
Tobias Adrian, Paolo Colla, Hyun Song Shin

September 2011

2011 Ely Lectures, Johns Hopkins University
Lecture 2
Credit to Non-Financial Firms (Flow of Funds, Table L102)
Changes in Non-Financial Corporate Liabilities: Corporate Bonds and Loans

![Graph showing changes in non-financial corporate liabilities over time, with specific years marked on the graph.]
Treasury and Baa Spread

Baa-10 yr Treasury Spread

Percent

Jan-86 Jul-87 Jan-89 Jul-90 Jan-92 Jul-93 Jan-94 Jul-95 Jan-96 Jul-97 Jan-98 Jul-99 Jan-00 Jul-01 Jan-02 Jul-03 Jan-04 Jul-05 Jan-06 Jul-07 Jan-08 Jul-09 Jan-10
Two groups of creditors

- Banks who borrow in order to lend

- Mean-variance investors who hold portfolio of (i) cash (ii) bank liabilities (iii) risky loans
Demand for Credit

Entrepreneurs endowed with projects which need investment of 1, but has no wealth

All projects are debt financed, and there is limited liability

Project indexed by $q$ has payoff

$$
\begin{cases}
1 + q & \text{with prob } 1 - \varepsilon \\
0 & \text{with prob } \varepsilon
\end{cases}
$$

Population density of projects given by $h(q)$. Demand for credit is the measure of projects with index $q$ higher than notional interest rate $r$

$$
K(r) = \int_r^\infty h(z) \, dz
$$
Credit Risk


Project $j$ succeeds when $Z_j > 0$, where

$$Z_j = -\Phi^{-1}(\varepsilon) + \sqrt{\rho Y} + \sqrt{1 - \rho X_j}$$

$\Phi(\cdot)$ c.d.f. of standard normal, $Y$ and $\{X_j\}$ independent standard normals

$$\Pr(Z_j < 0) = \Pr\left(\sqrt{\rho Y} + \sqrt{1 - \rho X_j} < \Phi^{-1}(\varepsilon)\right)$$

$$= \Phi\left(\Phi^{-1}(\varepsilon)\right) = \varepsilon$$
Credit Supply

Notation for balance sheet of bank

\[ C \quad E \quad L \]

\[ 1 + r \quad 1 + f \]
Bank diversifies away idiosyncratic risk

Conditional on $Y$, defaults are independent.

Keep $C$ fixed but diversify: increase number of borrowers, reduce face value of individual loans

In the limit, realized value of assets is function of $Y$ only

$$w(Y) \equiv (1 + r) C \cdot \Pr (Z_j \geq 0 | Y)$$

$$= (1 + r) C \cdot \Pr \left( \sqrt{\rho} Y + \sqrt{1 - \rho} X_j \geq \Phi^{-1} (\varepsilon) | Y \right)$$

$$= (1 + r) C \cdot \Phi \left( \frac{Y \sqrt{\rho} - \Phi^{-1} (\varepsilon)}{\sqrt{1 - \rho}} \right)$$

(*)
Figure 1: Asset realization densities for three values of $\rho$ [$\varepsilon = 0.1$, $C(1 + r) = 1$]
Figure 2: Asset realization densities for three values of $\varepsilon$ [$\rho = 0.2$, $C(1 + r) = 1$]
Bank Credit Supply and Value-at-Risk

Density over repayments

\[ w(Y) \]

Probability density over asset realizations

\[ 0 \quad (1+f)L \quad (1+r)C \]
• Turn credit risk model on its head and think of it as credit supply model
  – Fix $E$. Determine credit supply $C_S$

$$C_S = \frac{E}{1 - \frac{1+r}{1+f} \varphi (\rho, \alpha, \varepsilon)}, \quad \varphi \in (0, 1)$$

$\varphi$ is ratio of notional assets to notional debt to be derived below.
From (*), the c.d.f. of \( w \) is

\[
F(z) = \Pr (w \leq z) = \Pr (Y \leq w^{-1}(z)) = \Phi (w^{-1}(z))
\]

\[
= \Phi \left( \frac{1}{\sqrt{\rho}} \left( \Phi^{-1} (\varepsilon) + \sqrt{1 - \rho} \Phi^{-1} \left( \frac{z}{(1 + r) C} \right) \right) \right)
\]

Common risk factor \( \rho \) determines shape of the density, with larger \( \rho \) implying fatter tail.

**Value-at-Risk (VaR) rule:** keep enough equity to limit insolvency probability to \( \alpha > 0 \)
Bank credit supply $C$ determined from

$$\Pr (w < (1 + f) L) = \Phi \left( \frac{\Phi^{-1}(\varepsilon) + \sqrt{1 - \rho}\Phi^{-1}\left(\frac{(1+f)L}{(1+r)C}\right)}{\sqrt{\rho}} \right) = \alpha$$

$$\frac{\text{Notional liabilities}}{\text{Notional assets}} = \frac{(1 + f) L}{(1 + r) C} = \Phi \left( \frac{\sqrt{\rho\Phi^{-1}(\alpha) - \Phi^{-1}(\varepsilon)}}{\sqrt{1 - \rho}} \right) \quad (1)$$

where

$$\varphi (\alpha, \varepsilon, \rho) \equiv \Phi \left( \frac{\sqrt{\rho\Phi^{-1}(\alpha) - \Phi^{-1}(\varepsilon)}}{\sqrt{1 - \rho}} \right)$$
Figure 3: **Plot of notional debt to assets ratio** $\varphi(\alpha, \varepsilon, \rho)$. This chart plots $\varphi$ as a function of $\rho$ with $\alpha = 0.001$. Dark line is when $\varepsilon = 0.01$. Light line is when $\varepsilon = 0.005$. 
Supply of Credit by Bank

Credit supply $C$ and demand for funding $L$ is obtained from (1) and balance sheet identity $C = E + L$

$$C = \frac{E}{1 - \frac{1+r}{1+f} \cdot \varphi}, \quad L = \frac{E}{\frac{1+f}{1+r} \cdot \frac{1}{\varphi} - 1}$$

Aggregation holds due to proportionality

$$\text{Leverage} = \frac{1}{1 - \frac{1+r}{1+f} \cdot \varphi}$$

Risk premium is well-defined

$$\text{Risk premium} = (1 - \varepsilon)(1 + r) - 1$$
\[
C(r) = \frac{1 + f}{\varphi} - 1
\]

Supply of credit

Credit Supply

\[
\frac{E}{\varphi} \frac{1}{1 - \varepsilon(1 + f)}
\]
Bank Deleveraging and Credit Supply

Suppose bank liabilities are fully guaranteed by the government and so earn risk-free rate.

Let risk-free rate be zero, so that \( f = 0 \). Credit supply by banks is

\[
C_B = \frac{E}{1 - (1 + r) \varphi}
\]

Consider an increase in \( \varepsilon \) or in \( \rho \) which lowers \( \varphi \). Provided notional rate \( r \) rises high enough, credit supply remains unchanged.

Define **break-even notional rate**

\[
r(\varphi)
\]

as rate that leaves bank credit supply unchanged as function of \( \varphi \)
Break Even Notional Rate for Banks

From $C_B = E/ (1 - (1 + r) \varphi)$

$$(1 + r) \varphi = 1 - \frac{E}{C_B}$$

Taking log difference for $\varphi$ and $\varphi'$

$$r (\varphi') - r (\varphi) \simeq \ln \varphi - \ln \varphi' \quad (*)$$

(*) implies very sharp rise in break-even notional rates even for small changes in $\varphi$. For $r (\varphi) = 5\%$ and $10\%$ decline in $\varphi$, the notional rate $r$ has to rise to $15\%$ to leave bank credit supply unchanged.
Mean-Variance Investors

Loans are packaged into bonds that diversify away idiosyncratic risk.

Demand for bonds (supply of credit) by mean-variance investor with risk tolerance $\tau$

$$\frac{\tau [(1 - \varepsilon)(1 + r) - 1]}{\sigma^2 (1 + r)^2}$$

where $\sigma^2$ is variance of $w(Y)$. There are $N$ mean-variance investors, and $T = \tau N$. Aggregate supply of credit from mean-variance sector is

$$C_H = \frac{T [(1 - \varepsilon)(1 + r) - 1]}{\sigma^2 (1 + r)^2}$$

We need to work out $\sigma^2$. 
Moments

Let \( k = \Phi^{-1}(\varepsilon) \) and \( X_1, X_2, \ldots, X_n \) be i.i.d. standard normal.

\[
E[w^n] = E \left[ \left( \Phi \left( \frac{Y \sqrt{\rho} - k}{\sqrt{1-\rho}} \right) \right)^n \right]
\]

\[
= E \left[ \prod_{i=1}^{n} \Pr \left[ \sqrt{\rho}Y + \sqrt{1-\rho}X_i > k \middle| Y \right] \right]
\]

\[
= E \left[ \Pr \left[ \sqrt{\rho}Y + \sqrt{1-\rho}X_1 > k, \ldots, \sqrt{\rho}Y + \sqrt{1-\rho}X_n > k \middle| Y \right] \right]
\]

\[
= \Pr \left[ \sqrt{\rho}Y + \sqrt{1-\rho}Z_1 > k, \ldots, \sqrt{\rho}Y + \sqrt{1-\rho}Z_n > k \right]
\]

where \((Z_1, \ldots, Z_n)\) is multivariate standard normal with correlation \( \rho \)
Mean and Variance

\[ E[w] = 1 - \varepsilon \]

\[
\begin{align*}
\text{var}[w] &= \text{var}[1 - w] = \text{var}[L] = E[L^2] - E[L]^2 \\
&= \Pr[1 - Z_1 \leq k, 1 - Z_2 \leq k] - \varepsilon^2 \\
&= \Phi_2(k, k; \rho) - \varepsilon^2 \\
&= \Phi_2(\Phi^{-1}(\varepsilon), \Phi^{-1}(\varepsilon); \rho) - \varepsilon^2
\end{align*}
\]

where \( \Phi_2(\cdot, \cdot; \rho) \) cumulative bivariate standard normal with correlation \( \rho \).
Market Clearing Condition

The market clears when the demand for credit $K(r)$ is met by the aggregate supply of credit from banks and households

$$K(r) = C_B + C_H$$

$$= \frac{E}{1 - (1 + r) \varphi} + \frac{T [ (1 - \varepsilon) (1 + r) - 1]}{\sigma^2 (1 + r)^2}$$

Denote risk premium by $\pi \equiv (1 - \varepsilon) (1 + r) - 1$. Then market clearing condition can be written as

$$K \left( \frac{\pi + \varepsilon}{1 - \varepsilon} \right) = \frac{E (1 - \varepsilon)}{1 - \varepsilon - (1 + \pi) \varphi} + \frac{T \pi (1 - \varepsilon)^2}{\sigma^2 (1 + \pi)^2}$$
Comparative Statics of Risk Premium

Let

\[ G(\pi, \varepsilon) \equiv K \left( \frac{\pi + \varepsilon}{1 - \varepsilon} \right) - \frac{E(1 - \varepsilon)}{1 - \varepsilon - (1 + \pi) \varphi} - \frac{T\pi(1 - \varepsilon)^2}{\sigma^2(1 + \pi)^2} \]

We have

\[ \frac{\partial G}{\partial \pi} < 0 \]

Also, provided \(|K'(.)|\) is small relative to \(\left| \frac{d}{d\varepsilon} (C_B + C_H) \right|\), we have

\[ \frac{\partial G}{\partial \varepsilon} > 0 \]

so that

\[ \frac{d\pi}{d\varepsilon} = -\frac{\partial G/d\varepsilon}{\partial G/d\pi} > 0 \]
Benchmark Case

Let demand for credit be constant up to $\bar{r}$, so that $K'(r) = 0$ for all $r < \bar{r}$.

Consider shock where $\varepsilon$ increases. Provided equilibrium $r$ does not exceed $r(\varphi)$, empirical prediction is:

- Bank loans decline in absolute amount
- Bond financing increases in absolute amount
- The sum of the two remains constant
- Notional rate $r$ increases (but stays below $r(\varphi)$)
- Risk premium $\pi$ increases
Closer Look at Evidence

- Sample: U.S. public firms 1998-2010

- Intersection between
  - Compustat
  - Loan Pricing Corporation (LPC) Dealscan database
  - Securities Data Corporation (SDC) New Bond Issuances database

- 4,902 firms with new financing between 1998 and 2010 (out of 11,856 in Compustat sample)
<table>
<thead>
<tr>
<th>Tercile</th>
<th>Before crisis</th>
<th>After crisis</th>
<th>t-statistic (z-statistic)</th>
<th>After-Before</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Sorting variable: Assets</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.336 (0.182)</td>
<td>0.412 (0.252)</td>
<td>1.912** (3.171***))</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.993 (0.533)</td>
<td>0.799 (0.479)</td>
<td>-1.714* (-0.908)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3.338 (2.004)</td>
<td>3.329 (1.233)</td>
<td>-0.020 (-2.836***))</td>
<td></td>
</tr>
<tr>
<td>t-stat 3-1 (z-stat)</td>
<td>9.733*** (14.176***))</td>
<td>7.334*** (12.819***))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4: Total Financing Sorted by Total Assets
<table>
<thead>
<tr>
<th>Tercile</th>
<th>Before crisis</th>
<th>After crisis</th>
<th>t-statistic (z-statistic) After-Before</th>
<th>Before crisis</th>
<th>After crisis</th>
<th>t-statistic (z-statistic) After-Before</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel A: Sorting variable: Assets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.303 (0.142)</td>
<td>0.353 (0.223)</td>
<td>1.272 (2.143**)</td>
<td>0.033 (0.000)</td>
<td>0.059 (0.000)</td>
<td>2.901*** (2.924*** )</td>
</tr>
<tr>
<td>2</td>
<td>0.823 (0.437)</td>
<td>0.571 (0.264)</td>
<td>-2.694*** (-3.498**)</td>
<td>0.169 (0.000)</td>
<td>0.228 (0.186)</td>
<td>1.509 (4.720*** )</td>
</tr>
<tr>
<td>3</td>
<td>2.833 (1.627)</td>
<td>2.217 (0.485)</td>
<td>-1.520 (-5.650*** )</td>
<td>0.505 (0.000)</td>
<td>1.112 (0.525)</td>
<td>4.984*** (5.974*** )</td>
</tr>
<tr>
<td>t-stat 3-1 (z-stat)</td>
<td>9.455*** (13.497*** )</td>
<td>5.389*** (2.838*** )</td>
<td>6.890*** (7.988*** )</td>
<td>8.632*** (11.924*** )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5: Loan and bond financing sorted by total assets
Panel A: Total amount of loans issued

Figure 6:
Panel B: Average amount of loans issued

Figure 7:
Figure 8: Panel C: Number of loans issued

Source: LPC Dealscan
Figure 9:
Panel B: Total amount of loans issued (by purpose)

Figure 10:
Figure 11: Panel D: Cost of loans issued
Panel F: Maturity of loans issued

Figure 12:
Panel A: Total amount of bonds issued

Figure 13:
Panel B: Average amount of bonds issued

Figure 14:
Figure 15: Panel C: Number of bonds issued

Source: SDC New Bond Issues
Panel D: Cost of bonds issued

Figure 16:
Panel E: Maturity of bonds issued

Figure 17:
Figure 18:
Total amount of bonds issued (by purpose)