = \bar{v} - M \). Thus we have the following distribution of the value of the asset to the agents:

<table>
<thead>
<tr>
<th>state</th>
<th>probability</th>
<th>value to sellers</th>
<th>value to buyers</th>
</tr>
</thead>
<tbody>
<tr>
<td>lemon</td>
<td>( \delta )</td>
<td>( \bar{v} - M - c )</td>
<td>( \bar{v} - M + c )</td>
</tr>
<tr>
<td>normal</td>
<td>( 1 - 2\delta )</td>
<td>( \bar{v} - c )</td>
<td>( \bar{v} + c )</td>
</tr>
<tr>
<td>peach</td>
<td>( \delta )</td>
<td>( \bar{v} + M - c )</td>
<td>( \bar{v} + M + c )</td>
</tr>
</tbody>
</table>

There is adverse selection, with proportion \( q \) of both buyers and sellers informed of the true state, and proportion \( 1 - q \) uninformed. We will implicitly assume throughout our analysis that \( qN \), the number of informed agents, is an integer.

We have in mind the archetypal example where the asset in question is an asset-backed security backed by subprime mortgages, but where the quality of the mortgage pool depends sensitively on the region and date of their origination. The two groups of traders are equally well-informed most of the time, but we allow the possibility that one or other of the traders is better informed than his trading partner, and that the information could be positive or negative. The highly skewed payoffs associated with subprime CDOs motivates payoffs in the trading game, where for most of the time there are (small) gains from trade, but for a few states of the world, there
are large payoff consequences of trade. See Coval, Jurek and Stafford (2009) for an introduction to the economics of structured finance.

2.1 Loss Ratio

Throughout the paper we will fix the gains from trade $c$ as a constant. A convenient feature of our basic set up is that ex ante welfare (the sum of the agents’ ex ante expected utility from trade) is simply $2c$ times the probability of trade.

$$\text{Ex ante welfare} = 2c \times \text{Probability of trade}$$

Buyers and sellers are randomly matched with each other. Each buyer and each seller are asked if they would like to trade at price $\overline{v}$. If both say yes, they trade. If either says no, they do not. Clearly, informed buyers will buy if and only if the asset is normal or a peach, while informed sellers will sell if and only if the asset is normal or a lemon. We first consider the case where there is common knowledge of the structure of the model and ask if there is an equilibrium where uninformed agents participate in the market and trade.

If faced with an uninformed agent on the other side of the market (a probability $1 - q$ event), an uninformed trader has an expected gain from trade of $c$. Thus we have the expression $(1 - q) c$ for an agent’s expected gains from trade with an uninformed agent.

If faced with an informed agent on the other side of the market (a probability $q$ event), an uninformed agent still hopes to get a gain from trade of $c$; but with probability $\delta$, the informed agent will not participate in trade, and the agent will lose $M + c$. Thus expected losses from trade with an informed agent are $q (\delta (M + c) - c)$. An important parameter for us will be the agents’ loss ratio defined as

$$\psi = \frac{\text{expected losses}}{\text{expected gains}} = \frac{q (\delta (M + c) - c)}{(1 - q) c} \approx \frac{q \delta M}{c},$$

where the approximation is good when $q$ is small (i.e. the incidence of informed traders is low) and $\delta M$ is much larger than $c$ - that is, when the
possible loss to holding a defective security is much larger than the possible underlying gains from trade. Arguably, these are precisely the attributes that were involved in the most toxic of the mortgage-backed securities such as CDOs and CDO-squareds.

The loss ratio \( \psi \) plays a pivotal role in our paper. Consider the decision problems of the informed and uninformed traders. Informed traders have a dominant strategy. Informed buyers will offer to buy if and only if the asset is normal or a peach, while informed sellers will offer to sell if and only if the asset is normal or a lemon.

Consider the reasoning of an uninformed buyer. For uninformed buyers, their optimal strategy will be determined by their beliefs about the outcome and the proportion of uninformed agents who trade (denoted by \( p \)). Gains from trade conditional on the outcome are depicted in the following table.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Probability</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>lemon</td>
<td>( \delta )</td>
<td>( (q + (1 - q) p) (-M + c) )</td>
</tr>
<tr>
<td>normal</td>
<td>( 1 - 2\delta )</td>
<td>( (q + (1 - q) p) c )</td>
</tr>
<tr>
<td>peach</td>
<td>( \delta )</td>
<td>( (1 - q) p (M + c) )</td>
</tr>
</tbody>
</table>

For the buyer, the unfavorable outcome is when the good is a lemon, which happens with probability \( \delta \). Trade happens with the insider with probability \( q \) and with an uninformed trader with probability \( (1 - q) p \). Conditional on trade taking place, the payoff is \(-M + c\). Hence, the expected payoff for the buyer when the good is a lemon is given by the top right hand cell of the table above. The other cells in the table are derived through analogous reasoning. For the uninformed trader, the ex ante expected payoff when offering to trade is then given by

\[
\delta (q + (1 - q) p) (-M + c) + (1 - 2\delta) (q + (1 - q) p) c + \delta (1 - q) p (M + c) \\
= (1 - q) c p - \delta q (M + c) + q c \\
= (1 - q) c \left( p - \frac{q (\delta (M + c) - c)}{(1 - q) c} \right) \\
= (1 - q) c (p - \psi) \tag{1}
\]

From (2), we see the important role played by the loss ratio \( \psi \). The expected gain from trade is positive or negative depending on whether the loss ratio is smaller or larger than the probability \( p \) that the uninformed traders participate in the market. If the loss ratio \( \psi \) is less than or equal to
one, and if other uninformed traders offer to trade (so \( p \) equals 1), then (2) is positive. Hence the best reply is to offer to trade, meaning that everyone offering to trade is sustainable as an equilibrium. However, if the loss ratio \( \psi \) is strictly greater than one, the payoff to trading is guaranteed to be negative irrespective of the actions of others. Hence, the only equilibrium is the no trade equilibrium where all uninformed traders refuse to trade. We summarize our finding with the following preliminary result.

**Lemma 1.** Suppose \( \psi \) is common knowledge and \( \psi \leq 1 \). Then there is an equilibrium where trade always takes place. If \( \psi \) is common knowledge and \( \psi > 1 \), the only equilibrium is the no trade equilibrium.

In this sense, when \( \psi \) is common knowledge, the possibility of trade hinges on whether the loss ratio is greater than or less than one. However, the introduction of incomplete information can drastically reduce the possibility of trade. Exactly how incomplete information reduces the possibility of trade is the topic of our analysis. We will start by reporting an example showing how lack of common knowledge of the loss ratio can have a large impact and prevent trade even when \( \psi \) is much below 1. We will then explain the logic of this sensitivity in words before reporting formal analysis in the next Section.

### 2.2 Incomplete Information

We turn to the case where \( \psi \) is not common knowledge. Suppose instead that \( M \) is a random variable with possible support on \((c, \infty)\) and each informed and uninformed agent has different information about \( M \), and hence \( \psi \). We begin with an example in the spirit as Rubinstein’s (1989) email game. Suppose there is a set of possible states

\[
\Omega = \{1, 2, \ldots, 2K + 1\}
\]

There is a uniform prior over \( \Omega \). The loss ratio \( \psi \) now depends on the realized state in \( \Omega \), and takes two values - high or low. The loss ratio at state \( k \) is denoted by \( \psi_k \), and is given by

\[
\psi_k = \begin{cases} 
\psi_H & \text{if } k = 1 \\
\psi_L & \text{if } k > 1 
\end{cases}
\]

where \( \frac{1}{2} < \psi_L < 1 \) and \( \psi_H > \frac{3}{2} \). The uninformed sellers all have information about the state described by the following partition:

\[
\{\{1\}, \{2, 3\}, \ldots, \{2K - 2, 2K - 1\}, \{2K, 2K + 1\}\}
\]
while the uninformed buyers’ information partition is

\[ \{\{1, 2\} \cup \{3, 4\} \cup \ldots \cup \{2K - 1, 2K\} \cup \{2K + 1\} \}. \]

In this way, all traders have very good information on the fundamental value of the loss ratio \( \psi \). Indeed, all types - except buyers with information set \( \{1, 2\} \) - know the true value of \( \psi \) perfectly. Moreover, for all states other than state 1 - and thus with high ex ante probability - the loss ratio \( \psi \) is strictly below 1, which is the pre-condition for mutually beneficial trade. However, the incomplete information drastically curtails the possibility of trade so that the only equilibrium is the no trade equilibrium, as we will now show.

For an uninformed seller with information \( \{1\} \), the dominant action is to refuse to trade, since the loss ratio is known to be greater than 1. For an uninformed buyer with information \( \{1, 2\} \), the loss ratio is uncertain, but the average loss ratio is

\[ \frac{1}{2} \psi_L + \frac{1}{2} \psi_H > 1 \]

so that it is optimal not to participate (even if he expects uninformed sellers to participate).

Now consider the seller with information \( \{2, 3\} \). This trader knows that the loss ratio is \( \psi_L < 1 \). From (2), the expected payoff to participating in trade is

\[ (1 - q) c (p - \psi_L) \]

where \( p \) is the probability of meeting an uninformed buyer. However, the buyer with information \( \{1, 2\} \) is not participating in the market, and so the probability of meeting an uninformed buyer is 0.5 if the buyers with information \( \{3, 4\} \) are in the market, and is zero if these buyers are not in the market. In any case, the probability of meeting an uninformed buyer is at most 0.5. This means that \( \psi_L > p \) for sure, so that the gain from trade is negative. Thus, any seller with information \( \{2, 3\} \) refuses to participate in the market.

With all \( \{2, 3\} \) sellers out of the market, the \( \{3, 4\} \) buyers refuse to participate. With the \( \{3, 4\} \) buyers out of the market, the \( \{4, 5\} \) sellers refuse to participate, and so on. By iteration, all buyers and all sellers refuse to participate. Hence, the only equilibrium is the no trade equilibrium.

The example shows that even if the loss ratio is strictly below 1 for almost all states, a small “contamination” of a high loss ratio in one state can undermine market confidence through contagious adverse selection where the
uninformed traders pull out of the market, so that the only equilibrium outcome is for there to be no trade everywhere.

This is a result that echoes the result in Rubinstein’s (1989) email game. Morris, Rob and Shin (1995) discuss the underlying game-theoretic structure of Rubinstein’s email game, and how the possibility of coordination depends on the interaction of the payoff function and degree of common knowledge derived from the individual information structures.

What is the general logic of this example? If there is uncertainty about $M$ (and thus $\psi$) a trader will participate in the market if and only if his expectation of the proportion of others participating (given by $p$) exceeds his expected loss ratio. If we assume symmetry between sellers and buyers in their beliefs and higher order beliefs, a trader will participate in the market if and only if all the following statements hold.

1. his expectation of the loss ratio is less than 1,

2. (1) is true and his expectation of the proportion of agents on other side of market for whom (1) is true is greater than his expectation of the loss ratio,

3. (2) is true and his expectation of the proportion of agents on other side of market for whom (2) is true is greater than his expectation of the loss ratio,

4. and so on...

While stated informally, this turns out to be an exact characterization of when trade is possible in equilibrium. To see this, let us first give an informal argument on the necessity and sufficiency of the above list of statements for a trader to participate in the market. We will follow in the next section with a more formal argument associated with the notion of market confidence.

First, consider the “only if” part of proof. We need to show that if any one of the above list of statements is false, then the trader will not participate. If (1) is false, then the decision to participate is dominated by the decision not to participate. If (2) is false, then the decision to participate is not the best reply to any strategy profile of other traders that is undominated. In other words, if (2) is false, the decision to participate is knocked out at the second round of the iterated deletion of strictly dominated strategies. We can then proceed by induction. If (3) is false, then the decision to participate
is eliminated in the third round of the iterated deletion of strictly dominated strategies. In this way, if any one of the above list of statements is false, then the decision to participate does not survive iterated dominance, and hence is not part of any equilibrium.

In the example above, we see the iterated dominance argument in action. The seller with information \{1\} and buyer with information \{1, 2\} refuse to trade as a dominant action. Thus, for the seller with information \{2, 3\}, participating in the market is eliminated in the second round of deletion in the iterative deletion of dominated strategies. In this way, for any trader, one of the statements in the above list turns out to be false.

The “if” part of the proof rests on a convergence argument, where the list of statements above eventually picks out a set of traders for whom all the statements are true, and the decision to participate in the market is a best reply to all others in the set participating. As we will prove formally in the next section, the infinite list of statements above is equivalent to the following self-referential "fixed point," statement. Say that an agent has market confidence if and only if his expectation of the proportion of agents with market confidence is greater than his expected loss ratio. An agent trades if and only if he has market confidence.

3 Market Confidence

We will now formalize the self-referential idea of having "confidence in the market" - namely, that an agent has confidence in the market if his expectation of the proportion of traders with confidence in the market is greater than his expected loss ratio. We do this in two steps. First, we introduce a formal language to talk about an agent’s beliefs, an agent’s expectation of proportions of other agents holding certain beliefs, and so on, reporting some important properties of such higher order expectations of proportions. Second, we show how the properties of beliefs we have introduced characterize equilibria in the trading game.

Suppose that we have a collection of \(N\) agents. Each agent has a set of possible types \(T_i\), where \(T_i\) is a measurable set. We write \(\pi_i : T_i \to \Delta(T_{-i})\) for agent \(i\)’s beliefs about other agents’ types, where \(\Delta(T_{-i})\) is the notation for the set of all probabilities distributions for types of all agents other than \(i\).

We will be interested in rectangular events on the type space. A rectan-
gular event $E$ is the $n$-tuple

$$E = (E_1, E_2, \cdots, E_N)$$

where each $E_i \subseteq T_i$. Now let $\psi = (\psi_1, \psi_2, \cdots, \psi_N)$ where $\psi_i : T_i \to \mathbb{R}$ is the mapping that associates each type with a number for that type. The $n$-tuple $\psi$ is the profile of such functions - one for each agent. Later on we will associate these functions with variable loss ratios but for now they will simply be state dependent threshold beliefs that we want to introduce a language to reason about.

For a given rectangular event $E$, say that agent $i$ “$\psi_i$-believes $E$” if his expectation of the proportion of agents whose type $t_j$ belong in $E_j$ is at least as large as $\psi_i$. We use the notation $B_{\psi_i}^i (E)$ to denote the set of $i$’s types that $\psi_i$-believe $E$. In other words,

$$B_{\psi_i}^i (E) = \left\{ t_i \in T_i \Bigg| \int_{T_{-i}} \frac{1}{N} \# \{j | t_j \in E_j\} d\pi(t_i) \geq \psi_i \right\}.$$

Also write

$$B^\psi (E) = \left( B_{\psi_1}^1 (E), B_{\psi_2}^2 (E), \cdots, B_{\psi_N}^N (E) \right)$$

Thus, if the $n$-tuple of types $t$ belong in $B^\psi (E)$, then every agent $\psi_i$-believes $E$. For any rectangular event $E$, we see that $B^\psi (E)$ is also a rectangular event, since $B_{\psi_i}^i (E)$ is a subset of agent $i$’s types. Also, note that the $\psi$-belief operator $B^\psi (\cdot)$ has the closure property under logical implication - namely

$$E \subseteq E' \implies B^\psi (E) \subseteq B^\psi (E') \quad (3)$$

In other words, if a profile of types believe $E$ and $E$ implies $E'$, then they believe $E'$. The iterated event $B^\psi \left( B^\psi (E) \right)$ is defined as

$$B^\psi \left( B^\psi (E) \right) = \left( B_{\psi_1}^1 \left( B^\psi (E) \right), B_{\psi_2}^2 \left( B^\psi (E) \right), \cdots, B_{\psi_N}^N \left( B^\psi (E) \right) \right)$$

We use the notation $(B^\psi)^n (E)$ to denote the $n$-fold iteration of $B^\psi (\cdot)$ on the event $E$. Define $C^\psi (E)$ as

$$C^\psi (E) = B^\psi (E) \cap B^\psi \left( B^\psi (E) \right) \cap B^\psi \left( \left( B^\psi \right)^2 (E) \right) \cap \cdots \quad (4)$$
Say that there is common $\psi$-belief of rectangular event $E$ among the profile of agent types $t$ if

$$t \in C^\psi(E)$$

(5)

Say that rectangular event $E$ is $\psi$-evident if

$$E = C^\psi(E)$$

(6)

In other words, an event $E$ is $\psi$-evident when it is a fixed point of the common $\psi$-belief operator $C^\psi(\cdot)$. The definition of common $\psi$-belief and $\psi$-evident events are extensions of the definitions introduced by Monderer and Samet (1989), who in turn generalized Aumann’s (1976) notion of common knowledge. We have the following characterization of common $\psi$-belief.

**Proposition 2.** Rectangular event $E$ is common $\psi$-belief at $t$ if and only if there exists a $\psi$-evident event $E_0$ such that $t \in E_0 \subseteq E$.

**Proof.** For the ‘if’ direction, note that since $E'$ is $\psi$-evident, we have $E' \subseteq B^\psi(E') \subseteq B^\psi(B^\psi(E')) \subseteq \cdots$. From (3), we then have $E' \subseteq E \subseteq B^\psi(E) \subseteq B^\psi(B^\psi(E)) \subseteq \cdots$. Hence, $E$ is common $\psi$-belief at $t$. For the ‘only if’ direction, if $E$ is common $\psi$-belief at $t$, then $C^\psi(E) = B^\psi(C^\psi(E))$, so that $C^\psi(E)$ is $\psi$-evident.

The equivalence of the fixed-point definition and the iterative definition of common $\psi$-belief is in the spirit of the characterization of common knowledge by Aumann (1976), of common $p$-belief by Monderer and Samet (1989) and of state dependent $p$-belief in Morris and Shin (2007b). However, there is an added ingredient to our definition of common $\psi$-belief, which links with our earlier discussion of the possibility of trade.

There is tight connection between the notion of common $\psi$-belief and equilibrium in the trading game. To make the connection, suppose that there are $N$ buyers and $N$ sellers in the trading game that we discussed earlier. Each buyer may have different beliefs. But make a simplifying symmetry assumption: for each buyer, there is a seller who has identical beliefs about the loss ratio. Now note that rectangular events can be interpreted as strategy profiles in a binary action game. For rectangular event $E = (E_1, E_2, \cdots, E_N)$, we interpret type $t_i \in E_i$ as a type of trader $i$ who participates in the market. Thus, we may interpret $E$ as a full list of all types of all traders who participate in the market. Now, consider the belief operator $B^\psi(\cdot)$ where $\psi = (\psi_1, \psi_2, \cdots, \psi_N)$ is the profile of the expected loss ratios of the traders.
Then the rectangular event $B^\psi (E)$ has the following interpretation. When all types in $E$ participate in the market, then $B^\psi (E)$ consists of all types $t_i$ of all traders $i$ who puts probability at least $\psi_i$ to meeting an uninformed trader who is participating in the market. Hence, $B^\psi (E)$ consists of all types $t_i$ of all traders $i$ whose best reply is to participate in the market when all types in $E$ participate in the market. We thus have:

**Proposition 3.** There is an equilibrium in which all types in $E$ participate in the market if and only if $E$ is $\psi$-evident.

**Proof.** The proof follows straightforwardly from the fact that $E$ being $\psi$-evident means $E = B^\psi (E)$, so that strategy profile $E$ is a fixed point of the best reply mapping $B^\psi (.)$.

Propositions 2 and 3 show the usefulness of our definition of common belief. The belief operator $B^\psi (.)$ has two faces. On the one hand, it has the interpretation as a belief operator. However, it also has the interpretation as the best reply mapping. It is this dual face of the belief operator that allows us to define the notion of market confidence that links beliefs directly with equilibrium of the trading game.

Say that trader $i$ has confidence in the market if

$$t_i \in B_i^\psi (T) \cap B_i^\psi (B^\psi (T)) \cap B_i^\psi \left( \left( B^\psi \right)^2 (T) \right) \cap \ldots$$

(7)

where $T$ is the set of all types. Finally, say that there is market confidence if all traders have confidence in the market. In other words, there is market confidence if

$$t \in C^\psi (T)$$

(8)

When there is market confidence, the following list of statements are all true.

1. The loss ratios of all traders is less than 1 (since $B^\psi (T)$ is non-empty)
2. For every agent $i$, his expectation of the proportion of other agents whose loss ratios are less than 1 is at least $\psi_i$
3. For every agent $i$, his expectation of the proportion of agents for whom (1) and (2) are true is at least $\psi_i$
4. and so on.

Since $T$ is the set of all types, it has the interpretation of the strategy profile where every uninformed trader participates in the market. Hence, $C^\psi(T)$ has the interpretation of the “largest” equilibrium in the trading game which maximizes market participation of all traders. Therefore, we have the following succinct characterization of equilibrium with trade.

**Proposition 4.** There is an equilibrium in which type profile $t$ participate in the market if and only if there is market confidence at $t$.

This proposition links the iterative definition of common belief (which is also the iteration of the best reply mapping) with the self-referential, fixed point definition of market confidence. It enables us to diagnose the failure of the traders to exploit opportunities for mutually beneficial trade in terms of a failure of market confidence. The mere fact that mutually beneficial trade exists is not enough for trade to take place. Nor is it enough for all traders to believe that mutually beneficial trade is possible. Instead, we need a more stringent condition that ensures that there is sufficient common understanding of the existence of mutually beneficial trade. It is this common understanding that is captured by our definition of market confidence.

### 3.1 A Global Games Example

We can illustrate our notion of market confidence, why it characterizes trade and why it is surprisingly hard to achieve, with a global game example (Carlsson and van Damme (1993), Morris and Shin (1998, 2003)). We emphasize that Proposition 4 holds more generally than the global games information structure, but use this example to highlight why market confidence may be hard to achieve.

Suppose that a variable $\theta$ is distributed on the real line. There are $N$ buyers and $N$ sellers. Buyer $i$ and seller $i$ both observe a signal $t_i = \theta + \varepsilon_i$, where $\varepsilon_i$ is independently distributed normally with mean 0 and standard deviation $\sigma$. Thus as above, we assume that for every buyer, there is a seller with the same signal about $\theta$: this simplifies the exposition and, because we will focus on the case of large $N$, does not effect the qualitative conclusions. Also for simplicity, we will assume that $\theta$ is distributed uniformly on the real line, but the argument will go through for any smooth prior if $\sigma$ is small.
enough. Suppose that the loss ratio takes a high value \( \psi_H > \frac{3}{2} \) if \( \theta \) is less than 0 but takes a low value \( \psi_L \in \left( \frac{1}{2}, 1 \right) \) if \( \theta \) is greater than 0. Each trader \( i \)'s expected loss ratio is close to \( \psi_L \) for positive \( t_i \) and close to \( \psi_H \) for negative \( t_i \). Specifically,

\[
\psi_i(t_i) = \Pr(\theta < 0 | t_i) \psi_H + \Pr(\theta > 0 | t_i) \psi_L \\
= \left( 1 - \Phi \left( \frac{t_i}{\sigma} \right) \right) \psi_H + \Phi \left( \frac{t_i}{\sigma} \right) \psi_L \\
\geq \psi_L
\]

This example now fits the framework introduced at the beginning of this Section; in particular, each trader \( i \) will have a multinomial normal distribution about the signals of other \( N - 1 \) traders.

We will argue that no matter how accurate the traders' signals are (i.e., no matter how small \( \sigma \) is as long as it strictly positive) and no matter how large the state \( \theta \), and thus how confident traders are that the loss ratio is less than 1, there is never non-trivial common \( \psi \)-belief as defined above for sufficiently large \( N \), and thus there is never market confidence, and thus (by Proposition 4) there is no trade. To show that common \( \psi \)-belief does not arise, it is enough, by Proposition 2, to show that no rectangular event is \( \psi \)-evident. To see why, consider the event that each trader observes a signal above some high threshold \( t^* \), i.e.,

\[
E = E_1 \times \ldots \times E_N \\
= \{ t \in \mathbb{R}^N | t_i \geq t^* \text{ for each } i \} \tag{9}
\]

When is this event \( \psi \)-evident? A trader observing signal \( t_i \) thinks that \( \theta \) is normally distributed with mean \( t_i \) and standard deviation \( \sigma \) and that that any other trader \( j \)'s signal - equal to \( t_j = \theta + \varepsilon_j \) - is normally distributed with mean \( t_i \) and standard deviation \( \sqrt{2}\sigma \). Thus a trader observing signal \( t_i \geq t^* \) will assign probability \( 1 - \Phi \left( \frac{t^*-t_i}{\sqrt{2}\sigma} \right) \) to any other trader observing a signal above \( t^* \). Thus his expected proportion of traders with signals above \( t^* \) is

\[
\frac{1}{N} + \frac{N - 1}{N} \left( 1 - \Phi \left( \frac{t^*-t_i}{\sqrt{2}\sigma} \right) \right)
\]

For this to exceed \( \psi_i(t_i) \), it must exceed \( \psi_L > \frac{1}{2} \). Thus we must have

\[
1 - \Phi \left( \frac{t^*-t_i}{\sqrt{2}\sigma} \right) > \frac{N\psi_L - 1}{N - 1}
\]
and so
\[ t_i > t^* - \sqrt{2} \sigma \Phi^{-1} \left( \frac{N}{N-1} \right) (1 - \psi_L) \]
Thus the event \( B^\psi \left( \{ t \in \mathbb{R}^N | t_i \geq t^* \text{ for each } i \} \right) \) is a subset of the event:
\[ \left\{ t \in \mathbb{R}^N \left| t_i > t^* - \sqrt{2} \sigma \Phi^{-1} \left( \frac{N}{N-1} \right) (1 - \psi_L) \right. \text{ for each } i \right\} \]
Since \( \psi_L > \frac{1}{2} \), for sufficiently large \( N \), \( \left( \frac{N}{N-1} \right) (1 - \psi_L) < \frac{1}{2} \) and thus
\[ -\sqrt{2} \sigma \Phi^{-1} \left( \frac{N}{N-1} \right) (1 - \psi_L) > 0. \]
Thus any rectangular event \( E \) of the form (9) is not \( \psi \)-evident for any \( t^* \), since there is always some type \( t_i > t^* \) included in the event whose expected proportion of traders observing signals above \( t^* \) is less than \( \psi_L \). But this in turn implies that no event that includes an event of the form (9) is \( \psi \)-evident and thus no non-trivial event is \( \psi \)-evident. There is never market confidence and thus there is never trade.

The intuition for this result is as follows. The loss ratio never falls below \( \frac{1}{2} \). While this would be enough to sustain trade if the loss ratio were common knowledge, with noisy signals, a marginal uninformed trader - who is just willing to trade - will anticipate that about \( \frac{1}{2} \) the traders on the other side of the market will have observed lower signals and will not be trading. With half the uninformed traders not trading and a loss ratio more than \( \frac{1}{2} \), trade is not optimal for that marginal trader, a contradiction.

## 4 General Asset Returns and Trading Games

We now turn to how our result applies to more general contexts with general asset payoff distributions and trading games.

### 4.1 Asset Returns

Consider the model as before but assume that the common value component of the asset \( v = \bar{v} + \eta \), where \( \eta \) is smoothly distributed according to symmetric density \( f_\theta (\cdot) \), so \( \theta \) parameterizes asset returns. Informed sellers will trade
only if \( v - c = \overline{v} + \eta - c < \overline{v} \), i.e., if \( \eta < c \). Informed buyers will trade only if \( v + c = \overline{v} + \eta + c > \overline{v} \), i.e., if \( \eta > -c \). Thus a key parameter will be the probability under the distribution \( \theta \) that returns in the tails will be more than \( c \) from the mean:

\[
\delta (\theta) = \text{Prob}_\theta (\eta \geq c) = \int_{\eta=c}^{\infty} f_\theta (\eta) \, d\eta = 1 - F_\theta (c)
\]

Another key parameter will be the expected deviation of the common value of the asset from its mean if returns are in one of the tails:

\[
M (\theta) = E_\theta (\eta|\eta \geq c) = \frac{1}{\delta (\theta)} \int_{\eta=c}^{\infty} \eta f_\theta (\eta) \, d\eta
\]

These will be the only parameters of returns that will matter in our trading game. In particular, for each distribution \( \theta \), there is a corresponding loss ratio defined as above:

\[
\psi (\theta) = \frac{\text{expected losses}}{\text{expected gains}} = q (\delta (\theta) (M (\theta) + c) - c) \left/ \frac{1 - q}{c} \right.
\]

Maintaining symmetry between beliefs and higher order beliefs of sellers and buyers, the analysis of Section 2 goes through exactly as before where an agents’ beliefs and higher order beliefs about \( \theta \) determine his beliefs and higher order beliefs about his and other agents’ loss ratio.

Our ability to derive an exact characterization of when trade occurs relies on a couple of features of the model. We assume that traders are informed or uninformed (there is nothing in between). This is crucial to the analysis, since it means that adverse selection translates into a pure coordination problem, and agents’ assessment of the loss ratio is not correlated with their assessment of the proportion of agents on the other side of the market trading. A more realistic modelling would allow for intermediate types. But as we explained in the introduction, we wanted to focus on the coordination element of adverse selection.
4.2 Losing Market Confidence

How is market confidence lost? Suppose that initially there was common knowledge or approximate common knowledge of a loss ratio greater than $\frac{1}{2}$ but less than 1. There could be an equilibrium where everyone trades and all ex ante gains from trade are realized. Now suppose that there is a shock. Perhaps the loss ratio increases or perhaps it decreases, but crucially there is no longer approximate common knowledge of the loss ratio. Then a crisis will hit. It is the loss of approximate common knowledge of the loss ratio (which we label "market confidence") and not changes in the loss ratio that generate the crisis.

We already illustrated in Section 3.1 how the noisy private signals of global games can formalize the idea that agents have very accurate information without attaining common knowledge or approximate common knowledge. Here, we generalize this point - appealing to arguments used in the global games literature - to show how the critical loss ratio sustaining trade is reduced to $\frac{1}{2}$ from 1 if there is noisy private information.

In Section 3.1, we discussed the case where the common value component of the asset’s payoffs was parameterized by $\theta$ and the loss ratio corresponding to $\theta$ was written as $\psi(\theta)$. Let $g(\cdot)$ be a prior probability density on $\theta$ and suppose now that $\psi(\theta)$ were decreasing in $\theta$, so higher states $\theta$ correspond to a lower loss ratio. Suppose each agent (buyers and sellers) observes a signal $t_i = \theta + \sigma \varepsilon_i$, where $\sigma$ is a parameter measuring the size of noise and $\varepsilon_i$ is a noise term distributed according to density $f(\cdot)$ in the population. This information structure describes a type space as in Section 3, parameterized by $\sigma$, where each $T_i = \mathbb{R}$,

$$
\pi_i^\sigma(t_{-i}|t_i) = \frac{\int_0^\infty \prod_{j=1}^N f_j \left( \frac{t_j - \theta}{\sigma} \right) g(\theta) d\theta}{\int_0^\infty f_i \left( \frac{t_i - \theta}{\sigma} \right) g(\theta) d\theta}
$$
and

\[ \psi_i^\sigma(t_i) = \frac{\int_{\theta} \psi(\theta) f_i \left( \frac{\theta - \theta_i}{\sigma} \right) g(\theta) d\theta}{\int_{\theta} f_i \left( \frac{\theta - \theta_i}{\sigma} \right) g(\theta) d\theta} \]

Now let \( \theta^* \) be the point where the loss ratio is reduced to \( \frac{1}{2} \). Generalizing the argument in Section 3 and adapting arguments in the global games literature (see, e.g., Morris and Shin (2003, section 4.2)), one can show that, as \( \sigma \to 0 \), agent \( i \) has market confidence (i.e., \( \psi_i^\sigma \)-belief) if and only if \( t_i \geq \theta^* \). Thus with small noisy signals, there is market confidence only when the loss ratio is half as much as when there is common knowledge of the loss ratio. Such a reduction in the threshold level of the loss ratio could result in a drastic curtailment of trading, as we saw in the example earlier in the spirit of Rubinstein’s email game.

### 4.3 General Trading Mechanisms

We have assumed that agents made a yes or no decision whether to trade at some suggested price. This seems like a realistic assumption in the setting of over-the-counter (OTC) markets where mortgage-backed securities are traded. However, there are arguments that suggest that our results will be robust to generalized trading mechanisms.

To see this, consider a double auction setting where each trader proposes a price to trade and trade takes place at the average of the proposed prices only if the sell price is less than the buy price. Consider the following strategy profile in the double auction. All agents propose trade at price \( \overline{v} \) if they have market confidence. If they do not have market confidence, informed sellers propose trade at \( v + c \), informed buyers propose trade at \( v - c \), uninformed buyers propose trade at \( \overline{v} - M + c \) and uninformed sellers propose trade at \( \overline{v} + M - c \).

To check that this is an equilibrium, observe that an uninformed agent will never get any surplus if his trading partner does not have market confidence, and he will maximize his surplus if his opponent has market confidence if he proposes trade at price \( \overline{v} \). Thus under this strategy profile, he has essentially the same payoffs as in the simple trading game. Now consider a seller with a
lemon. His expected gain from proposing price $\bar{v}$ is $(c + M)$ times the probability he attaches to having an uninformed partner who believes in market confidence. His expected gain from proposing price $\bar{v} - M + c$ is $2c(1 - q)$. So his strategy is optimal if there is a lower bound on the probability he attaches to his uninformed partner believing in market confidence and $M$ is sufficiently large relative to $c$.

5 Lessons on the Crisis

Our model is stylized but informed by the recent financial crisis, and it addresses some puzzling questions that are raised by the financial crisis of 2007 - 2009. The problem of “toxic assets” first hit the headlines when the subprime crisis heralded the beginning of the global financial crisis in August 2007. The market for certain asset-backed securities, especially those backed by subprime residential mortgages was the first to suffer extreme illiquidity, as trading slowed to a trickle and market-clearing prices became virtually impossible to establish.\(^1\) The opaqueness of the asset-backed securities market and the attendant potential for adverse selection has frequently been blamed for the sudden drying up of liquidity. Yet, there is a puzzle at the heart of the crisis. Uncertainty about the true value of an asset should not invariably lead to the breakdown of trade. The stock market is a good illustration of how financial markets are normally well adapted to aggregating the diverse information of traders and arriving at a market-clearing price.

Our framework gives a possible avenue to resolving this puzzle. The starting point of our analysis was adverse selection resulting from information asymmetries on the true value of the asset. For asset-backed securities, the heterogeneity of the underlying loan pools that back the securities gives ample scope for greater expertise and information in ascertaining the fundamental value of the securities. When overall economic fundamentals are strong, such asymmetric information need not matter for the value of the particular asset-backed security, since such securities are debt claims that are insensitive to the value of the underlying claims, as noted by Gorton and Pennacchi (1990). However, when a shock impacts the economy (such as reversal of the housing market that ultimately underpins the value of the security), then the true

\(^1\)See Gorton (2010) for a detailed description of the subprime securitization process and the initial phase of the crisis. See also Adrian and Shin (2010), Greenlaw et al. (2008) and Brunnermeier (2009).
value of the debt security becomes more sensitive to private information and the asymmetric information begins to exert an influence in the trading decisions.

Moreover, the new breed of asset-backed securities such as collateralized debt obligations (CDOs) written on subprime mortgages have skewed payoffs in which they retain their value close to face value in most states of nature, but suffer catastrophic loss in extremely bad states (see Coval, Jurek and Stafford (2009)). Theoretically, the skewness of payoffs that combines small gains from trade in normal states with large losses in bad states is captured by the parameter $M$. Recall that the loss ratio $\psi$ can be approximated as

$$\psi \approx \frac{q\delta M}{c}$$

where the approximation holds good when $q$ is small (i.e. the incidence of informed traders is low) and $\delta M$ is much larger than $c$ - that is, when the possible loss to holding a defective security is much larger than the possible underlying gains from trade. $M$ being large is arguably a good description of the most toxic of the mortgage-backed securities such as CDOs and CDO-squareds.

In addition, note that our results do not rely on $\psi$ going up uniformly across all types of traders in the market. The breakdown of market confidence can result from the $\psi$ loss ratios going up for a small number of traders, and it is these traders leaving the market that sets off the vicious circle of greater illiquidity inducing more traders to leave the market.

6 Concluding Remarks

Our result on the importance of approximate common knowledge in enabling mutually beneficial trade reiterates the importance of shared understanding as in many other areas of economic life. Arguably, credit ratings and accounting numbers also derive part of their importance from common understanding. Holmstrom (2009) argues that the coarse nature of credit ratings serve this importance purpose, and that misguided attempts to enhance “transparency” by making finer distinctions may undermine this useful purpose. Elsewhere (Morris and Shin (2007a)), we have argued that accounting numbers also serve the important role of generating shared understanding. There are inevitable tradeoffs. The imperative for common understanding
can sometimes detract from the precision of accounting numbers. Common understanding is predicated on the lowest common denominator — the coarsest shared framework among a set of disparate individuals. So, the coarser is the information, the greater is the chance that the information can be understood by all. However, coarse information is also imprecise information. When communication is based on the coarsest individual information, there will be many individuals who are capable of handling more finely nuanced and complex usage. Hence there may be welfare losses when the opportunity to utilize the greater sophistication is forgone in favor of simplicity.

When common understanding is important, it is possible that greater precision of information can be detrimental to welfare if the greater precision comes at the expense of greater fragmentation, or if the greater precision of information leads to an exacerbation of externalities in the use of information that detracts from overall welfare. Accountants make the distinction between disclosure of information (e.g., reporting of numbers in a footnote) and recognition (e.g., inclusion in profit and loss statement) and observe that the latter has a larger empirical impact than the former (Barth, et al. 2003; Espahbodi, et al. 2002). The greater impact of recognized numbers presumably reflects greater common understanding of that information.

In this paper, we have seen the interaction between asymmetric information and the coordination motive generated by that asymmetric information. Our theme has been the corrosive effect of even small amounts of adverse selection in an asset market and how it can lead to the total breakdown of trade. In our model, there is common knowledge that an asset is worth strictly more to the buyer than the seller at every state of the world, and yet there can be a breakdown of trade. The problem is the failure of common understanding, and in particular what we have termed “market confidence”, defined as approximate common knowledge of an upper bound on expected losses.
References


