

# A Theory of Arbitrage Capital<sup>1</sup>

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# A Theory of Arbitrage Capital

## Abstract

Fire sales that occur during crises beg the question of why *sufficient* outside capital does not move in quickly to take advantage of fire sales, or in other words, why outside capital is so “slow-moving”. We propose an answer to this puzzle in the context of an equilibrium model of capital allocation. There are states of the world in which asset prices fall low enough that it is profitable to carry liquid capital to acquire assets in such states. Set against this, keeping capital in liquid form entails costs in terms of foregone profitable investments. We show that a robust consequence of this trade-off between making investments today and waiting for arbitrage opportunities in future is the combination of occasional fire sales and limited stand-by capital. When there are learning-by-doing effects, such stand-by capital moves in to acquire assets only if fire-sale discounts are sufficiently deep. An extension of our model to several types of investments gives rise to a novel channel for contagion where sufficiently adverse shocks to one type can induce fire sales in other types that are fundamentally unrelated, provided these investments are arbitrated by a common pool of capital.

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# 1 Introduction

Our understanding of financial crises has been enhanced by a large and rapidly growing empirical literature that has documented the incidence and severity of fire sales by distressed parties in a wide range of asset classes.<sup>1</sup> Indeed, it would not be too much of an exaggeration to say that fire-sales have been a defining feature of most financial crises, including the most recent crisis of 2007-09.

The term “fire sale” carries the connotation that assets are being sold at prices that are below some benchmark, fair fundamental price that would prevail in the absence of a crisis. However, the notion that assets are being sold at prices below their fundamental value begs an important question. How can fire sales take place in a world where arbitrage capital waits on the sidelines waiting to take advantage of artificially low prices? If there were such arbitrageurs who wait on the sidelines, would they not compete with each other as soon as the crisis erupts, providing a cushion for prices? As well as these “positive” questions on the nature of the equilibrium outcome, there are also important normative questions on the social value of arbitrage capital. Is the equilibrium provision of arbitrage capital at the efficient level? If not, is the efficient level higher or lower than the equilibrium level? Our paper sets out to answer these questions in a setting that is simple and transparent enough so that the answers uncover the deep economic mechanisms at work.

We show that the answers to both the positive and normative sets of questions rely on the interplay between two underlying allocative mechanisms in the economy. One operates at the ex post stage, and has to do with the efficient allocation of assets to those economic agents that can generate most value from them. The second operates at the ex ante stage, and has to do with how much of the economy’s resources are set aside in the form of “idle” arbitrage capital that waits on the sidelines.

In our baseline set-up, engaging in production entails greater expertise to insiders through learning-by-doing effects, and these insiders are the natural holders of the assets in the sense of being able to generate greater value from them compared to outsiders who take over distressed assets of failed insiders. This feature of our model is motivated by the substantial empirical evidence that outside investors enter only when price discounts become very large. In particular, the evidence from major financial crises suggest that assets acquired by private equity firms and other outside investors are subsequently re-sold, or “flipped”, to insiders

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<sup>1</sup>Fire sales have been shown to exist in distressed sales of aircrafts by Pulvino (1998), in cash auctions in bankruptcies by Stromberg (2000), in creditor recoveries during industry-wide distress especially for industries with high asset-specificity by Acharya, Bharath and Srinivasan (2007), in equity markets when mutual funds engage in sales of similar stocks by Coval and Stafford (2007), and in an international setting where foreign direct investment increases during emerging market crises to acquire assets at steep discounts in the evidence by Krugman (1998), Aguiar and Gopinath (2005), and Acharya, Shin and Yorulmazer (2007).

once the crisis subsides. We return to this point in the body of our paper.

The greater is the discount in the value realized by outsiders, the more severe must be the fire sale before outsiders enter to cushion the distress. Thus, for a finite pool of arbitrage capital at the ex post stage, there are states of the world with possibly steep price discounts, which then create the incentive to hold unproductive “idle” arbitrage capital at the ex ante stage. In equilibrium, the two choices – whether to invest in profitable activities or to set aside funds for arbitrage in the future – earns the same rate of return when viewed *ex ante*. As a consequence, limited provision of arbitrage capital and fire sales emerge as robust features of the equilibrium.

There are two important normative consequences of fire sales. First, there is ex post inefficiency due to assets being held by outsiders in some states of the world. Second, there is also ex ante inefficiency due to the ex ante allocation of resources to arbitrage capital. In particular, the two inefficiencies are closely related, since the ex ante profitability of holding unproductive arbitrage capital is greater when the ex post inefficiencies are greater. In particular, the two allocative mechanisms that generate the inefficiencies may reinforce each other, rather than mitigate each other. Thus, if the learning-by-doing effect is large, the fire sales are more severe, which tends to increase the attractiveness of holding arbitrage capital. In this way, greater ex post inefficiency generates more ex ante inefficiency.

There are also implications for the depth of fire sales following long periods of booms. During boom periods, risky projects are more attractive relative to holding cash balances. Hence, during the upturn of the business cycle, a higher fraction of agents choose to become insiders and there is less liquid capital put aside in anticipation of the downturn. As a result, when adverse shocks hit insiders during boom periods, fire-sale effects in asset prices are more severe. This result explains why crises that erupt after a long boom are associated with sharper, more severe downturns.<sup>2</sup>

The picture that emerges from our analysis is that private incentives lead to the over-provision of arbitrage capital relative to the first best. Intuitively, in states where there are few surviving insiders, asset prices must fall so that the market clears. Nevertheless, there is allocative efficiency as assets remain in the hands of insiders. The presence of arbitrage capital interferes with the efficient allocation. Ex post, there is inefficiency if arbitrageurs are less efficient than insiders in deploying assets. Importantly, even if arbitrageurs are as

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<sup>2</sup>Conversely, as economic times worsen, more capital is set aside for arbitrage. For instance, according to the article titled “Cashing in on the crash” in the *Economist* on August 23, 2007, vulture funds raised \$15.1 billion in the first seven months of 2007, more than the \$13.9 in all of 2006, to take advantage of fire sales due to expected distress in financial markets. The same article points out that while some hedge funds suffered, the others, such as Citadel, Ellington, and Marathon Asset Management had the ready cash. The article highlights the strategy of Citadel to keep more than a third of its assets in cash or liquid securities, allowing it to take advantage of fire sales when opportunities arise.

efficient as insiders, setting aside arbitrage capital ex ante implies passing over profitable investment opportunities. Somewhat counter-intuitively perhaps, relative to the competitive outcome, the first best features *lower* asset prices ex post and *greater* profitable investments ex ante.

Since our welfare results arise from the interplay between the ex post asset allocation and the ex ante provision of arbitrage capital, one important message from our paper is that the opening of capital markets in the ex post period where surviving insiders can raise new funding from outsiders does not eliminate the inefficiency, although there is some mitigation of the inefficiency. We demonstrate this feature in an extension of our framework where we allow insiders to raise additional funding from outsiders by selling financial claims. Since outsiders have the choice between acquiring physical assets or buying financial claims sold by insiders, arbitrage capital is allocated in such a way that the returns are equalized. The result is that fire-sale discounts in prices for acquisition of assets must equal that for provision of external finance, giving rise to a spillover or contagion from illiquidity in the market for real assets to that for financing of these assets. From the welfare standpoint, ex post efficiency of allocation is restored as arbitrageurs simply fund asset purchases of insiders, but they make same profits ex ante due to discounts they charge in funding insiders. As a result, it remains profitable for there to be some arbitrage capital in equilibrium and there continues to be some ex ante inefficiency in investment decisions.

In another extension of our benchmark model, we show that contagion can result across different sectors of the economy or between different asset classes whenever the provision of arbitrage capital in these markets is from a common pool; returns on different investments in the portfolio of an arbitrageur must be the same. The fact that the quantity of arbitrage capital is limited implies that these returns are positive in all markets where arbitrageurs allocate capital.<sup>3</sup> This channel of contagion operates even though the fundamentals of two sectors are independent and the existence of fire sales in one sector can give rise to fire sales in the other.

## 1.1 Related literature

Fire sales are, of course, not new to our paper. The idea that asset prices may contain liquidity discounts when potential buyers are financially constrained and assets are not easily redeployable were discussed by Williamson (1988) and Shleifer and Vishny (1992). This

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<sup>3</sup>For (apparent) “dislocations” between different capital markets and the effect of liquidations in one market on prices in another, see an excellent discussion of the large body of extant empirical evidence in Duffie and Struvolici (2008). For similar evidence in an international setting, see Rigobon (2002) and Kaminsky and Schmukler (2007), and the discussion in Pavlova and Rigobon (2007). The literature by and large attributes such dislocations to investment-style restrictions or limited arbitrage capital.

early literature suggests that firms, whose assets tend to be *specific* (that is, whose assets cannot be readily redeployed by firms outside of the industry) are likely to experience lower liquidation values because they may suffer from fire-sale discounts in cash auctions for asset sales, especially when firms within an industry get simultaneously into financial or economic distress. Since then, fire sales have often figured in models of crises (Allen and Gale, 1994, 1998, among others). Intimately tied to the notion of fire sales is the idea that arbitrageurs wanting to buy assets at steep discounts may also face financing frictions due to principal-agent problems. The resulting “limits of arbitrage” (Shleifer and Vishny, 1997) can entrench fire-sale prices for a period of time once they materialize.<sup>4</sup>

Our contribution relative to this earlier literature is to focus on the *ex ante* decisions of investors, which much of the literature takes as given, and thereby to explain the origins of the limited nature of arbitrage capital as an equilibrium phenomenon. In this sense, our work is closest to the analysis by Allen and Gale (2004) of the portfolio choice of banks between holding safe versus risky assets. Gorton and Huang (2004), another closely related paper, also considers the equilibrium portfolio choice of firms, deriving that it is socially inefficient to hold large quantities of safe assets required to avoid fire sales, and studying in this context the role of government bailouts during crises. Our framework is tractable and facilitates crisp conclusions on key comparative statics and welfare questions. In particular, our result that arbitrage capital is endogenously lower in good times, and therefore, that crises arising in good times feature deeper fire-sale discounts, is a noteworthy result, which (to our knowledge) has not been discussed so far in the literature.

Another advantage of our tractable framework is to open up for scrutiny the arbitrageurs’ access to different real and financial markets, and thereby identify a channel of contagion that relies purely on the limited nature of arbitrage capital. Our results on this front are closest to Gromb and Vayanos (2007) and Duffie and Struvolici (2008). Gromb and Vayanos (2007) consider arbitrageurs exploiting fire-sale opportunities across markets and this equilibrates returns they can earn in different markets. Duffie and Struvolici (2008) study a similar setting taking the financing friction of arbitrageurs as given. In both of these papers, the quantity of equilibrium arbitrage capital is exogenous, whereas our central theoretical concern is to endogenize the quantity of arbitrage capital and illustrate that its limited quantity as well as its limited expertise make fire sales a robust equilibrium phenomenon.<sup>5</sup>

Rampini and Viswanathan (2007) consider a dynamic contracting set-up for exploring which firms make investments and which preserve debt capacities for the future. This question is related to our analysis. While we model fire sales as an outcome from market clearing when

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<sup>4</sup>Mitchell, Pedersen and Pulvino (2007) provide compelling episodic evidence for the fact that capital appears to be “slow moving” when it enters markets affected by fire-sale discounts in prices.

<sup>5</sup>Note that contagion has been derived in many other settings through portfolio flows (Kodres and Pritsker, 2002) or utility-based assumptions (Kyle and Xiong, 2001).

buyers are financially constrained, Rampini and Viswanathan model asset prices as being temporarily low due to low cash flow realizations. Acharya, Shin and Yorulmazer (2010) also consider gains from acquiring assets at fire-sale prices which make it attractive for firms (in their model, banks) to hold liquid assets ex ante, but that when such fire-sale states are not too likely, banks hold too little liquidity due to the asset-substitution problem. Their focus is on analyzing how government or central bank interventions to resolve banking crises (that may be desirable ex post) affect ex-ante liquidity in potentially adverse ways.<sup>6</sup> Bolton, Santos, and Scheinkman (2008) and Diamond and Rajan (2009) present models wherein once an adverse state of the world arises, decisions of individual banks affect when assets get sold - right away or with delay. Immediate sales can increase returns to holding cash and lead to potentially excessive cash hoarding ex ante. In contrast to these models, our paper is more in the spirit of Allen and Gale (1994, 1998) and Gorton and Huang (2004) in that once the adverse state arises, there is an immediate (fire) sale of assets.

Our study is also related to the seminal work of Kiyotaki and Moore (1997) on credit cycles. In Kiyotaki and Moore (1997) and Krishnamurthy (2003), the underlying asset cannot be pledged because of inalienable human capital. Krishnamurthy (2003) differs from Kiyotaki and Moore (1997) in that all contingent claims on aggregate variables are subject to collateral constraints. However, land can be pledged and has value both as a productive asset and as collateral. Caballero and Krishnamurthy (2001) employ a Holmstrom-Tirole approach with exogenous liquidity shocks and allow firms to post collateral in a manner similar to Kiyotaki and Moore.

Finally, there are models in which there are pecuniary externalities from fire sales of assets. Lorenzoni (2008), for example, considers a competitive model of intermediaries in which ignoring of these externalities leads to excessive borrowing ex ante and excessive volatility ex post. In our model, there are no externalities from fire sales and the result is that the competitive equilibrium features too much setting aside of idle capital ex ante for the purpose of undertaking arbitrage ex post.

## 2 Model

The timeline of the model is provided in Figure 1. There are three dates - initial, interim and final. We index the three dates by  $t \in \{0, 1, 2\}$ . There is a unit measure of risk-neutral agents. Each agent is endowed with 1 unit of the consumption good at the initial date. Consumption takes place only at the final date (date 2).

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<sup>6</sup>Huang and Wang (2010) also show that competitive market forces fail to lead to an efficient supply of liquidity. The market provision of liquidity is generally too low when the probability of a liquidity event is small and is too high when the probability of a liquidity event is large.

There are two types of assets in the economy. There is a riskless storage technology that allows any agent to transfer one unit of the consumption good from date  $t$  to date  $t + 1$ . In this sense, the risk-free interest rate is zero.

There is also a risky investment opportunity that agents can undertake at date 0. The investment opportunity is indivisible and needs the full unit of the consumption good as the input. The outcome of the investment is random, and known by date 1. The payoff from the risky investment for agent  $i$  depends on the outcome of a binary random variable  $z_i$ . Specifically, the payoff from  $i$ 's risky investment is:

$$\begin{cases} R & \text{if } z_i = 1 \\ 0 & \text{if } z_i = 0 \end{cases} \quad (1)$$

where  $R > 1$ .

The probability that  $z_i = 0$  is given by  $k$ . However, we allow aggregate uncertainty in the economy. When viewed from date 0, the probability  $k$  itself is uncertain. Nature first draws  $k$  from a known density  $f(k)$  over  $[0, 1]$ , and then determines the realizations of individual projects  $\{z_i\}$  as independent and identically distributed (i.i.d.) draws from coin tosses where the probability of failure is fixed at  $k$ . By the law of large numbers, the proportion of risky investments that fail is exactly  $k$ , but this proportion is uncertain at the time of the investment. This aggregate uncertainty plays a key role in our model.

## 2.1 Insiders and Arbitrageurs

The agents decide whether or not to undertake the risky investment at date 0. Those who decide to invest are known as *insiders*. Those who choose to keep their wealth in the storage technology are known as *arbitrageurs*. We will denote the proportion of agents who become arbitrageurs by  $w$ .

We assume that, by the nature of the investment, the insiders benefit from learning-by-doing, so that the insiders become proficient in managing the asset and minimizing its depreciation. This is an important feature of our model, as the role of arbitrageurs is double-sided. Although they stand on the sidelines ready to purchase the assets of failed insiders, they are not natural holders of the asset, and there are social costs as a result of their ownership of assets, as we will describe below.

At date 1, the outcomes of all the insiders' projects are known. Proportion  $k$  of the projects fail, and by date 1, the identity of the successful and unsuccessful insiders is known. We assume that the owners of the failed investments are forced to put up their asset for sale in the market. In a richer model with debt financing, we could give further micro-foundations for such forced sales as the consequence of the inability to service debt following the failure

of the project. Here, we simply adopt as a reduced-form assumption that failed insiders are forced to put up their asset for sale at whatever is the ruling price for the asset at date 1.

We allow the possibility that arbitrageurs purchase the assets from the failed insiders at the market clearing price. Also, the insiders whose projects have succeeded also have the financial resources to purchase additional assets being sold by the failed insiders. Thus, at date 1, the assets of failed insiders are bought by successful insiders and arbitrageurs.

However, the learning by doing matters for the terminal value of the asset. Between date 1 and date 2 (i.e. after the realization of the investment, but before consumption at the final date), the depreciation of the risky asset depends on who holds the asset. If the asset is held by an insider, the asset can be managed well due to the expertise gained by the insider in the initial period of production. In the hands of the insider, the terminal value of the asset at date 2 is given by  $\bar{p} > 1$ . However, if the asset is bought by an arbitrageur at date 1, the upkeep of the asset is not as good, so that the terminal value of the asset is given by  $\underline{p}$  where

$$\underline{p} = \bar{p} - \Delta \tag{2}$$

and  $\Delta \in [0, \bar{p}]$  is the assumed additional depreciation of the asset value in the hands of the arbitrageurs.

The terminal values  $\{\underline{p}, \bar{p}\}$  could be interpreted as the continuation market price at date 2 for a new generation of potential investors, where the productivity of the asset depends on who has been managing the asset in the recent past. For the purpose of our model, however, we take  $\underline{p}$  and  $\bar{p}$  as given constants.

Our assumption that  $\underline{p} < \bar{p}$  is motivated by the empirical evidence that during major financial crises, the entry of outside investors takes place only when incumbents face severe financial distress and lack the resources to take over failing rivals. Elsewhere, in a separate study of corporate acquisitions during the Asian financial crisis of 1997-8 (Acharya, Shin and Yorulmazer (2009)) we show that even as foreign portfolio inflows came to a sudden halt and reversed with the onset of the crisis, there is a concurrent surge in foreign direct investment (FDI) inflows associated with foreign takeovers of distressed local firms. Tellingly, a large proportion of the FDI flows are subsequently reversed as the foreign investors re-sell, or “flip”, the assets back to local insiders once the crisis abates.

The resolution of distressed banks is perhaps the clearest illustration of the comparative advantage of insiders in managing the assets. When faced with the imminent failure of a bank, regulators turn to other (healthier) banks to take over the ailing rival. It is only when potential acquiring banks are themselves distressed that regulators turn to outside investors, such as private equity firms. Once the crisis abates, the private equity firms who had acquired stakes in ailing banks will exit their investment by selling their stake to another bank. For these reasons, we believe that the case where  $\underline{p} < \bar{p}$  is the natural one to examine in our

model.

Nevertheless, it should be emphasized that our framework is rich enough to accommodate the alternative scenario where  $\underline{p} > \bar{p}$ , so that the outsiders have greater expertise and are able to generate more value than the insiders. Appendix II below examines this case, where we show that the welfare results are made more subtle. The important point is that even with greater expertise ex post, the overall welfare calculation must take account also of the ex ante inefficiency associated with “idle” arbitrage capital, and the greater expertise of outsiders may not eliminate the inefficiency of arbitrage capital.

Returning to our benchmark model, we continue under the assumption that  $\underline{p} < \bar{p}$ . A natural regularity condition in this context is that the ex ante productivity of investment justifies the cost of investment, namely

$$E_k [(1 - k)R + k\bar{p}] > 1, \quad (3)$$

where  $E_k[\cdot]$  is the expectations operator with respect to the random variables  $k$ . This is a condition on the density over aggregate shocks  $k$ , which states that the aggregate shocks are not so bad that investment is unjustified even under the most optimistic scenario where all the assets of failed insiders end up in the hands of other insiders. We assume throughout that (3) holds.

At date 1 (after the realization of the random variable  $k$ , but before the reallocation of assets), each successful insider holds  $R$  units of the consumption good. He can either hold this to the terminal date in the storage technology and consume  $R$ , or he can buy the assets of the failed insiders.

We denote the market price of the risky asset at date 1 by  $p$ . With  $R$  units of resources, the successful insider can buy  $R/p$  units of the asset, which he can sell for  $\bar{p}$  each at the terminal date. Thus, if the insider buys the failed insiders’ assets, his final consumption is:

$$\frac{R}{p} \times \bar{p} \quad (4)$$

So, as a function of the realization of the project, the insider  $i$ ’s final period consumption is

$$\begin{cases} R & \text{if } i\text{'s project is successful but no assets purchased} \\ \frac{R}{p}\bar{p} & \text{if } i\text{'s project is successful and assets purchased} \\ p & \text{if } i\text{'s project is unsuccessful} \end{cases} \quad (5)$$

Since the market for the asset has to clear at date 1, the price should fall by enough so that the insider is not made better off just storing his wealth from date 1 to date 2. Hence,  $R \leq \frac{R}{p}\bar{p}$  or,

$$p \leq \bar{p} \quad (6)$$

Since the probability of  $i$ 's investment success is  $1 - k$ , the ex ante payoff of being an insider is

$$E_k \left[ (1 - k) \frac{R}{p} \bar{p} + kp \right] \quad (7)$$

where  $p$  satisfies (6).

We now turn to the payoff of the arbitrageur. The arbitrageur starts off by storing the consumption good, and his terminal consumption depends on the market price  $p$  ruling at date 1. If the price  $p$  is higher than the terminal value of the asset  $\underline{p}$  in the hands of the arbitrageurs, then the arbitrageur is better off keeping his wealth in the storage technology. However, if the price  $p$  falls sufficiently so that  $p \leq \underline{p}$ , then the arbitrageur will enter the market. With one unit of wealth, he can buy  $1/p$  units of the asset, each of which can be sold at date 2 for  $\underline{p}$ . Since  $p$  is a function of  $k$ , we can write the arbitrageur's ex ante payoff as

$$E_k \left[ \max \left\{ 1, \frac{\underline{p}}{p} \right\} \right] \quad (8)$$

## 2.2 Welfare Function

The welfare function is defined as the unweighted sum of the ex ante payoffs of all agents. In our framework, the welfare function is the expectation at date 0 of the total consumption across all agents - both insiders and arbitrageurs. From (7) and (8), we can write the welfare function as the weighted average of the insiders' payoff and the arbitrageurs' payoffs, with weights  $1 - w$  and  $w$  respectively.

$$\Pi = E_k \left[ (1 - w) \left( (1 - k) \frac{R}{p} \bar{p} + kp \right) + w \times \max \left\{ 1, \frac{\underline{p}}{p} \right\} \right] \quad (9)$$

We can simplify the expression for the welfare function using the fact that welfare takes account only of the total consumption rather than the distribution of consumption across agents. Thus, the market price  $p$  is irrelevant for welfare if the asset is transferred from one insider to another. Whatever is gained by the seller is lost by the buyer, and vice versa. The market price becomes relevant only when the asset changes hands from an insider to an arbitrageur, since the terminal value of the asset depends on the identity of the purchaser at date 1.

Consider two cases:  $p > \underline{p}$  and  $p \leq \underline{p}$ . First, if  $p > \underline{p}$  then the arbitrageurs prefer not to participate in the purchase of the asset at date 1, and keep all their wealth in the storage technology. Then total consumption across the population is the sum of the output of the

successful insiders and the terminal value of the aggregate asset stock. Thus, interim welfare when  $p > \underline{p}$  is given by

$$\Pi(w, k) = (1 - w) [(1 - k) R + \bar{p}] + w \quad (10)$$

where the notation  $\Pi(w, k)$  makes clear that interim welfare is a function of the realized aggregate shock  $k$  and the initial career choices  $w$ .

However if  $p \leq \underline{p}$ , then the aggregate shock is too large to be absorbed simply by reallocation of the asset between insiders. The arbitrageurs then enter the market at date 1 to participate in the purchase of the asset. Denote by  $y_A$  the mass of the failed insiders' assets purchased by arbitrageurs. Then, the total assets held by the insiders consist of the assets of successful insiders  $((1 - w)(1 - k))$ , and the assets of failed insiders bought by other insiders  $((1 - w)k - y_A)$ . Thus, when  $p \leq \underline{p}$ , interim welfare is

$$\Pi(w, k) = (1 - w) [(1 - k) R + k\bar{p}] + w - (\bar{p} - \underline{p}) y_A \quad (11)$$

Since (11) subsumes (10), we can write ex ante welfare as the expectation of (11) with respect to the realization of the aggregate shock  $k$ .

$$\Pi = E_k [(1 - w) [(1 - k) R + k\bar{p}] + w - \Delta \cdot y_A] \quad (12)$$

where we have appealed to the definition  $\Delta = \bar{p} - \underline{p}$ . From (12) we see that ex ante welfare is decreasing in  $E_k(y_A)$  for any fixed value of  $w$ . The key question is how  $\Pi$  behaves as a function of  $w$ .

However, we can prove a benchmark welfare result on the global optimum for  $w$ . Namely,  $w = 0$  is the unique global optimum for  $w$ . Nevertheless, we can also show that the global optimum is never an equilibrium outcome. Thus, there is a strict separation between the *socially optimal outcome* and the *equilibrium outcome*.

We prove two results, the first on the welfare optimum, and the second on the equilibrium outcome. The equilibrium in our model refers to the profile of choices at date 0 on whether to become an insider or not such that, given the choices of other agents, one's own choice maximizes one's individual payoff function, either as the insider given by (7) or as the arbitrageur, given by (8). Given the symmetry of our model, the proportion of insiders and arbitrageurs is a sufficient statistic for the strategy profile of the whole population.

**Proposition 1** *Under the regularity condition (3),  $w = 0$  is the unique global welfare optimum.*

The proof of Proposition 1 can be given briefly, and appeals to the aggregate outcome. As noted already, any outcome in which the asset ends date 2 in the hands of an arbitrageur

entails a welfare cost of  $\Delta$ , and hence is not socially optimal. Two conditions are necessary and sufficient for the social optimum - namely, that the interim output at date 1 is maximized, and no assets end up at date 2 in the hands of arbitrageurs. When  $w = 0$ , both conditions are satisfied. There are no arbitrageurs at date 1, and so all the assets end up in the hands of insiders only. Meanwhile, from the regularity condition (3), the interim output is maximized when  $w = 0$ . Hence, the outcome with no arbitrageurs is the unique global optimum for welfare.

### 3 Benchmark Equilibrium

Having characterized the first-best outcome, we now consider the competitive equilibrium. In the benchmark model, we assume that the surviving insiders cannot raise additional funding, and must rely solely on the resources from the successful project. Thus, until further notice, we operate under the following assumption.

**Assumption 1.** The surviving insiders cannot raise outside capital and each only has  $R$  units of the consumption good that can be used to purchase distressed assets.

The payoffs of the model are determined through a competitive auction of failed insiders' assets at the interim date. We will then solve the model backward, by first considering the sale of failed insiders' assets and the resulting asset prices, and next, analyzing the ex ante choice to become insiders or arbitrageurs.

#### 3.1 Asset Sales and Liquidation Prices

We keep track of two key features in the purchase of failed insiders' assets. First, arbitrageurs, using arbitrage capital, and the surviving insiders, using their first-period return, compete to purchase these assets. Second, surviving insiders may not have enough resources to acquire all failed insiders' assets. To focus on the interplay between these two features, we model asset sales as follows.

(i) All failed insiders' assets are pooled and competitively auctioned to the surviving insiders and arbitrageurs as described below.

(ii) The surviving insiders and arbitrageurs submit a demand schedule  $y_i(p)$  that specifies the quantity demanded for failed investors' assets for each price  $p$ . The index  $i$  belongs in  $[0, (1 - w)(1 - k)]$  if  $i$  is a surviving insider, while  $i \in [1 - w, 1]$  if  $i$  is an arbitrageur.

(iii) We assume that insiders cannot raise additional financing.<sup>7</sup> Hence, the resources

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<sup>7</sup>We relax this assumption in Section 4.

available to each surviving insider for purchasing failed insiders' assets is the payoff  $R$  from the risky investment.

(iv) The price  $p$  clears the market, where assets allocated to surviving insiders and arbitrageurs add up at most to the proportion of failed firms:<sup>8</sup>

$$\int_0^{(1-w)(1-k)} y_i(p) di + \int_{1-w}^1 y_i(p) di \leq (1-w)k. \quad (13)$$

(v) We pin down the price  $p$  by focusing on the symmetric case where all surviving insiders submit the same schedule, that is,  $y_i(p) = y(p)$  for all  $i \in [0, (1-w)(1-k)]$ , and all arbitrageurs submit identical schedules, that is,  $y_i(p) = y_a(p)$  for all  $i \in [1-w, 1]$ .

To solve for the competitive allocation, we first derive the demand schedule for surviving insiders. The expected profit of a surviving insider from the asset purchase is  $y(p)[\bar{p} - p]$ . The surviving insider wishes to maximize this profit subject to the resource constraint:

$$y(p) \cdot p \leq R. \quad (14)$$

Hence, for  $p < \bar{p}$ , surviving insiders are willing to purchase the maximum amount of assets using their resources. Thus, the optimal demand schedule for surviving insiders is

$$y(p) = \frac{R}{p}. \quad (15)$$

For  $p > \bar{p}$ , the demand is  $y(p) = 0$ , and for  $p = \bar{p}$ ,  $y(p)$  is infinitely elastic. In words, as long as purchasing assets is profitable, a surviving insider wishes to use up all its resources to purchase assets.

We can derive the demand schedule for arbitrageurs in a similar way. Note that, arbitrageurs value these assets at  $\underline{p}$ . For  $p < \underline{p}$ , arbitrageurs are willing to supply all their funds for the asset purchase. Thus, their demand schedule is

$$y_a(p) = \frac{1}{p}. \quad (16)$$

For  $p > \underline{p}$ , the demand is  $y_a(p) = 0$ , and for  $p = \underline{p}$ ,  $y_a(p)$  is infinitely elastic.

Next, we characterize how failed insiders' assets are allocated and the price function that results. The equilibrium price function is also illustrated in Figure 2.

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<sup>8</sup>Since no insider asset is scrapped, the equation holds with equality in equilibrium.

**Lemma 1** *The equilibrium price function for liquidated assets  $p^*(k)$  is given as follows:<sup>9</sup>*

$$p^*(k) = \begin{cases} \bar{p} & \text{for } k \leq \underline{k} \\ \frac{(1-k)R}{k} & \text{for } k \in (\underline{k}, \bar{k}] \\ \underline{p} & \text{for } k \in (\bar{k}, \bar{\bar{k}}] \\ \frac{(1-k)R}{k} + \frac{w}{(1-w)k} & \text{for } k > \bar{\bar{k}} \end{cases} . \quad (17)$$

where  $\underline{k}$ ,  $\bar{k}$ , and  $\bar{\bar{k}}$ , are given respectively by equations (32), (33), and (34), in Appendix I.

Arbitrageurs acquire assets whenever  $k > \bar{k}$ .

Finally, as arbitrage capital  $w$  increases, the price  $p^*$  weakly increases, that is,  $\frac{dp^*}{dw} \geq 0$ .

We know that in the absence of financial constraints, the efficient outcome is to sell all assets to surviving insiders. However, surviving insiders may not be able to pay the threshold price of  $\underline{p}$  for all assets. If price falls further, buying these assets becomes profitable for arbitrageurs and they participate in the auction, resulting in misallocation of assets whenever  $\Delta > 0$ .

Specifically, in the first region, that is, for  $k \leq \underline{k}$ , the number of failures is small and surviving insiders have enough liquidity to acquire assets at the full price  $\bar{p}$ .

For moderate proportion of failures, that is, for  $k \in (\underline{k}, \bar{k}]$ , however, surviving insiders can no longer pay the full price for all assets but can still pay at least the threshold value of  $\underline{p}$ , below which arbitrageurs have a positive demand. In this region, surviving insiders use all available funds and the price falls as the proportion of failures increases. This effect comes from cash-in-the-market pricing, as in Allen and Gale (1994, 1998), and is akin to the industry equilibrium hypothesis of Shleifer and Vishny (1992) who argue that when industry peers of a firm in distress are financially constrained, the peers may not be able to pay a price for assets of the distressed firm that equals the value of these assets to them.

However, as the proportion of failures increases even further, surviving insiders cannot pay the threshold price of  $\underline{p}$  for all assets and profitable options emerge for arbitrageurs. Hence, arbitrageurs are willing to supply their funds for the asset purchase. With the injection of arbitrageurs' funds, prices can be sustained at  $\underline{p}$  until a critical proportion of failures  $\bar{\bar{k}}$ .

In the extreme, the number of failures may be so large that even the injection of arbitrageur capital is not enough to sustain the price at  $\underline{p}$ . This can be considered as an aggregate shortage

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<sup>9</sup>Note that  $p^*$  depends on  $w$  as well as  $k$ , that is, we have  $p^*(k, w)$ . To simplify notation we use  $p^*(k)$  throughout the paper.

of liquidity in that there is cash-in-the-market pricing even when all liquidity in the economy is channeled for asset purchases.

Note that the resulting price function is downward-sloping in the proportion of failed firms  $k$  in two separate regions. In the first downward-sloping region, arbitrageurs have not yet entered the market ( $k \in (\underline{k}, \bar{k}]$ ) and there is cash-in-the-market pricing given the limited funds of surviving insiders. In the second downward-sloping region ( $k > \bar{k}$ ), even the funds of arbitrageurs are not enough to sustain the price at  $\underline{p}$ , their highest valuation of assets.

### 3.2 Inefficiency of Equilibrium

Insiders' expected profit, denoted by  $E(\pi_i)$ , consists of profit from their own investments, profit from asset purchases, and the amount they recover for their assets when they fail, which can be derived using the price in equation (17). In particular, we have that the profit of each insider is

$$E(\pi_i) = E \left[ (1 - k)R \frac{\bar{p}}{p^*} + kp^* - 1 \right], \quad (18)$$

where  $E$  denotes expectation over  $k$ .

In contrast, the only source of profit for arbitrageurs is the asset purchase at fire-sale prices. Therefore, we have that the profit of each arbitrageur is

$$E(\pi_a) = E \left[ \max \left\{ 0, \left( \frac{\underline{p}}{p^*} - 1 \right) \right\} \right], \quad (19)$$

where  $E(\pi_a)$  denotes the expected profit for arbitrageurs. In competitive equilibrium, the capital allocation (characterized by arbitrage capital  $w$ ) must be such that the two payoffs are equalized at the ex ante stage, so that

$$E(\pi_i) = E(\pi_a), \quad (20)$$

as otherwise, there is an incentive for some insiders to become arbitrageurs instead or vice-versa. Then, the following proposition formally characterizes agents' choices. Under a technical condition characterized in Appendix I, which qualitatively amounts to "the distribution  $f(k)$  not converging to zero too rapidly as  $k$  goes to 1" (or in other words, that there is a sufficiently "thick tail" that there will be a large number of failures and arbitrageurs will make profits), we obtain that

**Proposition 2** *In the competitive equilibrium, a proportion  $w^* \in \left(0, \frac{\underline{p}}{1+\underline{p}}\right)$  of agents choose to become arbitrageurs, where  $w^*$  satisfies the indifference equation in (20).*

*Further,  $\bar{k} < 1$ , so that there are states of the world where  $p^* < \underline{p}$ .*

Hence, in any equilibrium, the fractions of agents who choose to become insiders and arbitrageurs are bounded away from 0. An important implication is that cash-in-the-market prices are robust to the endogenous choice of arbitrage capital. That is, there will always be states of nature where the price falls not only below the fundamental value of  $\bar{p}$  but also below  $\underline{p}$ , the value arbitrageurs attach to these assets. In these states, there is an aggregate shortage of liquidity as all capital with insiders and arbitrageurs is not sufficient to keep the asset prices above or equal to  $\underline{p}$  which is necessary for efficient ex-post allocation of assets. This is a robust feature of our model. In order for there to be arbitrage capital in equilibrium, there must be states of the world where arbitrageurs make profits. In these states prices are below the arbitrageurs' valuation of assets. And, this is indeed the case in equilibrium.

We have the stark contrast between Proposition 2, which states that the equilibrium level of arbitrage capital is strictly positive, and Proposition 1 which states that the socially optimal level of arbitrage capital is zero. In fact, the result that zero arbitrage capital is first-best holds even when arbitrageurs are as efficient as insiders ( $\Delta = 0$ ) in running failed insiders' assets. This is because even though there is no ex-post allocation inefficiency in this case, profitable opportunities are passed ex ante as capital remains idle waiting for arbitrage opportunities that do not create any social welfare.

Thus, for arbitrage capital to have social value, there has to be appeal to other rationales. One candidate is risk aversion, which would introduce a motive to reduce the price fluctuations across states of the world (Allen and Gale, 2004, 2005). Nevertheless, even with risk aversion, the ex ante gains from risk-sharing have to be sufficiently large that it swamps the productive inefficiency. Any *presumption* that arbitrage capital has social value must thus be justified.

An alternative channel through which arbitrage capital may have value is to moderate amplifying effects of financial distress whenever some fragility exists in the economic system that triggers snowball effects (e.g., due to marking-to-market constraints as in Cifuentes, Ferrucci and Shin, 2005). In such a context, mitigating the initial shocks through the cushioning effect of arbitrage capital could have substantial welfare benefits. However, as with the case for risk-aversion, the final assessment should be based on a comparison of the magnitudes, and any presumption one way or the other would be unjustified.

Finally, another rationale for arbitrage capital is that the arbitrageurs are experts in managing distressed assets. In such a set-up, arbitrageurs are willing to pay a higher price for failed firms' assets and they will be the first to acquire these assets. In this case, while investing in the liquid asset yields lower returns compared to the risky investment, it allows arbitrageurs – as take-over experts – to acquire failed insiders' assets and generate higher returns from these distressed assets compared to the insiders. We sketch this version of our model in Appendix II.

Although it is plausible in some contexts that arbitrageurs are able to generate more

value than the insiders, such an assumption runs counter to the intuition that insiders gain by “learning-by-doing” so that they are the natural holders of the assets. As mentioned already, Acharya, Shin and Yorulmazer (2009) exhibit evidence that during emerging market crises, foreign arbitrageurs re-sell, or “flip”, the assets they acquired during the fire-sale to local insiders, suggesting they were temporary owners of assets due to aggregate shortage of liquidity rather than permanent owners due to expertise. There is also a growing body of empirical work in corporate finance that has focused on whether private equity firms create value by “governance arbitrage” of managerial inefficiencies, “multiple arbitrage” of buying low and selling high (as in our model), or purely through taking on cheap leverage during credit booms (see Acharya, Hanh and Kehoe, 2008, who suggest all of these contributing factors are present in private equity returns). Recent evidence suggests that the superior returns of private equity investors was associated with greater leverage during periods of lax credit (Axelson, Stromberg and Weisbach (2009)). Both these pieces of evidence – on emerging market crises and on contributors to private equity returns – lends weight to our maintained hypothesis that insiders can (in many cases) realize more value than the arbitrageurs.

### 3.3 Comparative Statics

Before generalizing our analysis to the more general context with a capital market, we note some comparative statics features of our model in the benchmark case under Assumption 1.

We have the following relationships between the level of arbitrage capital and (i) the business cycle proxied by the aggregate distribution of successful investments, and (ii) asset specificity.

**Proposition 3** *Equilibrium level of arbitrage capital  $w^*$  satisfies two features:*

(i) *Suppose  $f$  and  $g$  are two probability densities for  $k$ , where  $f$  dominates  $g$  in the sense of first-order stochastic dominance. Let  $w_f^*$  and  $w_g^*$  be the equilibrium level of arbitrage capital under densities  $f$  and  $g$ , respectively. Then,  $w_f^* > w_g^*$ .*

(ii) *Let  $\hat{w} = \frac{Rp\Delta}{Rp\Delta + R\bar{p} + p^2}$ . For  $w < \hat{w}$ , as the difference of expertise between insiders and arbitrageurs widens the equilibrium proportion of arbitrageurs decreases. That is,  $\frac{dw^*}{d\Delta} < 0$ .*

Consider (ii) first. As the difference between the expertise levels of insiders and arbitrageurs widens (i.e., as insiders’ assets become more specific), the return arbitrageurs make from these assets decreases. In turn, the region over which arbitrageurs enter the market shrinks. Thus, asset specificity reinforces fire-sale discounts in prices further.

Next, consider (i). During boom periods, it is more likely that risky projects perform well. The increased probability of the high return from the risky investment has two effects on agents' choice that go in the same direction. First, the expected return from being an insider increases. Also, the proportion of failed insiders decreases, which limits the fire-sale opportunities for arbitrageurs. Hence, during boom periods, we would expect a higher fraction of agents to become insiders and take risky projects and a smaller fraction to set aside capital for arbitrage.

Furthermore, from the price function in equation (17), we know that as the fraction of arbitrageurs  $w^*$  decreases, we observe bigger deviations in the price of failed insiders' assets from the fundamental value of  $\bar{p}$ . Hence, a corollary of Proposition 3 is that when adverse shocks arise during boom periods, fire-sale effects in asset prices are more severe, resulting in lower asset prices and higher price volatility. This result is a novel contribution of our analysis and provides one explanation for why crises that follow boom periods are associated with greater asset price deterioration.<sup>10</sup>

**Corollary 1** *Adverse shocks during boom periods measured by high values of  $k$  result in bigger deviations in the price of failed insiders' assets from the fundamental value of  $\bar{p}$ , that is,  $(\bar{p} - p^*(k))$  increases.<sup>11</sup>*

## 4 Introducing a Capital Market

The inefficiency identified in the benchmark equilibrium reflects Assumption 1, which stated that the surviving insiders cannot raise outside capital and each only has  $R$  units of the consumption good that can be used to purchase distressed assets. It might appear that the inefficiency is somehow fragile to the introduction of a capital market, where we relax Assumption 1 and allow surviving insiders to sell claims to the outsiders. However, this is

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<sup>10</sup>Acharya and Viswanathan (2011) build an alternative explanation in a model where there is greater entry of poorly-capitalized institutions when fundamentals are stronger, but in their model insiders serve as arbitrageurs and there is no arbitrage capital set aside in equilibrium.

<sup>11</sup>An interesting question is what happens when the crisis is over and the insiders build up capital again. In a set-up with more periods, one would expect outsiders to sell the assets back to the insiders, that is, flip the assets they acquired at fire-sale prices once the insiders build up sufficient capital. Acharya, Shin and Yorulmazer (2007) provide a theoretical model and empirical evidence for such flipping in the context of foreign direct investment (FDI). The authors provide empirical evidence for such flipping for the Asian crisis episode for 1996-2000. In particular, FDI acquisitions made during crisis times are subsequently flipped by the foreign acquirer - that is, re-sold quickly - to domestic buyers once financial conditions improve in the crisis-stricken country. In contrast, there is no systematic evidence of flipping of FDI acquisitions made during normal times. This difference arises because acquisitions in normal times do not feature fire-sale discounts and are thus unlikely to be made by inefficient foreign or out-of-industry acquirers.

not the case. It turns out that even when insiders can pledge all of their cash flows fully to arbitrageurs and raise “external financing” at date 1, the inefficiency persists. On the other hand, the introduction of a capital market mitigates the inefficiency, in a way to be made more precise below.

To see this, we relax Assumption 1 and allow insiders to generate funds from arbitrageurs against the assets they acquire. Formally, we allow insiders to generate funds at  $t = 1$  against the assets they acquire:

**Assumption 2.** The surviving insiders can raise outside capital by issuing shares to arbitrageurs and deploy it along with  $R$  units of the consumption good that each insider has to purchase distressed assets.

In particular, surviving insiders issue shares, which is a claim on a unit of failed insiders’ assets they acquire, to generate funds per unit of share issued. In general, we can assume that due to various imperfections such as asymmetric information, moral hazard, etc., surviving insiders may not be able to fully pledge their future cash flows (à la Holmstrom and Tirole (1998)). However, we consider here the case with full pledgeability as this gives surviving insiders the best chance to purchase liquidated assets and stacks the odds against arbitrage capital being attractive ex ante. Details of the case with partial pledgeability are available upon request.

We denote as  $q(k)$  the price of equity share in surviving insiders, purchased by arbitrageurs. Hence, when a proportion  $k$  of insiders fail, the amount of funding available with the surviving insiders for the purchase of assets, including funds that can be generated against returns from purchased assets, is given as:

$$L(k) = (1 - w)(1 - k) [R + sq(k)], \tag{21}$$

where  $s$  is the units of shares issued by each surviving insider, which must be less than or equal to  $m$ , the units of assets acquired by each surviving insider. Clearly, this total liquidity available with the surviving insiders for asset purchases is higher compared to the benchmark case. As a result, the region over which we observe cash-in-the-market pricing is smaller, i.e., it starts at a larger proportion of failures, compared to the benchmark case (Lemma 1).

Now, we have two markets: one for assets of failed insiders and one for shares of surviving insiders. To find the equilibrium prices and allocations in these two markets, we formally state the optimization problem that surviving insiders and arbitrageurs face.

If a surviving insider issues  $s$  units of shares at the price  $q(k)$  and purchases  $m$  units of assets at the price  $p(k)$ , it makes an expected profit of  $m(\bar{p} - p(k)) - s(\bar{p} - q(k))$ .

Note that in any equilibrium,  $q(k)$  cannot exceed  $\bar{p}$ . Thus, we have  $q(k) \leq \bar{p}$ , and surviving insiders issue equity just enough for the asset purchase, not more. Using this, we can state a

surviving insider's maximization problem as:

$$\begin{aligned}
\max_{m,s} \quad & m(\bar{p} - p(k)) - s(\bar{p} - q(k)) \\
\text{s.t.} \quad & s \cdot q(k) + R \geq m \cdot p(k) \\
& s \leq m.
\end{aligned} \tag{22}$$

For  $q(k) \leq p(k)$ , surviving insiders cannot make positive profits by issuing equity to purchase assets. Thus, when  $q(k) \leq p(k)$ ,  $s = 0$  and  $m = \frac{R}{p(k)}$ . And when  $q(k) > p(k)$ , surviving insiders make positive profits from asset purchase using the funds they generate by issuing equity. Hence, they would like to issue as much equity as possible, that is,  $s = m$ .

We can state each arbitrageur's maximization problem in a similar way:

$$\begin{aligned}
\max_{x,z} \quad & x(\underline{p} - p(k)) + z(\bar{p} - q(k)) \\
\text{s.t.} \quad & x \cdot p(k) + z \cdot q(k) \leq 1
\end{aligned} \tag{23}$$

where  $x$  and  $z$  represent the units of assets and the proportion of shares in surviving insiders purchased by arbitrageurs, respectively.

When the share price of surviving insiders,  $q(k)$ , is low compared to the price of failed insiders' assets,  $p(k)$ , arbitrageurs prefer to purchase shares of surviving insiders. However, if  $p(k)$  instead becomes low compared to  $q(k)$ , then arbitrageurs may prefer to acquire the assets directly.

When  $p(k) > \underline{p}$ , arbitrageurs do not want to purchase assets and  $x(q, p) = 0$ . When  $p(k) < \underline{p}$ , arbitrageurs choose  $x$  to maximize:

$$\begin{aligned}
& x(\underline{p} - p(k)) + \left( \frac{1 - xp(k)}{q(k)} \right) (\bar{p} - q(k)) \\
= & x \left( \underline{p} - \frac{p(k)\bar{p}}{q(k)} \right) + \left( \frac{\bar{p}}{q(k)} - 1 \right).
\end{aligned} \tag{24}$$

Thus, if  $p(k) < \underline{p}$  and  $\underline{p} q(k) > \bar{p} p(k)$ , then arbitrageurs use all their funds for the asset purchase, that is  $x = \frac{1}{p(k)}$ . When  $p(k) < \underline{p}$  and  $\underline{p} q(k) < \bar{p} p(k)$ , arbitrageurs use all their funds for the equity purchase, that is  $z = \frac{1}{q(k)}$ , and when  $\underline{p} q(k) = \bar{p} p(k)$ , arbitrageurs are indifferent between the equity and the asset purchase.

In equilibrium, demand for shares of surviving insiders and assets of failed insiders should equal their supply. Hence, we have the market clearing conditions:

$$(1 - w)(1 - k)s = wz \quad (\text{equity market}) \tag{25}$$

$$(1 - w)(1 - k)m + wx = (1 - w)k \quad (\text{asset market}) \tag{26}$$

We concentrate on the equilibrium where the participation of arbitrageurs in the equity market is maximum, which results in the maximum price for assets. However, even in this case, we show that for a large proportion of failures, the share price of surviving insiders falls below  $\underline{p}$ .

The price functions for failed insiders' assets and for shares of surviving insiders are given as follows (illustrated in Figure 3):

**Lemma 2** *In equilibrium, prices for real assets and financial shares are respectively:*

$$p^*(k) = \begin{cases} \bar{p} & \text{if } k \leq \hat{k} \\ \frac{(1-w)(1-k)R+w}{(1-w)k} & \text{if } k > \hat{k} \end{cases} \quad (27)$$

and

$$q^*(k) = \begin{cases} \bar{p} & \text{if } k \leq \bar{\bar{k}} \\ \mu p^*(k) & \text{if } k > \bar{\bar{k}} \end{cases}, \quad (28)$$

where  $\mu = \frac{\bar{p}}{\underline{p}}$ ,  $\hat{k} = \frac{(1-w)R+w}{(1-w)(R+\bar{p})}$ , and  $\bar{\bar{k}} = \frac{(1-w)R+w}{(1-w)(R+\underline{p})}$ .

As Lemma 2 shows, the price of shares of surviving insiders follows a pattern that reflects aggregate shortage of liquidity. When the proportion of failures is large ( $k > \bar{\bar{k}}$ ), cash-in-the-market pricing results in the price of assets falling even below the threshold value of arbitrageurs,  $\underline{p}$ . Since purchasing assets at such prices becomes profitable for arbitrageurs, in equilibrium they need to be compensated for purchasing shares of surviving insiders. As a result, share price of surviving insiders falls below their fundamental value,  $\bar{p}$ . In other words, surviving insiders can raise equity financing only at discounts. Thus, limited funds within the whole system and the resulting cash-in-the-market pricing affects not only the price of real assets but also the price of shares of surviving insiders. Furthermore, the discount that surviving insiders need to suffer in issuing equity is higher when the crisis is more severe (high  $k$ ).

In our model all the return generated by the risky and the liquid assets is shared by arbitrageurs and insiders (surviving and failed). Hence, the sum of expected profits of all insiders and arbitrageurs is equal to the expected net output, that is,

$$(1-w)E(\pi_i) + wE(\pi_a) = \Pi - 1 \quad (29)$$

In the benchmark case without the capital market arbitrageurs acquire assets in states where  $k > \bar{\bar{k}}$ . In particular, the total amount of failed insiders' assets acquired by arbitrageurs is

given as:

$$y_A(k) = \begin{cases} 0 & \text{if } k \leq \bar{k} \\ (1-w) \left[ k - \frac{(1-k)R}{\underline{p}} \right] & \text{if } k \in (\bar{k}, \bar{k}] \\ \frac{w}{p^*(k)} & \text{if } k > \bar{k} \end{cases} \quad (30)$$

where  $p^*(k)$  is given in equation (17). Note that with the introduction of capital markets, all arbitrageur funds can be obtained by surviving insiders through sale of shares, even though such sales can be at a discount. Hence, with the introduction of capital markets  $y_A = 0$ . Since, all the assets are acquired by insiders, who are the efficient users of these assets, expected output increases with the introduction of capital markets.

One important observation is that the introduction of capital markets do not affect arbitrageurs' expected profit. The reason for this is that even though arbitrageurs can acquire shares of surviving insiders, in equilibrium, arbitrageurs make the same profit from asset and share purchases. Further, as the region where there is aggregate shortage of liquidity remains the same ( $k > \bar{k}$ ), for the same level of arbitrageur capital  $w$ ,  $E(\pi_a)$  is the same as in the case with no capital markets. However, the introduction of capital markets increases the expected output so that insider profits,  $E(\pi_i)$ , are higher now. Since insider profits are greater but those of arbitrageurs are the same (for a given level of arbitrage capital), the indifference equation (20) implies that the equilibrium allocation to arbitrage capital falls with greater access of insiders to external financing.

These results are summarized in the proposition below.

**Proposition 4** *In the competitive equilibrium with external financing market, a proportion  $w^{**} > 0$  of agents choose to become arbitrageurs, where  $w^{**}$  is smaller than  $w^*$ , the equilibrium proportion of arbitrageurs when there is no access to external financing (as characterized in Proposition 2).*

In other words, the inefficiency of arbitrage capital allocation in the competitive equilibrium does not change qualitatively when capital markets are considered. Profitable opportunities are still bypassed ex ante as some capital remains on standby waiting for purchase of equity issues of insiders at fire-sale prices. However, there is full ex post allocation of liquidated assets to insiders and the equilibrium level of arbitrage capital is lower (and welfare is higher) compared to the case without market for external financing.

## 5 Contagion via Limited Arbitrage

In this section, we extend the benchmark model to examine how limited arbitrage capital generates contagion. We introduce another sector to our benchmark model which can variously be interpreted as another country or asset category. The objective is to analyze how illiquidity and the allocation of funds between the two countries' assets can lead to contagion from one country to the other, resulting in excessive co-movement across assets that have independent fundamentals. Even though the returns from the risky investments in these two countries are independent, contagion results from the fact that when arbitrage capital for different assets and markets comes from a common pool of investors, their equilibrium capital allocation requires that they earn the same rate of return across different assets and markets.<sup>12</sup>

Suppose that there are two ex-ante identical countries, denoted by  $i \in \{1, 2\}$ , each with a measure  $(1 - w_i)$  of insiders. The total arbitrageur capital is  $w_1 + w_2$ . To simplify notation, we assume that these two countries have identical features to the economy introduced in the benchmark model except that their shocks are independent. Therefore, we consider the symmetric case where equilibrium allocation of arbitrage capital in each country is the same:  $w_1 = w_2 = w$ . We allow insiders in one country to access markets in the other country, where we assume that insiders in country  $i$  are as efficient as insiders in country  $j$  in running the assets in country  $j$ .

Insiders in country  $i$  are willing to pay a maximum price of  $\bar{p}_i = \bar{p}$ , whereas arbitrageurs are willing to pay a maximum price of  $\underline{p}_i = \underline{p}$ , for failed firms' assets in country  $i = 1, 2$ . Both surviving insiders (of either country) and arbitrageurs can acquire assets put up for sale in each of the countries. In essence, the market for asset sales is fully integrated across the two countries. The implication of this is the following. Suppose that a fraction  $k_i$  of insiders in country  $i$  fails at  $t = 1$ . Whether that is sufficient to induce cash-in-the-market pricing for assets depends also upon the quantity of assets being put up for sale by the other country, in other words, on  $k_j$ , the fraction of insiders in country  $j$  that fails at  $t = 1$ .

Formally, we obtain the following lemma that characterizes the contagion effects on asset price in country  $i$  from country  $j$  (also illustrated in Figure 4).

**Lemma 3** *The price of assets as a function of the proportion of failed insiders in both coun-*

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<sup>12</sup>Other types of contagion explored in the literature include contagion through inter-linkages and direct exposures (Allen and Gale, 2000), information contagion (Chen, 1999), to cite a few.

tries ( $i$  and  $j$ ,  $i \neq j$ ) is as follows:

$$p_i^*(k_i, k_j) = \begin{cases} \bar{p} & \text{for } k_i \leq \underline{k}_i(k_j) \\ \frac{(2-k_1-k_2)R}{k_1+k_2} & \text{for } k_i \in (\underline{k}_i(k_j), \bar{k}_i(k_j)] \\ \underline{p} & \text{for } k_i \in (\bar{k}_i(k_j), \bar{\bar{k}}_i(k_j)] \\ \frac{(2-k_1-k_2)R}{k_1+k_2} + \frac{2w}{(1-w)(k_1+k_2)} & \text{for } k_i > \bar{\bar{k}}_i(k_j) \end{cases} \quad (31)$$

where  $\underline{k}_i(k_j)$ ,  $\bar{k}_i(k_j)$  and  $\bar{\bar{k}}_i(k_j)$  are given in equations (55), (56), and (59), respectively, in Appendix I.

The key aspects of the contagion are as follows: (i) The threshold  $\underline{k}_i$  is decreasing in  $k_j$ , so that country  $i$  enters the cash-in-the-market region relatively sooner when country  $j$  experiences higher proportion of failures; and, (ii) For  $k_i > \bar{\bar{k}}_i(k_j)$ , total liquidity of insiders and the arbitrageurs is not sufficient to keep the asset prices above  $\underline{p}$ . In equilibrium, arbitrageurs allocate their funds in these two countries such that they make the same profit from asset purchases in the two countries, which implies that  $p_i = p_j$ . Now, the threshold  $\bar{\bar{k}}_i$  is decreasing in  $k_j$  so that country  $i$  enters the second cash-in-the-market region also relatively sooner when country  $j$  experiences a more severe crisis.

In this case when countries are perfectly integrated in terms of mobility of insider as well as arbitrage capital, what is the equilibrium level of arbitrage capital allocation at  $t = 0$  (denoted as  $2w^I$ , so  $w^I$  per country)? And how does it relate to the equilibrium level of arbitrage capital in the ‘‘autarky’’ case (denoted as  $w^A$ , characterized earlier in Proposition 2) when the two countries are completely segmented in terms of mobility of capital?

Suppose that the arbitrage capital in case of integrated countries is  $w^I = w^A$ . Then, it can be shown that the integrated case maps one-for-one into the autarky case with one important difference that the number of failures,  $k$ , of each country in autarky case is replaced by the average number of failures,  $\frac{k_1+k_2}{2}$ , in the integrated case. In particular, the thresholds of failures at which the two cash-in-the-market regions obtain are also exactly identical. These are the thresholds  $\underline{k}$ ,  $\bar{k}$ , and  $\bar{\bar{k}}$ , of Proposition 2. The key difference, however, is in the likelihood of these outcomes. Clearly, the distribution  $f^A(k)$  of the number of failures per country is different from the distribution (denoted as)  $f^I(k)$  of average number of failures across countries.

This mapping leads to the following characterization of when arbitrage capital per country falls when capital markets are integrated across countries relative to the case when markets are segmented. We show that this is the case when the probability of events, wherein the average number of failures is high enough that arbitrageurs enter asset markets, is (event by

event) smaller than the probability of seeing the same number of failures per country. That is, if the diversification effect of independent outcomes across countries makes it less likely that the average number of failures will exceed a critically high threshold, then equilibrium arbitrage capital in integrated case is smaller per country than in autarky case.

**Proposition 5** *If  $f^A(k) \geq f^I(k)$ ,  $\forall k > \bar{k} \equiv \frac{R}{R+p}$  (and strictly greater for at least some  $k$  greater than  $\bar{k}$ ), then the arbitrage capital per country in the integrated capital markets case,  $\frac{w^I}{2}$ , is greater than the arbitrage capital per country in autarky case,  $w^A$ .*

The above condition is always met, for example, in the case of uniform distribution for  $f^A(k)$  and  $\bar{k} > \frac{1}{2}$ . By implication, when arbitrage capital per country falls, we can also show that overall welfare is improved due to a lower expectation of the ex-post misallocation cost. We stress, however, that this is in general not always the case. Shaffer (1994) and Winton (1999) explain that when the critical threshold for failures is not too high “pooling intensifies joint failure risk”. Intuitively, the contagion effect, namely that even when one country has small number of failures that it can experience cash-in-the-market pricing due to large number of failures in the other region, can now potentially dominate the diversification effect. If this is the case, equilibrium arbitrage capital per country may be greater in the integrated case than in the autarky case, and, in turn, overall welfare may be lower upon market integration due to greater ex-post misallocation of assets.

## 6 Conclusion

Our framework sheds light on fire sales as an equilibrium phenomenon when investors can choose ex ante how much arbitrage capital to hold. The joint occurrence of fire sales and limited arbitrage capital that moves in “slowly” to acquire assets (that is, only when price discounts are sufficiently steep) is a robust feature arising from the fundamental trade-off faced by investors. Arbitrage capital can take advantage of depressed prices in crisis states, but entails costs in the form of foregone profitable investments and not investing in expertise. Equalizing the ex ante return from the two activities leads to the interior nature of the equilibrium. Equilibrium arbitrage capital is limited and fire sales during crises become a robust phenomenon.

We also demonstrated how this equilibrium construction can be used to good effect in two applications. First, we showed that (perhaps surprisingly) setting aside of arbitrage capital can be inefficient from the standpoint of ex-ante investment. Although arbitrage capital cushions financial distress in crisis states, it leads to foregoing of ex-ante profitable investments. Our second application of the equilibrium construction was to examine a novel

channel of contagion between fundamental sectors that have independent fundamentals. The contagious link arises from the fact that arbitrageurs must earn the same rate of return on capital from different markets to which they supply liquidity. In particular, this contagion also carries over from real asset markets to markets for financing asset purchases.

It would be interesting in future research to examine a dynamic setting in which one can study how arbitrage capital allocation shifts over time, in particular, as crises approach, and calibrate the resulting prices and contagion across markets to empirically observed patterns.

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## Appendix I: Proofs

**Proof of Lemma 1:** The price cannot be greater than  $\bar{p}$  since in this case we have  $y_i(p) = y_a(p) = 0$ . If  $p \leq \bar{p}$ , and the proportion of failed insiders is sufficiently small, surviving insiders have enough funds to pay the full price  $\bar{p}$  for all assets. More specifically, this is the case when surviving insiders' liquidity  $(1 - w)(1 - k)R$  is adequate to purchase all failed insiders' assets  $(1 - w)k$  at the full price  $\bar{p}$ . Thus, for  $k \leq \underline{k}$ , where

$$\underline{k} = \frac{R}{R + \bar{p}}, \quad (32)$$

the auction price is  $p^* = \bar{p}$ . At this price, surviving insiders are indifferent between any quantity of assets purchased. Hence, each surviving insider is allocated a share  $y_i(\bar{p}) = k / (1 - k)$ .

For moderate values of  $k$ , surviving insiders cannot pay the full price for all assets but can still pay at least the threshold value of  $\underline{p}$ , below which arbitrageurs have a positive demand. Formally, for  $k \in (\underline{k}, \bar{k}]$ , where

$$\bar{k} = \frac{R}{R + \underline{p}}, \quad (33)$$

the price is set at  $p^* = (1 - k)R/k$ , and again, all assets are acquired by surviving insiders.

For  $k > \bar{k}$ , surviving insiders cannot pay the threshold price of  $\underline{p}$  for all assets and profitable options emerge for arbitrageurs. Hence, for  $k > \bar{k}$ , arbitrageurs have a positive demand and are willing to supply their funds for the asset purchase. With the injection of arbitrageurs' funds, prices can be sustained at  $\underline{p}$  until some critical proportion of failures  $\bar{\bar{k}} \geq \bar{k}$ . However, for  $k > \bar{\bar{k}}$ , even the injection of arbitrageur capital is not enough to sustain the price at  $\underline{p}$ .

Formally, for  $k > \bar{k}$ , the amount of arbitrage capital  $x$  needed to maintain the price at  $\underline{p}$ , is given by the market-clearing condition:  $(1 - w)(1 - k)R + x = (1 - w)k\underline{p}$ . This gives

$x = (1 - w)[k\underline{p} - (1 - k)R]$ . In turn, we obtain the threshold  $k$  above which the price cannot be sustained at  $\underline{p}$  even with entry of all arbitrageur funds  $w$ . That is, for  $k \in (\bar{k}, \bar{\bar{k}}]$ , where

$$\bar{\bar{k}} = \min \left\{ 1, \frac{(1 - w)R + w}{(1 - w)(R + \underline{p})} \right\}, \quad (34)$$

the price is set at  $\underline{p}$ . At this price, arbitrageurs are indifferent between any quantity of assets purchased. Hence, each surviving insider receives a share of  $y_i(\underline{p}) = \frac{R}{\underline{p}}$ , and the rest,  $y_a(\underline{p}) = \frac{1-w}{w} \left( k - \frac{(1-k)R}{\underline{p}} \right)$ , is allocated to the arbitrageurs.

For  $k > \bar{\bar{k}}$ , the price is again strictly decreasing in  $k$  and is given by

$$p^*(k) = \frac{(1 - k)R}{k} + \frac{w}{(1 - w)k}, \quad (35)$$

and  $y_i(p^*) = \frac{R}{p^*}$ , and  $y_a(p^*) = \frac{1}{p^*}$ .

Note that the proportion  $w$  of agents that choose to become arbitrageurs affects the price  $p^*$  only in the fourth region where  $k > \bar{\bar{k}}$ , as well as the boundary  $\bar{\bar{k}}$  of the fourth region itself. In particular, for higher values of  $w$ ,  $p^*$  is higher in this region. Furthermore, the region itself shifts to the left as the fraction of entrepreneurs increases, that is,

$$\frac{d\bar{\bar{k}}}{dw} = \frac{1}{(1 - w)^2 (R + \underline{p})} > 0. \quad \diamond \quad (36)$$

**Proof of Proposition 2:** We prove the results that  $w^* \in (0, 1)$  and  $\bar{\bar{k}} < 1$  jointly by analyzing all the four possible regions for  $k$  that are given in equation (17). While the price depends on  $w$  and  $k$ , for simplicity of notation we use  $p$  instead of  $p(k, w)$ .

From the indifference equation (20), we have

$$E \left[ (1 - k) \left( \frac{R\bar{p}}{p} \right) + kp - 1 \right] = E \left[ \frac{1}{p} (\underline{p} - p)^+ \right], \quad (37)$$

which implicitly gives the equilibrium level of  $w^*$  as

$$E [h(k, w^*)] = 0, \quad \text{where} \quad (38)$$

Hence,

$$h(k, w) = \frac{1}{p} (\underline{p} - p)^+ - (1 - k) \left( \frac{R\bar{p}}{p} \right) - kp + 1. \quad (39)$$

Next, we show that  $E[h(k, w)]$  is weakly decreasing in  $w$ . Note that the price  $p^*(k)$  given in equation (17) is continuous in  $w$ . Hence,  $h(k, w)$  is continuous in  $w$ . Thus, using Leibnitz's rule, we can show that

$$\frac{\partial E(h)}{\partial w} = \int_{k=0}^1 \left( \frac{\partial h}{\partial w} \right) f(k) dk. \quad (40)$$

Note that  $w$  affects  $h(k, w)$  only through the price  $p$ . From equation (17), for  $k \leq \bar{k}$ , price  $p$  is independent of  $w$ , so that  $\frac{\partial h}{\partial w} = 0$ .

For  $k > \bar{k}$ , we have

$$h(k, w^*) = \frac{p - (1 - k)R\bar{p}}{p} - kp, \quad (41)$$

which gives us

$$\frac{\partial h}{\partial w} = - \left( \frac{p - (1 - k)R\bar{p}}{p^2} + k \right) \underbrace{\left( \frac{\partial p}{\partial w} \right)}_{>0}. \quad (42)$$

Hence, for  $k > \bar{k}$ ,  $\frac{\partial h}{\partial w}$  has the opposite sign as the expression  $[p - (1 - k)R\bar{p} + kp^2]$ . Hence, in this region, for  $[p - (1 - k)R\bar{p} + kp^2] > 0$ ,<sup>13</sup> we have  $\frac{\partial h}{\partial w} < 0$ , which means that there is a unique  $w^*$  that satisfies the indifference equation (20).

Next we show that  $w^* \in \left(0, \frac{p}{1+p}\right)$ .

First, we show that  $w^* \geq \frac{p}{1+p}$  cannot be an equilibrium. In that case, price never falls below  $\underline{p}$  and  $E(\pi_a) = 0$ , and  $E(\pi_i) = E[(1 - k)R + kp - 1] > 0$ . Hence,  $w^* \geq \frac{p}{1+p}$  cannot be an equilibrium as some arbitrageurs would deviate and become insiders.

For  $w = 0$ , we have

$$p^*(k) = \begin{cases} \bar{p} & \text{for } k \leq \underline{k} \\ \frac{(1-k)R}{k} & \text{for } k > \underline{k} \end{cases}, \text{ and} \quad (43)$$

$$E(\pi_i) = E \left[ \frac{(1 - k)R\bar{p}}{p} + kp - 1 \right] = E[(1 - k)R + k\bar{p} - 1], \quad (44)$$

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<sup>13</sup>A sufficient condition for this inequality to hold is  $\bar{p} < \frac{R-\Delta}{R-1}$ . For  $k = 1$ , we have  $[p - (1 - k)R\bar{p} + kp^2] = p + p^2 > 0$ . Note that  $[p - (1 - k)R\bar{p} + kp^2] > [p - (1 - k)R\bar{p}]$ , which are both increasing in  $k$ , and the inequality  $[p - (1 - \bar{k})R\bar{p}] > 0$  is a sufficient condition. Note that the minimum value  $\bar{k}$  can take is  $\frac{R}{R+p}$ , which is when  $w = 0$ . We have  $p - (1 - \bar{k})R\bar{p} \geq p - \left(\frac{p}{R+p}\right)R\bar{p}$ . And, we can show that  $p - \left(\frac{p}{R+p}\right)R\bar{p} > 0$ , for  $\bar{p} < \frac{R-\Delta}{R-1}$ . Note that for  $r < 1$  and  $\Delta \in [0, r)$ , we always have  $\bar{p} < \frac{R-\Delta}{R-1}$ .

where  $\pi_i = (1 - k)R + k\bar{p} - 1$  for  $k \in [0, 1]$ . Note that  $\frac{d\pi_i}{dk} > 0$  since  $R > \bar{p}$ . Furthermore,  $E(\pi_i) < R$ , and hence bounded.

For  $w = 0$ , we have

$$E(\pi_a) = E\left[\frac{1}{p}(\underline{p} - p)^+\right]. \quad (45)$$

Note that  $\lim_{k \rightarrow 1} p = 0$  so that  $\lim_{k \rightarrow 1} (\underline{p}/p) = +\infty$ . Hence, if the probability distribution  $f(k)$  is such that it does not converge to 0 “too fast” as  $k$  converges to 1, then  $\lim_{k \rightarrow 1} (\underline{p}/p) f(k) = +\infty$  so that  $E(\pi_a) = +\infty$ . For example, for all continuous  $f(k)$  that converge to a positive value as  $k$  converges to 1, we have  $\lim_{k \rightarrow 1} (\underline{p}/p) f(k) = +\infty$  so that  $E(\pi_a) = +\infty$ . Formally, for  $w = 0$ , we have  $\underline{p}/p = (\underline{p}/R) \left(\frac{1}{1-k} - 1\right)$ , which converges to  $+\infty$  as  $k$  converges to 1. Even though  $f(k)$  can converge to 0 as  $k$  converges to 1, as long as  $f(k)$  has an order of  $k$  less than 1, we have  $\lim_{k \rightarrow 1} (\underline{p}/p) f(k) = +\infty$ . For  $f$  that has an order of  $k$  greater than 1 and  $\lim_{k \rightarrow 1} f(k) = 0$ , we have  $\lim_{k \rightarrow 1} (\underline{p}/p) f(k) = 0$ , so that, for  $w = 0$ , we have  $E(\pi_a) < +\infty$ . Under such probability distributions, it is possible to have no arbitrage capital ( $w = 0$ ) in equilibrium. For example  $f(k) = c(1 - k)^a$ , for  $k \in [0, 1]$  and  $a > 1$ , would give such a result.

Hence  $w = 0$  cannot be an equilibrium as someone would deviate and take advantage of the potential profits from fire sales. Hence, in equilibrium, we have a unique in equilibrium  $w^* \in (0, 1)$ .  $\diamond$

**Proof of Proposition 3:** Recall that the equilibrium level of arbitrage capital  $w^*$  is implicitly given by the equation  $E[h(k, w^*)] = 0$ , where  $h(k, w)$  is as defined in equation (39). Using this implicit condition, we prove the two parts of the proposition as follows.

**Part (i):** Note that  $\frac{\partial h(k, w)}{\partial k} > 0$  is a sufficient condition for  $E_f[h(k, w)] > E_g[h(k, w)]$  when  $f$  FOSD  $g$ , where  $E_f$  and  $E_g$  represent expectations over probability distributions  $f$  and  $g$ , respectively. We already showed that  $\frac{\partial E[h(k, w)]}{\partial w} < 0$ , so that it is sufficient to show  $\frac{\partial h(k, w)}{\partial k} > 0$  to prove the result. To do that, we look at the four possible regions for  $k$ .

(1) For  $k \leq \underline{k}$ , we have  $p = \bar{p}$ , which gives us  $h(k, w) = -(1 - k)R - k\bar{p} + 1$ . Hence,  $\frac{\partial h}{\partial k} = R - \bar{p} > 0$ .

(2) For  $k \in (\underline{k}, \bar{k}]$ , we have  $p = \frac{(1-k)R}{k}$ , which gives us

$$h(k, w) = -(1 - k) \left(\frac{R\bar{p}}{p}\right) - kp + 1 = -k\bar{p} - (1 - k)R + 1. \quad (46)$$

Hence,  $\frac{\partial h}{\partial k} = R - \bar{p} > 0$ .

(3) For  $k \in (\bar{k}, \bar{\bar{k}}]$ , we have  $p = \underline{p}$ , and

$$h(k, w) = -(1 - k) \left(\frac{R\bar{p}}{\underline{p}}\right) - k\underline{p} + 1, \text{ which gives us } \frac{\partial h}{\partial k} = \frac{R\bar{p}}{\underline{p}} - \underline{p} > R - \bar{p} > 0. \quad (47)$$

(4) For  $k > \bar{k}$ , we have  $p = \frac{(1-w)(1-k)R+w}{(1-w)k}$ , and

$$h(k, w) = \frac{(1-w)kp - (1-w)k(1-k)R\bar{p} - (1-w)(1-k)R - w}{(1-w)(1-k)R + w} - (1-k)R - \frac{w}{(1-w)} + 1. \quad (48)$$

Note that, in the above expression, the denominator of the first term is decreasing in  $k$ , whereas the second term is increasing in  $k$ . Hence, if the numerator of the first expression is increasing in  $k$ , then it is sufficient for  $\frac{\partial h(k, w)}{\partial k} > 0$ . The derivative of the numerator of the first expression with respect to  $k$  is given as:

$$(1-w) [p - (1-k)R\bar{p} + kR\bar{p} + R] = (1-w) [p - R\bar{p} + 2kR\bar{p} + R]. \quad (49)$$

Next, we show that, for  $R > \underline{p}$ , we have  $[p - R\bar{p} + 2kR\bar{p} + R] > 0$ .

Let  $A = p - R\bar{p} + 2kR\bar{p} + R$ . We have  $\frac{\partial A}{\partial k} = 2R\bar{p} > 0$ . Hence, if  $A > 0$  for  $k = \bar{k}$ , we have  $A > 0$  for all  $k > \bar{k}$ .

We have  $\frac{d\bar{k}}{dw} = \frac{1}{(1-w)^2(R+\underline{p})} > 0$ . Hence, if we can show that  $A > 0$  for  $k = \bar{k}$  and  $w = 0$ , we are done. For  $w = 0$ , we have  $\bar{k} = \frac{R}{R+\underline{p}}$ , which gives us  $A = \underline{p} - R\bar{p} + 2\left(\frac{R}{R+\underline{p}}\right)R\bar{p} + R$ . For  $R > \underline{p}$ , we have  $\frac{R}{R+\underline{p}} > \frac{1}{2}$  so that  $A > \underline{p} + R > 0$ .

Hence, we have  $\frac{\partial h(k, w)}{\partial k} > 0$ .

**Part (ii):** From the indifference equation (20), we have  $E_k [h(k, w^*)] = 0$ . Thus, we have

$$\frac{\partial E [h(k, w^*)]}{\partial w} \cdot \frac{dw^*}{d\Delta} + \frac{\partial E [h(k, w^*)]}{\partial \Delta} = 0. \quad (50)$$

We already showed that  $\left(\frac{\partial E [h(k, w^*)]}{\partial w}\right) < 0$ , so that

$$\text{sign} \left( \frac{dw^*}{d\Delta} \right) = \text{sign} \left( \frac{\partial E [h(k, w^*)]}{\partial \Delta} \right). \quad (51)$$

Hence, we need to show that  $\left(\frac{\partial E [h(k, w^*)]}{\partial \Delta}\right) < 0$ . Note that the price  $p^*(k)$  given in equation (17) is continuous in  $\Delta$ . Hence,  $h(k, w^*)$  is continuous in  $\Delta$ . Thus, using Leibnitz's rule, we can show that

$$\frac{\partial E (h)}{\partial \Delta} = \int_{k=0}^1 \left( \frac{\partial h}{\partial \Delta} \right) f(k) dk. \quad (52)$$

Next, we analyze each of the four regions of  $k$  given in equation (17). Note that for  $k \leq \bar{k}$ , price  $p(k, w)$  is independent of  $\Delta$ . This, in turn, implies that for  $k \leq \bar{k}$ ,  $h(\cdot)$  is independent of  $\Delta$ .

For  $k \in (\bar{k}, \bar{\bar{k}}]$ , we have  $p(k, w) = \underline{p}$ , where  $\frac{\partial p}{\partial \Delta} = -1 < 0$ . For  $k \in [\bar{k}, \bar{\bar{k}}]$ , we have

$$h(k, w^*) = -(1-k) \left( \frac{R\bar{p}}{\underline{p}} \right) - k\underline{p} + 1, \text{ which gives us } \frac{\partial h}{\partial \underline{p}} = (1-k) \left( \frac{R\bar{p}}{\underline{p}^2} \right) - k. \quad (53)$$

Note that  $\frac{\partial h}{\partial \underline{p}}$  is decreasing in  $k$ . Hence, if  $\frac{\partial h}{\partial \underline{p}} \geq 0$  for  $k = \bar{\bar{k}}$ , then  $\frac{\partial h}{\partial \underline{p}} > 0$  for all  $k \in [\bar{k}, \bar{\bar{k}}]$ . Furthermore, we have  $\frac{\partial h}{\partial \underline{p}} = (1-k) \left( \frac{R\bar{p}}{\underline{p}^2} \right) - k = 0$  when  $k = \hat{k} = \frac{R\bar{p}}{R\bar{p} + \underline{p}^2}$ . Hence, if  $\bar{\bar{k}} < \hat{k}$ , then  $\frac{\partial h}{\partial \underline{p}} > 0$  for all  $k \in [\bar{k}, \bar{\bar{k}}]$ . We have  $\bar{\bar{k}} < \hat{k}$  if and only if

$$\frac{(1-w)R + w}{(1-w)(R + \underline{p})} < \frac{R\bar{p}}{R\bar{p} + \underline{p}^2},$$

which holds if and only if  $w < \hat{w} = \frac{R\underline{p}\Delta}{R\underline{p}\Delta + R\bar{p} + \underline{p}^2}$ .

This, combined with the fact that  $\frac{\partial p}{\partial \Delta} < 0$ , gives us  $\frac{\partial h}{\partial \Delta} < 0$ , for  $k \in (\bar{k}, \bar{\bar{k}}]$ .

For  $k > \bar{\bar{k}}$ , the price  $p^*$  is independent of  $\Delta$  as all the funds within the surviving insiders and arbitrageurs are exhausted. Hence, for  $k > \bar{\bar{k}}$ ,  $\frac{\partial h}{\partial \Delta} = 0$ .

Combining these results, we get  $\left( \frac{\partial E(h)}{\partial \Delta} \right) < 0$ .  $\diamond$

**Proof of Lemma 2:** The steps of the proof are organized in a way that lays down the results for different regions of the proportion ( $k$ ) of failed insiders.

(1) For  $k \leq \hat{k}$ , liquidity within the surviving insiders and the liquidity they can raise by issuing shares to arbitrageurs is sufficient to sustain the price for the failed insiders' assets at  $\bar{p}$ . Since  $p^*(k) = \bar{p} > \underline{p}$ , we have  $x = 0$  and  $m = \frac{k}{1-k}$ . Each surviving insider issues enough equity, at  $q(k) = \bar{p}$ , to purchase  $\frac{k}{1-k}$  units of failed insiders' assets at  $p^*(k) = \bar{p}$ . Thus, we have

$$R + s\bar{p} = \left( \frac{k}{1-k} \right) \bar{p}, \text{ which gives us:}$$

$$s = \frac{k}{1-k} - \frac{R}{\bar{p}} \quad \text{and} \quad z = \frac{1-w}{w} \left[ k - \frac{(1-k)R}{\bar{p}} \right].$$

(2) For  $\hat{k} < k \leq \bar{\bar{k}}$ , liquidity within the surviving insiders and the liquidity they can raise through equity issuance from arbitrageurs is sufficient to sustain  $p^*(k)$  at least at  $\underline{p}$ .

Since  $p^*(k) \geq \underline{p}$ , we have  $x = 0$  and  $m = \frac{k}{1-k}$ . Each surviving insider issues enough equity,

at  $q(k) = \bar{p}$ , to purchase  $\frac{k}{1-k}$  units of failed insiders' assets at  $p^*(k) = \frac{(1-w)(1-k)R+w}{(1-w)k}$ , that is,

$$R + s\bar{p} = \left(\frac{k}{1-k}\right) p^*(k), \text{ which gives us}$$

$$s = \frac{w}{(1-w)(1-k)\bar{p}} \quad \text{and} \quad z = \frac{1}{\bar{p}}.$$

(3) For  $k > \bar{k}$ , total liquidity within the surviving insiders and the liquidity they can raise through equity issuance from arbitrageurs is no longer sufficient to sustain  $p^*(k)$  at  $\underline{p}$ . Since  $p^*(k) < \underline{p}$ , arbitrageurs may prefer to participate in the market for failed insiders' assets.

If  $p^*(k) < \underline{p}$  and  $\underline{p} q(k) > \bar{p} p^*(k)$ , then arbitrageurs use all their funds for the asset purchase, that is  $x = \frac{1}{p^*(k)}$ .

If  $p^*(k) < \underline{p}$  and  $\underline{p} q(k) < \bar{p} p^*(k)$ , then arbitrageurs use all their funds for the equity purchase, that is  $z = \frac{1}{q(k)}$ , and if  $\underline{p} q(k) = \bar{p} p^*(k)$ , arbitrageurs are indifferent between the purchase of surviving insiders' shares and the failed insiders' assets.

Now, let  $\mu = \frac{\bar{p}}{\underline{p}}$ . Whether arbitrageurs buy shares of the surviving insiders or the assets of the failed insiders, their entire funds  $w$  eventually end up in the asset market. Hence, for  $k > \bar{k}$ , the price for failed insiders' assets is given as:

$$p^*(k) = \frac{(1-w)(1-k)R+w}{(1-w)k}. \quad (54)$$

If the price  $q(k)$  of a share is higher than  $\mu p^*(k)$ , then arbitrageurs are better off buying the assets of failed insiders, rather than buying shares of the surviving insiders, that is,  $z = 0$  and  $x = \frac{1}{p^*(k)}$ . Hence, we cannot have an equilibrium where  $q(k) > \mu p^*(k)$  and  $z > 0$ .

Next, we show that surviving insiders need to suffer some discount when they generate funds in the capital market. Note that arbitrageurs are willing to purchase shares of surviving insiders, that is,  $z > 0$ , only when  $q(k) \leq \mu p^*(k)$  and surviving insiders are willing to issue equity, that is,  $s > 0$ , only when  $q(k) \geq p^*(k)$ . Suppose that the market-clearing mechanism works in a way that allows the maximum possible funds to go to insiders through equity issuance, that is  $q(k) = \mu p^*(k)$ . Note that this allows for the highest price  $q(k)$  for shares. However, even in this case, surviving insiders need to suffer some discount when they generate funds in the capital market. Hence, in equilibrium, for  $k > \bar{k}$ , we have  $q(k) = \mu p^*(k) < \bar{p}$ .  $\diamond$

**Proof of Lemma 3:** For small proportion of failures in the two countries, that is, for  $[(1-w_1)(1-k_1) + (1-w_2)(1-k_2)]R \geq [(1-w_1)k_1 + (1-w_2)k_2]\bar{p}$ , there is enough liquidity within the insiders and the asset prices in the two countries are above the fundamental

value  $\bar{p}$ . Given  $w_1 = w_2$ , this holds for  $k_i \leq \underline{k}_i(k_j)$  for  $i \neq j$ , where

$$\underline{k}_i(k_j) = \frac{2R}{R + \bar{p}} - k_j, \quad (55)$$

the prices in both countries equal  $\bar{p}$  and all assets are acquired by surviving insiders.

As the proportion of failures in the two countries increase, that is, for  $[(1 - w_1)k_1 + (1 - w_2)k_2] \underline{p} \leq [(1 - w_1)(1 - k_1) + (1 - w_2)(1 - k_2)] R \leq [(1 - w_1)k_1 + (1 - w_2)k_2] \bar{p}$ , the liquidity within the insiders cannot sustain the prices at  $\bar{p}$  and we observe cash-in-the-market prices in the two countries. Note that the threshold  $\underline{k}_i$  is decreasing in  $k_j$ . Hence, country  $i$  enters the cash-in-the-market region relatively sooner when country  $j$  experiences higher proportion of failures. That is, given  $w_1 = w_2$ , for  $\underline{k}_i(k_j) < k_i \leq \bar{k}_i(k_j)$  for  $i \neq j$ , where

$$\bar{k}_i(k_j) = \frac{2R}{R + \underline{p}} - k_j, \quad (56)$$

even though surviving insiders can pay a price higher than  $\underline{p}$  for all failed insiders' assets in both countries and therefore can "beat" arbitrageurs in the auction, their liquidity is not sufficient to sustain the prices at the fundamental value  $\bar{p}$ .

In equilibrium, surviving insiders allocate their funds in these two countries such that they make the same profit from asset purchases in the two countries, which implies that

$$\frac{\bar{p} - p_i}{p_i} = \frac{\bar{p} - p_j}{p_j}, \text{ that is, } p_i = p_j. \quad (57)$$

Hence, for  $\underline{k}_i(k_j) < k_i \leq \bar{k}_i(k_j)$ , we obtain

$$p_i^*(k_i, k_j) = \frac{(2 - k_1 - k_2)R}{k_1 + k_2}. \quad (58)$$

For  $k_i > \bar{k}_i(k_j)$ , liquidity within the insiders cannot sustain the prices at  $\underline{p}$  and profitable opportunities emerge for arbitrageurs. With the injection of arbitrageurs' funds, prices can be sustained at  $\underline{p}$  until some critical proportion of failures  $\overline{\overline{k}}_i(k_j)$ , where

$$\overline{\overline{k}}_i(k_j) = \frac{2R}{R + \underline{p}} + \frac{2w}{(1 - w)(R + \underline{p})} - k_j. \quad (59)$$

For  $k_i > \overline{\overline{k}}_i(k_j)$ , total liquidity of insiders and the arbitrageurs is not sufficient to keep the asset prices above  $\underline{p}$ . In equilibrium, arbitrageurs allocate their funds in these two countries such that they make the same profit from asset purchases in the two countries, which implies that in equilibrium  $p_i = p_j$ . Note that the threshold  $\overline{\overline{k}}_i$  is decreasing in  $k_j$ , that is, country

$i$  enters the second cash-in-the-market region relatively sooner when country  $j$  experiences a more severe crisis. For  $k_i > \bar{\bar{k}}_i(k_j)$ , we obtain

$$p_i^*(k_i, k_j) = \frac{(2 - k_1 - k_2)R}{k_1 + k_2} + \frac{2w}{(1-w)(k_1 + k_2)}. \quad \diamond \quad (60)$$

**Proof of Proposition 5:** Let  $w^I = w^A = w$ . Define  $n$  as a random variable equal to  $\frac{k_1 + k_2}{2}$ . Then, the equilibrium price in the case of integrated markets (Lemma 3) can be rewritten as:

$$p^*(n) = \begin{cases} \bar{p} & \text{for } n \leq \underline{k} \\ \frac{(1-n)R}{n} & \text{for } n \in (\underline{k}, \bar{k}] \\ \underline{p} & \text{for } n \in (\bar{k}, \bar{\bar{k}}] \\ \frac{(1-n)R}{n} + \frac{w}{(1-w)n} & \text{for } n > \bar{\bar{k}} \end{cases}, \quad (61)$$

where  $\underline{k}$ ,  $\bar{k}$ ,  $\bar{\bar{k}}$  are given by the same equations as in the autarky case (see equations (32), (33), and (34), respectively, in the proof of Lemma 1). Thus, the price for assets in the integrated case is the same as in the autarky case except that the state variable is  $n$  instead of  $k$ .

If we denote as  $y_a^i$  the per country assets acquired per unit of arbitrage capital, then we obtain that

$$y_a(n) = \begin{cases} 0 & \text{for } n \leq \bar{k} \\ \frac{(1-w)}{w} \left[ n - \frac{(1-n)R}{\underline{p}} \right] & \text{for } n \in (\bar{k}, \bar{\bar{k}}] \\ \frac{(1-w)n}{(1-w)(1-n)R + w} & \text{for } n > \bar{\bar{k}} \end{cases}. \quad (62)$$

Again, this is exactly the same as arbitrageurs' acquisition of assets in the autarky case with  $n$  (average number of failures) replacing  $k$  (per country failures).

Then, each arbitrageur's profits per country can be expressed as

$$E(\pi_a^I) = \int_{\bar{k}}^1 y_a(n) [\underline{p} - p^*(n)] f^I(n) dn. \quad (63)$$

Substituting for variable of integration  $z$  as  $k$ , and comparing to arbitrageur profits in autarky case, we obtain that

$$E(\pi_a^A) - E(\pi_a^I) = \int_{\bar{k}}^1 y_a(k) [\underline{p} - p^*(k)] \cdot [f^A(k) - f^I(k)] dn. \quad (64)$$

Since  $p^*(k) = \underline{p}$  for  $k \in (\bar{k}, \bar{\bar{k}}]$ , it follows that whenever  $f^A(k) \geq f^I(k)$  for  $k > \bar{k}$  (and strictly greater for some  $k$ ), we obtain that for  $w^I = 2w^A$ ,  $E(\pi_a^A) > E(\pi_a^I)$ . In words, arbitrageur

profits are greater in autarky than in integrated case if total arbitrage capital per country is the same in two cases.

Next, we compare overall welfare in the two cases. Note that overall welfare can be stated intuitively as arbitrage capital plus the expected cash flow from insiders' assets if they were to be in hands of efficient users (namely insiders) also in second period, minus the expected misallocation costs on assets acquired and run by arbitrageurs. Denote the expected cash flows per country in integrated case as  $E(\Pi^I)$  and in autarky as  $E(\Pi^A)$ . Then,

$$E(\Pi^A) - E(\Pi^I) = - \int_{\bar{k}}^1 w y_a(k) (\bar{p} - \underline{p}) \cdot [f^A(k) - f^I(k)] dn, \quad (65)$$

since misallocation per asset is  $(\bar{p} - \underline{p})$  and it arises whenever arbitrageurs acquire assets, which is when  $n > \bar{k}$  in the integrated case and  $k > \bar{k}$  in the autarky case (even though arbitrageurs make profits only above the threshold  $\bar{k}$ ). Then, it follows that whenever  $f^A(k) \geq f^I(k)$  for  $k > \bar{k}$  (and strictly greater for some  $k$ ), we obtain that for  $w^I = w^A$ ,  $E(\Pi^A) < E(\Pi^I)$ . In words, overall welfare per country is smaller in autarky than in integrated case if total arbitrage capital per country is the same in two cases.

Denoting insider profits in the two cases as  $E(\pi_i^A)$  and  $E(\pi_i^I)$ , and since  $\Pi - 1 = (1 - w)E(\pi_i) + wE(\pi_a)$ , it also follows that  $E(\pi_i^A) < E(\pi_i^I)$ , when the condition in Proposition 5 is satisfied. That is, insider profits are smaller in autarky than in integrated case if total arbitrage capital per country is the same in two cases.

Now, at equilibrium level of arbitrage capital in the integrated case, we must have  $E(\pi_a^I) = E(\pi_i^I)$ . Similarly, at equilibrium level of arbitrage capital for autarky,  $E(\pi_a^A) = E(\pi_i^A)$ . But if  $w^I = w^A$ , we just showed that  $E(\pi_a^I) < E(\pi_a^A)$  and  $E(\pi_i^I) > E(\pi_i^A)$ , so that  $E(\pi_a^I) < E(\pi_i^I)$ . Thus, it must be the case that in equilibrium of the integrated case  $w^I < w^A$ , whenever the condition of Proposition 5 is satisfied.  $\diamond$

## Appendix II: Arbitrageurs as take-over experts

One potential interpretation of arbitrageurs can be that they may be experts in taking over and managing distressed assets, in which case, they value failed firms' assets higher than surviving firms. Hence, arbitrageurs are willing to pay a higher price for failed firms' assets and they will be the first to acquire these assets. However, when arbitrageur funds are limited, for sufficiently large proportion of failures, prices fall and even though insiders are inefficient in managing distressed assets, they will acquire some of these assets.

Formally, let arbitrageurs generate a return of  $\hat{p} = \bar{p} + \delta$ , with  $\delta > 0$ , from failed firms' assets. Hence, for  $p < \hat{p}$ , arbitrageurs are willing to supply all their funds for the asset

purchase and their demand schedule is  $y_a(p) = \frac{1}{p}$ . For  $p > \hat{p}$ , their demand is  $y_a(p) = 0$ , and for  $p = \hat{p}$ ,  $y_a(p)$  is infinitely elastic.

In the absence of financial constraints, the efficient outcome is to sell the assets to arbitrageurs. However, when arbitrage capital is limited, arbitrageurs may not be able to pay the price of  $\hat{p}$  for all assets and some of the failed insiders' assets get acquired by surviving insiders.

If the proportion of failed insiders is sufficiently small, arbitrageurs have enough funds to pay the full price  $\hat{p}$  for all assets. More specifically, for  $k \leq \underline{k}_a$ , where

$$\underline{k}_a = \frac{w}{(1-w)\hat{p}}, \quad (66)$$

the auction price is  $p_a^* = \hat{p}$  and each arbitrageur is allocated a share  $y_a(\hat{p}) = \frac{(1-w)k}{w}$ .

For moderate values of  $k$ , arbitrageurs cannot pay the price  $\hat{p}$  for all assets but can still pay at least  $\bar{p}$ , below which insiders have a positive demand. Formally, for  $k \in (\underline{k}_a, \bar{k}_a]$ , where

$$\bar{k}_a = \frac{w}{(1-w)\bar{p}}, \quad (67)$$

the price is set at  $p_a^* = \frac{w}{(1-w)k}$ , and all assets are acquired by arbitrageurs.

For  $k > \bar{k}_a$ , arbitrageurs cannot pay  $\bar{p}$  for all assets, and surviving insiders supply their funds for the asset purchase. With the injection of insiders' funds, prices can be sustained at  $\bar{p}$  until some critical proportion of failures  $\bar{k} \geq \bar{k}_a$ . We obtain the resulting price function (also see Figure 5):

$$p_a^*(k) = \begin{cases} \hat{p} & \text{for } k \leq \underline{k} \\ \frac{w}{(1-w)k} & \text{for } k \in (\underline{k}, \bar{k}] \\ \bar{p} & \text{for } k \in (\bar{k}, \bar{\bar{k}}] \\ \frac{(1-k)R}{k} + \frac{w}{(1-w)k} & \text{for } k > \bar{\bar{k}} \end{cases}. \quad (68)$$

Note that as the proportion  $w$  of agents that choose to become arbitrageurs increases, the boundaries  $\underline{k}_a$ ,  $\bar{k}_a$  and  $\bar{\bar{k}}$  increase, as well as the price  $p_a^*$  in the second and the fourth regions. Hence, as  $w$  increases, the price  $p^*$  weakly increases, that is, we have  $\frac{dp_a^*}{dw} \geq 0$ .<sup>14</sup>

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<sup>14</sup>Not that the equilibrium level of arbitrage capital is determined by the same condition as in the benchmark model, with the difference that the expected profit for arbitrageurs' is  $E(\pi_a) = E\left[\frac{1}{p}(\hat{p} - p)\right]$ , and in equilibrium, a proportion  $w^* \in \left(0, \frac{\hat{p}}{1+\hat{p}}\right)$  of agents choose to become arbitrageurs, where  $w^*$  satisfies the indifference equation in (20).

The social planner maximizes the expected total output generated by the economy:

$$\Pi = E [w + (1 - w) (1 - k) R + y_I \bar{p} + y_A \widehat{p}], \quad (69)$$

where  $y_I$  and  $y_A$  represent the units of failed insiders' assets acquired by insiders and arbitrageurs, respectively, and  $y_I + y_A = (1 - w)k$ .<sup>15</sup>

Note that the condition  $E[(1 - k)R + k\bar{p}] > 1$  is sufficient for the risky investment to have a higher expected return than the investment in the safe asset. While investing in the liquid asset yields lower returns compared to the risky investment, it allows arbitrageurs – take-over experts – to acquire failed insiders' assets and generate higher returns from these distressed assets compared to the insiders. Hence, for sufficiently high values of  $\delta$ , that is, when arbitrageurs are sufficiently more efficient in running distressed assets compared to insiders, the forgone expected output from investing in the liquid assets is compensated by the efficiency gain from the take-over expertise of arbitrageurs. In that case, it would be socially optimal to set aside some arbitrage capital to acquire distressed assets.

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<sup>15</sup>Furthermore, we obtain:

$$y_A = \begin{cases} (1 - w)k & \text{for } k \leq \underline{k} \\ (1 - w)k & \text{for } k \in (\underline{k}, \bar{k}] \\ \frac{w}{\bar{p}} & \text{for } k \in (\bar{k}, \bar{\bar{k}}] \\ \frac{w(1-w)k}{(1-w)(1-k)R+w} & \text{for } k > \bar{\bar{k}} \end{cases} \quad \text{and} \quad y_I = \begin{cases} 0 & \text{for } k \leq \underline{k} \\ 0 & \text{for } k \in (\underline{k}, \bar{k}] \\ (1 - w)k - \frac{w}{\bar{p}} & \text{for } k \in (\bar{k}, \bar{\bar{k}}] \\ \frac{(1-w)^2 k(1-k)R}{(1-w)(1-k)R+w} & \text{for } k > \bar{\bar{k}} \end{cases}.$$

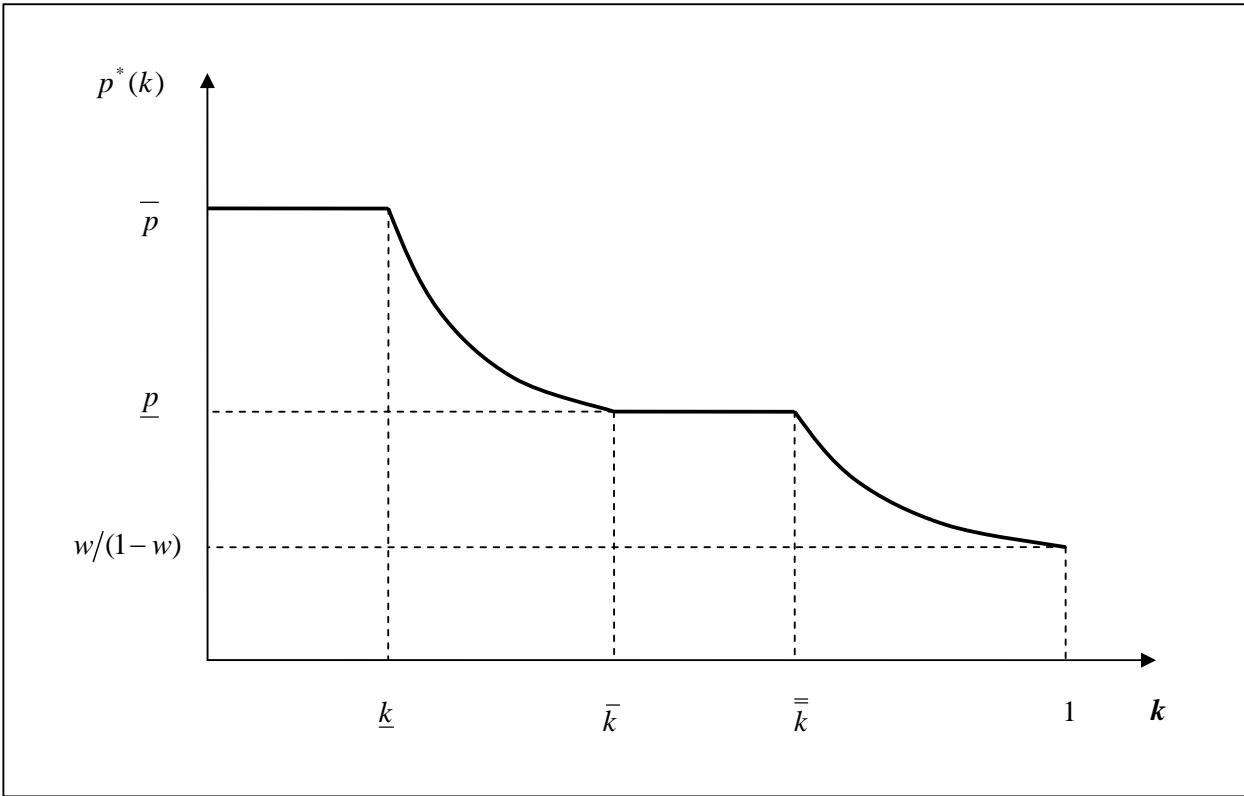


Figure 2: Price function (Lemma 1)

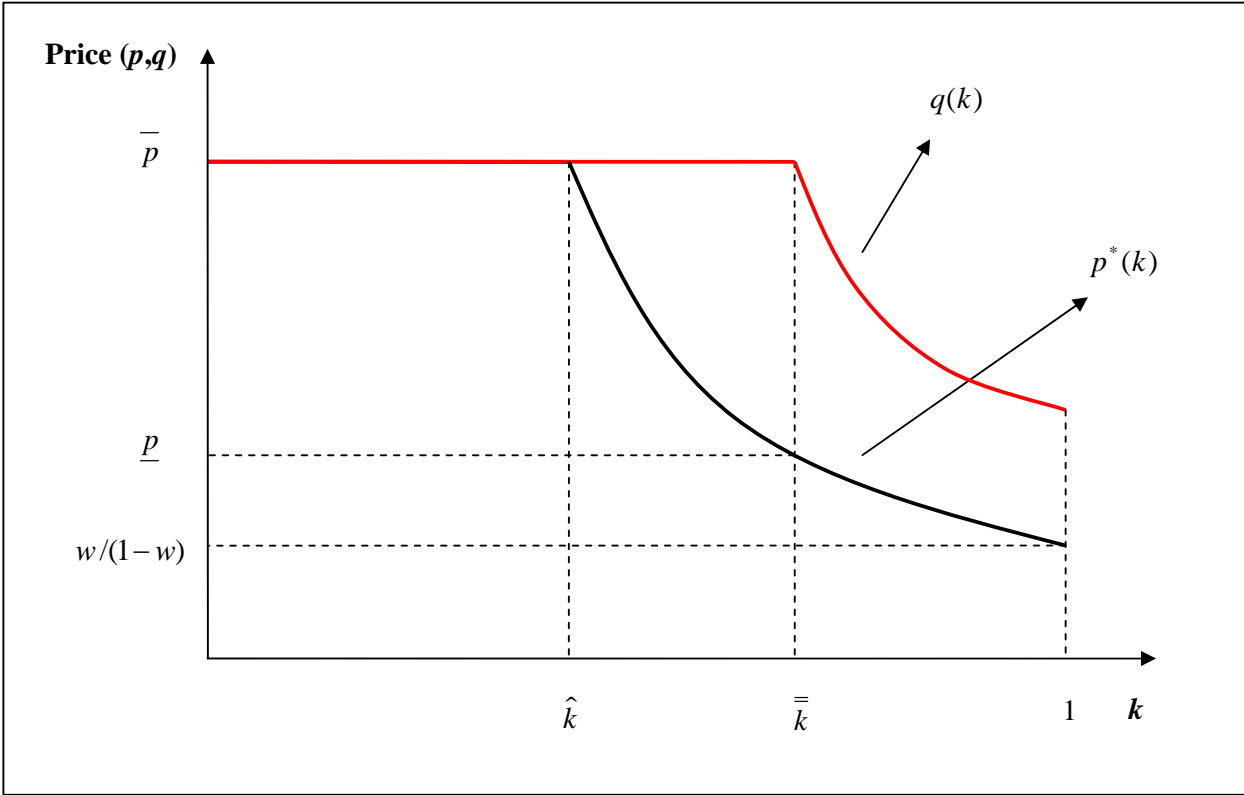


Figure 3: Prices with the capital market (Lemma 2)

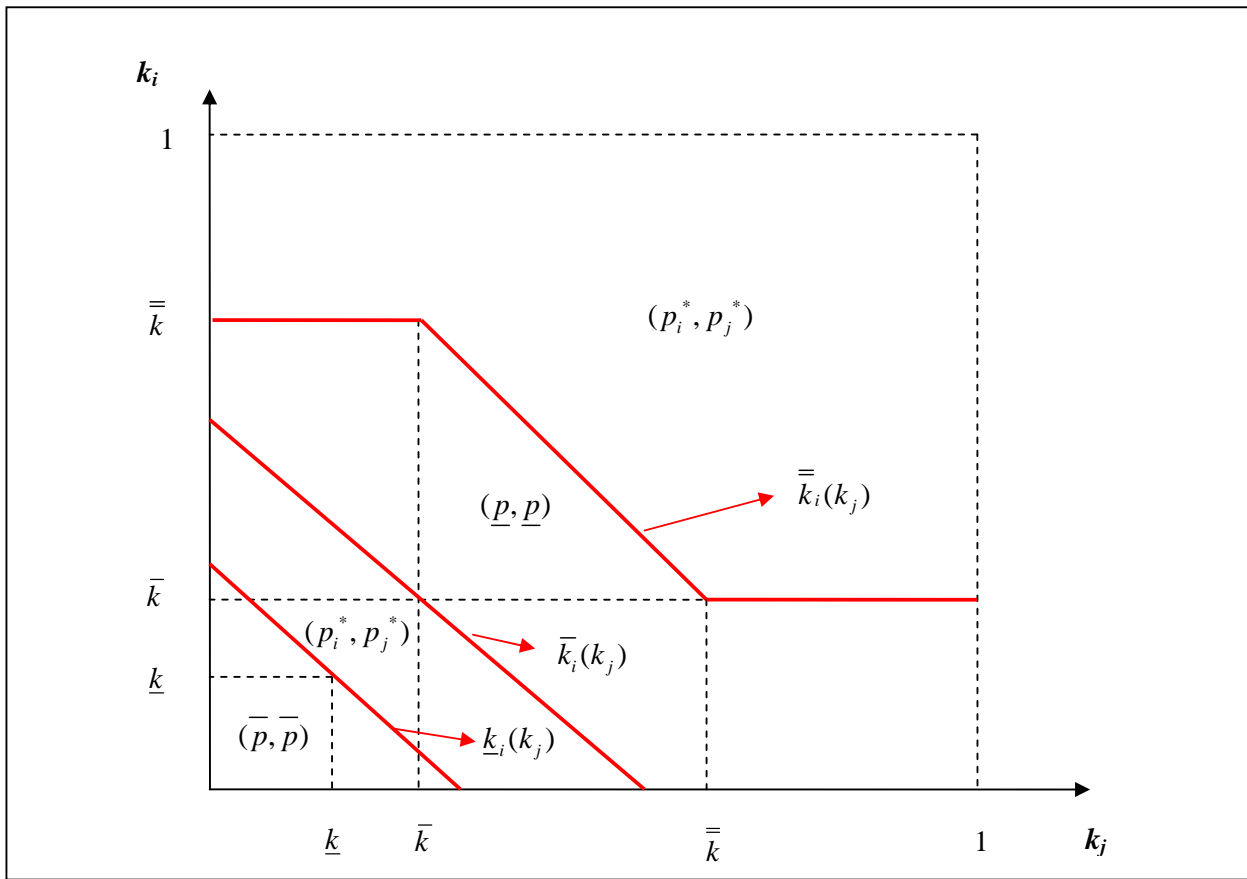


Figure 4: Prices  $(p_i^*, p_j^*)$  with integrated markets (Lemma 3).

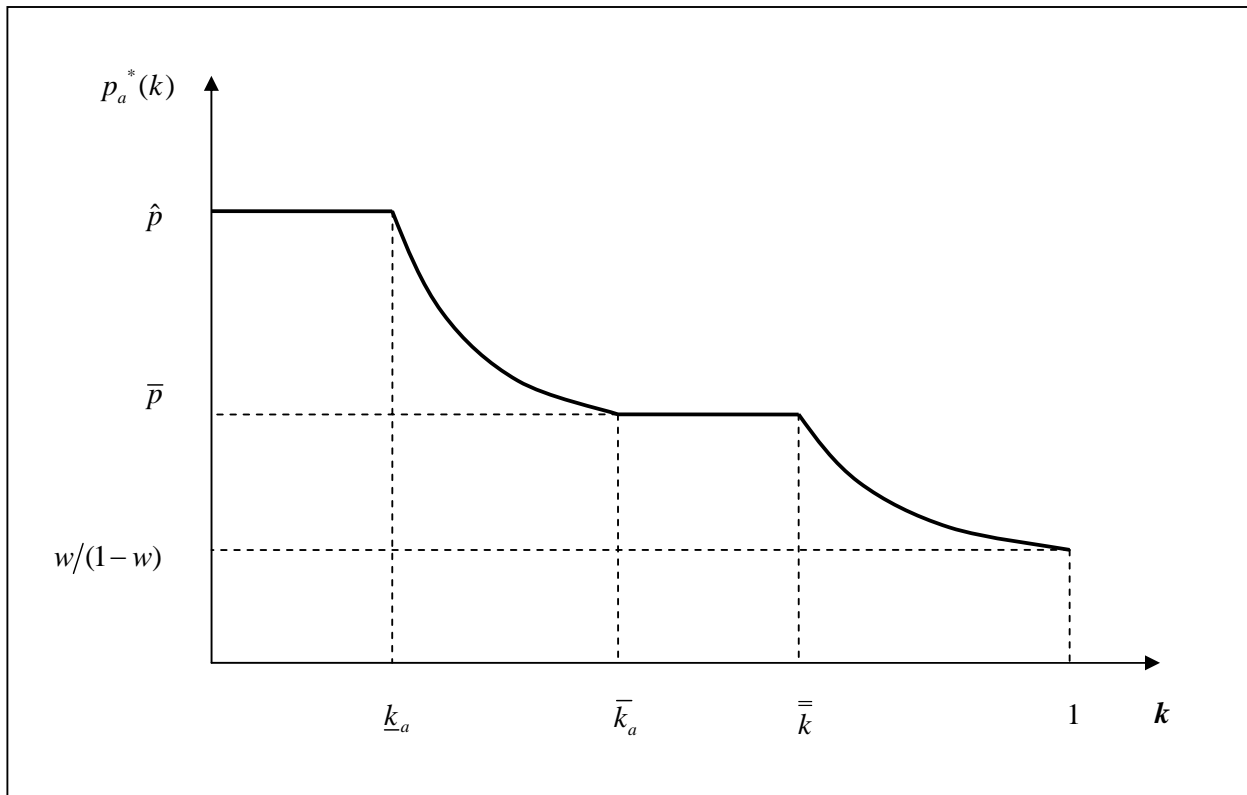


Figure 5: Price with arbitrageurs as take-over experts (Appendix II)