Lecture 1
Endogenous Risk

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Acknowledgements


Millennium Bridge
**Diagnosis**

Trouble was at 1 hertz (one complete cycle per second)

Walking pace is approximately two steps per second (2 hertz)

Although most force exerts down when walking, there is small sideways force every two steps (1 hertz)
Diagnosis
Probability of Coordination

What is the probability that a thousand people walking at random end up walking exactly in step, and remain in lock step thereafter?

• If individual steps are independent, then probability is close to zero.

• But if there is a coordination mechanism, the probability is close to 1 under the right conditions.

Bridge moves $\rightarrow$ Adjust stance

$\uparrow$
Further adjust stance

$\downarrow$
$\leftarrow$ Push bridge
Millennium Bridge Analogy

Bridge moves

Pedestrians adjust stance
Millennium Bridge Analogy

Prices and measured risks change

Banks adjust balance sheet
Dual Role of Market Prices

Market prices play two roles

- Reflection of fundamentals
- Imperative for actions

Sometimes, reliance on market prices can distort market prices
**Endogenous Risk**

Endogenous risk refers to the risk from shocks generated and amplified within the system (feedback effects).

It stands in contrast to exogenous risk, which refer to the risk from shocks that originate from outside the system (storms, earthquakes, heavy loads).

Preconditions for feedback

- Marking to market
  - Sensitivity of equity cushion to price changes

- Risk management systems that depend on market variables
  - Sensitivity of required equity cushion to shifts in risk measures
• Sensitivity of price changes and risk measures to portfolio adjustments
  – Uniformity of trading positions among active market participants
Overview of Lectures

Lecture 1.
Market booms and crashes

Lecture 2.
How does endogenous risk explain the subprime crisis?

Lecture 3.
How should financial regulation and monetary policy be formulated to take account of all this?
Value at Risk

Informally speaking, value at risk is motivated by the question:

“What (realistically) is the worst that could happen over one day, one week, or one year?”

Definition. Let $W$ be a random variable. The value at risk at confidence level $c$ relative to base level $W_0$ is the smallest non-negative number denoted by VaR such that

$$\text{Prob}(W < W_0 - \text{VaR}) \leq 1 - c$$

Example. $W$ is the market value of assets of the firm at some fixed date in the future, and $W_0$ is today’s assets. If the firm has capital (equity) equal to value at risk, it will remain solvent with probability $c$. 
Long Short Strategy Hedge Fund

A hedge fund can hold any combination of:

- Cash
- Security 1
- Security 2

Negative holding of cash means that the hedge fund has borrowed the money.

Negative holding of a security means that the hedge fund has borrowed that security and so appears on the liabilities side of the balance sheet.

Consider some examples.
The leverage of the hedge fund is defined as the ratio of total assets to equity.

\[
\text{leverage} = \frac{\text{total assets}}{\text{equity}}
\]

- \(a_1\) is the holding of security 1 (in $ millions)
• $a_2$ is the holding of security 2 (in $\text{\$ millions}$)

• $c$ is the holding of cash (in $\text{\$ millions}$)

• Hedge fund has equity of $e$

• Risky security $i$ has return variance $\sigma_i^2$, correlation $\rho$

Balance sheet constraint:

$$a_1 + a_2 + c = e$$

• The hedge fund is risk neutral, but faces a value at risk constraint
  
  – haircut imposed by prime broker in repurchase agreement
- internal risk constraint

Hedge fund’s problem is

$$\max_{a_1, a_2} E(r) \quad \text{subject to} \quad VaR \leq e$$

Value at risk is some multiple of $\sigma_r$, standard deviation of portfolio return

$$VaR = \alpha \sigma_r$$

Constraint can be written as

$$\sigma_r^2 \leq \left( \frac{e}{\alpha} \right)^2$$
Lagrangian is

\[ \mathcal{L} = E(r) - \lambda \left( \sigma_r^2 - \left( \frac{e}{\alpha} \right)^2 \right) \]

First order condition

\[
\begin{bmatrix}
a_1 \\
a_2
\end{bmatrix} = \frac{1}{2\lambda} \begin{bmatrix} \Sigma^{-1} & \\
\end{bmatrix} \begin{bmatrix}
\mu_1 \\
\mu_2
\end{bmatrix} \tag{\ast}
\]

where \( \Sigma \) is covariance matrix of returns. The Lagrange multiplier can be
solved from

\[
\sigma_r^2 = a' \Sigma a \\
= \frac{1}{4\lambda^2} \mu' \Sigma^{-1} \mu \\
= \left( \frac{e}{\alpha} \right)^2 \quad \text{by constraint}
\]

Lagrange multiplier is

\[
\lambda = \frac{\alpha}{2e} \sqrt{\mu' \Sigma^{-1} \mu}
\]

Optimal portfolio is

\[
\begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix}
= \frac{e}{\alpha} \cdot \frac{1}{\sqrt{\mu' \Sigma^{-1} \mu}} \begin{bmatrix}
  \Sigma^{-1}
\end{bmatrix} \begin{bmatrix}
  \mu_1 \\
  \mu_2
\end{bmatrix}
\]
Example. $\sigma^2 = 1, \mu_1 = 0.1, \mu_2 = 0.05, \alpha = 2.33$

Lagrange multiplier is

\[
\lambda = \frac{\alpha}{2e\sigma} \cdot \sqrt{\frac{\mu_1^2 + \mu_2^2 - 2\rho\mu_1\mu_2}{1 - \rho^2}}
\]

From (*), solve for optimal portfolio

\[
\begin{bmatrix}
a_1 \\
a_2
\end{bmatrix} = \frac{e}{\alpha\sigma ((1 - \rho^2)(\mu_1^2 + \mu_2^2 - 2\rho\mu_1\mu_2))} \begin{bmatrix}
\mu_1 - \rho\mu_2 \\
\mu_2 - \rho\mu_1
\end{bmatrix}
\]
For $\rho \in \left[ \frac{1}{2}, 1 \right]$, leverage is

\[
L = 1 - \frac{a_2}{e} = 1 + \frac{1}{\alpha \sigma} \cdot \frac{\rho \mu_1 - \mu_2}{(1 - \rho^2) (\mu_1^2 - 2 \rho \mu_1 \mu_2 + \mu_2^2)}
\]

Leverage is increasing in $\rho$, decreasing in $\sigma$
Portfolio weights

Leverage

\( \rho \)

\( a_1 \)

\( a_2 \)

leverage

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LTCM

Convergence trades exploiting positive correlation

- Treasuries and risky debt (corporate, mortgage-backed, swaps)
- On-the-run and off-the-run treasuries
- German and Italian government bonds
- Currencies
- Equities (mergers, Shell, HSBC)

In practice, the covariance matrix $\Sigma$ must be estimated from historical data, but historical data is backward looking.
Amplified feedback effect

\[
\text{Shock } \begin{cases} r_1 & \downarrow \\ r_2 & \uparrow \end{cases} \Rightarrow \rho \text{ down, } \mu_1 \text{ down, } \mu_2 \text{ up}
\]

\[\downarrow\]

\[\uparrow\]

\[r_1 \downarrow\]

\[r_2 \uparrow\]

\[\Leftrightarrow \text{ Sell 1, Buy 2}\]

This is just like the Millennium bridge. A shock is amplified through the actions of market participants.
Belief Updating

- Moving Average (with window $M$)

$$\mu_t = \frac{1}{M} \sum_{i=1}^{M} r_{t-i}$$

The forecast variance and covariance approximated by

$$\hat{\sigma}_t^2 = \frac{1}{M} \sum_{i=1}^{M} r_{t-i}^2$$

$$\hat{\sigma}_{12,t} = \frac{1}{M} \sum_{i=1}^{M} r_{1,t-i}r_{2,t-i}$$
Exponential Moving average (with “decay” parameter $\lambda$)

$$\hat{\mu}_t = \lambda \hat{\mu}_{t-1} + (1 - \lambda) r_{t-1}$$

$$\hat{\sigma}^2_t = \lambda \hat{\sigma}^2_{t-1} + (1 - \lambda) r^2_{t-1}$$

$$\hat{\sigma}_{12,t} = \lambda \hat{\sigma}_{12,t-1} + (1 - \lambda) r_{1,t-1}r_{2,t-1}$$

This is the method popularized by RiskMetrics (formerly part of JP Morgan) who suggests values of $\lambda$ as follows.

Daily data: $\lambda = 0.94$

Monthly data: $\lambda = 0.97$
Numerical Example

• Traders face upward-sloping supply curve and downward-sloping demand curve.

• Trades have price impact.

• Price change consists of “fundamental” component and a trade-induced component.

• Fundamental returns \((y_1, y_2)\) jointly normally distributed with \(\sigma_1^2 = \sigma_2^2 = 0.5\), \(\rho = 0.99\)

• Realised return is

\[
    r_{it} = y_{it} + \phi \cdot \Delta a_{it}
\]

(Danielsson and Shin (2008))
**Feedback from Trades to Price Changes to Trades**

- Beliefs \((\mu_{t-1}, \Sigma_{t-1})\)
- Desired holdings \(a_t\)
- Fundamental returns \((y_{1t}, y_{2t})\) and trades determine realised returns \((r_{1t}, r_{2t})\)
- Update beliefs from exponential moving average with \(\lambda = 0.94\)

\[
\mu_{t-1}, \Sigma_{t-1}, r_t \quad \overset{\lambda = 0.94}{\longrightarrow} \quad \mu_t, \Sigma_t
\]

- Desired holdings \(a_{t+1}\),
- ...

...
Correlations

Correlations and normalized Lagrange multiplier

$\text{corr}(y \sim) \quad \text{corr}(y) \quad \text{Lagrange}$

$\text{corr}(y \sim) \quad \text{corr}(y) \quad \text{Lagrange}$
Adverse shock of 3.0 on both assets at date 20, adverse shock of 3.0 on asset 2 at date 90
Two requirements for dynamic hedging

- Since a put option pays out more when price is low, this means maintaining a short position in the underlying asset.

- Since the slope of the put option’s value becomes steeper as the price
falls, this means taking an even larger short position when the underlying asset falls in price. In other words, dynamic hedging dictates that when the price **falls**, you **sell** more of the asset.

Replicating a put option through dynamic trading entails a “sell cheap, buy dear” strategy.

The delta $\Delta$ of an option is the change of the option price with respect to the change in the price of underlying asset. For a put,

$$-1 \leq \Delta \leq 0$$
Suppose a trader starts with a cash balance of \( P \), which also happens to be the price of the put option that the trader wishes to replicate. With this wealth, the trader can either purchase the put option itself, or purchase the portfolio given by:

\[
\begin{align*}
\{ & \Delta \\
\} & -S\Delta + P \\
\end{align*}
\]  

underlying asset cash

(1)
The value of this portfolio is also $P$, since the $\Delta$ units of the underlying asset has price $-S\Delta$. Since the trader wishes to replicate a put option, $\Delta$ is negative.

The portfolio given by (1) is financed by selling short $|\Delta|$ units of underlying asset at price $S'$, and adding the proceeds to the cash balance.

Now, suppose price changes to $S'$. The value of the portfolio at the new price is

$$
\Delta \cdot S' + P - S\Delta = P + \Delta (S' - S) \approx P'
$$

where $P'$ is the price of the put option given price $S'$. 

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Thus, the trader manages to approximate the wealth of a trader who starts out by holding the put option itself. Since the approximation is linear, the accuracy of the approximation is greater the smaller is the price change. The investor then forms the new portfolio:
\[
\begin{cases}
\Delta' & \text{underlying asset} \\
-S'\Delta' + P' & \text{cash}
\end{cases}
\] (2)

which is affordable given his/her wealth of \( P' \).

Proceeding in this way, the trader reaches the expiry date of the option. There are two cases we need to consider.
If option expires in the money (i.e. when the price $S$ is below the exercise price $X$), $\Delta = -1$, so that the portfolio given by (2) is

\[
\left\{ \begin{array}{ll}
-1 & \text{underlying asset} \\
S + (X - S) & \text{cash}
\end{array} \right.
\]

If the option expires out of the money, (i.e. when price $S$ is above $X$), the portfolio (2) is

\[
\left\{ \begin{array}{ll}
0 & \text{underlying asset} \\
0 & \text{cash}
\end{array} \right.
\]

Either way, the final value of the trader’s portfolio is

\[\max\{X - S, 0\}\]

which is the payoff to buying and holding one put at beginning.
1987 Stock Market Crash.

S&P 500 Index, 1987
Dynamic hedging strategy dictates selling of the underlying asset when its price falls, and dictates buying the underlying asset when its price rises.

It is a “sell cheap, buy dear” strategy.

This is because the delta of a put option becomes more negative as the price of the underlying asset falls.

When the trader is small relative to the market as a whole, or when the active traders in the market hold diverse positions, one would expect little or no feedback of the traders’ decisions on the market dynamics itself. However, when a large segment of the market is engaged in such trading strategies, the market dynamics may be affected by the trading strategy itself, and hence lead to potentially destabilising price paths.

The Brady Commission found that around $100 billion in funds were following formal portfolio insurance programs, representing around 3 percent of the pre-crash market value. However, this is almost certainly an
underestimate of total selling pressure arising from other funds that were also following similar strategies, albeit informally.

It noted that whereas some portfolio insurers rebalanced several times per day, many others followed the strategy of rebalancing their portfolios once a day - at the open, based on prior day’s close. The favored hedging instrument were the futures contracts in the underlying stock market indices, rather than the constituent stocks themselves. The futures contract for the S&P index was traded (as is today) at the Chicago Mercantile Exchange (CME). If any gap opened up between the futures contract price and the underlying constituent stocks of the S&P index, then index arbitrageurs would step in and trade profitably until the gap closed again.

The implicit selling pressure arising from the dynamic hedging rule of the traders had the potential of influencing the price of the underlying asset itself, thereby introducing further rounds of selling. In other words, the price is influenced by the selling.
During the days leading up to the crash of October 19th, the U.S. stock market had experienced sharp falls.

In the period from Wednesday October 14th to Friday October 16th, the market declined around 10%.

The sales dictated by dynamic hedging models amounted to around $12
billion (either in cash or futures contracts), but the actual sales up to Friday had only been around $4 billion. This was due mainly to the fact that much of the price declines on Friday happened in the last hour or so of trading.

This meant that by the time of the open on Monday morning, there was a substantial amount of pent-up selling pressure. Over the weekend, experienced market observers knew that the opening of the market on Monday would bring very substantial sales.

The market opened on Monday, 19th October with a large gap in the price. The price continued to fall during the day. But no-one quite imagined how large the price falls would be. Both the S&P and Dow Jones fell over 20% on October 19th.

At times, the imbalance between purchases and sales meant that much of the underlying market for stocks did not function. Instead, traders
attempted to use the index futures market to hedge their exposures. The S&P index futures sold at large discounts to the cash market on Monday 19th and Tuesday 20th for this reason.

The important lesson to emerge from the 1987 stock market crash is that uncertainty governing stock returns is better described as being endogenous rather than exogenous. The returns are generated partly by the increased selling pressure from the traders, which then interacts with price falls.
Duration Matching by Pension Fund

The market price of an asset or liability reflects the current terms of trade between willing parties, and hence provides a signal of the underlying fundamentals.

For this reason, the practice of *marking to market* (i.e. evaluating all components of a balance sheet at the current market price) is a way to incorporate the most up to date information in evaluating the true position of a firm. As such, the benefits include:

- better information for outside investors, who do not have any privileged information on the firm
- tighter discipline of managers by shining a bright light into the dark corners of a firm’s operations that would otherwise be hidden from view.
However, marking to market is not without its problems. The problem emanates from the double-edged role of market prices.

- **Reflection of fundamentals:** market prices serve as a signal of the underlying economic fundamentals.

- **Imperative to action:** market prices dictate certain actions for firms and institutions that have decided to follow price-sensitive decision rules.

One example of the second bullet point is the delta-hedging. There, prices dictate your trading decisions (“sell cheap, buy dear”).

When feedback took hold, the decision rule *contaminated* the price signal. In this sense, reliance on market prices can sometimes *distort* market prices. Paradoxically, the more one relies on market prices, the greater will be the distortion.
The stylized balance sheet of a pension fund looks as follows.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Securities</td>
<td>Annuities</td>
</tr>
<tr>
<td>Property</td>
<td>Pensions</td>
</tr>
</tbody>
</table>

Both sides of the balance sheet are quite sensitive to interest rate movements, since the items in the balance sheet involve long-term cashflows. For this reason, the managers of pension funds attempt to hedge their balance sheets against changes in interest rates.

One way to do this is through duration matching: i.e. matching the duration of assets with the duration of liabilities.

In some countries, pension funds are required by their regulators to mark their liabilities to market.
In the U.K., the accounting rule FRS 17 (financial reporting standard 17) requires pension funds to mark their liabilities to market using the discount rates implicit in the prices of high grade corporate bonds, rated at single A or above.

When liabilities are marked to market and pension funds attempt to match duration, the demand curve for some long-maturity fixed income securities can become perverse. Rather than the demand curve being downward-sloping, the demand curve can slope *upward*. That is, as price rises, the demand for the asset rises. Here is an example

**Example.** A pension fund is responsible for running a final salary pension plan. It has liabilities that grow at the rate $g$, due to the growing wages of each successive cohort of retirees who joined the pension plan. The liabilities of the pension fund is a growing perpetuity with cashflows

\[ C, (1 + g) C, (1 + g)^2 C, \cdots \]
By regulation, the pension fund must mark its liabilities to market by using the discount rate derived from the price of a riskless perpetuity that pays a constant 1 each period. Let $p$ be the price of the perpetuity that pays 1 each period. Then

$$p = \frac{1}{r}$$

where $r$ is the yield. The (modified) duration of this perpetuity is

$$-\frac{dp/dr}{p} = \frac{1/r^2}{1/r} = \frac{1}{r} = p$$

So, the modified duration of the perpetuity is just its price.

Denote by $L$ the market value of the pension fund’s liability, as prescribed
by regulation. Then

\[ L = \frac{C}{r - g} \]

The modified duration of the liability is then

\[-\frac{dL}{dr} \left(\frac{dL}{L}\right) = \frac{C/(r-g)^2}{C/(r-g)} = \frac{1}{r-g}\]

So, in order to match duration, the pension fund must hold \( y \) units of the perpetuity that pays 1 unit each period, where \( y \) satisfies

\[ py \cdot \frac{1}{r} = L \cdot \frac{1}{r-g} \]
Hence

\[ y = C \left( \frac{r}{r-g} \right)^2 \]
\[ = C \left( \frac{1/p}{1/p - g} \right)^2 \]
\[ = C \left( \frac{1}{1-gp} \right)^2 \]

The following chart plots \( y \) as a function of \( p \) when \( g = 3\% \) and \( C = 100 \).
The demand $y$ for the perpetuity is increasing in its price $p$. The demand curve is \textit{upward}-sloping.$^1$

$^1$Another question to ponder: What happens to the solvency of the pension fund as $r$ continues to fall?
What is the intuition for this result? Note that the duration of the liability is rising much faster than the duration of assets when $r$ falls. As the yield falls, more of the simple perpetuity is demanded so as to keep pace with the growing duration of the liabilities $L$.

In many countries, the market for long-dated government bonds is affected by the demand for long-duration assets that can play a role in meeting the duration of long-term liabilities. Very long-dated fixed income securities provide more “bang for the buck” in the sense of providing more duration per dollar spent.

The yield curve for U.K. government bonds (“gilts”) was inverted until recently. Below is the yield curve for April 27th, 2007.
UK Gilt Yield Curve (Apr 27, 2007)
Leverage Targeting

Initial balance sheet

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Securities, 100</td>
<td>Equity, 10</td>
</tr>
<tr>
<td></td>
<td>Debt, 90</td>
</tr>
</tbody>
</table>

Assume price of debt approximately constant. Suppose the security price increases by 1% to 101.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Securities, 101</td>
<td>Equity, 11</td>
</tr>
<tr>
<td></td>
<td>Debt, 90</td>
</tr>
</tbody>
</table>
Leverage falls to

\[
\frac{101}{11} = 9.18
\]

**The demand curve is upward-sloping.**

The new balance sheet looks like this.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Securities, 110</td>
<td>Equity, 11</td>
</tr>
<tr>
<td></td>
<td>Debt, 99</td>
</tr>
</tbody>
</table>

The leverage is now back up to 10.

The mechanism works in reverse, too. Suppose there is shock to the
security price so that

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Securities, 109</td>
<td>Equity, 10</td>
</tr>
<tr>
<td></td>
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</tr>
</tbody>
</table>

Leverage is too high \((109/10 = 10.9)\).

Sell securities worth 9, paydown debt of 9.

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</table>

Back to leverage of 10.
Supply curve is downward-sloping.
Amplification

Adjust leverage

Stronger balance sheets

Increase B/S size

Asset price boom

Weaker balance sheets

Reduce B/S size

Asset price decline
**Example.** Auction of the 3G mobile phone licenses

Our final example is a cautionary tale on relying too much on market prices, although there is not the same type of feedback as in earlier examples.

Beginning in early 2000, many European governments conducted formal auctions to sell the exclusive rights to particular frequencies of the radio spectrum to operate so-called “third generation” (3G) mobile phones (cell phones).

Technically, the third generation of mobile phones were a leap ahead of existing technology in terms of its capacity, enabling download speeds that were similar to broadband internet connections. In the boardrooms of the telecommunications companies in Europe all manner of exciting prospects were discussed.

The auctions just happened to coincide with the peak of the technology boom in the stock market of the late 1990s. Some of the ideas that
were floated by the internet entrepreneurs seem rather far-fetched today. They did not seem so far-fetched to some advocates at the time. The big argument these advocates had was that the market was pricing technology assets at very high prices.

The same argument was influential in the board rooms of the major telecoms companies. For the incumbent firms, getting a license was seen not only as a matter of seizing a profitable opportunity, but a matter of survival. For an incumbent firm not to win a license would have relegated it to the second tier of telecom companies.

With hindsight, 3G licenses were not the critical next generation technology after all (newer technologies such as WiMAX is considered as a credible alternative). It is now clear (with hindsight) that many companies overpaid for their licenses. Many of the highest payers experienced severe financial distress subsequently.
Let’s first see what happened at the time.

The series of auctions kicked off in the U.K., right at the peak of the Nasdaq bubble. Over the next two years, eight other European countries conducted auctions for the 3G licenses. Some countries (like France) chose not to conduct a formal auction, but instead relied on “beauty contests”, where bidding companies were asked to write proposals for why they should be a recipient of a license.

The chart below plots the revenues raised in the auctions in each country, where the revenue is measured in Euros per capita (i.e. total sum raised divided by the country’s population)\(^2\). The blue series is the Nasdaq stock index.

\(^2\)Data are from Paul Klemperer’s paper “How (Not) to Run Auctions: the European 3G Telecom Auctions” http://www.nuff.ox.ac.uk/users/klemperer/hownot.pdf
Suppose the Nasdaq index represents the value of assets that grow at the rate $g$. The price of a growing perpetuity with initial cash flow of 1 is

$$p = \frac{1}{r - g}$$

Let’s treat $p$ as the value of the Nasdaq index. If we knew the growth rate $g$, we could infer what discount rate the market was applying to the Nasdaq index as a whole.

Suppose that the 3G licenses will yield a faster growing stream of cash flows net of set-up costs. Say that the cash flows are growing at the rate $\gamma$, where

$$\gamma > g$$
Then the net present value of a 3G license (denoted by $q$) will be

$$q = \frac{1}{r - \gamma}$$

$$= \frac{1}{(g + \frac{1}{p}) - \gamma}$$

$$= \frac{1}{\frac{1}{p} - (\gamma - g)}$$

Then, $q$ is increasing in $p$, but in a non-linear way. Below is a plot of $q$ as a function of $p$ when $\gamma - g = 0.01$. 
If the telecom executives (and their consultants from investment banks running their spreadsheets) looked to market prices to ascertain the true value of the licenses, then they would be willing to pay more as the value of the Nasdaq rises. In fact, the willingness to pay would be increasing in a non-linear way.

Below is a scatter chart plotting the relationship between the revenue raised per capita and the squared value of the Nasdaq at the beginning of the
month in which the auction was conducted.
The scatter chart suggests that the square of the Nasdaq does a reasonably good job of explaining the revenue generated in the auctions. The $R^2$ from the ordinary least squares regression is 82%.
Preview of Lectures 2 and 3

Lecture 2.
How does endogenous risk explain the subprime crisis?

Lecture 3.
How should financial regulation and monetary policy be formulated to take account of all this?