Working Capital, Trade and Macro Fluctuations*

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Abstract

In addressing the precipitous drop in trade volumes in the recent crisis, the real and financial explanations have sometimes been juxtaposed as competing explanations. However, they can be reconciled by appeal to the time dimension of production and the working capital demands associated with offshoring and vertical specialization of production. We explore a model of manufacturing production chains with offshoring where firms choose their time profile of production and where inventories, accounts receivable, and productivity are procyclical and track financial conditions.

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1 Introduction

Production takes time, especially when conducted through long production chains. Working capital is the financing that a firm needs to deal with the mismatch between incurring costs and receiving payment from sales. In this paper we revisit the issue of the time accounting of firms’ working capital in a bid to understand better the role of financial conditions on macro fluctuations and in trade.

A useful perspective in understanding working capital is from the simplified balance sheet of a firm, as depicted in (1). When the firm has several production stages, inventories include the intermediate goods that will ultimately lead to sales, and inventories enter as assets on the firm’s balance sheet. The longer is the production process, the larger are the inventories on the firm’s balance sheet, and the greater is the funding need for the firm. If the production chain crosses the boundary of the firm, then the firm will keep score on the (as yet unrealized) future cash flows by entering the accounts receivables from their customers as part of the assets of the firm.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>Equity</td>
</tr>
<tr>
<td>Inventories</td>
<td>Short term debt</td>
</tr>
<tr>
<td>Receivables</td>
<td>Payables</td>
</tr>
<tr>
<td>Long-term assets</td>
<td>Long-term liabilities</td>
</tr>
</tbody>
</table>

(1)

If the firm’s equity capital is limited, it must obtain outside funding to carry the short-term assets on the firm’s balance sheet. This is equivalent to saying that the firm must obtain funding to keep production going until cash flows are finally realized. If the financing is obtained from banks in the form of short-term debt, then overall credit conditions ruling in the economy will affect the terms of the tradeoff between lengthening the production chain to
Figure 1. Inventories of a firm with a three-stage production process. At date 3, the firm has three vintages of inventories, and older vintages have higher value reflecting greater inputs in the past.

reap efficiency gains in production versus the greater financing costs entailed in carrying larger current assets on the balance sheet.

There is, however, a twist to the time accounting of current assets, which makes the impact of the time dimension of production more potent than meets the eye. Consider Figure 1 which depicts the inventories of a firm with a three-stage production process. The firm undertakes the first production stage at date 1, sends the intermediate good to stage 2 in date 2. At date 3, the firm has three vintages of inventories. The oldest inventory (3 periods old) has the highest value reflecting greater inputs in the past. If the unit value of the inventory is of the same order as its age, then the total stock of inventories carried by the firm is increasing at the rate of the square of the length of its production chain. Thus, the time accounting of working capital is highly sensitive to the length of the chain, necessitating much greater incremental financing needs as production chains become longer. As such, financial conditions will impact the firm’s corporate finance decisions more sensitively in periods when firms use long production chains.

Inventory investment is known to be highly procyclical, and accounts for a large proportion of the change in GDP over the business cycle. In their survey for the Handbook of Macroeconomics, Ramey and West (1999) show that over the nine post-war recessions in the United States up to 1991, the
average peak-to-trough decline in inventories is nearly 70% of the peak-to-trough decline in GDP. Schwartzman (2010) shows that the pattern for emerging economies is even more procyclical. Since output is the sum of sales and the change in inventories, the procyclicality of inventory investment sits uncomfortably with the textbook treatment of inventories as a buffer stock used to smooth sales. Blinder (1986) and Blinder and Maccini (1991) note that far from inventories serving to smooth sales to keep pace with production, production is more volatile than sales.

The rapid growth of trade in intermediate goods and offshoring provides the perfect setting for the study of the time dimension of production. Grossman and Rossi-Hansberg (2006, 2008) argue that offshoring is now so prevalent that the classical theory of trade in finished goods should be augmented by the theory of the trade in tasks. In the same spirit, recent advances in our understanding of offshoring have focused on their technological and informational determinants, such as the specialization between routine and complex tasks (Antras, Garicano and Rossi-Hansberg (2006)), robustness to quality variability (Costinot, Vogel and Wang (2011)) or the complementarity of production processes (Baldwin and Venables (2010)).

In this paper, we take a different tack and explore the time dimension of offshoring and its consequences for the management of working capital. The time accounting discussion for inventories given above apply with even greater force when applied to offshoring and trade.

Although the production process is largely determined by technological realities, the firm nevertheless has considerable scope to choose its production time profile. Offshoring provides a good illustration of the discretion that firms have in this regard. Figure 2 illustrates the effect of offshoring for a firm with a three-stage production process, with each stage taking one unit of time. The left panel depicts the firm without offshoring, while the right panel shows the time profile when the second production stage is located offshore, entailing a lengthening of the production process due to time taken
for transport of intermediate goods to the remote location and back. For the purpose of illustration, we suppose that the transport stage takes the same length of time as is necessary for a single production stage. Amiti and Weinstein (2011) argue that the transport stage could be as long as two months when taking account of the paperwork involved in shipping.

In Figure 2, offshoring extends the firm’s production chain from three periods to five. Before the offshoring, the firm holds three vintages of inventories, reflecting the three stages of production. Under offshoring, the firm holds inventories of five vintages, including inventories that are in transit (grey-shaded cells).

What is clear from this example is that inventories should not simply be considered as buffer stock that enables a firm to smooth production. Tom Friedman’s (2005) popular book on globalization (“The World is Flat”) has a revealing quote from the chief executive officer of UPS in this respect. The UPS CEO is quoted as follows.

“When our grandfathers owned shops, inventory was what was in the back room. Now it is a box two hours away on a package car, or it
might be hundreds more crossing the country by rail or jet, and you
have thousands more crossing the ocean” [Friedman (2005, p. 174)]

Rather than being a buffer stock, inventories reflect the choice of the
length of the supply chain, and will be highly sensitive to the factors that
affect that choice. We will see below that financial conditions will be chief
among them. During periods when financing is easily obtained, we would
expect firms to lengthen their production chains to reap the benefits of glob-
alization. However, an abrupt tightening of credit will exert disruptions to
the operation of the global supply chain and lead to a drop in offshoring
activity and trading volumes.¹ In the aftermath of the crisis, the rolling
back of offshoring (“onshoring” or “reshoring”) has become a staple offering
of management consultancies, who have emphasized the virtues of shorter
supply chains.²

The contractionary effect of financing constraints during the recent crisis
have been documented by Chor and Manova (2009) and Manova (2012),
who document how fluctuations in financing needs are associated closely
with changes in trading volume, and by Amiti and Weinstein (2011) who
use micro-level data from Japan to show that banks with tighter financing
constraints impose greater dampening effect on exports of firms reliant on
those banks. These findings are corroborated in studies of the terms of
show that cash-in-advance becomes more prevalent during the crisis, and
that existing customers reliant on outside financing reduce their orders more

¹The Financial Times headline “Crisis and climate force supply chain shift” on 9
August 2009 neatly summarizes the consolidation of globally extended supply chains. On
July 15, it carried a similar article with the title “Reaggregating the supply chain”.
²See Boston Consulting Group (2011) and Accenture (2011). According to the Ac-
centure report, “[c]ompanies are beginning to realize that having offshored much of their
manufacturing and supply operations away from their demand locations, they hurt their
ability to meet their customers’ expectations across a wide spectrum of areas, such as being
able to rapidly meet increasing customer desires for unique products, continuing to main-
tain rapid delivery/response times, as well as maintaining low inventories and competitive
total costs.”
than other customers.

Well before the crisis, Kashyap, Lamont and Stein (1994) had documented the sensitive nature of inventories to financial conditions, especially to shocks that reduced bank credit supply. The severe banking sector contraction associated with the 2008 crisis can be expected to have exerted very severe brakes on the practice of offshoring and the associated increase in trading volume. The impact on trade is especially large due to the growth in vertical specialization and trade in intermediate goods documented by Yi (2003). Bems, Johnson and Yi (2011) show that gross trade fell more than value-added trade, implying that the demand declines hit vertically specialized sectors harder, reinforcing the case for the role of production chains in explaining recent events.

In a vertically integrated production process, the units could belong to the same firm, or to different firms. To address the boundary of the firm, we must appeal to the contracting environment (as done, for example, by Antras and Chor (2011)). When the transactions are between firms rather
This table presents panel regressions with firm fixed effects for the annual log difference of receivables, inventories and payables of US manufacturing firms. \( \text{dln}(\text{BD leverage}) \) is the annual log difference of the leverage of the US broker-dealer sector from the US Flow of Funds. The sample is from 1990 to 2012. Standard errors are clustered at the firm level.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{dln}(\text{sales}) )</td>
<td>0.8645***</td>
<td>0.7344***</td>
<td>0.5321***</td>
</tr>
<tr>
<td></td>
<td>(87.76)</td>
<td>(62.69)</td>
<td>(53.12)</td>
</tr>
<tr>
<td>( \text{dln}(\text{BD leverage}) )</td>
<td>0.0510***</td>
<td>0.0452***</td>
<td>0.0354*</td>
</tr>
<tr>
<td></td>
<td>(3.04)</td>
<td>(2.91)</td>
<td>(1.94)</td>
</tr>
<tr>
<td>constant</td>
<td>-0.00686</td>
<td>0.0020**</td>
<td>0.02264***</td>
</tr>
<tr>
<td></td>
<td>(-0.0)</td>
<td>(2.14)</td>
<td>(29.89)</td>
</tr>
<tr>
<td>Observations</td>
<td>61484</td>
<td>60169</td>
<td>64886</td>
</tr>
<tr>
<td>Firms</td>
<td>6377</td>
<td>6192</td>
<td>6583</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.5423</td>
<td>0.5156</td>
<td>0.3674</td>
</tr>
</tbody>
</table>

than within firms, the time dimension of production will be reflected in the firms’ accounts receivable and accounts payable. Figure 3 plots the annual changes in receivables, payables and inventories of non-financial corporate businesses in the United States and shows clearly how receivables, payables and inventories move in unison with the business cycle.

The evidence from Figure 3 on aggregate fluctuations on working capital holds at the firm level, too. Table 1 reports panel regressions for the annual growth of receivables, inventories and payables for US manufacturing firms. Adjusting for the growth of sales, the growth in the components of working capital shows positive comovement with the leverage of financial intermediaries (given by the leverage of the aggregate US broker dealer sector). Thus the aggregate changes in Figure 3 reflect the changes at the firm level, also.

We see our paper as being complementary to technological explanations of the fluctuations in trade volumes over the crisis, such as Eaton, Kortum, Neiman and Romalis (2009) and Alessandria, Kaboski and Midrigan (2009, 2010). Not only are the financial and real explanations consistent they are arguably two sides of the same coin, as Alessandria et al. (2009, 2010) draw
attention to the role of inventories in the amplification of the downturn.

Before presenting our model of offshoring, we examine a benchmark model of supply chains without offshoring, where the only “friction” is that production takes time. Even so, fluctuations in credit conditions have large impact on output and productivity. Our model of offshoring builds on the benchmark model of supply chains by holding fixed the technology, but allowing the length of the supply chain to be the choice variable. The rationale is that the degree of “roundaboutness” of production (in the terminology of Böhm-Bawerk (1884)) cannot easily be varied in the short run, and the firm must adjust its supply chain by varying the extent of offshoring. We show that the optimal choice of supply chain length depends critically on financial conditions, yielding a credit demand function of firms for the purpose of financing the supply chain. Finally, we close the model by deriving a credit supply function, and conduct comparative statics exercises with respect to financial shocks. We find that tighter financial conditions will result in higher loan risk premiums and a contraction in the degree of offshoring undertaken by the firms.

2 Benchmark “Austrian” Model

We begin with an elementary model of supply chains without offshoring. Our model is deliberately stark in order to isolate the time dimension of production and the only substantial decision is the ex ante choice of the length of the production chain. There are no product or labor market distortions. The only friction is that production takes time. In this sense, our benchmark model has an Austrian theme that echoes the capital theory of Böhm-Bawerk (1884).

There is a population of $L$ workers and $L$ firms each owned by a penniless entrepreneur. Each firm is matched with one worker. Production takes place through chains of length $n$, so that there are $L/n$ production chains in the
economy. We assume \( L \) is large relative to \( n \), so that the economy consists of a large number of production chains.

Within each production chain, there is a downstream firm, labeled as firm 1, that sells the final output. The other firms produce intermediate inputs in the production of the final good. Firm \( n \) supplies its output to firm \( n - 1 \), who in turn supplies output to \( n - 2 \), and so on. Each step of the production process takes one unit of time, where time is indexed by \( t \in \{0, 1, 2, \cdots \} \).

Although each step of the production process is identified with a firm, this is for narrative purposes only. Our model is silent on where the boundary of the firm lies along the chain. Some of the consecutive production stages could lie within the same firm, while some consecutive stages could be in different firms. If the production chain lies within the firm, claims on intermediate goods will show up on a firm’s balance sheet as inventories. If the production chain lies across firms, then they show up as accounts receivable. What matters for us is the aggregate financing need, rather than the allocation of financing into inventories and accounts receivable.

The wage rate is \( w \) per period and wage cost cannot be deferred and must be paid immediately. Labor is provided inelastically, so that total labor supply is fixed at \( L \). There is no physical capital. The cashflow to the chain is given in the table below.

<table>
<thead>
<tr>
<th>date ( t )</th>
<th>Firms</th>
<th>cumulative cashflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n - 1 )</td>
<td>( -w ) ( \cdots ) ( -w )</td>
<td>( -3w )</td>
</tr>
<tr>
<td>( n )</td>
<td>( -w ) ( \cdots ) ( -w )</td>
<td>( -w )</td>
</tr>
<tr>
<td>( n + 1 )</td>
<td>( y(n) - w ) ( -w ) ( \cdots ) ( -w )</td>
<td>( -\frac{1}{2}n(n + 1)w )</td>
</tr>
</tbody>
</table>
At the beginning of date 1, firm $n$ begins production and sends the intermediate good to firm $n - 1$ at the end of date 1, who takes delivery and begins production at the beginning of date 2, and so on. Meanwhile, at the beginning of date 2, firm $n$ starts another sequence of production decisions by producing its output, which is sent to firm $n - 1$, and so on.

The first positive cashflow to the chain comes at date $n + 1$ when firm 1 sells the final output for $y(n)$. The cash transfer upstream is instantaneous, so that all upstream firms are paid for their contribution to the output.

Firms borrow by rolling over one period loans. The risk-free interest rate is zero, and is associated with a storage technology that does not depreciate in value. Although the risk-free rate is zero, the firms’ borrowing cost will reflect default risk and a risk premium in the credit market. Once the output is marketed from date $n + 1$, there is a constant hazard rate $\varepsilon > 0$ that the chain will fail with zero liquidation value so that lenders suffer full loss on their loans to the chain. Before date $n + 1$, there is no probability of failure, and firms can borrow at the risk-free rate of zero. But starting from the loan repayable at date $n + 1$, they must borrow at the higher rate $r > \varepsilon$, which reflects the default risk $\varepsilon$ as well as the risk premium, which will be endogenized by introducing a financial sector in Section 4. For now, we treat the borrowing rate $r$ as given. Firms have limited liability, so that once a production chain fails, the firms in the chain can re-group costlessly to set up another chain of same length by borrowing afresh.

Before the first cash flow materializes to the chain from the sale of the final product, the chain must finance the initial set-up cost of $\frac{1}{2}n(n + 1)w$. We can decompose this sum into the steady state inventory $nw$ that must be carried by the firm in steady state and the initial “triangle” of working capital of $\frac{1}{2}n(n - 1)w$. Firms start with no equity and all financing is done by raising debt. Thus, the total initial financing need of the production chain of length $n$ is given by

$$\frac{n(n + 1)w}{2}$$

(2)
From the lenders' perspective, the cash flow is negative until date $n$, but then they start receiving interest repayment on the outstanding stock of loans. Figure 4 compares the profile of lenders’ cash flows conditional on survival of the chain. The light line gives the cash flow profile by lending to a production chain of length $n$, while the dark line gives the profile from lending to a chain of length $n' > n$.

Note that the outstanding loan amount is of the order of the square of the length of the production chain, since the initial set-up cost of the chain is the “triangle” until the final product is marketed. For the firms, the choice of the length of the production chain trades off the marginal increase in productivity from lengthening the chain against the increased cost of financing working capital.

There are $L/n$ production chains, so that the aggregate working capital demand in the economy, denoted by $K$, is

$$K = \frac{1}{2}n(n + 1)w \times \frac{L}{n}$$

$$= \frac{n + 1}{2}wL$$

(3)
The production chain consisting of $n$ firms has output $y(n)$. The output per firm (equivalent to output per worker) is $y(n)/n$. We will adopt the following production function

$$\frac{y(n)}{n} = n^\alpha, \quad (0 < \alpha < 1) \tag{4}$$

The formulation of productivity in our model harks back to Böhm-Bawerk’s (1884) notion of “roundabout production”, where intermediate goods are used as inputs in further intermediate goods. Our assumption that $0 < \alpha < 1$ captures the feature that:

“[t]he indirect method entails a sacrifice of time but gains the advantage of an increase in the quantity of the product. Successive prolongations of the roundabout method of production yield further quantitative increases though in diminishing proportion.”

3 Bohm-Bawerk (1884), p 88 of 1959 English translation by G. Huncke, Libertarian Press.

3Bohm-Bawerk (1884), p 88 of 1959 English translation by G. Huncke, Libertarian Press.
which boils down to the problem of maximizing the per period surplus:

$$\Pi = n^\alpha L - wL - rK$$

$$= n^\alpha L - wL \left(1 + \frac{r(n+1)}{2}\right)$$  \hspace{1cm} (6)

The first-order condition for $n$ gives

$$n = \left(\frac{2\alpha}{w}\right)^{\frac{1}{1-\alpha}}$$  \hspace{1cm} (7)

We assume that firms bid away their surplus by competing for workers, so that the wage rate is determined by the zero profit condition:

$$n^\alpha = w \left(1 + \frac{r(n+1)}{2}\right)$$  \hspace{1cm} (8)

We can then solve the model in closed form. The wage $w$ is

$$w = 2 \left(\frac{\alpha}{r}\right)^\alpha \left(1 - \frac{\alpha}{2 + r}\right)^{1-\alpha}$$  \hspace{1cm} (9)

Optimal chain length is

$$n = \frac{\alpha}{1 - \alpha} \left(1 + \frac{2}{r}\right)$$  \hspace{1cm} (10)

so that productivity per worker is

$$\left(\frac{\alpha}{1 - \alpha}\right)^\alpha \left(1 + \frac{2}{r}\right)^{\alpha}$$  \hspace{1cm} (11)

and total output $Y$ is

$$Y = n^\alpha L = \left(\frac{\alpha}{1 - \alpha}\right)^\alpha \left(1 + \frac{2}{r}\right)^{\alpha} L$$  \hspace{1cm} (12)

Note that the wage, productivity and output are declining in the borrowing rate $r$, which incorporates the risk premium $r-\varepsilon$. The reason for the negative impact of the borrowing rate on real variables in spite of the absence of the
standard intertemporal savings decision arises from the decline in the length of production chains in the economy as financing cost increases.

Finally, the total credit used by all production chains in the economy is

\[
K = \frac{n + 1}{2} w L = \left( \frac{\alpha}{1 - \alpha} \right)^{\alpha} \left( 1 + \frac{2}{r} \right)^{\alpha} \left( \frac{\alpha}{r} + \frac{1 - \alpha}{2 + r} \right) L \tag{13}
\]

Note that total financing need is increasing linearly in chain length \( n \), since the financing need is the “triangle” whose size increases at the rate of the square of the chain length.

We may interpret \( K \) as the aggregate credit demand in the economy. Credit demand is declining in the borrowing rate \( r \). Once we introduce a financial sector in Section 4, the borrowing rate \( r \) can be solved as the market clearing rate that equates \( K \) with total credit supply.

The ratio \( K/Y \) could be interpreted as the credit to GDP ratio, and has the simple form as below, which also declines with the borrowing rate.

\[
\frac{K}{Y} = \frac{\alpha}{r} + \frac{1 - \alpha}{2 + r} \tag{14}
\]

Since credit is a stock while output is a flow, the choice of the time period is important in interpreting the ratio \( K/Y \). In our model, this ratio is given meaning by setting the unit time interval to be the time required to finish one stage of production.

### 2.1 Analogy with Fixed Capital

There is an analogy between working capital and fixed capital, but the analogy is not exact. If we treat working capital as a factor of production, we can give a reduced-form representation of total output, but where the total factor productivity term is not a constant, but instead depends on financial conditions.
Such an exercise is of interest given Valerie Ramey’s (1989) study of modeling inventories as a factor of production. Indeed, we can give a Cobb-Douglas representation as follows. Note from (12) that total output can be written as

\[ Y(K, L) = n^\alpha L \]

\[ = \left( \frac{2K}{wL} - 1 \right)^\alpha L \]

\[ = \left( \frac{2}{w} - \frac{L}{K} \right)^\alpha K^\alpha L^{1-\alpha} \]  \hspace{1cm} (15)

Imposing a Cobb-Douglas functional form for working capital will result in a misspecified production function, where total factor productivity depends on endogenous variables.

Figure 5 plots the TFP term in the production function as a function of the borrowing rate \( r \) when \( \alpha = 0.033 \). The TFP term is not well-defined when \( r = 0 \), since both expressions inside the brackets in (15) shoot off to infinity. However, for reasonable ranges for \( r \), the TFP term is decreasing in the borrowing rate.

To an outside observer who imposes a Cobb-Douglas production function
on the economy, they would observe that productivity undergoes shocks as financial conditions change. When financial conditions are tight and the risk premium in the borrowing rate increases, they will also observe that total factor productivity falls. This is in spite of the fact that our model has none of the standard distortions or frictions to product or labor markets, or indeed any intertemporal choice. The only “friction” in our model is that production takes time.

This feature of our model where the TFP depends on financial conditions is in a similar spirit to the finding in Buera and Moll (2011), who show that the aggregate TPF of an economy with heterogeneous firms that are differentially affected by collateral constraints will also exhibit sensitivity to financial conditions. Our mechanism is very different from that of Buera and Moll (2011), and the lesson from our paper is that vertical specialization of production may give rise to productivity effects that are not captured by a representative firm production function.

2.2 Sales and Value Added

The most distinctive feature of our “Austrian” model of production chains is the distinction between total sales and output as value-added. This distinction is meaningless when production is undertaken by atomistic firms, but is highly informative and relevant when production takes place in chains. The empirical significance of this distinction will become clear when we discuss the time accounting of offshoring and trade.

Consider the sales of each firm in the chain. From the zero profit condition, each firm’s sale is the cost of production, including the cost of working
capital.

\[ p_n = w + rw \]
\[ p_{n-1} = w + rw(n - 1) + p_n \]
\[ p_{n-2} = w + rw(n - 2) + p_{n-1} \]
\[ \vdots \]
\[ p_1 = w + rw + p_2 \]

By recursive substitution,

\[ p_n = w(1 + rn) \]
\[ p_{n-1} = 2w(1 + rn) - wr \]
\[ p_{n-2} = 3w(1 + rn) - wr(1 + 2) \]
\[ \vdots \]
\[ p_1 = nw(1 + rn) - wr(1 + 2 + \cdots + (n - 1)) \]

Therefore total sales are

\[ \sum_{k=1}^{n} p_k = w(1 + rn) \left( \sum_{k=1}^{n} k \right) - wr \sum_{k=1}^{n-1} k(n - k) \] (18)

Using the algebraic identity:

\[ \sum_{k=1}^{n-1} k(n - k) = \frac{1}{6} n(n - 1)(n + 1) \]

total sales in the chain are

\[ \sum_{k=1}^{n} p_k = \frac{1}{2} nw(nr + 1)(n + 1) - \frac{1}{6} nrw(n - 1)(n + 1) \] (19)

Total value added in the chain is

\[ p_1 = nw(1 + rn) - wr \sum_{k=1}^{n-1} k \]
\[ = nw(nr + 1) - \frac{1}{2} nrw(n - 1) \] (20)
Figure 6. Plot of the ratio of sales to value-added of the economy as a function of the borrowing rate $r$ ($\alpha = 0.033$)

Hence, the sales to value-added ratio is

$$\frac{\sum_{k=1}^{n} p_k}{p_1} = (n + 1) \frac{\frac{1}{3}r + \frac{2}{3}nr + 1}{r + nr + 2}$$

$$= \frac{(r + 2\alpha)(r + \alpha(1 + r) + 3)}{3r(1 - \alpha)(r + 2)}$$

(21)

Figure 6 plots the sales to value-added ratio given by (21) when $\alpha = 0.033$. We see that the sales to value-added ratio is decreasing in the borrowing rate $r$, reflecting the shorter production chains when financial conditions are tighter.

Although our model is not sufficiently developed to take to the data, it is illuminating to get some bearing on the empirical magnitudes for the sales to value-added ratio for US manufacturing firms. The U.S. Census Bureau publishes an annual survey of manufacturing firms and provides estimates of the total value of shipments and value-added of the manufacturing sector. Figure 7 plots the recent movements in total shipments and value-added, where both series have been normalized to be 1 in 2000. The total value of shipments for the manufacturing sector in 2000 was 4.21 trillion dollars, and
value-added was 1.97 trillion dollars.

We see that the two series do not always move in step. Total sales (value of shipments) is more procyclical than value-added, where sales overtake value-added from below in 2005, but then fall much more in 2009. The ratio of total shipments to value-added for the manufacturing sector lies in the range of 2.0 to 2.4. The ratio rises strongly in the period before the crisis but crashes in 2009, consistent with the basic picture given by our model in Figure 6.

3 Production Chains with Offshoring

We now develop our model of offshoring by changing some key features of the benchmark model above.

First, in line with the intuition that the degree of “roundaboutness” of production will not be easily changed in the short run, we fix the production chain length. Instead, the choice of the firm is to decide whether to perform a particular task at home or send it offshore to a destination where the task
can be done more effectively.

Assume there is a large number of multi-national firms, with a presence in all countries. Given the large number of multi-national firms, all markets are competitive and the profit of each multi-national firm is zero. There is no restriction on free trade and labor can move freely across countries.

Assume that there are \( \bar{n} \) stages to the production chain and \( \bar{n} \) countries. Each country has an absolute advantage in precisely one stage of the production process. The absolute advantage derives from the location, not the worker, so that if any worker moves to the country with absolute advantage in a particular task, the new worker is able to produce at the higher productivity for that task. There is a constant \( b > 0 \) such that the country with the absolute advantage in production stage \( \bar{n} \) has an effective labor input of \( 1 + b \) compared to the effective labor input of 1 in any other country for that task.

The output of a production chain depends on the amount of offshoring done to utilize the most effective inputs. Specifically, the output of a production chain is given by

\[
\left( \sum_{i=1}^{\bar{n}} x_i \right)^\alpha \quad (0 < \alpha < 1)
\] (22)

where \( x_i = 1 + b \) if the production of the \( i \)th stage takes place in the country with the absolute advantage in stage \( i \) while \( x_i = 1 \) if the production takes place anywhere else. Thus, if a firm offshores \( s \) stages of the production chain to the country with the absolute advantage in that process, output is given by

\[
y(s) = (\bar{n} + bs)^\alpha
\] (23)

The firm’s decision is to choose \( s \), the extent of offshoring.

Offshoring entails two costs - the cost of transport and the financing cost due to the lengthening of the production chain. We assume that transport requires labor services just as for production. Offshoring also incurs financing
costs due to the time needed to transport intermediate goods. Transportation by ship to a foreign country takes much longer than delivery within the same country. According to Amiti and Weinstein (2011), overseas shipping could take two months. To formalize this in the simplest way, we assume that if an intermediate good is transported to another country, transport takes one unit of time, which is the same as the time needed for production of an intermediate good. Within the same country, we assume that transport happens instantaneously.

As in the benchmark model, wages cannot be deferred and firms that engage in intermediate good production or overseas transport need working capital to pay wages. Assume wage for each stage of intermediate good and transportation service is paid at the beginning of the production stage. Wage per unit of time is $w$. We maintain the assumption that firms have no equity so that working capital is financed with debt at borrowing rate $r$.

The financing requirement depends on the extent of offshoring, as offshoring lengthens the production process. If all production happens within a country, the production process consists of $\bar{n}$ stages and takes $\bar{n}$ periods. If intermediate goods are always transported across borders to the next stage, and the final product returns to the home country, the production process takes $2\bar{n}$ periods in total. If offshoring takes place $\bar{s}$ times, the production process takes $\bar{n} + \bar{s}$ periods. Total financing requirement for the world economy, denoted by $K$, is then

$$K = \frac{1}{2}(\bar{n} + \bar{s})(\bar{n} + \bar{s} + 1)w \times \frac{L}{(\bar{n} + \bar{s})}$$

$$= \frac{\bar{n} + \bar{s} + 1}{2}wL$$

(24)

where $L$ is the world labor force. The per period interest cost for the world economy is

$$r \frac{\bar{n} + \bar{s} + 1}{2}wL$$

(25)

The profit of a multinational firm is given by
\[ \Pi = (\bar{n} + bs)^\alpha zL - wzL - rzK \]
\[ = (\bar{n} + bs)^\alpha zL - wzL \left(1 + \frac{r (\bar{n} + s + 1)}{2}\right) \quad (26) \]
where \( z \) is the proportion of the world workforce employed by firm. The firm maximizes profit by choosing \( s \). Since the firms’ profits are driven down to zero, the share \( z \) will not play any meaningful role in our model. The first-order condition for \( s \) yields
\[ \bar{n} + bs = \frac{1}{w^{\frac{1}{1-\alpha}} \left(\frac{2b\alpha}{r}\right)^{\frac{1}{1-\alpha}}} \quad (27) \]
and the zero profit condition is
\[ (\bar{n} + bs)^\alpha = w \left(1 + \frac{r (\bar{n} + s + 1)}{2}\right) \]
\[ = w \left(1 + \frac{r}{2} (1 + \bar{n}) + \frac{rs}{2}\right) \quad (28) \]
From (27) and (28) we can solve the model in closed form. The extent of off-shoring is
\[ s = \frac{\alpha}{1 - b\alpha} \left(1 + \bar{n} \left(1 - \frac{1}{b}\right) + \frac{2}{r}\right) - \bar{n} \quad (29) \]
The wage rate is
\[ w = \frac{2b\alpha}{r} \left(\frac{1 - b\alpha}{2 + r (1 + \bar{n}(1 - \frac{1}{b}))}\right)^{1-\alpha} \quad (30) \]
Note that both \( s \) and \( w \) are decreasing in the borrowing cost \( r \). Thus, the extent of offshoring depends on financial conditions, where a tightening of credit will reduce offshoring and result in a concomitant reduction in trade volume. The reduction is trade volume will be higher for more elaborate production processes with a greater vertical specialization. The empirical evidence in Bems, Johnson and Yi (2011) is consistent with such a prediction.

Finally, since total financing \( K \) is given by \((\bar{n} + s + 1) wL/2\) it is increasing in \( s \) and \( w \). Therefore, the global demand for credit is decreasing in the borrowing cost \( r \).
3.1 Trade Growth Accounting

Although the growth in offshoring is not easily observed directly, we describe an accounting framework which can be used to approximate it. We use the following notation. $Y$ is GDP, $Y_D$ is domestic value-added, $S_D$ is the domestic manufacturing sales and $M$ is total imports. Then, define $\beta$ and $\gamma$ so that

$$S_D = \beta \times \left( \frac{\text{imported intermediate goods}}{\text{imported intermediate goods} + \text{domestically produced intermediate goods}} \right)$$ (31)

$$M = \gamma \times \left( \frac{\text{imported intermediate goods}}{\text{domestically produced intermediate goods}} \right)$$ (32)

Meanwhile, we define our measure of offshoring $q$ as the ratio of imported intermediate goods to the total intermediate goods - both imported and domestically produced. Thus, we have:

$$q \equiv \frac{\text{imported intermediate goods}}{\text{imported intermediate goods} + \text{domestically produced intermediate goods}}$$ (33)

Then by using our definitions of $\beta$ and $\gamma$, we can write

$$M = q \times S_D \times \frac{\gamma}{\beta}$$

$$= q \times \frac{S_D}{Y_D} \times \frac{Y_D}{Y} \times Y \times \frac{\gamma}{\beta}$$ (34)

So, import/GDP ratio is

$$\frac{M}{Y} = q \times \frac{S_D}{Y_D} \times \frac{Y_D}{Y} \times \frac{\gamma}{\beta}$$ (35)

Finally, if we make the assumption that $\beta$ and $\gamma$ are constants, then the growth of offshoring can be obtained as

$$g(q) = g\left(\frac{M}{Y}\right) - g\left(\frac{S_D}{Y_D}\right) - g\left(\frac{Y_D}{Y}\right)$$
In long hand we have

\[
\text{Offshoring growth} = \text{Growth of Imports/GDP} - \text{Growth of mfg sales/value-added} - \text{Growth of mfg/GDP}
\]

Figures 8 and 9 plot the component series in obtaining our time series for \(q\). We see that the incidence of offshoring follows the tell-tale procyclical pattern around the recent financial crisis, following the patterns of fluctuating financial conditions.

4 Closing the Model with Credit Supply

Up to now, we have treated the borrowing rate \(r\) as given. We now close the model by building a financial sector and modeling the credit supply by banks. The borrowing rate \(r\) is then determined as the rate that clears the credit market.

Before describing the model of credit supply in more detail, it is useful to note the salient features of banking sector credit supply in order that our model may capture the financial frictions more faithfully.
Figure 9. Annual growth rates of offshoring, imports/GDP, manufacturing sales to value-added and share of manufacturing in GDP

We have already noted the findings of Kashyap, Lamont and Stein (1994) and Amiti and Weinstein (2009), who (in different contexts) have pointed to the pivotal role of the banking sector in determining the credit conditions for trade finance. Adrian, Colla and Shin (2011) investigate the nature of the financial frictions that operated in the recent crisis, where the banking sector behavior is described in more detail. Here, we will focus on adapting some key features of the banking sector into our model of offshoring and production chains.

The banking sector is special in several respects, compared to the non-financial corporate sector. In textbook discussions of corporate financing decisions, the set of positive net present value (NPV) projects is often taken as being exogenously given, with the implication that the size of the balance sheet is fixed. Leverage increases by substituting equity for debt, such as through an equity buy-back financed by a debt issue, as depicted by the left hand panel in Figure 10.

However, the left hand panel in Figure 10 turns out not to be a good
description of the way that the banking sector leverage varies over the financial cycle. For banks, however, leverage fluctuates through changes in the total size of the balance sheet with equity being the pre-determined variable. Hence, leverage and total assets tend to move in lock-step, as depicted in the right hand panel of Figure 10. A consequence of this feature is that equity should be seen as the pre-determined variable when modeling bank lending, and we can see banks as choosing their leverage given the fixed level of bank equity. This is the approach we will take here.4

4.1 Bank Credit Supply

Credit in the economy is intermediated through banks. We assume that workers (as investors) cannot lend directly to entrepreneurs, and must lend through the banking sector. Banking sector equity $E$ is fixed, with equity ownership evenly distributed among the worker population. Credit is short-term, and rolled over every period. The bank lends out amount $C$ (for “credit”) at date $t$ at the lending rate $r$, so that the bank is owed $(1 + r)C$ in date $t + 1$. The lending is financed from the combination of equity $E$ and

4Adrian and Shin (2012) discuss the reasons for the distinctive patterns in bank balance sheet management and its consequences for the financial cycle.
deposit funding $L$, raised from workers. The cost of debt financing is $f$ so that the bank owes $(1 + f)L$ at date $t + 1$ (its notional liabilities). We will show shortly that $f$ is the risk-free rate.

Recall that production chains are subject to a hazard rate $\varepsilon > 0$ of failure by of production chain. Entrepreneurs have limited liability, and so the failure of the chain results in credit losses for the bank. The correlation in defaults across loans follows the Vasicek (2002) model. Production chain $j$ survives into the next period (so that the loan is repaid) when $Z_j > 0$, where $Z_j$ is the random variable

$$Z_j = -\Phi^{-1}(\varepsilon) + \sqrt{\rho}S + \sqrt{1 - \rho}X_j$$

where $\Phi(\cdot)$ is the c.d.f. of the standard normal, $S$ and $\{X_j\}$ are independent standard normals, and $\rho$ is a constant between zero and one. $S$ has the interpretation of the economy-wide fundamental factor that affects all chains, while $X_j$ is the idiosyncratic factor for chain $j$. The parameter $\rho$ is the weight on the common factor, which limits the extent of diversification that investors can achieve. Note that the probability of default is given by $\Pr(Z_j < 0) = \Pr(\sqrt{\rho}S + \sqrt{1 - \rho}X_j < \Phi^{-1}(\varepsilon)) = \Phi(\Phi^{-1}(\varepsilon)) = \varepsilon$, consistent with our assumption that each chain has a constant hazard rate of failure of $\varepsilon$.

With bank equity fixed, total lending is determined by the leverage of the bank. Leverage is determined through the following contracting problem, which follows Bruno and Shin (2012). The bank chooses between two alterantive portfolios. The good portfolio consists of loans which have a probability of default $\varepsilon$, and $\rho = 0$. The bad portfolio consists of loans with a higher probability of default $\varepsilon + k$, for $k > 0$ and non-zero $\rho$. The bad portfolio generates greater dispersion in the outcome density for the loan portfolio, and is associated with a higher option value of limited liability.

Credit extended by the bank is $C$ at interest rate $r$ so that the notional value of assets is $(1 + r)C$. Conditional on $Y$, defaults are independent. Taking the limit where the number of borrowers becomes large while keeping the notional assets fixed, the realized value of the bank’s assets can be written
as a deterministic function of $Y$. If the bank chooses the bad portfolio, the realized value of assets at date $T$ is the random variable $w_B(Y)$ defined as:

$$w_B(Y) = (1 + r) C \cdot \Pr \left( \sqrt{\rho Y} + \sqrt{1 - \rho} X_j \geq \Phi^{-1}(\varepsilon + k) \mid Y \right)$$

$$= (1 + r) C \cdot \Phi \left( \frac{Y \sqrt{\rho} - \Phi^{-1}(\varepsilon + k)}{\sqrt{1 - \rho}} \right) \quad (37)$$

It is convenient to normalize $w_B$ by the face value of assets. We define $\hat{w}_B(Y) \equiv w_B(Y) / (1 + r) C$. The c.d.f. of $\hat{w}_B$ is then given by

$$F_B(z) = \Pr (\hat{w}_B \leq z)$$

$$= \Pr (Y \leq \hat{w}_B^{-1}(z))$$

$$= \Phi (\hat{w}_B^{-1}(z))$$

$$= \Phi \left( \frac{\Phi^{-1}(\varepsilon + k) + \sqrt{1 - \rho}}{\sqrt{\rho}} \right) \quad (38)$$

If the bank chooses the good portfolio, the default probability is $\varepsilon$ and correlation in defaults is zero. The outcome distribution for the good portfolio is obtained from (38) by setting $k = 0$ and letting $\rho \to 0$. In this limit, the numerator of the expression inside the brackets is positive when $z > 1 - \varepsilon$ and negative when $z < 1 - \varepsilon$. Thus, the outcome distribution of the good portfolio is

$$F_G(z) = \begin{cases} 0 & \text{if } z < 1 - \varepsilon \\ 1 & \text{if } z \geq 1 - \varepsilon \end{cases} \quad (39)$$

so that the good portfolio consists of i.i.d. loans all of which have a probability of default of $\varepsilon$, and the bank can fully diversify across the i.i.d. loans. Define:

$$\varphi \equiv (1 + f) L / (1 + r) C \quad (40)$$

$\varphi$ is the notional debt of the bank - the amount to be repaid - normalized by total notional assets. At the same time, $\varphi$ is the strike price of the embedded option for the bank from limited liability. The maximizes net worth, which can be written as

$$E(\hat{w}) - [\varphi - \pi(\varphi)] \quad (41)$$
where $E(\hat{w})$ is the expected realization of the (normalized) loan portfolio, and the expression in square brackets is the expected repayment by the bank to wholesale creditors, which can be decomposed following Merton (1974) as the repayment made in full in all states of the world minus the option value to default due to the limited liability of the bank. $\pi(\varphi)$ is the value of the put option when the strike price is given by $\varphi = (1 + f) L / (1 + r) C$.

The contracting problem takes equity $E$ as given and chooses $L, C$ and $f$ to maximize the bank’s expected payoff (41) subject to the incentive compatibility constraint for the bank to choose the good portfolio, and the break-even constraint for the wholesale creditors. The incentive compatibility constraint is

$$E_G(\hat{w}) - [\varphi - \pi_G(\varphi)] \geq E_B(\hat{w}) - [\varphi - \pi_B(\varphi)] \quad (42)$$

where $E_G(\hat{w})$ is the expected value of the good portfolio and $\pi_G(\varphi)$ is the value of the put option with strike price $\varphi$ under the outcome distribution for the good portfolio. $E_B(\hat{w})$ and $\pi_B(\varphi)$ are defined analogously for the expected outcome and option values associated with the bad portfolio. Writing $\Delta\pi(\varphi) = \pi_B(\varphi) - \pi_G(\varphi)$, (42) can be written more simply as

$$\Delta\pi(\varphi) \leq k \quad (43)$$

Incentive compatibility is maintained by keeping leverage low enough that the higher option value to default does not exceed the greater expected payoff of the good portfolio.

**Lemma 1** There is a unique $\varphi^*$ that solves $\Delta\pi(\varphi) = k$, where $\varphi^* < 1 - \varepsilon$.

The proof is as follows. From Breeden and Litzenberger (1978), the state price density is the second derivative of the option price with respect to its strike price, so that

$$\Delta\pi(\varphi) = \begin{cases} \int_0^\varphi F_B(s) \, ds & \text{if } \varphi < 1 - \varepsilon \\ \int_0^{1-\varepsilon} F_B(s) \, ds - \int_{1-\varepsilon}^\varphi [1 - F_B(s)] \, ds & \text{if } \varphi \geq 1 - \varepsilon \end{cases} \quad (44)$$
Thus $\Delta \pi (\varphi)$ is single-peaked, reaching its maximum at $\varphi = 1 - \varepsilon$. Since

$$
\int_0^1 [F_B (s) - F_G (s)] \, ds = \int_0^1 [1 - F_G (s)] \, ds - \int_0^1 [1 - F_B (s)] \, ds
= E_G (\hat{w}) - E_B (\hat{w}) = k
$$

(45)

$\Delta \pi (\varphi)$ approaches $k$ from above as $\varphi \to 1$. As $\varphi < 1$ for any bank with positive notional equity, there is a unique solution to $\Delta \pi (\varphi) = k$ in the range where $\Delta \pi (\varphi)$ is increasing. Therefore $\varphi^* < 1 - \varepsilon$. This proves the lemma.

We can now fully solve for credit supply. The good portfolio has payoff $1 - \varepsilon$ with certainty (as seen in (39)). Since the bank has zero probability of default whenever $\varphi < 1 - \varepsilon$, Lemma 1 implies that the bank’s probability of default is zero. From the break-even constraint of the wholesale creditors, the funding rate is therefore given by the risk-free rate. Finally, from the balance sheet identity $E + L = C$, we can solve for the bank’s supply of credit as

$$
C = \frac{E}{1 - \frac{1+r}{1+f} \varphi^*}
$$

(46)

where $\varphi^*$ is the unique solution in Lemma 1.

By combining the credit supply function given above with the credit demand functions for financing working capital, we can solve for the equilibrium borrowing rate $r$ as the rate that clears the credit market. Any shock that reduces banking sector credit, such as credit losses that reduce bank equity $E$ or a delveraging episode where banks reduce leverage and lending for given equity $E$, will result in a shift upward of the credit supply curve, leading to an increase in the borrowing rate $r$. The increased borrowing rate will then kick in motion the combination of reduced productivity, reduced wages and lower offshoring activity described in Sections 2 and 3. We summarize our main result as follows.

**Proposition 2** A reduction in banking sector credit results in (1) an increase in the borrowing rate $r$ (2) fall in output $Y$, (3) fall in productivity per worker, (4) fall in the wage $w$ and (5) fall in the offshoring activity of firms.
A corollary of (5) is that trade volumes will also fall, with the decline being magnified by the extent of vertical specialization of production as formalized by the length of production chains.

5 Further Avenues for Research

Financial shocks that raise the cost of financing can have a substantial impact on macro variables through their impact on the cost of working capital. Our results derive from the feature that production takes time and the operation of a production chain entails heavy demands on financing. One consequence of this feature is that long production chains are sustainable only when credit is cheap, and chains that have become over-extended are vulnerable to financial shocks that raise the cost of borrowing. The financial crisis of 2007-2009 fits this description well.

Our model has been deliberately stark so as to highlight the role of working capital. We have abstracted away from many of the standard ingredients that have been used to model financial frictions in the macro literature. We have no fixed capital, no savings decisions, nor labor supply decisions. Having turned off these intertemporal and labor supply choices, we can isolate the effect of working capital better.

Although much of the discussion of financial frictions in the economy has focused attention on fixed investment, the components of working capital have fluctuated in a much more volatile way in the recent crisis. Figure 11 plots the annual capital expenditure of non-farm, non-financial firms in the United States, taken from the Federal Reserve’s Flow of Funds series. Fixed investment fell in 2008 and 2009, but the percentage falls are small (especially in 2008). Inventories fell much more dramatically during the crisis, turning negative in 2008 and especially in 2009.

Our results also relate to the literature on financial frictions and their impact on macro activity. Gilchrist, Yankov, and Zakrajsek (2009) documents
evidence that credit spreads have substantial effect on macro activity measures, and Hall (2010, 2011) models fluctuations in fixed investment through financing costs that are amplified by distortions in the product market. Ohanian (2010) is more skeptical about the effect of financial frictions citing the large cash holdings in firms, and the fact that firms rely mostly on internal funds for fixed investment. The business cycle accounting literature in the manner of Chari, Kehoe and McGrattan (2007) finds lack of clear-cut quantitative evidence on deviations of fixed investment relative to the benchmark model. The contribution of our paper relative to this large literature is to highlight the working capital channel of financial frictions, and show how financing cost can impact output even in a model without physical capital or labor/product market distortions.

Working capital is more familiar to the literature on financial crises, especially those in emerging economies. Calvo, Izquierdo and Talvi (2006) document several stylized facts that appear consistently during financial crises, such as the fact that credit and total factor productivity drop sharply with the
onset of the crisis but that employment drops to a lesser extent. Our model addresses these features, and our deliberately stark modeling choices enable a relatively clean identification of the working capital channel of financial shocks. Neumeyer and Perri (2005) and Mendoza (2010) have emphasized working capital shortages in their models of fluctuations in emerging economies. although their modeling relies on quantitative constraints on firms’ financing.

Schwartzman (2010) takes the ratio of inventories to cost of goods sold as a measure of the “time to produce” for the firm, and shows that cross-section variation in the ratio is mirrored in the reduction in output during crisis periods. Raddatz (2006, 2010) also presents cross-section evidence using firm level data that financial shocks affect firm level financing needs as revealed through components of working capital. These cross-section empirical studies have the potential to provide the identification for empirical studies that attempt to quantify the impact of tighter financial conditions.

Our study suggests that our understanding will be expanded from the complementary effort to shed light on the micro-level contracting details of trade finance at the firm level. Antras and Foley (2011) use firm-level trade finance data to address the prevalence of various trade financing terms, and how such terms vary over the cycle in response to changes in financial conditions. They show, for instance, that cash in advance is prevalent when contractual enforcement is likely to be a problem, but interestingly, they also find that cash in advance becomes more prevalent during the crisis, especially for new customers who do not yet have established trade relationships. Such cyclical variation in trade terms presents opportunities for studying the impact of financial conditions on trade.
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