

# Illiquidity Component of Credit Risk

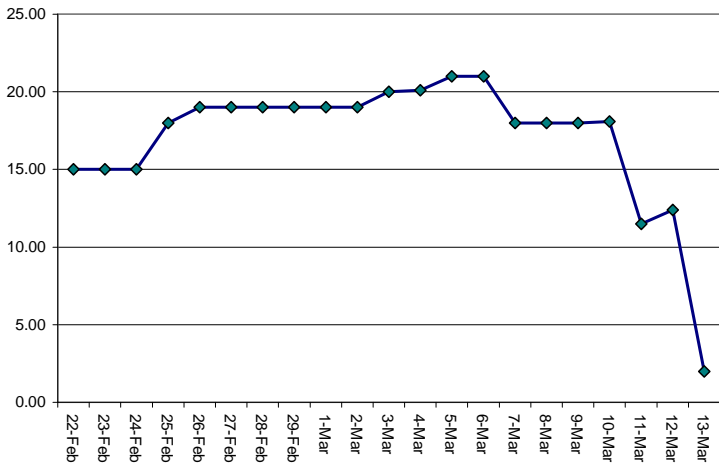
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# Conventional Approach to Credit Risk

- ▶ Problem is default by borrowers
- ▶ Focus on asset side of balance sheet
  - ▶ Problem is shortfall in asset values
- ▶ Capital is buffer to protect creditors, especially depositors
  - ▶ Basel-style approach to bank capital regulation

**Bear Stearns Liquidity Pool (US\$ billions)**  
(Source: SEC)



## Liquidity versus Solvency

Christopher Cox, (then) SEC chairman, on Bear Stearns in March 2008.

*“[T]he fate of Bear Stearns was the result of a lack of confidence, not a lack of capital. When the tumult began last week, and at all times until its agreement to be acquired by JP Morgan Chase during the weekend, the firm had a capital cushion well above what is required to meet supervisory standards calculated using the Basel II standard.*

*Specifically, even at the time of its sale on Sunday, Bear Stearns’ capital, and its broker-dealers’ capital, exceeded supervisory standards. Counterparty withdrawals and credit denials, resulting in a loss of liquidity - not inadequate capital - caused Bear’s demise.”*

# Liquidity versus Solvency

- ▶ An old and important distinction
- ▶ But hard to disentangle in practice
  - ▶ Did the run hasten failure of an already insolvent bank?
  - ▶ Or, did the run scupper an otherwise sound bank?
- ▶ Total credit risk is sum of *risk of insolvency* and *risk of run*
  - ▶ but the two are jointly determined
  - ▶ need to look at how they interact
  - ▶ need for a theory...

# This Paper

- ▶ Provides accounting framework to decompose credit risk into:
  - ▶ **Insolvency Risk:** probability that creditors do not get paid even in the absence of a run
  - ▶ **Illiquidity Risk:** probability that creditors do not get paid *because* of a run, when they would have been paid in the absence of a run
- ▶ Extends global game literature to address how the two interact
  - ▶ Existing literature focuses on (in)efficiency of coordination outcome (Morris and Shin (1998, 2004), Rochet and Vives (2004), Frankel and Pauzner (2005))
  - ▶ Focus instead on how *future* fundamental risk interacts with illiquidity and quantify total credit risk

# Balance Sheet Perspectives

Assets	Liabilities
Cash Risk Assets	Equity Short Debt Long Debt

- ▶ Conventional approach to credit risk (e.g. Basel capital rules) look at ratio:

$$\frac{\text{Risk Assets}}{\text{Equity}}$$

- ▶ But illiquidity risk highlights the ratios:

$$\frac{\text{Risk Assets}}{\text{Cash}}, \quad \frac{\text{Short Debt}}{\text{Cash}}$$

# Decline in Cash Holdings (until Lehman Weekend...)

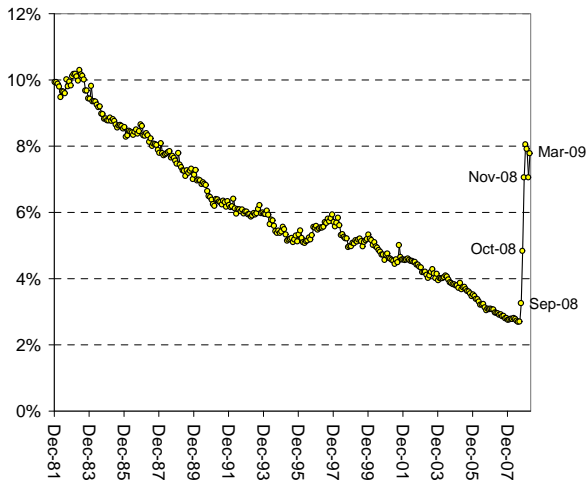
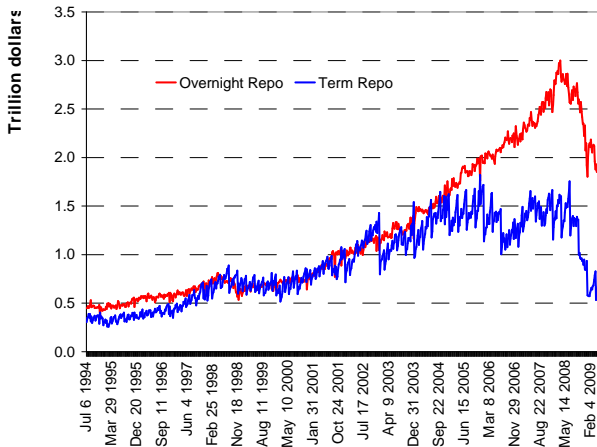


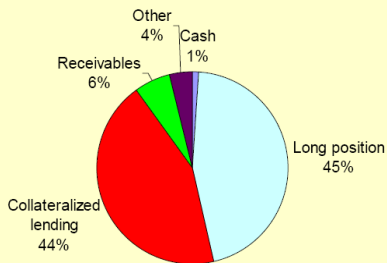
Figure: US Commercial Bank Cash Ratio (Source: Federal Reserve H8)

# Shortening Maturity

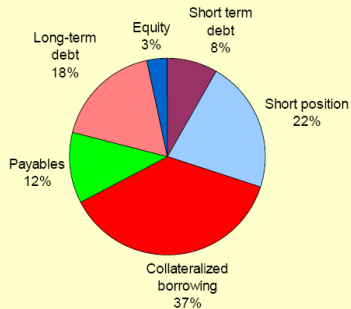


Outstanding Repurchase Agreements of US Primary Dealers  
(\$ Trillion) (Source: Federal Reserve Bank of New York)

# Lehman Brothers Balance Sheet, End-2007

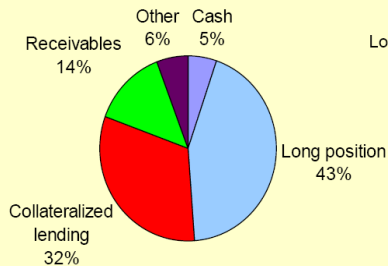


Assets

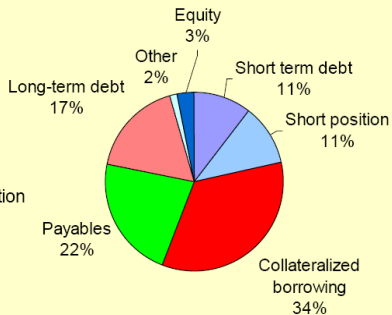


Liabilities

# Bear Stearns Balance Sheet, End-2007



Assets



Liabilities

## Benchmark Model (Assumptions Later Relaxed)

Three dates, ex ante (0), interim (1) and ex post (2).

The risky asset pays  $\theta_2$  in the final period 2.

$\theta_0$  ex ante value of one unit of risky asset

$\theta_1 = \theta_0 + \sigma_1 \varepsilon_1$ , interim value

$\theta_2 = \theta_1 + \sigma_2 \varepsilon_2$ , ex post value

$\varepsilon_1, \varepsilon_2$  independent, uniformly distributed on  $[-\frac{1}{2}, \frac{1}{2}]$ .

“Fundamental risk ratio”  $\rho = \frac{\sigma_2}{\sigma_1}$

## Ex Post (Date 2) Balance Sheet

$S_2$  is face value of short term debt,  $L_2$  is face value of long-term debt,  $E_2$  is ex post equity.

Assets	Liabilities
Cash, $M$ Risky Assets $\theta_2 Y$	Equity $E_2$ Short Debt $S_2$ Long Debt $L_2$

Bank is solvent if

$$M + \theta_2 Y \geq S_2 + L_2$$

Or

$$\theta_2 \geq \frac{S_2 + L_2 - M}{Y} \equiv \theta^{**}. \quad (1)$$

Defines **solvency point**  $\theta^{**}$ .

## Interim (Date 1) Cash Pool

Total cash available to the bank at the interim date

$$A^* = M + \psi Y$$

where  $\psi$  is pledgeable value of each unit of risky asset at date 1.

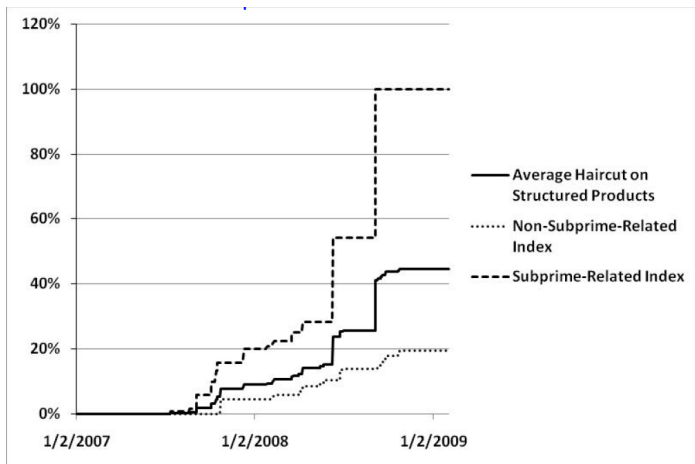
$\psi$  is outside the model (depends on secondary market)

Morris and Shin (2009, "Contagious Adverse Selection")

In benchmark case (later relaxed), assume:

- ▶ Successful run destroys all value for creditors who don't run (e.g. Lehman case)
- ▶ Unsuccessful run leaves final value intact (e.g. repo, not sell assets)

# Repo Haircuts on ABS



Source: Gorton and Metrick (2009)

Figure: Repo Haircut

## Liquidity Ratio and Outside Option Ratio

Bank fails if the proportion of short term debt holders who run exceeds *liquidity ratio*:

$$\lambda = \frac{A^*}{S},$$

where  $S$  is face value of the short-term debt at interim date.

- ▶ Short run creditors have alternative with gross return  $r^*$ .
- ▶ Notional return by staying invested is

$$r_S = \frac{S_2}{S}$$

Defines *outside option ratio*.

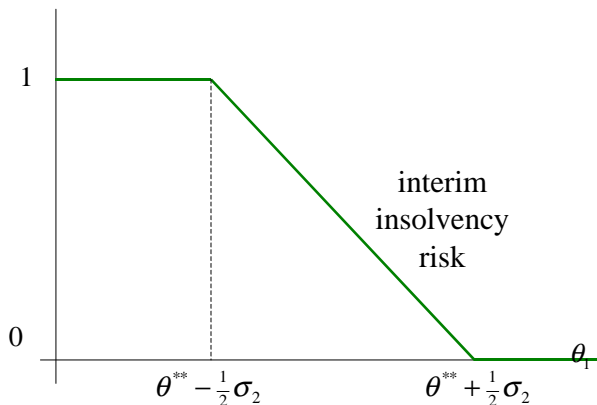
$$\mu = \frac{r^*}{r_S},$$

Liquidity ratio  $\lambda$ , the outside option ratio  $\mu$  and the fundamental risk ratio  $\rho$  are key parameters.

## Interim Insolvency Risk

**Interim insolvency risk** is the probability that the bank will fail *if there is no run*

$$\mathcal{N}_1(\theta_1) = \Pr(\theta_2 \leq \theta^{**} | \theta_1)$$



Expected return to rolling over if there is no run

$$r_S (1 - \mathcal{N}_1(\theta_1)) = \begin{cases} 0, & \text{if } \theta_1 \leq \theta^{**} - \frac{1}{2}\sigma_2 \\ \left(\frac{1}{2} + \frac{\theta_1 - \theta^{**}}{\sigma_2}\right) r_S, & \text{if } \theta^{**} - \frac{1}{2}\sigma_2 \leq \theta_1 \leq \theta^{**} + \frac{1}{2}\sigma_2 \\ r_S, & \text{if } \theta^{**} + \frac{1}{2}\sigma_2 \leq \theta_1 \end{cases}$$

**Indifference condition** between running and not running

(assuming global game strategic uncertainty where proportion who run is uniformly distributed over  $[0, 1]$ )

$$r^* = \begin{cases} 0, & \text{if } \theta_1 \leq \theta^{**} - \frac{1}{2}\sigma_2 \\ \left(\frac{1}{2} + \frac{\theta_1 - \theta^{**}}{\sigma_2}\right) \frac{A^*}{S} r_S, & \text{if } \theta^{**} - \frac{1}{2}\sigma_2 \leq \theta_1 \leq \theta^{**} + \frac{1}{2}\sigma_2 \\ \frac{A^*}{S} r_S, & \text{if } \theta^{**} + \frac{1}{2}\sigma_2 \leq \theta_1 \end{cases} . \quad (2)$$

“Run point”  $\theta^*$  is unique value of  $\theta_1$  that solves indifference condition

$$\theta^* = \theta^{**} + \sigma_2 \left( \frac{\mu}{\lambda} - \frac{1}{2} \right).$$

# Interim Total Credit Risk

$$C_1(\theta_1) = \mathcal{N}_1(\theta_1) + \mathcal{L}_1(\theta_1)$$

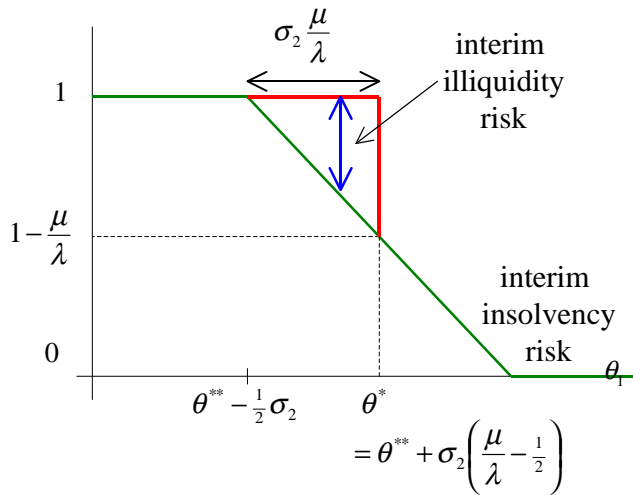


Figure: Total interim credit risk

## Ex Ante Credit Risk

Consider case where initial uncertainty is large (benchmark case giving separability result)

$$\begin{aligned}\mathcal{N}_0(\theta_0) &= \int_{\varepsilon=-\frac{1}{2}}^{\frac{1}{2}} \mathcal{N}_1(\theta_0 + \sigma_1 \varepsilon) \\ &= \frac{1}{\sigma_1} \left[ \theta^{**} - \left( \theta_0 - \frac{1}{2} \sigma_1 \right) \right] \\ &= \frac{1}{2} + \frac{\theta^{**} - \theta_0}{\sigma_1}.\end{aligned}\tag{3}$$

Ex ante illiquidity risk is

$$\begin{aligned}\mathcal{L}_0(\theta_0) &= \int_{\varepsilon=-\frac{1}{2}}^{\frac{1}{2}} \mathcal{L}_1(\theta_0 + \sigma_1 \varepsilon) \\ &= \frac{1}{2\sigma_1} \left[ \left[ \theta^{**} + \sigma_2 \left( \frac{\mu}{\lambda} - \frac{1}{2} \right) \right] - \left[ \theta^{**} - \frac{1}{2} \sigma_2 \right] \right] \frac{\mu}{\lambda} \\ &= \frac{\rho}{2} \left( \frac{\mu}{\lambda} \right)^2.\end{aligned}\tag{4}$$

## Ex Ante Credit Risk

$$\begin{aligned}C_0(\theta_0) &= \mathcal{N}_0(\theta_0) + \mathcal{L}_0(\theta_0) \\ &= \frac{1}{2} + \frac{\theta^{**} - \theta_0}{\sigma_1} + \frac{\rho}{2} \left(\frac{\mu}{\lambda}\right)^2\end{aligned}$$

So,

$$\begin{aligned}\text{Ex ante insolvency risk} &= \text{Prob.}(\theta_2 \leq \theta^{**}) \\ &= \frac{1}{2} + \frac{\theta^{**} - \theta_0}{\sigma_1}.\end{aligned}$$

$$\text{Ex ante illiquidity risk} = \frac{\rho}{2} \left(\frac{\mu}{\lambda}\right)^2,$$

- where
1.  $\rho$  is the fundamental risk ratio
  2.  $\mu$  is the outside option ratio
  3.  $\lambda$  is the liquidity ratio

## Additive Separability in Benchmark Case

$$\begin{aligned}C_0(\theta_0) &= \mathcal{N}_0(\theta_0) + \mathcal{L}_0(\theta_0) \\ &= \frac{1}{2} + \frac{\theta^{**} - \theta_0}{\sigma_1} + \frac{\rho}{2} \left(\frac{\mu}{\lambda}\right)^2\end{aligned}$$

Illiquidity risk is:

- ▶ increasing in the fundamental risk ratio  $\rho$
- ▶ Increasing in the outside option ratio  $\mu$
- ▶ Decreasing in the liquidity ratio  $\lambda$ .

## Other properties: How general?

- ▶  $\mathcal{N}_0(\theta_0) = \frac{1}{2} + \frac{\theta^{**} - \theta_0}{\sigma_1}$ .
  - ▶ ex ante insolvency risk is independent of  $\mu$  and  $\lambda$  once  $\theta^{**}$  is given.
- ▶  $\sigma_1 \frac{d\mathcal{N}_0}{d\theta^{**}} = 1$ .
  - ▶ Insolvency risk increases at a constant rate with  $\theta^{**}$ .
- ▶  $\mathcal{L}_0(\theta_0) = \frac{\sigma_2}{2\sigma_1} \left(\frac{\mu}{\lambda}\right)^2$ .
  - ▶ Illiquidity risk is independent of  $\theta^{**}$ .
  - ▶ Illiquidity risk disappears when  $\sigma_2 = 0$ .
- ▶  $\sigma_1 \frac{d\mathcal{L}_0}{d\lambda} = -\frac{\sigma_2 \mu^2}{\lambda^3}$ .
  - ▶ Illiquidity risk grows linearly in future uncertainty  $\sigma_2$ .

## General Distributions

Shocks  $\varepsilon_1$  and  $\varepsilon_2$  have densities  $f_1$  and  $f_2$  (c.d.f.s  $F_1$  and  $F_2$ )

$$\mathcal{N}_1(\theta_1) = F_2\left(\frac{\theta^{**} - \theta_1}{\sigma_2}\right).$$

Expected return to rolling over, conditional on no run

$$r_S \left(1 - F_2\left(\frac{\theta^{**} - \theta_1}{\sigma_2}\right)\right);$$

Indifference condition:

$$\frac{A^*}{S} \left(1 - F_2\left(\frac{\theta^{**} - \theta^*}{\sigma_2}\right)\right) r_S = r^*;$$

Run point  $\theta^*$  is

$$\theta^* = \theta^{**} - \sigma_2 F_2^{-1}\left(1 - \frac{\mu}{\lambda}\right). \quad (5)$$

Interim illiquidity risk

$$\mathcal{L}_1(\theta_1) = \begin{cases} 1 - F_2\left(\frac{\theta^{**} - \theta_1}{\sigma_2}\right), & \text{if } \theta \leq \theta^* \\ 0, & \text{if } \theta > \theta^* \end{cases}. \quad (6)$$

Total interim credit risk

$$\mathcal{C}_1(\theta_1) = \begin{cases} 1, & \text{if } \theta \leq \theta^* \\ F_2\left(\frac{\theta^{**} - \theta_1}{\sigma_2}\right), & \text{if } \theta > \theta^* \end{cases}. \quad (7)$$

Ex ante insolvency risk

$$\mathcal{N}_0(\theta_0) = \int_{\theta_1 = -\infty}^{\infty} F_2\left(\frac{\theta^{**} - \theta_1}{\sigma_2}\right) \frac{1}{\sigma_1} f_1\left(\frac{\theta_1 - \theta_0}{\sigma_1}\right) d\theta_1. \quad (8)$$

Ex ante illiquidity risk

$$\mathcal{L}_0(\theta_0) = \int_{\theta_1 = -\infty}^{\theta^{**} - \sigma_2 F_2^{-1}\left(1 - \frac{\mu}{\lambda}\right)} \left(1 - F_2\left(\frac{\theta^{**} - \theta_1}{\sigma_2}\right)\right) \frac{1}{\sigma_1} f_1\left(\frac{\theta_1 - \theta_0}{\sigma_1}\right) d\theta_1.$$

Total ex ante credit risk is the sum of the two

$$\begin{aligned} C_0(\theta_0) &= \mathcal{N}_0(\theta_0) + \mathcal{L}_0(\theta_0) \\ &= F_1\left(\frac{\theta^{**} - \theta_0 - \sigma_2 F_2^{-1}(1 - q)}{\sigma_1}\right) \\ &\quad + \int_{\theta_1 = \theta^{**} - \sigma_2 F_2^{-1}\left(1 - \frac{\mu}{\lambda}\right)}^{\infty} F_2\left(\frac{\theta^{**} - \theta_1}{\sigma_2}\right) \frac{1}{\sigma_1} f_1\left(\frac{\theta_1 - \theta_0}{\sigma_1}\right) d\theta_1. \end{aligned}$$

## Compare with benchmark case with uniform distributions

1.  $\mathcal{N}_0(\theta_0) = \frac{1}{2} + \frac{\theta^{**} - \theta_0}{\sigma_1}$ , and thus independent of  $\mu$  and  $\lambda$ .
2.  $\sigma_1 \frac{d\mathcal{N}_0}{d\theta^{**}} = 1$ .
3.  $\mathcal{L}_0(\theta_0) = \frac{\sigma_2}{2\sigma_1} \left(\frac{\mu}{\lambda}\right)^2$ , and thus independent of  $\theta^{**}$ .
4.  $\sigma_1 \frac{d\mathcal{L}_0}{d\lambda} = -\frac{\sigma_2 \mu^2}{\lambda^3}$ .

Qualitatively similar results hold with general distributions:

1.  $\mathcal{N}_0(\theta_0)$  is independent of  $\mu$  and  $\lambda$ .
2. as  $\sigma_1 \rightarrow \infty$ ,  $\sigma_1 \frac{d\mathcal{N}_0}{d\theta^{**}}$  is constant (i.e., independent of  $\theta^{**}$ ).
3. as  $\sigma_1 \rightarrow \infty$ ,  $\sigma_1 \mathcal{L}_0(\theta_0)$  is independent of  $\theta^{**}$ .
4. as  $\sigma_1 \rightarrow \infty$ ,  $\sigma_1 \frac{d\mathcal{L}_0}{d\lambda}$ 
  - 4.1 is negative;
  - 4.2 is linear in  $\sigma_2$ ;
  - 4.3 has absolute value decreasing in  $\lambda$  (assuming declining hazard rate condition on  $F(\cdot)$ )

## Replacing one dollar of risky asset with cash

Go back to benchmark case (uniform densities, large  $\sigma_1$ )

Recall

$$\theta^{**} = \frac{S_2 + L_2 - M}{Y}.$$

Hence,

$$\begin{aligned}\frac{d\theta^{**}}{dM} &= -\frac{1}{Y} \\ \frac{d\theta^{**}}{dY} &= -\frac{S_2 + L_2 - M}{Y^2} \\ &= \frac{1}{Y} \left( \frac{E}{Y} - 1 \right)\end{aligned}$$

where (hybrid) equity measure  $E$  is difference between ex ante asset values and ex post liabilities

$$E = M + Y - S_2 - L_2,$$

## Two Faces of Cash

Cash has two attributes:

- ▶ Safe asset (mitigates insolvency risk)
- ▶ Liquid asset (mitigates illiquidity risk)

In the benchmark case, two attributes enter additively.

Total effect on total credit risk of replacing one dollar of risky asset by cash is:

$$\begin{aligned} -\frac{dC_0}{dY} + \frac{dC_0}{dM} &= \overbrace{-\frac{dN_0}{dY} + \frac{dN_0}{dM}}^{\text{cash as safe asset}} - \overbrace{\frac{dL_0}{dY} + \frac{dL_0}{dM}}^{\text{cash as liquid asset}} \\ &= -\frac{E}{\sigma_1 Y^2} - \frac{\sigma_2 \mu^2 (1 - \psi)}{\sigma_1 \lambda^3 S} \\ &= -\frac{1}{\sigma_1} \left( \frac{E}{Y^2} + \frac{\sigma_2 \mu^2 (1 - \psi)}{\lambda^3 S} \right). \end{aligned}$$

Replacing risky asset by cash has big effect on credit risk when:

- ▶ equity ( $E$ ) is high relative to risky asset holdings  $Y$
- ▶ ex post uncertainty ( $\sigma_2$ ) is high
- ▶ the fire sale discount ( $\psi$ ) is high
- ▶ outside option ratio ( $\mu$ ) is high
- ▶ the liquidity ratio ( $\lambda$ ) is low

Arguably, all these features figured prominently for the case of highly leveraged financial intermediaries such as Bear Stearn and Lehman Brothers.

Result offers guidance in setting reserve requirements (required cash holdings at central bank) for financial institutions.

# Cash Pool Depends on Innovation in Fundamentals

Examine sensitivity of cash pool to innovation  $\theta_1 - \theta_0$  through parameter  $\delta$

$$A^*(\theta_1) = M + (\psi + \delta(\theta_1 - \theta_0)) Y,$$

(benchmark case is when  $\delta = 0$ ). For small  $\delta$ ,

$$\mathcal{L}_0(\theta_0) \approx \frac{\sigma_2}{2\sigma_1} \left(\frac{\mu}{\lambda}\right)^2 \left(1 - \frac{\delta Y}{\lambda S} (\theta^* - \theta_0)\right)^2$$

- ▶ Normally,  $\theta^* < \theta_0$ , so illiquidity risk is increasing in  $\theta_0$ .
  - ▶ Illiquidity risk is lower for riskier claims (low  $\theta_0$ ) for fixed  $\lambda$ . But  $\lambda$  is generally lower for riskier claims.
- ▶ Effect of innovation in  $\theta$  is amplified by low  $\lambda$  parameter

## Partial Liquidations

- ▶ When liquidity withdrawals run has to be met by asset sales, then solvency point  $\theta^{**}$  depends on outcome of run, even unsuccessful ones

$$\theta^{**}(\pi) = \begin{cases} \frac{S+L_2-M}{Y}, & \text{if } \pi \leq \frac{M}{S} \\ \frac{(1-\pi)S+L_2}{Y-\frac{\pi S-M}{\psi}}, & \text{if } \frac{M}{S} \leq \pi \leq \frac{A^*}{S} \end{cases}$$

where  $\pi$  is proportion of short-term creditors who run.

- ▶ With partial liquidation, there is ex ante illiquidity risk even when there is no fundamental uncertainty between dates 1 and 2 (Rochet and Vives (2004)).
- ▶ Illiquidity risk when  $\sigma_2 \rightarrow 0$  is

$$\frac{(\mu S - M)(S + L_2 - M - \psi Y)}{\sigma_1 Y (\psi Y - \mu S + M)}$$

# Other Variations

- ▶ General balance sheets
  - ▶ Many classes of risky assets with differing haircuts and cash raising attributes
  - ▶ TSLF (Term Securities Lending Facility, swapping MBSs for Treasuries)
- ▶ Small ex ante uncertainty
- ▶ Public information, optimism/pessimism

# Conclusion

- ▶ Funding of financial institutions by ultra short term credit and lack of liquid assets on balance sheet have played role in crisis
- ▶ Liquidity issues should be addressed in new regulatory regime; but we need to understand interaction between illiquidity and insolvency to do this
- ▶ We have developed model based distinction between insolvency and illiquidity risk
- ▶ Offers guidance on when re-liquification may be as important as re-capitalization (and when it won't)