Does widespread adoption of Value-at-Risk as a risk management tool enhance financial stability or undermine it? Financial regulation based on Value-at-Risk such as the Basel II rules for bank capital is founded on the assumption that making each bank safe makes the system safe.

Of course, it is a truism that ensuring the soundness of each individual institution ensures the soundness of the system as a whole. But for this proposition to be a good prescription for policy, actions that enhance the soundness of a particular institution should promote overall stability. However, the proposition is vulnerable to the fallacy of composition. It is possible, indeed often likely, that attempts by individual institutions to respond to the waxing and waning of measured risks can amplify the boom-bust cycle. The “boom” part of the boom-bust cycle is especially important as we will see below.

We began these lectures with the quote from the anonymous risk manager who insisted that the value added of a good risk management system is that one can take more risks. In this spirit, financial institutions have been encouraged to load up on exposures when measured risks are low, only to shed them as fast as it can when risks begin to materialise. Unfortunately, the recoiling from risk by one institution generates greater materialised risk for others. Put differently, there are externalities in the financial system where actions by one institution have spillover effects on others.

But even more important than the realisation that such externalities exist is the task of identifying the mechanism through which they operate.
Traditionally, financial contagion has been viewed through the lens of cascading defaults, where if A has borrowed from B and B has borrowed from C, then the default of A impacts B, which then impacts C, and so on. This line of reasoning usually leads to analyses of interbank claims and financial networks, which are shocked by some hypothetical default by one or more constituents of the network. We could dub this the “domino” model of financial contagion, and the domino model has been a staple of much research on financial stability.

However, the near-universal conclusion from these studies have been that the potential for systemic crisis is small\(^1\). In the models, it is only with implausibly large shocks that the simulations generate any meaningful contagion. The global financial crisis has exposed the weakness of the domino model, although the appeal of the domino model still exerts a resilient hold, as witnessed by the large volume of research based on the domino model that are still produced by central banks and policy organisations.

One objective of these lectures is to show that a more potent channel through which externalities in financial markets exercise their influence is through the pricing of risk, and the resulting portfolio decisions of market participants. Actual defaults need not even figure in the mechanism, and the effects operate even in a setting where the financial institutions have not borrowed and lent to each other. The rest of this chapter is devoted to backing up these claims. The general equilibrium example that follows is therefore deliberately stark. It has two features that deserve emphasis.

First, there is no default in the model. The debt that appears in the model is risk-free. However, as we will see, the amplification of the financial cycle is very potent. John Geanakoplos (2009) has highlighted how risk-free debt may still give rise to powerful spillover effects through fluctuations in leverage and the pricing of risk. Adrian and Shin (2007) exhibit empirical evidence that bears on the fluctuations in the pricing of risk from the balance sheets of financial intermediaries.

The fact that our example has no default is useful in illustrating how booms and busts result from actions in anticipation of defaults, rather than the defaults themselves. Rather like a Greek tragedy, it is the actions taken by actors who want to avoid a bad outcome that precipitates disaster. It also

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\(^1\)For a comprehensive discussion of the performance of domino models of contagion used at central banks, see the recent work of Elsinger, Lehar and Summer (2006a, 2006b, 2006c).
draws our attention to where it belongs - the boom phase of the boom-bust cycle. In this respect, the analysis below is in line with Andrew Crockett’s (2000) comment that risk increases in booms and materialises in busts.

Second, in our general equilibrium example, there is no lending and borrowing between financial institutions. So, any effect we see in the model cannot be attributed to the domino model of systemic risk. This is not to deny that interlocking claims do not matter. Far from it. We will see in a later chapter that financial networks and interlocking claims and obligations do amplify the boom-bust cycle.

However, the key is the pricing of risk, and how balance sheet management based on Value-at-Risk amplifies the fluctuations in the price of risk. In order to demonstrate this claim, we work with a deliberately stark model where there are no interlocking claims and obligations between financial institutions, and where there is no default. Instead, the spillover effects operate through market prices, in particular the price of risk. We will see that even such a simple setting generates large amplifications.

### 3.1 General Equilibrium with Value-at-Risk

Our example is set in a one period asset market. Today is date 0. A risky security is traded today in anticipation of its realised payoff in the next period (date 1). Since trade takes place only once, we can drop the time subscripts, simplifying the notation. The payoff of the risky security is known at date 1.

When viewed from date 0, the risky security’s payoff is a random variable \( \tilde{\omega} \), with expected value \( q > 0 \). The uncertainty surrounding the risky security’s payoff takes a particularly simple form. The random variable \( \tilde{\omega} \) is uniformly distributed over the interval:

\[
[q - z, q + z]
\]

where \( z > 0 \) is a known constant. The mean and variance of \( \tilde{\omega} \) is given by

\[
E(\tilde{\omega}) = q \\
\sigma^2 = \frac{z^2}{3}
\]

There is also a risk-free security, cash, that pays an interest rate of zero. Let \( p \) denote the price of the risky security. For an investor with equity \( e \) who
holds \( y \) units of the risky security, the payoff of the portfolio is the random variable:

\[
W \equiv \tilde{w}y + (e - py)
\]  

(3.1)

Let us now introduce two groups of investors - passive investors and active investors. The passive investors can be thought of as non-leveraged investors such as pension funds and mutual funds, while the active investors can be interpreted as leveraged institutions such as banks and securities firms who manage their balance sheets actively. The risky securities can be interpreted as loans granted to ultimate borrowers, but where there is a risk that the borrowers do not fully repay the loan. Figure 3.1 depicts the relationships. Under this interpretation, the market value of the risky securities can be thought of as the marked-to-market value of the loans granted to the ultimate borrowers. The passive investors’ holding of the risky security can then be interpreted as the credit that is granted directly by the household sector (through the holding of corporate bonds, for example), while the holding of the risky securities by the active investors can be given the interpretation of intermediated finance where the active investors are banks that borrow from the households in order to lend to the ultimate borrowers.

We assume that the passive investors have mean-variance preferences over the payoff from the portfolio. They aim to maximise

\[
U = E(W) - \frac{1}{2\tau}\sigma_W^2
\]  

(3.2)

where \( \tau > 0 \) is a constant called the investor’s “risk tolerance” and \( \sigma_W^2 \) is
3.1. GENERAL EQUILIBRIUM WITH VALUE-AT-RISK

the variance of $W$. In terms of the decision variable $y$, the passive investor’s objective function can be written as

$$U(y) = qy + (e - py) - \frac{1}{6\tau}y^2 z^2$$

(3.3)

The optimal holding of the risky security satisfies the first order condition:

$$q - p - \frac{1}{3\tau}z^2 y = 0$$

(3.4)

The price must be below the expected payoff for the risk-averse investor to hold any of the risky security. The optimal risky security holding of the passive investor (denoted by $y_P$) is given by

$$y_P = \begin{cases} 
\frac{3\tau}{z^2} (q - p) & \text{if } q > p \\
0 & \text{otherwise}
\end{cases}$$

(3.5)

These linear demands can be summed to give the aggregate demand. If $\tau_i$ is the risk tolerance of the $i$th investor and $\tau = \sum_i \tau_i$, then (3.5) gives the aggregate demand of the passive investor sector as a whole.

Now turn to the portfolio decision of the active (leveraged) investors. These active investors are risk-neutral but face a Value-at-Risk (VaR) constraint, as is commonly the case for banks and other leveraged institutions. The general VaR constraint is that the capital cushion be large enough that the default probability is kept below some benchmark level. Consider the special case where that benchmark level is zero. Then, the VaR constraint boils down to the condition that leveraged investors issue only risk-free debt.

Denote by VaR the Value-at-Risk of the leveraged investor. The constraint is that the investor’s capital (equity) $e$ be large enough to cover this Value-at-Risk. The optimisation problem for an active investor is:

$$\max_y E(W) \text{ subject to VaR} \leq e$$

(3.6)

If the price is too high (i.e. when $p > q$) the investor holds no risky securities. When $p < q$, then $E(W)$ is strictly increasing in $y$, and so the Value-at-Risk constraint binds. The optimal holding of the risky security can be obtained by solving VaR = $e$. To solve this equation, write out the balance sheet of the leveraged investor as
The Value-at-Risk constraint stipulates that the debt issued by the investor be risk-free. For each unit of the security, the minimum payoff is \( q - z \). In order for the investor’s debt to be risk-free, \( y \) should satisfy \( p y - e \leq (q - z) y \), or

\[
py - (q - z) y \leq e
\]  

(3.7)

The left hand side of (3.7) is the Value-at-Risk (the worst possible loss) relative to today’s market value of assets, which must be met by equity \( e \). Since the constraint binds, the optimal holding of the risky securities for the leveraged investor is

\[
y = \frac{e}{z - (q - p)}
\]  

(3.8)

and the balance sheet is

\[
\begin{array}{|c|c|}
\hline
\text{Assets} & \text{Liabilities} \\
\hline
\text{securities, } py & \text{equity, } e \\
\text{debt, } py - e & \text{debt, } (q - z) y \\
\hline
\end{array}
\]  

(3.9)

Since (3.8) is linear in \( e \), the aggregate demand of the leveraged sector has the same form as (3.8) when \( e \) is the aggregate capital of the leveraged sector as a whole.

Denoting by \( y_A \) the holding of the risky securities by the active investors and by \( y_P \) the holding by the passive investors, the market clearing condition is

\[
y_A + y_P = S
\]  

(3.10)

where \( S \) is the total endowment of the risky securities. Figure 3.2 illustrates the equilibrium for a fixed value of aggregate capital \( e \). For the passive investors, their demand is linear, with the intercept at \( q \). The demand of the leveraged sector can be read off from (3.8). The solution is fully determined as a function of \( e \). In a dynamic model, \( e \) can be treated as the state variable (see Danielsson, et al. (2009)).
3.1. GENERAL EQUILIBRIUM WITH VALUE-AT-RISK

Figure 3.2: Market Clearing Price

Figure 3.3: Amplified response to improvement in fundamentals $q$
Now consider a possible scenario involving an improvement in the fundamentals of the risky security where the expected payoff of the risky securities rises from \( q \) to \( q' \). In our banking interpretation of the model, an improvement in the expected payoff could result from an improvement in the macroeconomic outlook, lowering the probability that the borrowers would default on their loans. Figure 3.3 illustrates the scenario. The improvement in the fundamentals of the risky security pushes up the demand curves for both the passive and active investors, as illustrated in Figure 3.3. However, there is an amplified response from the leveraged institutions as a result of marked-to-market gains on their balance sheets.

From (3.9), denote by \( e' \) the new equity level of the leveraged investors that incorporates the capital gain when the price rises to \( p' \). The initial amount of debt was \( (q - z) y \). Since the new asset value is \( p'y \), the new equity level \( e' \) is

\[
e' = p'y - (q - z) y = (z + p' - q) y
\] (3.11)

Figure 3.4 breaks out the steps in the balance sheet expansion. The initial balance sheet is on the left, where the total asset value is \( py \). The middle balance sheet shows the effect of an improvement in fundamentals that comes from an increase in \( q \), but before any adjustment in the risky security holding. There is an increase in the value of the securities without any change in the debt value, since the debt was already risk-free to begin with. So, the increase in asset value flows through entirely to an increase in equity. Equation (3.11) expresses the new value of equity \( e' \) in the middle balance sheet in Figure 3.4.

The increase in equity relaxes the Value-at-Risk constraint, and the leveraged sector can increase its holding of risky securities. The new holding \( y' \) is larger, and is enough to make the VaR constraint bind at the higher equity level, with a higher fundamental value \( q' \). That is,

\[
e' = p'y' - (q - z) y' = (z + p' - q') y'
\] (3.12)

After the \( q \) shock, the investor’s balance sheet has strengthened, in that capital has increased without any change in debt value. There has been an erosion of leverage, leading to spare capacity on the balance sheet in the
In order to utilize the slack in balance sheet capacity, the investor takes on additional debt to purchase additional risky securities. The demand response is upward-sloping. The new holding of securities is now $y'$, and the total asset value is $p'y'$. Equation (3.12) expresses the new value of equity $\varepsilon'$ in terms of the new higher holding $y'$ in the right hand side balance sheet in Figure 3.4. From (3.11) and (3.12), we can write the new holding $y'$ of the risky security as

$$y' = y \left( 1 + \frac{q' - q}{z + p' - q'} \right) \quad (3.13)$$

From the demand of passive investors (3.5) and market clearing,

$$p' - q' = \frac{z^2}{3\tau} (y' - S)$$

Substituting into (3.13),

$$y' = y \left( 1 + \frac{q' - q}{z + \frac{z^2}{3\tau} (y' - S)} \right) \quad (3.14)$$

This defines a quadratic equation in $y'$. The solution is where the right hand side of (3.14) cuts the 45 degree line. The leveraged sector amplifies  

Figure 3.4: Balance sheet expansion from $q$ shock
booms and busts if \( y' - y \) has the same sign as \( q' - q \). Then, any shift in fundamentals gets amplified by the portfolio decisions of the leveraged sector. The condition for amplification is that the denominator in the second term of (3.14) is positive. But this condition is guaranteed from (3.13) and the fact that \( p' > q' - z \) (i.e. that the price of the risky security is higher than its worst possible realized payoff).

Note also that the size of the amplification is increasing in leverage, seen from the fact that \( y' - y \) is large when \( z \) is small. Recall that \( z \) is the fundamental risk. When \( z \) is small, the associated Value-at-Risk is also small, allowing the leveraged sector to maintain high leverage. The higher is the leverage, the greater is the marked-to-market capital gains and losses. Amplification is large when the leveraged sector itself is large relative to the total economy. Finally, note that the amplification is more likely when the passive sector’s risk tolerance \( \tau \) is high.

The price gap, \( q - p \) is the difference between the expected payoff from the risky security and its price. It is one measure of the price of risk in the economy. The market clearing condition and the demand of the passive sector (3.5) give an empirical counterpart to the price gap given by the size of the leveraged sector. Recall that \( y_A \) is the holding of the risky security by the leveraged sector. We have

\[
q - p = \frac{\zeta}{3\tau} (S - y_A)
\]

which gives our first empirical hypothesis.

**Empirical Hypothesis.** Risk premiums are low when the size of the leveraged sector is large relative to the non-leveraged sector.

We will explore alternative notions of risk premiums in the next section. The amplifying mechanism works exactly in reverse on the way down. A negative shock to the fundamentals of the risky security drives down its price, which erodes the marked-to-market capital of the leveraged sector. The erosion of capital induces the sector to shed assets so as to reduce leverage down to a level that is consistent with the VaR constraint. Risk premium increases when the leveraged sector suffers losses, since \( q - p \) increases.
3.2 Pricing of Risk and Credit Supply

We now explore the fluctuations in risk pricing in our model more systematically. For now, let us treat $S$ (the total endowment of the risky security) as being exogenous. Once we solve for the model fully, we can make $S$ endogenous and address the issue of credit supply with shifts in economic fundamentals.

Begin with the market-clearing condition for the risky security, $y_A + y_P = S$. Substituting in the expressions for the demands of the active and passive sectors, we can write the market clearing condition as

$$\frac{e}{z - (q - p)} + \frac{3\tau}{z^2} (q - p) = S$$

(3.16)

We also impose a restriction on the parameters from the requirement that the active investors have a strictly positive total holding of the risky security, or equivalently that the passive sector’s holding is strictly smaller than the total endowment $S$. From (3.5) this restriction can be written as

$$\frac{3\tau}{z^2} (q - p) < S$$

(3.17)

Our discussion so far of the amplification of shocks resulting from the leveraged investors’ balance sheet management suggests that a reasonable hypothesis is that the risk premium to holding the risky security is falling as the fundamental payoff of the risky security improves. This is indeed the case. We have:

**Proposition 1** The expected return on the risky security is strictly decreasing in $q$.

The expected return to the risky security is $(q/p) - 1$. It is more convenient to work with a monotonic transformation of the expected return given by

$$\pi \equiv 1 - \frac{p}{q}$$

(3.18)

We see that $\pi$ lies between zero and one. When $\pi = 0$, the price of the risky security is equal to its expected payoff, so that there is no risk premium in holding the risky security over cash. As $\pi$ increases, the greater is the
expected return to holding the risky security. Using the $\pi$ notation, the market-clearing condition (3.16) can be written as follows.

$$ F \equiv e + \frac{3\tau}{z^2} q \pi (z - q \pi) - S (z - q \pi) = 0 \quad (3.19) $$

We need to show that $\pi$ is decreasing in $q$. From the implicit function theorem,

$$ \frac{d\pi}{dq} = -\frac{\partial F/\partial q}{\partial F/\partial \pi} \quad (3.20) $$

and

$$ \frac{\partial F}{\partial q} = \pi \left( \frac{3\tau}{z} \left( 1 - \frac{2\pi q}{z} \right) + S \right) $$

Dividing this expression by $3\tau \pi / z^2 > 0$, we see that $\partial F/\partial q$ has the same sign as

$$ (z - \pi q) + \left( \frac{z^2}{3\tau} S - \pi q \right) $$

$$ = (z - (q - p)) + \left( \frac{z^2}{3\tau} S - (q - p) \right) \quad (3.21) $$

The left hand term in (3.21) is positive since price $p$ is above the minimum payoff $q - z$. The right hand term is positive from our parameter restriction (3.17) that ensures that the risky security holding by the leveraged sector is strictly positive. Hence, $\partial F/\partial q > 0$. Similarly, it can be shown that $\partial F/\partial \pi > 0$. Therefore, $d\pi/dq < 0$. This concludes the proof of Proposition 1.

The expected return on the risky security is falling as the fundamentals improve. We could rephrase this finding as saying that the risk premium in the economy is declining during booms. The decline in risk premiums is a familiar feature in boom times. Although the somewhat mechanical proof we have given for Proposition 1 is not so illuminating concerning the economic mechanism, the heuristic argument in the previous section involving the three balance sheets in Figure 3.4 captures the spirit of the argument more directly.

When fundamentals improve, the leveraged investors (the banks) experience mark-to-market gains on their balance sheets, leading to higher equity capital. The higher mark-to-market capital generates additional balance
3.2. PRICING OF RISK AND CREDIT SUPPLY

sheet capacity for the banks that must be put to use. In our model, the excess balance sheet capacity is put to use by increasing lending (purchasing more risky securities) with money borrowed from the passive investors.

**Shadow Value of Bank Capital**

Another window on the risk premium in the economy is through the Lagrange multiplier associated with the constrained optimisation problem of the banks, which is to maximise the expected payoff from the portfolio $E(W)$ subject to the Value-at-Risk constraint. The Lagrange multiplier is the rate of increase of the objective function with respect to a relaxation of the constraint, and hence can be interpreted as the shadow value of bank capital. Denoting by $\lambda$ the Lagrange multiplier, we have

$$\lambda = \frac{dE(W)}{de}$$

$$= \frac{dE(W)}{dy} \frac{dy}{de}$$

$$= (q - p) \cdot \frac{1}{z - (q - p)}$$

(3.22)

where we have obtained the expression for $dE(W)/dy$ from (3.2) and $dy/de$ is obtained from (3.8), which gives the optimal portfolio decision of the leveraged investor. We see from (3.22) that as the price gap $q - p$ becomes compressed, the Lagrange multiplier $\lambda$ declines. The implication is that the marginal increase of a dollar’s worth of new capital for the leveraged investor is generating less expected payoff. As the price gap $q - p$ goes to zero, so does the Lagrange multiplier, implying that the return to a dollar’s worth of capital goes to zero.

Furthermore, we have from (3.15) that the price gap $q - p$ is decreasing as the size of the leveraged sector increases relative to the whole economy. The shadow value of bank capital can be written as:

$$\lambda = (q - p) \cdot \frac{1}{z - (q - p)}$$

$$= \frac{z(S - y_A)}{3\tau + z(y_A - S)}$$

(3.23)

We have the following proposition.
Proposition 2  The shadow value of bank capital is decreasing in the size of the leveraged sector.

The leverage of the active investor is defined as the ratio of total assets to equity. Leverage is given by

\[
\frac{py}{e} = \frac{p}{e} \times \frac{e}{z - (q - p)} = \frac{p}{z - (q - p)}
\]

As \( q \) increases, the numerator \( p(q) \) increases without bound. Since the price gap is bounded below by zero, overall leverage eventually increases in \( q \). Thus, leverage is high when total assets are large. In the terminology of Adrian and Shin (2007), the leveraged investors exhibit procyclical leverage.

Proposition 3  For values of \( q \) above some threshold \( \bar{q} \), leverage is procyclical.

In the run-up to the global financial crisis of 2007 to 2009, the financial system was said to “awash with liquidity”, in the sense that credit was easy to obtain. Adrian and Shin (2007) show that liquidity in this sense is closely related to the growth of financial intermediary balance sheets. When taken in conjunction with the findings of Adrian and Shin (2007), Propositions 1 and 2 shed some light on the notion of liquidity. When asset prices rise, financial intermediaries’ balance sheets generally become stronger, and—without adjusting asset holdings—their leverage becomes eroded. The financial intermediaries then hold surplus capital, and they will attempt to find ways in which they can employ their surplus capital. In analogy with manufacturing firms, we may see the financial system as having “surplus capacity”. For such surplus capacity to be utilised, the intermediaries must expand their balance sheets. On the liability side, they take on more debt. On the asset side, they search for potential borrowers. When the set of potential borrowers is fixed, the greater willingness to lend leads to an erosion in risk premium from lending, and spreads become compressed.

Feedback

A tell-tale characteristic of investors driven by the Value-at-Risk constraint is that their demands chase the most recent price changes. As long as
the expected return is positive, the optimal policy is for the investor to buy the risky security up to the maximum permitted by his Value-at-Risk. In this sense, it makes sense to talk of an investor’s “risk budget”. In the market for the risky security with both active and passive traders, the market-clearing price is determined where the demands of both groups of traders sum to the total endowment of the risky security. In such a setting, an increase in the price of the risky security sets off an amplifying spiral of price increases and further purchases. The positive shock increases the marked-to-market capital of the VaR-constrained traders, relaxing the risk constraint and allowing the investor to buy more of the risky security. This pushes the demand up as a consequence. However, as more of the risky security ends up in the hands of the VaR-constrained investors, the market-clearing price is driven up further, which sets off another round of increase in the marked-to-market capital of the investors, pushing out the demands still further.

In this way, the presence of investors who manage their leverage actively have the potential to amplify shocks as price increases and balance sheet effects become intertwined. The Millennium Bridge analogy applies in this feedback process. Note the importance of marking to market, and the dual role of market prices. Purchases drive up prices, but price increases induce actions (further purchases) on the part of the investors. The mechanism works in reverse on the way down. A negative shock to the price of the risky security drives down its price, which erodes the marked-to-market capital of the leveraged investor. The erosion of capital is a cue for the investor to shed some of the assets so as to reduce leverage down to a level that is consistent with the VaR constraint. The two circular figures Figure 3.5 taken from Adrian and Shin (2007) and Shin (2005a) depict the feedback from prices to actions to back to prices, both on the “way up” and on the “way down”. Adrian and Shin (2007, 2008a) discuss the consequences of such balance sheet dynamics for the financial system as a whole and for monetary policy.

Supply of Credit

Up to now, we have treated the total endowment of the risky securities \( S \) as being fixed. However, as the risk spread on lending becomes compressed, the leveraged investors (the banks) will be tempted to search for new borrowers they can lend to. In terms of our model, if we allow \( S \) to be endogenously determined, we can expect credit supply to be increasing when the risk pre-
mium falls. Through this window, we could gain a glimpse into the way that credit supply responds to overall economic conditions.

To explore this idea further, we modify our model in the following way. Suppose there is a large pool of potential borrowers who wish to borrow to fund a project, from either the active investors (the banks) or the passive investors (the households). They will borrow from whomever is willing to lend.

Assume that the potential borrowers are identical, and each have identical projects to those which are already being financed by the banks and households. In other words, the potential projects that are waiting to be financed are perfect substitutes with the projects already being funded. Denote the risk premium associated with the pool of potential projects by the constant \( \pi_0 \). If the market risk premium were ever to fall below \( \pi_0 \), the investors in the existing projects would be better off selling the existing projects to fund the projects that are sitting on the sidelines. Therefore, the market premium cannot fall below \( \pi_0 \), so that in any equilibrium with endogenous credit supply, we have

\[
\pi \geq \pi_0 \quad (3.25)
\]

Define the supply of credit function \( S(q) \) as the function that maps \( q \) to the total lending \( S \). When \( \pi(q) \geq \pi_0 \), there is no effect of a small change in \( q \) on the supply of credit. Define \( q^* \) as the threshold value of \( q \) defined as \( q^* = \pi^{-1}(\pi_0) \). When \( q > q^* \), then the equilibrium stock of lending \( S \) is determined by the market clearing condition (3.19) where \( \pi = \pi_0 \). Hence, \( S \) satisfies

\[
F \equiv e + \frac{3\tau}{z^2} q \pi_0 (z - q \pi_0) - S(z - q \pi_0) = 0
\]
The slope of the supply of credit function is given by
\[ \frac{dS}{dq} = -\frac{\partial F/\partial q}{\partial F/\partial S} \] (3.26)

We know from (3.21) that the numerator of (3.26) is positive, while \( \partial F/\partial S = -(z - q\pi_0) = q - p - z < 0 \). Therefore \( dS/dq > 0 \), so that credit supply is increasing in \( q \). We can summarise the result as follows.

**Proposition 4** The supply of credit \( S \) is strictly increasing in \( q \) when \( q > q^* \).

The assumption that the pool of potential borrowers have projects that are perfect substitutes for the existing projects being funded is a strong assumption, and unlikely to hold in practice. Instead, it would be reasonable to suppose that the project quality varies within the pool of potential borrowers, and that the good projects are funded first. For instance, the pool of borrowers would consist of households that do not yet own a house, but would like to buy a house with a mortgage. Among the potential borrowers would be good borrowers with secure and verifiable income.

However, as the good borrowers obtain funding and leave the pool of potential borrowers, the remaining potential borrowers will be less good credits. If the banks’ balance sheets show substantial slack, they will search for borrowers to lend to. As balance sheets continue to expand, more borrowers will receive funding. When all the good borrowers already have a mortgage, then the banks must lower their lending standards in order to generate the assets they can put on their balance sheets. In the sub-prime mortgage market in the United States in the years running up to the financial crisis of 2007, we saw that when balance sheets are expanding fast enough, even borrowers that do not have the means to repay are granted credit—so intense is the urge to employ surplus capital. The seeds of the subsequent downturn in the credit cycle are thus sown.

### 3.3 Long-Short Strategy Hedge Fund

Some risk can be diversified away, but there are limits to diversification as long as there is aggregate risk. Some hedge funds claim to offer a market-neutral return in the sense that total return does not depend on whether the