Uncertainty and Trade Agreements*

Nuno Limão  Giovanni Maggi

This Draft: June 25, 2012

Abstract

We examine the links between trade agreements and economic/political uncertainty. We disentangle the "uncertainty-managing" motive for a TA from the more standard "mean motive," identify the conditions under which there is an uncertainty-reducing motive, and study what determines the importance of this motive relative to the mean motive. We show that a standard trade model with income-risk neutrality generates a "puzzle", namely that there tends to be an uncertainty-increasing motive for a TA. With income-risk aversion, on the other hand, the uncertainty-managing motive for a TA is determined by interesting trade-offs, which in turn can be linked to observable (or estimable) quantities. We highlight that the origin of the shocks (country-specific or global) and their type (shocks to economic fundamentals or to political preferences) is important in determining the direction and strength of the uncertainty-managing motive. We also examine the investment and trade impacts of the uncertainty-managing component of a TA, and show that these may not always go in the direction that policy practitioners highlight when describing the benefits of TAs.

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*Limao: Department of Economics, University of Maryland, College Park, MD 20742; email: limao@econ.umd.edu. Maggi: Department of Economics, Yale University, New Haven, CT 06511; email: giovanni.maggi@yale.edu. We thank Mostafa Beshkar, Swati Dhingra and the participants in the 2012 AEA meetings and in the 2012 Venice CAGE/CEP Workshop for helpful comments.
1. Introduction

Policy practitioners often argue that a central benefit of trade agreements is to reduce trade policy uncertainty. Indeed, the WTO and other trade agreements (TAs) explicitly state that one of their goals is to increase the predictability of the trade policy environment.\(^1\) The recent great recession illustrates the potential value of such agreements in reducing trade policy uncertainty. During that crisis there was widespread fear of disastrous trade wars, such as those in the 1930’s that preceded (and spurred) the creation of the GATT. But while applied protection increased, the worst fears were not realized (cf. Bown, 2011). It is at least conceivable that one of the reasons why trade wars did not happen is that countries are constrained by TAs that limit policy variation even under large shocks.

In spite of the importance that policy makers and international institutions attribute to the notion of an uncertainty-reducing role of TAs, we know little about its theoretical underpinnings. A large body of theory has explored the possible roles of a TA as a means to correct international policy externalities (cf. Grossman and Helpman [1995], Bagwell and Staiger [1999] and Ossa [2011]) and to allow governments to commit vis-a-vis domestic actors (cf. Maggi and Rodriguez-Clare [1998] and Limão and Tovar-Rodriguez [2011]). But this research focuses on the role of a TA in managing the level of trade barriers, not their uncertainty. A key objective of our paper is a formal investigation of the nexus between trade agreements and economic/political uncertainty.

More specifically, our aim is to explore the conditions under which there is a policy-uncertainty-reducing motive for a TA, analyze what determines the relative importance of the “uncertainty motive” versus the more standard “mean motive,” and investigate the impacts of the uncertainty-managing component of a TA on investment and trade. We believe that disentangling the uncertainty motive from the mean motive is important to gain a better understanding of where the gains from a TA come from.

We focus on a scenario where there are no frictions in contracting between governments, so that the TA is a complete contingent contract.\(^2\) The optimal TA will in general change the policy distribution relative to the one arising in the noncooperative scenario. These two distributions will have different means, as suggested by the literature on TAs under certainty. Thus, to isolate the uncertainty-managing role for a TA we perform the following thought experiment: we ask if there is a role for a “mean preserving agreement” (MPA), i.e. an agreement that keeps the average trade barriers at the noncooperative level. If the optimal MPA leads to a distribution that is different from the noncooperative one, we say that there is an “uncertainty-managing motive” (or simply an “uncertainty motive”) for a TA, and if the MPA distribution exhibits lower uncertainty than the noncooperative one we say that there is an “uncertainty-reducing

\(^1\)For example, the WTO’s web site states that “Just as important as freer trade – perhaps more important – are other principles of the WTO system. For example: non-discrimination, and making sure the conditions for trade are stable, predictable and transparent.” As another example, the 2001 Mexico-EU Free Trade Area states that its goals are “to create an expanded and secure market for goods and services” and “to ensure a stable and predictable environment for investment.”

\(^2\)If a TA reduces policy uncertainty solely because it is too costly to include contingencies in the contract, then it is hard to argue that the motive of the TA is to reduce policy uncertainty; rather, the reduction in uncertainty in this case would be a mechanical side effect of the contracting frictions.
motive” for the TA. 3

We start with a simple framework in which we specify reduced-form government objectives as functions of a trade policy and an underlying shock. Initially we focus on a setting where only the importing country (Home) chooses a trade policy, \( t \), and this policy exerts an international externality on the exporting country (Foreign). Home’s noncooperative choice of \( t \) is increasing in some shock, \( \lambda \), and both \( t \) and \( \lambda \) can potentially affect Foreign’s objective. This setting can be interpreted as one where Foreign is a small welfare-maximizing country that practices free trade. 4

We identify two key effects in determining whether there is an uncertainty motive for a TA. The first one is what we label as the “policy-risk preference” effect, 5 determined by the concavity/convexity of Foreign’s objective with respect to Home’s trade policy: when Foreign is policy-risk averse, this effect works in favor of an uncertainty-reducing motive. Intuition might suggest that this effect is all that matters for determining whether there is “too much” or “too little” risk in the noncooperative policy, since it determines the sign of the international “policy-risk externality.” And indeed this is the case when shocks affect Foreign only through the policy (“political economy” shocks). However, when the shocks affect Foreign directly, and not just via the policy (“economic” shocks), there is an additional effect that we label “policy-externality shifting” effect. This effect is given by the direct impact of the shock on the marginal externality imposed by the policy on Foreign: if an increase in \( \lambda \) strengthens this externality for a given level of the policy, then this effect works in favor of the uncertainty-reducing motive. Interestingly, therefore, in this case the sign of the international “policy-risk externality” is not sufficient to determine whether there is “too much” or “too little” risk in the noncooperative tariff.

Our next step is to examine how the sign of the uncertainty motive and its magnitude depend on economic fundamentals in the context of a specific trade model. We start by asking: what does a standard trade model predict regarding the uncertainty motive for a TA? To address this question we focus on a general equilibrium trade model with two goods, competitive markets and (initially) income risk neutral individuals.

We distinguish between shocks of two different types: importer-specific shocks, which originate in the importing country and affect the exporting country only through terms of trade (TOT); and common shocks, which affect both countries directly (e.g. global shocks or domestic shocks that are perfectly correlated).

In the case of importer-specific shocks, which we now discuss, the impact of the shocks on the exporting country is channeled through the TOT. As a consequence of this, the optimal MPA reduces TOT risk for the exporting country if and only if the latter is TOT-risk averse.

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3 We also consider an alternative thought experiment, which focuses on the tariff schedule that a government would unilaterally choose if it were constrained to deliver the same mean as the optimal agreement. If such “mean-preserving unilateral” choice exhibits more uncertainty than the optimal TA, we say that there is an uncertainty-reducing motive. In section 2.2 we will discuss the similarities and differences between the results under the two thought experiments, and the reasons we choose the MPA thought experiment as the base for our analysis.

4 While this is the natural interpretation given our focus on TAs, the setup can also be interpreted as applying to other policies with international externalities.

5 In this paper we use the words “risk” and “uncertainty” interchangeably.
Moreover, the optimal MPA changes \textit{trade policy} risk in the same (opposite) direction as TOT risk if the TOT move in the opposite (same) direction as the noncooperative tariff as a result of the shock. Intuitively, the possibility that trade policy risk may increase in spite of TOT-risk aversion arises when the policy can be used in a countercyclical way to offset the impact of economic shocks on TOT. In what follows, to fix ideas we focus the description of our results on the case in which the optimal MPA affects TOT risk and trade policy risk in the same direction.

When we examine the sign of the exporting country’s TOT-risk preference, we find that in the presence of income-risk neutral individuals the country tends to be TOT-risk \textit{loving}. As a consequence, the optimal MPA tends to \textit{increase} trade policy/TOT risk under importer-specific shocks. We regard this as an interesting “puzzle” arising from the standard trade model: this model seems at odds with the often-heard informal argument regarding the uncertainty-reducing role of TAs. For this reason we next consider a natural modification of the standard model, namely introducing individual income-risk aversion.

In the presence of income-risk aversion, the uncertainty-managing motive for a TA is determined by interesting trade-offs. We find that the optimal MPA reduces trade-policy/TOT risk if income-risk aversion is sufficiently high; and for a given degree of income-risk aversion, the optimal MPA is more likely to decrease trade policy/TOT risk when (i) the economy is more open; (ii) the export supply elasticity is lower; and (iii) the economy is more specialized. We also highlight that the degree of openness has a non-monotonic effect on Foreign’s attitude toward TOT risk.

We then focus on the case of common shocks. In this case, knowing whether a country is averse to TOT risk is no longer sufficient to determine whether the optimal MPA reduces TOT risk. The reason lies in the TOT-externality shifting effect mentioned above. This effect works in favor of the uncertainty-reducing motive if Foreign’s openness (adjusted by a factor reflecting the marginal utility of income) increases with the shock \( \lambda \) for given TOT. Thus, even if the Foreign country is TOT-risk neutral there will be an uncertainty-managing motive for a TA. And as a further consequence, there \textit{may} be an uncertainty-reducing motive for a TA even with income-risk neutral individuals.

At a broad level, the results described above underscore a key point of this paper: the origin of the shocks (country-specific or global) and their type (shocks to economic fundamentals or to political preferences) is important in determining the direction and strength of the uncertainty-managing motive.

Regardless of the nature of the shocks, however, we show that, if one is willing to assume that the model is true, one can in principle check directly in the data whether there is an uncertainty-reducing motive for a TA. In particular, this is the case if the (adjusted) degree of openness co-varies with the noncooperative tariff as a result of the underlying shocks.

Next we provide a measure of the relative importance of the uncertainty motive for a TA versus its “mean motive”. We isolate the latter motive by focusing on “uncertainty-preserving agreements” (UPAs), meaning agreements that shift \( t(\lambda) \) uniformly for all shock levels.\textsuperscript{7} Then

\textsuperscript{6}This feature is essentially due to the convexity of indirect utilities in prices, reflecting the ability of firms and consumers to make decisions after uncertainty is resolved, and has already been noted by others, e.g. Flemming et al [1977] and Eaton [1979].

\textsuperscript{7}This type of parallel shift of \( t(\lambda) \) preserves all the higher central moments of the distribution of \( t \) (variance,
we consider a local approximation of the gain from an MPA relative to the gain from a UPA (starting from the noncooperative policy). We take this relative gain as a measure of the relative importance of the uncertainty motive for a TA, and show how it depends on the degree of openness, the export supply elasticity, the degree of income diversification and the degree of income-risk aversion, as well as the variance of the shocks.

We further observe that parameter changes that (weakly) increase both the value of an MPA and the value of the UPA will also (weakly) increase the value of an agreement that jointly improves policy mean and policy risk. In particular, we show that an increase in the variance of the shocks, which we interpret as an increase in the degree of uncertainty in the economic/political environment, increases the value of such an agreement. This in turn suggests that governments should have a higher propensity to sign a trade agreement when the environment is more uncertain.

In our basic model, factors can be allocated after uncertainty is resolved, but in section 3 we extend the model to allow for ex-ante investments, i.e. allocation decisions that must be made before the resolution of uncertainty. We focus on the ex-ante allocation of a single factor, “capital”, in the Foreign country. We show that the conditions that determine the uncertainty motive for a TA in the presence of ex-ante investments are similar to those derived in the static model, provided the market allocation of capital is efficient given Home’s trade policy. The absence of a separate uncertainty motive associated with investment arises because, even though the TA can affect capital allocation, this has no first order effect on the Foreign objective under efficient allocation.

Even if there is no separate “investment” motive for a MPA, such an agreement will affect equilibrium investment and trade relative to the noncooperative equilibrium. For the case of importer-specific shocks, we show that if income-risk aversion is sufficiently strong and the support of the shock sufficiently small, the optimal MPA will reduce trade policy uncertainty and increase investment in the export sector. Under those conditions we also find that expected trade increases if the export supply elasticity does not increase too rapidly with TOT. But we also highlight that these effects can go in the counterintuitive direction if the support of the shock is not small or if shocks are common: in these cases it is possible that an uncertainty-reducing TA leads to less investment in the export sector and less expected trade.

In section 4 we return to the basic model and extend the analysis to allow for two (symmetric) policy-active countries. The general condition for the optimal MPA to reduce trade policy uncertainty still includes the foreign-policy-risk aversion and externality-shifting effects. However, now there is an additional effect, which works in favor of an uncertainty-reducing motive if tariffs are strategic substitutes, and against it they are strategic complements. We also highlight that in this symmetric-countries setting, the externality-shifting effect will always be present independently of the nature of the shocks. Therefore, even if countries are TOT-risk neutral there will be an uncertainty-managing motive for a TA.

While most theoretical analyses of TAs abstract from uncertainty, some recent work explores how uncertainty of a specific type can explain certain design features of TAs. In particular, Amador and Bagwell (2011) argue that private information about political economy shocks can explain why TAs such as the WTO employ rigid tariff caps (as do Beshkar and Bond [2010] skeweness etc.).
by relying on costly verification of such shocks). In contrast to these recent papers, we isolate and examine the “pure” uncertainty-managing motive for a TA in the absence of contracting frictions. We also allow for a broader set of shocks and show how the nature of the shocks affects the uncertainty-managing motive for a TA.\(^8\)

Also, there is a small but growing empirical literature on TAs and uncertainty. Cadot et al. (2011) show evidence that regional trade agreements reduce trade-policy volatility in agriculture.\(^9\) Rose (2004) and Mansfield and Reinhardt (2008) empirically examine the effect of TAs on the volatility of trade flows, but this is different from the volatility of trade policies. Finally, the theoretical impact of uncertainty reducing features of TAs on trade flows and firms’ entry investments into foreign markets is modelled and tested by Handley and Limão (2011) and Handley (2011). But whereas they take the actual policy and agreement as exogenous we make it endogenous and employ different economic structure.

We structure the paper in the following way. In section 2 we lay out a basic framework with only one policy-active country, and we disentangle the uncertainty from the mean motive for an international agreement. In section 3 we impose a more specific trade structure and explore the economic determinants of the uncertainty motive for a TA under different types of shocks. In section 4 we extend the model to allow for ex-ante investments and examine how the uncertainty-managing component of a TA affects investment and trade. In section 5 we extend the analysis to a setting with two policy-active, symmetric countries. We conclude in section 6.

2. Basic framework

To make our points transparent, we start by focusing on a two-country setting where only one country chooses a trade policy, hence there is a one-way international policy externality.

There are two countries (Home and Foreign) and two sectors. The Home government chooses a trade barrier \(t\),\(^{10}\) while the Foreign government is passive. At this stage there is no need to impose any further economic structure, but in the following section we will interpret this setting as a perfectly competitive world where Home is a large country and Foreign is a small, welfare-maximizing country; and in section 5 we will extend the model to allow for two large policy-active countries.\(^{11}\)

We let \(\lambda\) denote a shock to the economic (or political-economy) environment, distributed according to the c.d.f. \(F_\lambda(\lambda)\). This shock can affect Home, Foreign, or both. In the first part of the paper we consider reduced-form government objectives as functions of the trade policy \(t\)

\(^{8}\)It is worth noting that in Amador and Bagwell (2011) the shock is importer specific and of the political economy type. Moreover the exporter’s objective is convex in foreign tariffs so in the absence of contracting frictions their model would generate an uncertainty-*increasing* motive for the TA. We also suspect that in their model (with contracting frictions), if one imposed a mean-preservation constraint, again the agreement would increase policy uncertainty.

\(^{9}\)They also find that the WTO’s agricultural agreement reduced agricultural trade-policy volatility, in spite of the weak disciplines involved, but the effect is only weakly identified

\(^{10}\)For now we interpret \(t\) as a general trade barrier, but when we introduce the general-equilibrium trade structure in the next section, we will define \(t\) as the logarithm of the ad-valorem tariff.

\(^{11}\)In the literature on trade agreements there is a small tradition of models with a small country and a large country, a prominent example being McLaren (1997).
and the shock parameter $\lambda$. In the next section we will “open up” the black box of government objectives and introduce a more specific trade structure.

The Home government objective is $G(t, \lambda)$. We assume $G$ is concave in $t$ and satisfies the single crossing property $G_{t\lambda} > 0$. The Foreign government objective is $G^*(t, \lambda)$, which we assume decreasing in $t$. The governments’ joint payoff is $G_W(t, \lambda) = G(t, \lambda) + G^*(t, \lambda)$. We assume $G_W$ is concave in $t$ and satisfies the single crossing property $G_{t\lambda}^W > 0$.

We start by describing the non-cooperative policy choice. We assume the Home government observes $\lambda$ before choosing its trade policy, hence the noncooperative policy is given by:

$$t^N(\lambda) = \arg\max_t G(t, \lambda).$$

The single crossing property $G_{t\lambda} > 0$ implies that $t^N(\lambda)$ is increasing. The distribution of the shock, $F(\lambda)$ and the shape of the $t^N(\cdot)$ schedule induce a distribution for the noncooperative policy $t^N$.

We now describe our assumptions regarding the trade agreement (TA). We assume that the agreement is signed ex ante, before $\lambda$ is realized, so the timing is the following: (0) the TA is signed; (1) $\lambda$ is realized and observed by both countries; (2) $t$ is implemented and payoffs are realized.

We assume the TA maximizes the governments’ expected joint payoff $EG^W$, so the optimal TA is given by

$$t^A(\lambda) = \arg\max_t EG^W(t, \lambda).$$

The single crossing property $G^W_{t\lambda} > 0$ implies that $t^A(\lambda)$ is increasing.

We assume that $\lambda$ is verifiable, so the agreement can be made contingent on $\lambda$. If the TA were non-contingent (i.e. "rigid") because of contracting imperfections (such as costs of specifying contingencies or imperfect verifiability), then the tariff would be fixed over some range of shocks. Thus contracting frictions tend to lower trade policy uncertainty as a mechanical side effect. This is surely a relevant effect in reality, but not a very interesting one, so we want to abstract from it, and ask instead whether there are deeper reasons why a TA might reduce trade policy uncertainty, rather than a side effect of contracting frictions. In other words, we want to examine whether there can be theoretical foundations to the informal argument that reducing trade policy uncertainty is a goal of trade agreements. For this reason we believe that a scenario of complete contingent contracting is the natural benchmark to consider.\textsuperscript{13}

2.1. Isolating the uncertainty motive in a trade agreement

Clearly, the noncooperative policy is inefficient because an international policy externality is present, and this motivates governments to sign a TA. When we introduce an explicit trade

\textsuperscript{12}This implicitly assumes that international transfers are available.

\textsuperscript{13}While, as we explained above, the reason for this assumption is to shut down contract rigidity as a possible cause of reductions in policy uncertainty, we note that the GATT-WTO includes a number of contingent clauses, for example the "escape clauses" in GATT Articles XIX and XXVIII. For a model that endogenizes the degree to which a trade agreement is contingent, based on the presence of contracting costs, see Horn, Maggi and Staiger (2010).
structure in the next section, this policy externality will operate via the terms-of-trade, but for now this can be interpreted as a more general international policy externality.

The international policy externality is transmitted through the whole distribution of \( t \), so it is natural to distinguish two dimensions of such externality, a "mean externality" and an "uncertainty externality". The latter is the impact of a change in the degree of uncertainty in \( t \) on Foreign’s expected welfare \( EG^* \). The role of a TA is to address both of these dimensions, so conceptually we distinguish between a "mean motive" and an "uncertainty motive" for a TA.

In order to isolate the uncertainty motive for a TA, we consider the following thought experiment: if we shut down the “mean motive” by constraining the TA to keep the average \( t \) at the noncooperative equilibrium level, is there any role left for a TA? This is the idea behind our notion of "mean preserving agreement" (MPA). If the optimal MPA changes the riskiness of \( t \) relative to the noncooperative tariff \( t^N(\lambda) \), we say that there is an uncertainty-managing motive for a TA. And in this case, if the optimal MPA decreases (increases) the riskiness of \( t \) relative to \( t^N(\lambda) \), we say that there is an uncertainty-reducing (-increasing) motive for a TA.

Formally, the optimal MPA is defined as

\[
t^{MPA}(\lambda) = \arg \max_{t(\lambda)} EG^W(t(\lambda), \lambda) \text{ s.t. } Et(\lambda) = Et^N(\lambda).
\]

where the operator \( E \) denotes an expectation over \( \lambda \).

Before we study the optimal MPA, we provide some intuition by considering a local argument for the simplest possible case. Consider the case where \( \lambda \) affects Foreign only through the policy \( t \), so that its payoff is simply \( G^*(t) \). This is the case if \( \lambda \) is a pure preference shock for the Home government, which could be interpreted as a political economy shock.

Start from the noncooperative tariff \( t^N(\lambda) \) and ask: how can we change the tariff schedule locally to achieve an increase in \( EG^W = EG + EG^* \), while preserving the mean of the tariff? Since \( t^N(\lambda) \) maximizes \( EG \), a small change from \( t^N(\lambda) \) will have a second-order effect on \( EG \) and a first-order effect on \( EG^* \). Clearly, then, to achieve an increase in \( EG^W \) we must increase \( EG^* \). Suppose \( G^* \) is convex in \( t \): then if we change the tariff schedule (slightly) such that the new tariff is a mean-preserving spread of \( t^N(\lambda) \) then this will increase \( EG^* \) (by the now standard Rothschild-Stiglitz, 1970, equivalence result) and thus \( EG^W \) will also increase. Likewise, if \( G^* \) is concave in \( t \), we can achieve an increase in \( EG^W \) by changing the tariff schedule in such a way that the new tariff is a mean-preserving compression of \( t^N(\lambda) \). Therefore this argument suggests that the key condition determining whether the optimal MPA increases or decreases policy uncertainty is the concavity/convexity of the exporter’s objective with respect to the policy.

Of course, the argument above suggests only a sufficient condition for local improvement over the noncooperative outcome; in particular, one can improve over the noncooperative outcome in many other ways, including by changing the tariff schedule in ways that are neither a mean-preserving compression nor spread of \( t^N(\lambda) \). But as we show below, this intuition does carry over to the globally optimal MPA (when the single crossing property is satisfied).

More importantly, however, the Rothschild-Stiglitz type argument above applies to the case of a pure “political-economy” shock in the importing country, but not if the shock affects the exporting country directly, not only through the policy \( t \). In this case, it is not enough to know
whether the exporter’s objective is concave or convex with respect to \( t \) to determine how the optimal MPA will change policy uncertainty. In what follows, we will first derive the result formally, and then we will provide some intuition to highlight its basic logic.

To derive the FOCs for the optimal MPA problem in (2.1) we set up the Lagrangian:

\[
L = EG^W(t, \lambda) + \psi \left( Et^N(\lambda) - Et(\lambda) \right)
\]

(2.2)

Since the multiplier \( \psi \) is constant with respect to \( \lambda \), we can rewrite the Lagrangian as follows

\[
L = \int \left[ G^W(t, \lambda) + \psi t^N(\lambda) - \psi t(\lambda) \right] dF_\lambda(\lambda)
\]

(2.3)

and since we can maximize this pointwise we obtain the following FOCs

\[
G_t^W(t(\lambda), \lambda) = \psi \text{ for all } \lambda \\
Et^N(\lambda) = Et(\lambda)
\]

Note that the FOC requires the marginal contribution of \( t \) to global welfare, \( G_t^W \), to be equalized across states (realizations of \( \lambda \)), and in particular \( G_t^W \) should be equal to the multiplier \( \psi \), which is easily shown to be negative. This will play a key role in our proofs below. Also note that the FOC for the unconstrained optimal agreement is given by \( G_t^W(t, \lambda) = 0 \), so both for the unconstrained optimum and for the optimal MPA, \( G_t^W \) is equalized across states, but in the former case it is equalized at zero, while in the latter case it is equalized at some negative constant.

Using the FOC we can prove:

**Lemma 1.** (a) If \( \frac{d}{d\lambda} G_t^*(t^N(\lambda), \lambda) < 0 \) for all \( \lambda \), then \( t^{MPA}(\lambda) \) intersects \( t^N(\lambda) \) once and from above. (b) If \( \frac{d}{d\lambda} G_t^*(t^N(\lambda), \lambda) > 0 \) for all \( \lambda \), then \( t^{MPA}(\lambda) \) intersects \( t^N(\lambda) \) once and from below. (c) If \( \frac{d}{d\lambda} G_t^*(t^N(\lambda), \lambda) = 0 \) for all \( \lambda \), then \( t^{MPA}(\lambda) = t^N(\lambda) \) for all \( \lambda \).

**Proof:**

We start by proving part (c). The schedules \( t^{MPA}(\lambda) \) and \( t^N(\lambda) \) are clearly continuous. The mean constraint and the continuity of \( t^{MPA}(\lambda) \) and \( t^N(\lambda) \) ensure the existence of at least one intersection. Consider one such intersection \( \hat{\lambda} \), so that \( t^{MPA}(\hat{\lambda}) = t^N(\hat{\lambda}) \). By the FOC, \( G_t^W(t^N(\hat{\lambda}), \hat{\lambda}) = \psi \). Since \( G_t(t^N(\hat{\lambda}), \hat{\lambda}) = 0 \) this implies \( G_t^*(t^N(\hat{\lambda}), \hat{\lambda}) = \psi \). Now if \( \frac{d}{d\lambda} G_t^*(t^N(\lambda), \lambda) = 0 \) then \( G_t^*(t^N(\lambda), \lambda) = \psi \) for all \( \lambda \), which in turn implies \( G_t^W(t^N(\lambda), \lambda) = \psi \) for all \( \lambda \). Therefore the schedule \( t^N(\lambda) \) satisfies the FOC, hence \( t^{MPA}(\lambda) = t^N(\lambda) \) for all \( \lambda \).

We next prove part (a). Again, \( t^{MPA}(\lambda) \) and \( t^N(\lambda) \) must intersect at least once. We now argue that if \( \frac{d}{d\lambda} G_t^*(t^N(\lambda), \lambda) < 0 \) then \( t^{MPA}(\lambda) \) can only intersect \( t^N(\lambda) \) from above. This, together with continuity, will also ensure the uniqueness of the intersection.

We argue by contradiction. Suppose \( t^{MPA}(\lambda) \) intersects \( t^N(\lambda) \) at some point \( \hat{\lambda} \) from below. Consider two values of \( \lambda \) on the opposite sides of this intersection, \( \lambda_1 < \hat{\lambda} < \lambda_2 \), such that \( t^{MPA}(\lambda_1) < t^N(\lambda_1) \) and \( t^{MPA}(\lambda_2) > t^N(\lambda_2) \).

Recalling that \( G_t(t^N(\lambda), \lambda) = 0 \) and \( \frac{d}{d\lambda} G_t^*(t^N(\lambda), \lambda) < 0 \) for all \( \lambda \), then

\[
G_t^W(t^N(\lambda_2), \lambda_2) = G_t^*(t^N(\lambda_2), \lambda_2) < G_t^*(t^N(\lambda_1), \lambda_1) = G_t^W(t^N(\lambda_1), \lambda_1)
\]
These inequalities and the concavity of $G^W$ in $t$ imply

$$G^W_t(t^{MPA}(\lambda_2), \lambda_2) < G^W_t(t^N(\lambda_2), \lambda_2) < G^W_t(t^N(\lambda_1), \lambda_1) < G^W_t(t^{MPA}(\lambda_1), \lambda_1)$$

This contradicts the FOC, which requires $G^W_t$ to be equalized across states.

The proof of part (b) is analogous to that of part (a).

QED

Figure 1 illustrates Lemma 1 graphically for the case $\frac{d}{d\lambda} G^*_t(t^N(\lambda), \lambda) < 0$. The basic intuition for the result can be conveyed by focusing on the case in which $\lambda$ can take only two values, say $\lambda^H$ and $\lambda^L$. Let us start from the noncooperative schedule $t^N(\lambda)$ and ask: how can we improve the ex-ante joint payoff? Given the mean-preservation constraint, there are only two ways to deviate from $t^N(\lambda)$: decreasing $t$ for $\lambda = \lambda^H$ and increasing $t$ for $\lambda = \lambda^L$; or vice-versa. Recall that $t^N(\lambda)$ is increasing, that the marginal joint payoff is $G^W_t = G_t + G^*_t$, and that $G_t = 0$ at $t^N(\lambda)$. Now, if $\frac{d}{d\lambda} G^*_t(t^N(\lambda), \lambda) < 0$ then the marginal joint benefit of lowering $t$ for $\lambda = \lambda^H$ is higher than the marginal cost of raising $t$ for $\lambda = \lambda^L$, and hence this deviation improves the objective. Similarly, if $\frac{d}{d\lambda} G^*_t(t^N(\lambda), \lambda) > 0$ the objective can be improved by making the opposite deviation, that is by making the $t(\lambda)$ schedule steeper than $t^N(\lambda)$. And if $\frac{d}{d\lambda} G^*_t(t^N(\lambda), \lambda) = 0$, the marginal benefit of lowering $t$ for $\lambda = \lambda^H$ is equal to the marginal cost of raising $t$ for $\lambda = \lambda^L$, so a (marginal) deviation cannot improve the objective, and hence $t^N(\lambda)$ is the optimum. The proof of Lemma 1 extends this basic logic to the case of continuous $\lambda$.

Note that Lemma 1 does not rely on the single crossing properties we assumed for $G$ and $G^W$, while the next result does.

From Lemma 1 it is a small step to derive our first proposition. In the proposition, we say that the optimal MPA reduces (increases) policy uncertainty if $t^{MPA}(\lambda)$ is a mean preserving compression (spread) of $t^N(\lambda)$. Also, we use a superscript $N$ to indicate that a function is evaluated at $t^N(\lambda)$.
Proposition 1. (a) If \( \frac{d}{d\lambda} G^*_t(t^N(\lambda), \lambda) < 0 \) (or equivalently \( \frac{G^*_t(t^N(\lambda), \lambda)}{G^*_t(\lambda^N, \lambda)} > \frac{G^*_t(\lambda^N, \lambda)}{G^*_t(\lambda^N, \lambda)} \)) for all \( \lambda \), the optimal MPA reduces policy uncertainty. (b) If \( \frac{d}{d\lambda} G^*_t(t^N(\lambda), \lambda) > 0 \) (or equivalently \( \frac{G^*_t(t^N(\lambda), \lambda)}{G^*_t(\lambda^N, \lambda)} < \frac{G^*_t(\lambda^N, \lambda)}{G^*_t(\lambda^N, \lambda)} \)) for all \( \lambda \), the optimal MPA reduces policy uncertainty. (c) If \( \frac{d}{d\lambda} G^*_t(t^N(\lambda), \lambda) = 0 \) for all \( \lambda \), the optimal MPA is given by the noncooperative tariff \( t^{MPA}(\lambda) = t^N(\lambda) \).

Proof:

First observe that \( G_{t\lambda} > 0 \) implies \( t^N(\lambda) \) is increasing, and \( G_{t\lambda}^W > 0 \) implies \( t^{MPA}(\lambda) \) is increasing (this can be proved by implicitly differentiating the FOC for the MPA problem and recalling that \( \psi \) is independent of \( \lambda \)).

Part (a). Note that \( \frac{dt^N}{d\lambda} = \frac{G_{t\lambda}(t^N(\lambda), \lambda)}{G_{t\lambda}(t^N(\lambda), \lambda)} \), so the condition \( \frac{G^*_t(t^N(\lambda), \lambda)}{G^*_t(\lambda^N, \lambda)} > \frac{G^*_t(\lambda^N, \lambda)}{G^*_t(\lambda^N, \lambda)} \) is equivalent to \( G^*_t \cdot \frac{dt^N}{d\lambda} + G^*_t(t^N(\lambda), \lambda) < 0 \), or \( \frac{d}{d\lambda} G^*_t(t^N(\lambda), \lambda) > 0 \).

By Lemma 1, in this case \( t^{MPA}(\lambda) \) intersects \( t^N(\lambda) \) once and from above. We show that the random variable \( t^N(\lambda) \) is a second order stochastic shift of the random variable \( t^{MPA}(\lambda) \), which together with the fact that these two random variables have the same mean implies that the former is a MPS of the latter. Let \( \lambda^N(t) \) denote the inverse of \( t^N(\lambda) \) and \( \lambda^{MPA}(t) \) the inverse of \( t^{MPA}(\lambda) \); these inverse functions exist because \( t^N(\lambda) \) and \( t^{MPA}(\lambda) \) are both increasing. Also, let \( \hat{t} \) be the value of \( t \) for which the two curves intersect.

The cdf of \( t^N \) is given by \( F_N(t) = Pr(t^N(\lambda) \leq t) = Pr(\lambda \leq \lambda^N(t)) \) and the cdf of \( t^{MPA} \) is given by \( F_{MPA}(t) = Pr(t^{MPA}(\lambda) \leq t) = Pr(\lambda \leq \lambda^{MPA}(t)) \). Lemma 1 implies that \( \lambda^{MPA}(t) < \lambda^N(t) \) for all \( t < \hat{t} \) and \( \lambda^{MPA}(t) > \lambda^N(t) \) for all \( t > \hat{t} \), which in turn implies that \( F_{MPA}(t) < F_N(t) \) for all \( t < \hat{t} \) and \( F_{MPA}(t) > F_N(t) \) for all \( t > \hat{t} \). This implies that \( t^N(\lambda) \) is a second order stochastic shift of \( t^{MPA}(\lambda) \), as claimed.

Part (b) can be handled with a similar argument as the one above, and part (c) was already proved in Lemma 1.

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Proposition 1 states that the presence of an uncertainty-managing motive for a TA is determined by how the shock \( \lambda \) affects the international policy externality \( G^*_t \), taking into account its direct effect and its indirect effect through the policy. In particular, if \( G^*_t(t^N(\lambda), \lambda) \) is decreasing (increasing) in \( \lambda \) there is an uncertainty-reducing (increasing) motive for a TA. Writing \( G^*_t(t^N(\lambda), \lambda) = G^*_t \cdot \frac{dt^N}{d\lambda} + G^*_t \), it is clear that the uncertainty motive for a TA depends on two key determinants: (i) Foreign’s policy-risk preference (captured by \( G^*_t \) and weighted by \( \frac{dt^N}{d\lambda} \)), and (ii) the direct impact of the shock \( \lambda \) on the policy externality holding \( t \) constant (as captured by \( G^*_t \)). Recalling that \( G^*_t < 0 \), when \( G^*_t < 0 \) the shock shifts “up” the marginal externality for given \( t \) and works in favor of an uncertainty-reducing motive, and shifts it “down” if \( G^*_t > 0 \), so we refer to this as the policy-externality shifting effect.

Proposition 1 makes clear that the source of the uncertainty matters a great deal. In particular, we can distinguish between two types of shock: (1) a “policy preference” (or “political economy”) shock, which affects the Foreign country only through policy uncertainty (in which case \( G^* = G^*(t) \)); and (2) an “economic” shock, which affects the Foreign country not only indirectly through the policy \( t \) but also directly (in which case \( G^* = G^*(t, \lambda) \)).
Let us focus first on case (1). In this case, Proposition 1 says that the optimal MPA reduces policy risk if and only if $G_{tt}^* < 0$, so the uncertainty motive for a TA is determined solely by the Foreign government’s attitude toward foreign-policy risk. This confirms the initial intuition (described above) which is based on Rotschild and Stiglitz’s (1970) result: when Foreign’s objective is concave in $t$, a MPS in $t$ reduces $EG^*$, so there is a negative “risk externality” in trade policy, hence the noncooperative tariff is "too risky," and therefore there is value to an agreement that reduces that risk (with the opposite logic holding if Foreign’s objective is convex in $t$). We should highlight however that, even in this simpler case where the result is intuitive, it is far from self-evident. In principle the optimal MPA could lead to any mean-preserving change in the policy distribution relative to the noncooperative scenario, and since the MPS risk criterion is a partial ordering, it was not a priori obvious that the MPA distribution could be ranked in a MPS sense relative to the noncooperative distribution.$^{14}$

Next consider case (2), where the shock is of the “economic” type. In this case, the Rotschild-Stiglitz type intuition is no longer adequate: Proposition 1 states that the sign of $G_{tt}^N$ is no longer sufficient to determine whether the optimal MPA reduces or increases trade-policy risk. Interestingly, even though the only source of inefficiency in the noncooperative tariff is the international externality generated by the tariff, and the sign of the “risk externality” is given by the sign of $G_{tt}^{*N}$, the sign of this externality does not uniquely determine whether there is “too much” or “too little” risk in the noncooperative tariff.$^{15}$

To highlight further how this case differs from the previous one and provide an alternative line of intuition, suppose that Foreign’s objective is linear in $t$ (i.e. $G_{tt}^* = 0$), so that Foreign is trade-policy-risk neutral. Suppose further that $G_{t\lambda}^{*N} < 0$, so that the (negative) international externality is stronger when $\lambda$ is higher. Then intuitively, since the MPA tariff must have the same mean as the noncooperative tariff, it is jointly preferable for the two countries to reduce the tariff in high-$\lambda$ states, where the noncooperative tariff is higher, and increase it in low-$\lambda$ states, where the noncooperative tariff is lower, thus the optimal MPA lowers trade policy uncertainty.

To summarize, if shocks are of the political-economy type, the sign of $G_{tt}^N$ is sufficient to determine the direction of the uncertainty-managing motive for a TA, but in the case of “economic” shocks this is no longer true, and the externality-shifting effect ($G_{t\lambda}^*$) also matters.

2.2. An alternative thought experiment

One could consider an alternative thought experiment to isolate the uncertainty motive for a TA. Suppose the Home government can choose a tariff schedule $t(\lambda)$ subject to the constraint $Et(\lambda) = Et^A(\lambda)$ (i.e. the chosen tariff must have the same mean as the optimal TA), and

$^{14}$Moreover, in the proof we show that if $G_{tt}^* < 0$ ($> 0$) then the optimal MPA tariff is a “simple” mean preserving spread (compression) of the noncooperative tariff, meaning that the respective cdf’s cross only once.

$^{15}$Mathematically, the reason why the Rotschild-Stiglitz result cannot be applied here (so it is not necessarily true that if $G_{tt}^* > 0$ a mean preserving spread in $t$ increases $EG^*$), is that the Rotschild-Stiglitz result applies when utility is a function of a single random variable (in our case $G^*(t)$), while here $G^*$ is a function of two random variables, $t$ and $\lambda$, which are correlated.
consider the resulting "mean preserving unilateral" choice:

\[ t^{MPU}(\lambda) = \arg \max_{t(\lambda)} EG(t, \lambda) \text{ s.t. } Et(\lambda) = Et^A(\lambda). \]

If \( t^{MPU}(\lambda) \) is more risky than \( t^A(\lambda) \), then the noncooperative equilibrium involves "too much" policy risk, so we say that there is an uncertainty-reducing motive for a TA.

Applying a similar logic as for the MPA analysis, one can show that \( t^{MPU}(\lambda) \) is a mean preserving spread of \( t^A(\lambda) \), and hence there is an uncertainty-reducing motive for a TA, if

\[
\frac{d}{d\lambda} G_t^*(t^A(\lambda), \lambda) < 0 \tag{2.4}
\]

This condition is qualitatively similar to the one derived in the MPA analysis, and it confirms that the key determinant of the uncertainty motive for a TA is how the international externality \( G_t^* \) varies with the shock \( \lambda \), taking into account its direct effect and its indirect effect through the policy. But there is one important difference, namely that this condition is evaluated at the optimal agreement tariff \( t^A(\lambda) \) rather than the noncooperative tariff \( t^N(\lambda) \).

Note that if \( \lambda \) affects Foreign’s objective only through the policy \( t \), as in the case of a “political economy” shock, the condition becomes \( G_{tt}^*(\cdot) t^A(\lambda) < 0 \), or equivalently \( G_{tt}^*(\cdot) < 0 \) (since \( t^A(\lambda) > 0 \)). Thus (recalling the assumption that \( G_{tt}^* \) does not change sign) the two thought experiments in this case yield the same answer, namely that there is an uncertainty-reducing motive for a TA if and only if the Foreign government is averse to foreign-policy risk.

If \( \lambda \) is an “economic” shock, so that it affects Foreign’s objective not only through the policy \( t \) but also directly, the condition becomes \( G_{tt}^*(\cdot) t^A(\lambda) + G_{t\lambda}^*(\cdot) < 0 \). In this case, both thought experiments suggest that the uncertainty motive for a TA depends on Foreign’s policy-risk preference \( G_{tt}^* \) and on the externality shifting effect \( G_{t\lambda}^* \), but the relative weight of these two terms differs: in the MPA case \( G_{tt}^* \) is weighted by \( t^N(\lambda) \), whereas in the MPU case it is weighted by \( t^A(\lambda) \). This may or may not imply different answers across the two thought experiments, depending on whether the economic shock originates in the Home country or hits directly both countries. As we will show in the next section, if the shock originates in the Home country, so that it affects Foreign only through terms of trade (and since by assumption \( G_{tt}^* \) and \( G_{t\lambda}^* \) do not change sign), then \( \frac{d}{d\lambda} G_t^*(t(\lambda), \lambda) \) has the same sign regardless of whether it is evaluated at \( t^N(\lambda) \) or at \( t^A(\lambda) \). So with importer-specific shocks the two thought experiments yield the same qualitative answer. But if the shock affects both countries directly (common shock), then the two thought experiments may give different answers.

In what follows we base our analysis on the MPA thought experiment. The main reason is that, as we will show later, focusing on the MPA allows us to characterize the relative importance of the uncertainty motive in terms of quantities that can in principle be observed or estimated, while the MPU thought experiment does not share this desirable property.\(^{16}\)

\(^{16}\)To be more specific, we approximate the value of a small MPA starting from the noncooperative tariff, which is in principle observable. But we do not observe the MPU tariff so in order to approximate the value of a small mean preserving change around an observable situation we would need to do so around the optimal agreement. However, small changes around this tariff have no first-order effect on the joint payoff.
2.3. “Uncertainty motive” vs “mean motive”

Proposition 1 highlights that there is an uncertainty-managing motive for a TA as long as the policy externality $G_t^*$ is sensitive to the shock $\lambda$. Next we would like to characterize how important the uncertainty motive is in relation to the “mean motive”.

It seems reasonable to focus on the value of an MPA relative to the noncooperative tariff choice as a measure of the importance of the uncertainty motive in a TA. Define the value of an agreement $a$ as

$$V^a = EG^W(t^a(\lambda), \lambda) - EG^W(t^N(\lambda), \lambda).$$

Then one possible measure of the importance of the uncertainty motive would be $V^{MPA}/V^A$ (recall that $t^A(\lambda)$ denotes the optimal agreement). The problem with this “decomposition” is that the remaining share of the value of the agreement may not purely reflect a mean motive, since changing policy mean and policy uncertainty may interact (i.e. may be complementary or substitutable) in increasing the expected global payoff. Therefore we proceed in the following way: we isolate the mean motive by focusing on an “uncertainty-preserving agreement” (UPA) and consider the ratio $V^{MPA}/V^{UPA}$ as our measure of the relative importance of the uncertainty vs. the mean motive.

We define an uncertainty-preserving agreement in the following way. Let $t^{UPA} = t^N(\lambda) - \Delta$ be a parallel shift of the noncooperative tariff schedule (where we can focus on positive values of $\Delta$). The optimal UPA is then $t^{UPA}(\lambda) = t^N(\lambda) - \Delta^{UPA}$ where $\Delta^{UPA}$ is given by

$$\Delta^{UPA} = \arg \max_{\Delta} EG^W(t^N(\lambda) - \Delta, \lambda)$$

Note that $t^{UPA}(\lambda)$ has the same central higher moments as $t^N$ (variance, skewness, kurtosis), but a lower mean. Therefore the UPA as defined above seems the appropriate complement to the MPA, which keeps the mean constant but allows the rest of the distribution to change.

Rather than attempting to derive the exact value $V^{MPA}$, we take a shortcut and approximate the effect on $EG^W$ of a small mean-preserving change in the tariff, starting from the noncooperative tariff:

$$\frac{\partial EG^W(t^N(\lambda) - \varepsilon(\lambda - \bar{\lambda}), \lambda)}{\partial \varepsilon} \bigg|_{\varepsilon=0} = -E[G_t^N(\lambda)(\lambda - \bar{\lambda})]$$

$$\approx -E \left[ \left( G_t^{*N}(\bar{\lambda}) + \frac{dG_t^{*N}(\bar{\lambda})}{d\lambda} \cdot (\lambda - \bar{\lambda}) \right) (\lambda - \bar{\lambda}) \right]$$

$$= -\frac{dG_t^{*N}(\bar{\lambda})}{d\lambda} \cdot \sigma^2$$

where $\frac{dG_t^{*N}(\bar{\lambda})}{d\lambda}$ is shorthand for $\left( \frac{dG_t^{*N}(\lambda)}{d\lambda} \right)_{\lambda=\tilde{\lambda}}$, and we let $\bar{\lambda}$ and $\sigma^2$ denote respectively the mean and the variance of $\lambda$. The first line is due to $G_t^N = 0$ and the second results from a first order approximation of $G_t^{*N}(\lambda)$ around $\bar{\lambda}$.

Similarly, the value of a small UPA starting from the noncooperative tariff can be approxi-
mated as
\[
\frac{\partial E G^W (t^N (\lambda) - \Delta, \lambda)}{\partial \Delta} \bigg|_{\Delta=0} = -E[G_t^N (\lambda)]
\]
(2.6)

Therefore we obtain
\[
\frac{\partial}{\partial \varepsilon} \frac{\partial E G^W (t^N (\lambda) - \varepsilon (\lambda - \bar{\lambda}), \lambda)_{\varepsilon=0}}{\partial \Delta} \bigg|_{\Delta=0} \approx \left| \frac{d \ln G_t^N (\bar{\lambda})}{d \lambda} \right| \cdot \sigma^2
\]
(2.7)

Intuitively, the relative importance of the uncertainty motive for the TA is higher when (i) the shock \( \lambda \) has a larger impact on the intensity of the international externality \( (G_t^N) \), and (ii) when there is a higher degree of “fundamental” uncertainty, as captured by the variance of \( \lambda \).

One further question one may ask is: Under what conditions are the uncertainty and mean motives for a TA complements or substitutes? That is, when is the optimal TA more valuable than the sum of its two parts, or \( V^A > V^{MPA} + V^{UPA} \)? This in general can go either way depending on third derivatives, and in particular on \( G_{ttt}^W \) and \( G_{tt \lambda}^W \). Given that most model structures have no clear implications for third derivatives, we will not focus on this effect in the specific trade structure below.

Finally, one may ask whether the local approximation analysis presented above has something to say regarding the value of the optimal trade agreement. More specifically, if one asks what is the value of a partial move towards the optimal agreement starting from the noncooperative outcome, the answer is a simple corollary of the analysis above: the value of a (small) joint improvement in tariff mean and tariff uncertainty is a weighted average of expressions (2.5) and (2.6) above, with the weights determined by the relative change in \( \varepsilon \) and \( \Delta \). Thus, if a certain parameter change increases (weakly) both the value of an MPA and the value of a UPA, it will also increase (weakly) the value of the overall agreement. In the next section we will use this observation in the context of the specific trade structure.

### 3. Determinants of uncertainty and mean motives in a trade agreement

We now open up the black box of government objectives in order to examine how the uncertainty and mean motives for a TA depend on economic fundamentals.

We will consider in sequence two salient cases that differ in the nature of the underlying uncertainty. First, we will consider a shock that originates in the importing country (Home); this includes both the case of a pure policy preference shock and the case of an economic shock originating in Home (we will refer to this as the case of “importer-specific” shocks). Then we will consider a shock that affects directly both countries (we will refer to this as the case of “common” shocks); this could be a global shock or two perfectly correlated domestic shocks. As we will show, there is an important qualitative difference between these two types of uncertainty.

\[\text{Notice that, since we are approximating the gains from a small change starting from the noncooperative policy, we can ignore the complements/substitutes question mentioned in the previous paragraph.}\]
3.1. Importer-specific shocks

In keeping with the reduced-form model above, we consider a standard two-country, two-good trade model with competitive markets. Assume Home is the natural exporter of the numeraire good, indexed by 0, while Foreign (the small country) is the natural exporter of the other good, which has no index.

The Home country can choose an ad-valorem tariff on imports of the non-numeraire good. Let \( \pi (\text{resp. } \pi^*) \) denote the log of the price of the nonnumeraire good in Home (resp. Foreign), and \( t \) the log of the ad-valorem tariff rate. Clearly, by the usual arbitrage condition, if the tariff is not prohibitive then we must have \( \pi^* = \pi - t \). Since Foreign has no policy of its own, we can refer to \( \pi^* \) as the “terms of trade” (TOT). We will interpret \( \lambda \) as the log of the underlying shock so \( t' (\lambda) \) can be interpreted as the elasticity of the tariff factor with respect to the shock.

The reason our notation uses the log of the tariff rate and of the relative price is the following. In general equilibrium settings with uncertainty about relative prices, the conventional notion of risk based on arithmetic mean-preserving spread of relative prices leads to predictions that are sensitive to the choice of numeraire (see for example Flemming et al, 1977). These scholars have argued that a more robust approach is to define an increase in relative-price risk as a geometric mean preserving spread of the relative price, which is an arithmetic mean preserving spread of the log of the relative price (in our notation, \( \pi^* \)). For analogous reasons we define the trade policy as the log of the tariff rate.

We let \( x^*(\pi^*) \) denote Foreign’s export supply curve and \( m(\pi, t, \lambda) \) denote Home’s import demand function. Note that \( m \) depends not only on the domestic price \( \pi \) but also on \( t \), since the tariff affects revenue and hence income. Also note that we allow the shock to affect directly only the Home country. The Foreign country will be affected by the shock only indirectly through its TOT. The market clearing condition \( x^*(\pi^*) = m(\pi^* + t, t, \lambda) \) determines the equilibrium TOT as a function of the policy and the shock, \( \pi^* (t, \lambda) \). Note that since Foreign is small, the incidence of the tariff falls entirely on Foreign, hence \( \pi^* (t, \lambda) = \pi (\lambda) - t \).

Before we introduce any structure on preferences and technology, it is useful to highlight a corollary of Proposition 1 that applies when the shock is specific to the importing country, as we are assuming here, regardless of the nature of preferences and technology.

As we mentioned above, the Foreign country is affected by \( t \) and \( \lambda \) only through TOT, so we can write Foreign’s objective as \( G^* (\pi^* (t, \lambda)) = G^* (\pi (\lambda) - t) \). It is useful to note that we can think of Home as choosing the TOT \( \pi^* \) rather than the tariff, and rewrite Home’s objective as \( G (t, \lambda) = G (\pi (\lambda) - \pi^*, \lambda) \). We continue to assume \( G_{\pi t} < 0 \) and \( G_{\pi \lambda} > 0 \), but in order to ensure that Home’s noncooperative choice of \( \pi^* \) is monotonic in \( \lambda \) we make the additional regularity assumption that \( \frac{\partial^2 G}{\partial \pi^2} \) has the same sign for all \((\pi^*, \lambda)\); it is direct to verify that this assumption is satisfied if the elasticity of the TOT wrt the shock at the Nash, i.e. \( \frac{d \pi^*}{d \lambda} = \pi' (\lambda) - t N' (\lambda) \), does not change sign with \( \lambda \).

In the Appendix we prove:

**Corollary 1.** Suppose uncertainty originates in the importing country, so that it affects the exporting country only through TOT. Then the optimal MPA (i) reduces (increases) TOT uncertainty if \( G_{\pi^* \pi^*}^N < 0 \) (\( > 0 \)) and (ii) reduces (increases) trade policy uncertainty if \( G_{\pi^* \pi^*}^N \cdot (t N' (\lambda) - \pi' (\lambda)) < 0 \) (\( > 0 \)).
Corollary 1(i) is intuitive and states that the impact of the optimal MPA on TOT risk is determined solely by the Foreign country’s preference for TOT risk, as captured by the sign of $G_{\pi N}^*$. 

On the other hand, Corollary 1(ii) states that the impact of the MPA on trade policy risk is determined by whether the TOT risk preference and the total impact of the shock on TOT go in the same direction. Recall that the shock $\lambda$ is defined as one that increases the noncooperative tariff, and that the total impact of the shock on TOT (taking into account the direct effect and the indirect effect through the policy) is given by $\frac{dx^*}{d\lambda} = \pi' (\lambda) - t^N (\lambda)$. Then Corollary 1 says that the optimal MPA reduces trade policy risk if either (i) Foreign is averse to TOT risk ($G_{\pi N}^* < 0$) and TOT move in the opposite direction as the noncooperative tariff as a result of the shock, or (ii) Foreign is TOT risk loving ($G_{\pi N}^* > 0$) and TOT move in the same direction as the noncooperative tariff.

Intuitively, why does the sign of the impact of the shock on TOT matter? Suppose that Foreign is TOT-risk averse, so that it is globally optimal to reduce TOT risk. There is a single instrument, the tariff, to address two different sources of TOT risk with potentially offsetting effects: a “policy” risk (captured by $t^N (\lambda)$) and an “economic” risk (captured by $\frac{dx^*}{d\lambda}$). Without economic risk (e.g. in the case of a pure political economy shock), a mean preserving compression in $t$ clearly reduces TOT risk. And the same is true whenever policy risk is not offset by economic risk, so that $\frac{dx^*}{d\lambda}$ and $t^N (\lambda)$ have the opposite sign. But if the economic risk offsets the policy risk, so that TOT risk is reduced by increasing policy risk, then the optimal MPA will require an increase in policy risk to offset the economic risk in order to lower TOT risk.

One point that is already apparent here, and will be strengthened further in the next section, is that the source of the shock matters. As we noted above, if the shock is of a political-economy type, so that only “policy risk” is present, then the MPA decreases trade policy uncertainty if Foreign is TOT-risk averse; but if the shock is of an “economic” kind, it may happen that the MPA increases trade policy uncertainty even though Foreign is TOT-risk averse.

### 3.1.1. Preferences and technology

Our next step is to impose some standard assumptions on preferences and technology, in order to explore how the condition highlighted in Corollary 1 depends on economic fundamentals.

To make the key points we only need to specify the economic structure in the Foreign country. On the technology side, we assume CRS with a strictly concave PPF, so that supply functions are strictly increasing. This allows us to represent technology through a GDP (or revenue) function. Letting $p^*$ be the domestic relative price and $(q^*_0, q^*)$ the outputs, we define $R^*(p^*) \equiv \max_{q^*_0, q^*} \{ q^*_0 + p^* q^* \} \text{ s.t. } (q^*_0, q^*) \in Q^*$, where $Q^*$ is the set of feasible outputs.

Note that we allow for any number of production factors. But also note that, by assuming a strictly concave PPF, we are implicitly assuming that at least some production factors are flexible, in the sense that they can be allocated ex-post after the shock $\lambda$ is realized.

On the preference side, we assume that all citizens have identical and homothetic preferences. This implies that indirect utility takes the form $U \left( \frac{y}{\phi(p^*)} \right)$, where $y$ is income in terms of
numeraire and $\phi(p^*)$ a price index. It is natural to refer to $\frac{y_j}{\phi(p)}$ as individual $j$’s "real income."\(^{18}\) We assume $U(\cdot)$ exhibits constant relative risk aversion (CRRA), indexed by the parameter $\theta$. We further assume that all citizens have identical factor endowments and normalize the population measure to one, so that we can write the aggregate indirect utility as $V^*(p^*) = U\left(\frac{R^*(p^*)}{\phi(p^*)}\right)$. We assume that the Foreign government maximizes social welfare, so $G^* = V^*$. Finally, we assume that Foreign exports the nonnumeraire good for all values of $\lambda$ in its support (and hence for all values of $p^*$ in its support), or in other words, the trade pattern cannot switch as a result of the shock.

We start by focusing on the benchmark case of income-risk neutrality.

Since we are adopting the GMPS notion of risk (as we discussed above), it is natural to define risk neutrality as indifference with respect to a GMPS of real income, which corresponds to the case: $V^* = \ln \left(\frac{y^*}{\phi}\right)$, or $\theta \to 0$ in the CRRA specification. Thus in this case the welfare function is $V^*(p^*) = \ln \left(\frac{R^*(p^*)}{\phi(p^*)}\right)$.

The key step to apply Corollary 1 is to examine the Foreign country’s attitude toward TOT risk, as captured by $G^*_{\pi^*\pi^*}$. Writing $G^* = \ln R^*(p^*) - \ln \phi(p^*)$ and differentiating twice,

$$G^*_{\pi^*\pi^*} = \frac{\partial^2 \ln R^*}{\partial (\ln p^*)^2} - \frac{\partial^2 \ln \phi^*}{\partial (\ln p^*)^2}$$

Next note that the elasticity of $R^*(p^*)$ is given by the production share of the good exported by Foreign: $\frac{\partial \ln R^*}{\partial \ln p^*} = \frac{\nu^* q^*}{R^*}$. Differentiating this elasticity with respect to $\ln p^*$ and simplifying, we obtain:

$$\frac{\partial^2 \ln R^*}{\partial (\ln p^*)^2} = \frac{p^* q^*}{R^*} \cdot (1 - \frac{p^* q^*}{R^*}) + \frac{p^* q^*}{R^*} > 0.$$  

The expression for $\frac{\partial^2 \ln R^*}{\partial (\ln p^*)^2}$ is composed of two terms. The term $\frac{\nu^* q^*}{R^*}$ is related to output substitutability, and is higher when the supply function $q^*(p^*)$ is steeper. The term $\frac{\nu^* q^*}{R^*} \cdot (1 - \frac{\nu^* q^*}{R^*})$ captures the semi-elasticity of the share of income from exports with respect to prices at given quantities; it is zero if the economy is completely specialized and reaches a maximum of 0.25 when the export sector is half of GDP ($\frac{\nu^* q^*}{R^*} = \frac{1}{2}$).

Next note that employing Roy’s identity we obtain $\frac{\partial \ln \phi^*}{\partial \ln p^*} = \frac{\phi^*}{\phi^*}$, hence the elasticity of $\phi^*(p^*)$ is given by the consumption share of the export good: $\frac{\partial \ln \phi^*}{\partial \ln p^*} = \frac{\nu^* c^*}{R^*}$. It follows that

$$\frac{\partial^2 \ln \phi^*}{\partial (\ln p^*)^2} = \frac{\partial \left(\frac{\nu^* c^*}{R^*}\right)}{\partial p^*} \cdot p^*,$$

which is negative if the consumption share of Foreign’s export is decreasing in the price of this good. Adding things up and simplifying, we find

$$G^*_{\pi^*\pi^*}\mid_{\theta \to 0} = \Omega^* (\varepsilon^* + D^*)$$

\(^{18}\)Here we are implicitly making the standard assumption that consumption occurs after prices are observed.
where $\varepsilon_x^*$ is the export supply price elasticity; $D^* \equiv 1 - \frac{p^*q^*}{R^*}$ is the degree of income diversification (GDP share of import-competing sector); and $\Omega^* \equiv \frac{p^*x^*}{R^*}$ is the degree of openness (export share in GDP).

We will assume throughout that $\varepsilon_x^*$ is nonnegative.\(^{19}\) Given this assumption, it follows that $G^*_x|_{\theta = 0} > 0$: with income-risk neutrality, the Foreign country enjoys an increase in TOT risk. The intuition for this feature is that the ability to adjust production and consumption ex-post tends to make the indirect utility convex in prices. This feature in itself is not new to our model, and was pointed out for example by Eaton (1979). This feature, coupled with the result of Corollary 1, in turn implies

**Proposition 2.** Assume the shock is importer-specific. If individuals are income-risk neutral, then the optimal MPA (i) increases TOT uncertainty, and (ii) increases trade-policy uncertainty if and only if the policy impact of the shock is not offset by its direct economic effect, or $t^{N'}(\lambda) > \pi^t(\lambda)$.

This result highlights a “puzzle” generated by the standard trade model with income-risk neutrality. It states that under plausible conditions (positive export supply elasticity), in this model the optimal MPA always increases TOT uncertainty. Moreover, the MPA also increases trade-policy risk unless the dominant source of TOT risk is not policy related; or viewed from a different perspective, if the MPA reduces policy uncertainty it does so only for the purpose of increasing TOT uncertainty.

Evidently, if one wants to make economic sense of the WTO-type informal arguments discussed in the introduction, which state that one of the goals of TAs is to reduce uncertainty in the trading environment, one must depart from the standard trade model with income-risk neutrality. The most natural modification of the model that one can consider is to introduce income-risk aversion ($\theta < 0$), which is what we do next.

Let us re-examine the Foreign country’s preference for TOT risk, that is $G^*_x|_{\pi^*}$. Recall that

$$G^* = \frac{1}{\theta} \left( \frac{R^*}{\phi^*} \right)^{\theta}.$$

Note that, even with income risk aversion, in the Foreign country there is still no motive for trade protection, so our assumption that this country practices free trade continues to be without loss of generality given the representative-citizen assumption.\(^{20}\)

Letting $R^*/\phi^* = v^*$, we obtain

$$G^*_x|_{\pi^*} = (v^*)^{\theta} \left[ \theta \left( \frac{\partial \ln v(p^*)}{\partial \ln p^*} \right)^2 + \frac{\partial^2 \ln v(p^*)}{\partial (\ln p^*)^2} \right].$$

\(^{19}\)There is considerable empirical evidence that this is the case in reality for most sectors and most countries (see for example Tockarick, 2010).

\(^{20}\)As Eaton and Grossman (1985) made clear, in a small country an insurance motive for trade protection can arise only if citizens have heterogenous incomes, at least ex-post. In our setting, Foreign citizens are always homogenous, even ex-post. This will be true also in the next section, where we consider a dynamic setting with ex-ante investments.
Plugging in the derivatives of the revenue and price-index functions derived above, and simplifying, we obtain

$$G^*_{\pi^+\pi^*} = v^*\Omega^* (\theta\Omega^* + \varepsilon^*_x + D^*)$$  \hspace{1cm} (3.1)$$

This expression, together with the result of Corollary 1, leads to the next result. To reduce the number of cases to consider, from now on we will focus on the case $t^N (\lambda) > \pi' (\lambda)$, that is the case in which the dominant source of TOT risk is policy related.

**Proposition 3.** Assume the shock is importer-specific, and suppose $t^N (\lambda) > \pi' (\lambda)$. Then the optimal MPA reduces (increases) trade-policy and TOT uncertainty if $\theta\Omega^* + \varepsilon^*_x + D^* < 0$ ($> 0$).

Several aspects of this result are worth highlighting. Let us start by focusing on the impact of the MPA on TOT uncertainty. Expression (3.1) indicates that this depends crucially on four variables. First, if income risk aversion is sufficiently high (namely when $\theta < -\varepsilon^*_x + D^* \Omega^*$) then the optimal MPA decreases TOT uncertainty relative to the noncooperative equilibrium. Second, for given $\theta < 0$, the optimal MPA is "more likely" to decrease TOT uncertainty when (a) the economy is more specialized ($D^*$ is lower); (b) the export supply elasticity $\varepsilon^*_x$ lower; and (c) the economy is more open ($\Omega^*$ is higher).\(^{21}\)

Next note that the degree of openness $\Omega^*$ has a non-monotonic effect on the magnitude of $G^*_{\pi^+\pi^*}$, that is on Foreign’s attitude toward TOT or policy risk. If $\Omega^*$ is small, the optimal MPA has a negligible effect on TOT uncertainty; if the country is moderately open, TOT uncertainty increases as a result of the agreement; and if the country is very open, TOT uncertainty decreases as a result of the agreement (provided individuals are sufficiently risk averse).\(^{22}\)

### 3.2. Common shocks

Now consider the case of a common shock, meaning that $\lambda$ affects Foreign not just through TOT but also directly. To avoid confusion, we let $G^*(t, \lambda) \equiv \bar{G}^* (\pi^* (t, \lambda), \lambda)$ denote Foreign’s reduced-form objective function. The shock $\lambda$ could represent a global shock or two perfectly correlated domestic shocks.

Recall the general condition from the previous section: $G^*_t \cdot \frac{dt^N}{d\lambda} + G^*_\lambda < 0$. To apply this condition, we start by deriving the TOT externality exerted on the Foreign country by an increase in $t$. This can be written as

$$G^*_t = -v^*\theta \Omega^*$$

\(^{21}\)Stated more precisely: as $\varepsilon^*_x$ increases, the sign of $\frac{\partial^2 G^*}{\partial (\ln \pi)^2}$ can switch from negative to positive, but not vice-versa. And similar statements apply to the impact of $D^*$ and $\Omega^*$.

\(^{22}\)In the text we talk about changes in $\Omega^*, D^*$ and $\varepsilon^*_x$ as if these variables were exogenous, but of course they are not, so here we make those statements more precise. Let $\xi$ denote the vector of all technology and preference parameters (excluding $\theta$). We can think of the key endogenous variables $\Omega^*, D^*$ and $\varepsilon^*_x$ as functions of $\xi$. Note that $\theta$ does not affect these variables. Next note that the feasible range of $\Omega^*$ and $D^*$ is as follows: $\Omega^* \in [0, 1], D^* \in [0, 1]$, while $\varepsilon^*_x \geq 0$ by assumption. In the text, when we refer to a change in an endogenous variable, we mean that the parameter vector $\xi$ is being changed in such a way that the variable of interest changes while the others do not. If we include in $\xi$ the whole technology and preference structure, by varying $\xi$ we can span the whole feasible range of $\Omega^*, D^*$ and $\varepsilon^*_x$, so this "all else equal" thought experiment can be performed.
Intuitively, the degree of openness $\Omega^*$ captures the impact of a change in $t$ on real income through TOT, and the factor $v^*$ is related to the marginal utility of income: with $\theta < 0$, holding the income effect constant the externality is stronger when real income ($v^*$) is lower, because the marginal utility of income is then higher. In what follows we will refer to $v^*\Omega^*$ as the “adjusted” degree of openness.

Letting $\frac{\partial \ln (v^*\Omega^*)}{\partial \lambda}$ denote the elasticity of adjusted openness wrt the shock holding TOT constant, we obtain:

**Proposition 4.** Assume the shock is common, and suppose $t^{N'}(\lambda) > \pi'(\lambda)$. Then the optimal MPA reduces (increases) trade-policy and TOT uncertainty if the following is negative (positive):

$$
(\theta \Omega^* + \varepsilon_x^* + D^*)(t^{N'}(\lambda) - \pi'(\lambda)) - \frac{\partial \ln (v^*\Omega^*)}{\partial \lambda}
$$

(3.2)

The first term of (3.2) is analogous to the case of importer-specific shock, and captures Foreign’s preference for TOT risk. The second term is new, and is due to the policy-externality shifting effect. When this effect is positive, it works in favor of the uncertainty-reducing motive for a TA. Recalling that we defined $\lambda$ as a shock that leads to a higher noncooperative tariff $t^N$; this effect should be interpreted as follows: if shocks that increase the noncooperative tariff also increase the adjusted degree of openness when holding the tariff constant, this strengthens the uncertainty-reducing motive. To summarize, in the case of common shocks, the policy uncertainty motive for a TA is determined by two factors: Foreign’s preference for TOT risk, and the co-variation of the adjusted degree of openness and the TOT (in the noncooperative equilibrium).

It is worth emphasizing that, unlike in the case of importer-specific shocks considered in the previous section, with common shocks the impact of the optimal MPA on TOT uncertainty is not determined solely by Foreign’s preference for TOT risk, i.e. by the sign of $G_{\pi,\pi}$. One implication is that even if the Foreign country is TOT-risk neutral ($G_{\pi,\pi} = 0$), generically there will be an uncertainty motive for a TA, and the optimal MPA will affect TOT uncertainty. And by a similar token, it is possible that even if individuals are risk-neutral ($\theta \to 0$) and hence Foreign is TOT-risk loving ($G_{\pi,\pi} > 0$), there may be an uncertainty-reducing motive for a TA. We do not necessarily view this possibility as a resolution of the “puzzle” highlighted in the previous section, since the sign of the TOT-externality-shifting effect depends on the exact nature of the shock and economic structure, and hence there is no presumption that this effect will offset the preference for TOT risk. But our theory highlights the potentially important role of this effect in determining the uncertainty motive for a TA.

At a broader level, one key point that emerges from our analysis and is clearly illustrated by the comparison between the case of importer-specific shock and the case of common shock, is that the source of the shocks matters greatly for the direction and intensity of the uncertainty motive for a TA.

We conclude this section with an observation. Expression 3.2 is useful because it highlights the key effects that determine the uncertainty motive for a TA. But note that, if one is willing to assume that the model is true, one can in principle check directly in the data whether there is an uncertainty-reducing motive for a TA. Such a motive requires $\frac{d}{dx}(-v^*\Omega^*)^N < 0$. Recalling
that $t^N(\lambda)$ is increasing, letting $\lambda(t^N)$ denote its inverse, and noting that $d\lambda = dt^N/t^N(\lambda)$, the condition can be rewritten as

$$\frac{d\ln(v^*\Omega^*)}{dt^N} > 0$$

We can interpret the term $\frac{d\ln(v^*\Omega^*)}{dt^N}$ as an induced elasticity, that is the elasticity of $(v^*\Omega^*)^N$ wrt the tariff (recall $t^N$ is the log of the tariff factor) induced by the variation in the underlying shock. In principle, one can use information on openness, real income per capita ($v^*$) and estimates of $\theta$ to construct a measure of the adjusted degree of openness. The analysis above then implies that if this measure is positively correlated with noncooperative tariffs there is an uncertainty reducing motive for a TA.

### 3.3. Gains from managing policy uncertainty and policy mean

Next we apply the formulas derived in the previous section to gauge the relative importance of the uncertainty motive versus the mean motive. Note that the case of common shocks in effect encompasses the case of importer-specific shock, which obtains as a special case if $\lambda$ affects $G^*$ only through TOT, so we focus the analysis below on the case of common shocks.

Using $G^*_t = -v^*\Omega^*$, we can apply the formulas derived in section 2.3 which approximate the value of an MPA and of a UPA starting from the noncooperative outcome. The approximate value of an MPA is

$$\frac{\partial EG^W(t^N(\lambda) - \varepsilon(\lambda - \bar{\lambda}), \lambda)}{\partial \varepsilon} \bigg|_{\varepsilon=0} \approx \left(\theta\Omega^* + \varepsilon_x^* + D^*\right) \left(t^N(\lambda) - \pi'(\lambda)\right) - \frac{\partial \ln \left(v^*\Omega^*\right)}{\partial \lambda} \left(v^*\Omega^*\right) \cdot \sigma^2$$

(3.3)

and that of a UPA is

$$\frac{\partial}{\partial \Delta} \frac{\partial EG^W(t^N(\lambda) - \Delta, \lambda)}{\partial \Delta} \bigg|_{\Delta=0} \approx v^*\Omega^*$$

(3.4)

where the above expressions are evaluated at the noncooperative tariff. Thus the relative value of an MPA vs. a UPA can be approximated as:

$$\left(\theta\Omega^* + \varepsilon_x^* + D^*\right) \left(t^N(\lambda) - \pi'(\lambda)\right) - \frac{\partial \ln \left(v^*\Omega^*\right)}{\partial \lambda} \left(v^*\Omega^*\right) \cdot \sigma^2$$

(3.5)

Note that this expression is composed of two factors: the first one is identical to expression 3.2, which determines the direction of the uncertainty motive, and the second captures the degree of underlying uncertainty in the environment.

Finally, we can use the analysis above to approximate the value of a partial move towards the optimal agreement (that improves both policy mean and policy risk). Applying the observation made in section 2.3, such value is given by a weighted average of expressions (3.3) and (3.4) above, thus a parameter change that increases (weakly) both the value of the MPA and the value of the UPA will also increase (weakly) the value of the overall agreement.

There is one such parameter change that is of particular interest and has a clear-cut effect: an increase in $\sigma^2_\lambda$, which we can interpret as an increase in the degree of uncertainty in the
economic/political environment, implies larger gains from a trade agreement.\textsuperscript{23} This in turn suggests that governments should have stronger incentives to sign trade agreements when the trading environment is more uncertain.

3.4. The effect of the MPA on trade volume

Next we ask: how does the uncertainty-managing component of the optimal TA affect trade? More specifically, we ask what is the impact of the optimal MPA on the expected volume of trade and on the volatility of trade. We start by focusing on the case of importer-specific shocks, and to fix ideas we suppose that the optimal MPA reduces TOT risk.

Note that trade volume can be written as \( x^* (\pi^*) \) and so the change in expected log trade due to the MPA is \( \int \ln x^* (\pi^*) \, d (F_{MPA} (\pi^*) - F_N (\pi^*)) \). By standard Rotschild-Stiglitz logic, it is immediate to conclude that expected trade increases as a result of the MPA if and only if Foreign’s export supply elasticity \( \varepsilon^*_x (\pi^*) \equiv \frac{\partial \ln x^*}{\partial \pi^*} \) is decreasing in \( \pi^* \). Also note that the same conclusion applies the (log) trade value \( \pi^* + \ln x^* (\pi^*) \), since an MPA keeps \( E(t) \) and thus \( E(t) \) unchanged.

In the type of trade model we are currently considering there is no presumption that an export supply function has increasing or decreasing elasticity, so this is ultimately an empirical question. But while in this static model the impact of the MPA on expected trade can go in either direction, in the next section we will argue that, under some conditions, the presence of ex-ante investments introduces a definite effect in the direction of increasing expected trade.

Next, focus on the impact of the MPA on trade volatility. It is easy to show that in the “neutral” case of constant export supply elasticity, an MPA that reduces TOT uncertainty also reduces trade volatility. Thus there is an intuitive tendency for the optimal MPA to impact trade policy uncertainty and trade volume (and value) uncertainty in the same direction.

It is interesting to contrast our results on the trade impact of the MPA with a central result of the TOT theory of trade agreements, highlighted in Bagwell and Staiger (1999) and in a number of other papers by the same authors: a fundamental and very general effect of trade agreements is to increase the volume of trade between countries. Thus the mean motive for TAs has an unambiguous expanding impact on trade. In contrast, the uncertainty motive for TAs may impact trade in either direction, and this applies not only to the mean of trade volume but also to trade volatility: as we know the optimal MPA may reduce or increase uncertainty depending on parameters, and so obviously the impact on trade volatility can also go in either direction.

Finally, we note that in the case of common shocks the above analysis of the impact of the MPA on trade is not sufficient. In particular, focus on the impact of the MPA on expected

\textsuperscript{23} This point can also be made by looking at the full gains from the optimal trade agreement, if one is willing to approximate the payoff functions globally with quadratic functions. To see this, write the value of the optimal trade agreement as \( E[G^W (t^A (\lambda), \lambda) - G^W (t^N (\lambda), \lambda)] \). Clearly, a mean preserving spread in \( \lambda \) increases this value iff \( G^W (t^A (\lambda), \lambda) - G^W (t^N (\lambda), \lambda) \) is convex in \( \lambda \). Assuming that all third derivatives of \( G \) and \( G^W \) are zero, this is the case if \( G^W_{tt} (t^A) ^2 + 2G^W_{tA} t^A - \left( G^W_{tt} (t^N) ^2 + 2G^W_{tA} t^N \right) > 0 \). Using \( t^A = \frac{G^W_{tA}}{G^W_{tt}} \) and simplifying, this condition we obtain \( (t^A - t^N)^2 > 0 \), which is satisfied if \( t^A \neq t^N \).
trade volume. The (log) trade volume can be written as \( \ln x^* (\pi^*, \lambda) \), and expected trade is \( \int \int \ln x^* (\pi^*, \lambda) \, dF (\pi^*, \lambda) \). The MPA changes not only the marginal distribution of \( \pi^* \) but also its correlation with \( \lambda \), so it changes \( F (\pi^*, \lambda) \) in non-trivial ways, so we cannot apply the Rotschild-Stiglitz result as we did in the case of importer-specific shock. Thus there will be an extra effect of the MPA on expected trade, whose direction depends on the exact nature of the shock.

4. Uncertainty and trade agreements with ex-ante investments

Thus far we have assumed that all allocation decisions occur ex post, after the shock is realized. But in reality there are a variety of production factors that cannot be flexibly shifted in response to policy and economic shocks. In this section we consider the implications of allowing for allocation decisions that must be made ex-ante, before the shock is realized, or “ex-ante investments”. As we noted in the introduction, the often-heard informal arguments about the motives for TAs claim that they should increase investment and trade by reducing uncertainty. Allowing for ex-ante investments in our model seems compelling if one wants to formally examine this issue.

Recall that the model allows for an arbitrary number of factors that are mobile ex-post. We now assume that one of these, “capital,” is mobile ex ante but fixed ex post.\(^{24}\) We normalize the endowment of capital to one and let \( k^* \) denote the fraction of capital allocated to the export sector. To simplify the analysis we assume that all factors in the Home country are perfectly flexible so they can be allocated after the shock \( \lambda \) is realized. This allows us to keep the economic structure for Home in the background, as we did in the static model.

We assume the following timing: (1) A tariff schedule is selected (cooperatively or non-cooperatively); (2) capital is allocated; (3) \( \lambda \) is realized; (4) the trade policy is implemented and markets clear.

In the noncooperative scenario, we let the Home country choose a contingent schedule \( t^N (\lambda) \) to maximize its expected payoff. In the cooperative scenario, again we let countries choose a schedule \( t^{MPA} (\lambda) \) to maximize expected joint welfare subject to the constraint \( Et^{MPA} (\lambda) = Et^N (\lambda) \). Thus we keep the timing constant across the cooperative and noncooperative scenarios. The reason for this choice is to abstract from domestic-commitment motives for TAs. And of course, if we want a TA to be able to affect investment decisions by managing policy uncertainty, we need to assume that policy choices come before investment decisions, and this explains our choice of timing.\(^{25}\)

The first step of the analysis is to extend Proposition 1 from the “static” model to this “dynamic” environment. Since capital is allocated after observing the trade policy schedule,

\(^{24}\)We could allow for a higher number of factors that are mobile ex ante but fixed ex post, but the notation would get more cumbersome. And of course, the model also allows for factors that are fully fixed (immobile both ex ante and ex post).

\(^{25}\)While the assumption is made to provide a clean thought experiment, we note that in some cases countries are able to choose contingent protection programs in ways that represent long-term commitments. For example the U.S. and the E.U. have contingent protection laws that apply in the absence of trade agreements.
and the allocation may depend on whether or not an agreement is in place, we write Foreign’s objective as \( G^*(t(\lambda), \lambda, k^*) \). We continue to write Home’s objective as \( G(t(\lambda), \lambda) \), which reflects the small country assumption.\(^\text{26}\)

In keeping with our assumption that there is no role for trade policy intervention in Foreign, assume that capital is perfectly divisible, so that the citizens of the small country are not only identical ex ante, but also ex post, and thus there is no redistribution motive for a tariff. Moreover, if Foreign maximizes the representative agent’s welfare then capital in Foreign is efficiently allocated given Home’s trade policy, that is, \( k^* \) also maximizes \( EG^*(t(\lambda), \lambda, k^*) \).

We now argue that Proposition 1 extends to this setting, in the sense that we only need to determine if \( dG_t^*(t^N(\lambda), \lambda, k^*)/d\lambda < 0 \) to know if there is an uncertainty-reducing role for a TA. The following local argument provides some intuition for this result when the shock is importer-specific, that is when Foreign’s payoff can be written as \( G^*(t^N(\lambda), k^*) \). Starting at \( t^N(\lambda) \), a small mean-preserving compression has no first order effect on \( G \) since this objective is maximized by \( t^N(\lambda) \). Therefore, the new schedule will only increase \( EG^W \) if it increases \( EG^W \).

This policy change will have no first-order effect on \( EG^W \) via \( k^* \) because \( k^* \) is socially efficient given the policy schedule. So any impact of the policy change on \( EG^W \) must be due to the “static” effect, i.e. to \( G^N(t(\lambda), \lambda, k^*) < 0 \).

We now consider the full MPA program, extending it to the more general setting where shocks can be “common”. Recalling that, for a given \( t(\lambda), k^* \) maximizes \( EG^*(t(\lambda), \lambda, k^*) \) and has no effect on \( EG^W \), then \( k^* \) maximizes \( EG^W(t(\lambda), \lambda; k^*) \). Thus we can write the MPA program as

\[
\begin{align*}
  t^{MPA}(\lambda) & = \arg \max_{t(\lambda)} EG^W(t(\lambda), \lambda, k^{MPA}) \\
  \text{s.t. } Et(\lambda) & = Et^N(\lambda) \\
  \text{where } k^{MPA} & = \arg \max_k EG^W(t^{MPA}(\lambda), \lambda, k^*)
\end{align*}
\]

or equivalently

\[
\begin{align*}
  \{t^{MPA}(\lambda), k^{MPA}\} & : \arg \max_{\tau(\lambda), k^*} EG^W(t(\lambda), \lambda, k^*) \\
  \text{s.t. } Et(\lambda) & = Et^N(\lambda)
\end{align*}
\]

Assuming an interior optimum, we obtain the following FOCs:

\[
\begin{align*}
  G^W_t(t, \lambda, k^*) & = \psi \text{ for all } \lambda \\
  Et^N(\lambda) & = Et(\lambda) \\
  EG^W_{k^*}(t(\lambda), \lambda, k^*) & = 0
\end{align*}
\]

\(^{26}\)Suppose for example the Home country maximizes the welfare of a representative individual (similar argument would apply if Home maximized some weighted average of individuals’ indirect utilities, reflecting political economy concerns): \( G = G(v(\pi, y, \lambda)) \). The indirect utility \( v \) is a function of the domestic relative price \( \pi \) and income, with income itself a function of \( \pi \), Home endowments and tariff revenue. Tariff revenue is in turn a function of \( t \) and trade volume, where the latter is again function of \( \pi \). So we can write \( G(v(\pi, t, \lambda)) \), and if the Foreign country is small then \( \pi \) is independent of \( t \) and Foreign endowments, so we can simply write \( G(t, \lambda) \).
We can now apply a similar argument as in the case without investments, using the first two of the FOC above. The only difference is that the derivative \( \frac{d}{dk} \mathcal{G}_{t}^{*N} \) is evaluated at the optimal level of \( k^{*} \), but as long as the sign of this derivative does not change with \( k^{*} \), Proposition 1 extends. We can state the following:

**Proposition 5.**

(a) If \( \frac{d}{d\lambda} \mathcal{G}_{t}^{*}(t^{N}(\lambda), \lambda, k^{*}) < 0 \) for all \((k^{*}, \lambda)\), then there is an uncertainty-reducing motive for a TA.

(b) If \( \frac{d}{d\lambda} \mathcal{G}_{t}^{*}(t^{N}(\lambda), \lambda, k^{*}) > 0 \) for all \((k^{*}, \lambda)\), then there is an uncertainty-increasing motive for a TA.

(c) If \( \frac{d}{d\lambda} \mathcal{G}_{t}^{*}(t^{N}(\lambda), \lambda, k^{*}) = 0 \) for all \((k^{*}, \lambda)\), then there is no uncertainty motive for a TA.

**Proof:** See Appendix.

Proposition 5 highlights that the uncertainty motive for the TA is driven by the static effect, i.e. the impact of the shock on the policy externality conditional on the capital level. In a broad sense, we can interpret this result as indicating that there is no separate “investment motive” in the context of a TA that addresses policy uncertainty.

This conclusion, as we highlighted, relies on the competitive allocation of capital being socially efficient given the importer’s trade policy, which is ensured in our setting by the assumptions of welfare maximizing government and perfectly divisible capital. While these assumptions are somewhat restrictive, we note that the same result would obtain in a setting where capital is not divisible, provided the government can use an entry subsidy/tax to effectively control the allocation of capital.\(^{27}\) Of course one could consider reasonable alternative scenarios where capital allocation is not efficient, and in such scenarios there could be an “investment motive” for an uncertainty-managing TA, or in other words, there could be scope for a TA to “correct” the capital allocation through changes in policy uncertainty, but it is important to keep in mind that this would be a second-best argument for such TAs, as the first-best way to address such inefficiency would be the use of more targeted policies.

Given that the condition for an uncertainty-reducing motive for a TA is similar to the static model, the results of the previous sections apply as stated, with the only difference that the relevant expressions are evaluated at a given capital allocation. Moreover, the expression for the approximate value of an MPA relative to a UPA is also unchanged, since there is no first order effect on Foreign welfare due to capital re-allocation.

\(^{27}\)To see this note that whether the competitive allocation is efficient or not depends crucially on whether capital is divisible or indivisible. If capital is divisible, all citizens will have identical incomes ex-post, and as a consequence, there is no idiosyncratic risk, hence no scope for insurance markets, which implies the competitive allocation is efficient. There is aggregate risk in this economy, but it cannot be diversified away (since we are not considering international insurance markets). If capital is indivisible, so that each citizen must choose ex-ante whether to allocate her capital to the export sector or the import-competing sector, then ex-post there will be heterogenous agents that will fare differently in different states. In this situation, the competitive equilibrium is not efficient, and in general capital will not be allocated efficiently. Of course, this inefficiency can in principle be remedied by government intervention. The first-best policy is a contingent transfer scheme that provides insurance to citizens. If such a policy is available, then the inefficiency in the capital allocation will be removed. If the first-best policy is not available, but the government can use an entry subsidy/tax to effectively control the allocation of capital, then again the inefficiency in the capital allocation will be removed, and hence \( \frac{d\Delta V}{d\lambda} = 0 \).
But even if there is no separate “investment” motive for a MPA, such an agreement in general does affect equilibrium investment and trade levels relative to the noncooperative equilibrium, as we show in the next two sub-sections.

4.1. Investment effects of the MPA

In this section we ask how the optimal MPA, which isolates the uncertainty motive in a trade agreement, affects ex-ante investments.

We focus on the case in which shocks are importer-specific and Foreign is TOT-risk averse \((G_{\pi^*} < 0)\), so that the optimal MPA reduces TOT risk. To simplify the exposition we also assume that the trade pattern does not switch as \(k^*\) changes, that is, Foreign exports the nonnumeraire good for all \(k^* \geq 0\).

Recall that we can write Foreign’s objective as \(G^*(\pi^*(t, \lambda), k^*)\) and that efficient capital allocation implies \(\frac{\partial E G^*(\pi^*, k^*)}{\partial k^*} = 0\). By standard results (Rotschild and Stiglitz, 1971), the equilibrium \(k^*\) increases as a result of a mean-preserving compression in \(\pi^*\) if \(\frac{\partial}{\partial k^*} G_{\pi^*}^* (\pi^*, k^*) < 0\) for all \(\pi^*\) in its support. Thus the effect depends on the impact of \(k^*\) on the TOT risk preference. In general this is ambiguous, but we now highlight interesting sufficient conditions under which it is negative. We will show that this is the case if uncertainty is sufficiently small, in the sense that the support of \(\lambda\) is small, and risk aversion sufficiently strong.\(^{28}\)

Note that the result of Proposition 3 extends directly to this dynamic setting, in the sense that the expression for \(G_{\pi^*}^*\) is just the same as in (3.1), provided its various components are re-interpreted as conditional on the capital allocation \(k^*\). Subject to this re-interpretation, we have

\[
\frac{\partial}{\partial k^*} G_{\pi^*}^* (\pi^*, k^*) = \frac{\partial}{\partial k^*} \left[ \pi^* \Omega^* (\theta \Omega^* + \varepsilon_x^* + D^*) \right]
\]

As a first step, we argue that an increase in \(k^*\) leads to a decrease in the degree of diversification \(D^*\). We can write

\[
D^* = 1 - \frac{p^* q^*}{p^* q^* + q^*_0} = 1 - \frac{1}{1 + \frac{q^*_0}{p^* q^*}}
\]

An increase in \(k^*\) (holding \(\pi^* = \ln p^* \) constant) leads to an increase in \(q^*\) and a decrease in \(q^*_0\), hence \(D^*\) falls.

Next focus on \(\Omega^*\). We have

\[
\Omega^* = \frac{p^* x^*}{R^*} = \frac{p^* q^* - p^* c^*}{R^*} = \frac{1}{1 + \frac{q^*_0}{p^* q^*}} - \frac{c^*}{R^*}
\]

As \(k^*\) increases, the first term in the above expression increases, as we argued above. Next note that \(k^*\) affects the consumption share \(p^* c^*/R^*\) only through \(R^*\). In principle \(\frac{\partial R^*}{\partial k^*}\) has an ambiguous

\(^{28}\)The general ambiguity of the impact of mean-preserving changes in prices on investment decisions is well known. In the literature this ambiguity is resolved in different ways, e.g. assuming decreasing absolute risk aversion, positing a specific shock distribution, restricting the economic environment or, as we do, considering cases with small uncertainty. We also note that we could prove the result under the alternative assumption that the probability mass is sufficiently concentrated, rather than the support being sufficiently small, but in this case the notation and the analysis would be more cumbersome.
sign, but note that under certainty \( k^* \) maximizes \( R^* \), hence \( \frac{\partial R^*}{\partial k^*} = 0 \) under certainty. If \( p^* \) is uncertain but has a small support, \( \frac{\partial R^*}{\partial k^*} \) will be small in absolute value, and hence \( \frac{\partial}{\partial k^*} \left( \frac{p^* \cdot \varepsilon^*_x}{h} \right) \) will also be small in absolute value. This ensures that if the support is small enough, \( \Omega^* \) is increasing in \( k^* \).

Next note that a change in \( k^* \) in general has an ambiguous effect on the export supply elasticity \( \varepsilon^*_x \), so in principle the effect of \( k^* \) on \( \left( \Omega^* \left( \theta \Omega^* + \varepsilon^*_x + D^* \right) \right) \) is ambiguous, however if risk aversion is sufficiently strong, i.e. if \( \theta \) is sufficiently negative, then clearly the effect is negative.

Finally, consider the sign of the whole expression (4.6). Letting \( \Omega^* \left( \theta \Omega^* + \varepsilon^*_x + D^* \right) \equiv h(p^*, k^*) \), we can rewrite (4.6) as

\[
\frac{\partial}{\partial k^*} \left[ v^* \theta (p^*, k^*) h(p^*, k^*) \right] = \frac{\partial v^* \theta}{\partial k^*} \cdot h + v^* \theta \cdot \frac{\partial h}{\partial k^*} = \left( \theta \frac{v^*_k}{v^*} + \frac{h_{k^*}}{h} \right) \cdot h \cdot v^* \theta \quad (4.7)
\]

Note that the term \( \frac{v^*_k}{v^*} \) is the relative change in real income due to a capital re-allocation. This is zero under certainty, and under uncertainty it necessarily changes sign over the range of \( k^* \), since if it was always positive or negative there would be an incentive to re-allocate capital. We now argue that if \( \theta \) is sufficiently negative and the support of \( p^* \) is small enough, the expression above is negative. Fix \( \theta \) at some level \( \tilde{\theta} \) such that \( h < 0 \) and \( \frac{h_{k^*}}{h} > A > 0 \) under certainty (where \( A \) is some positive constant). The arguments above ensure that such \( \tilde{\theta} \) must exist. Next recall that \( k^* \) satisfies \( v^*_k = 0 \) under certainty. Then, as the support of \( p^* \) shrinks to zero, \( \frac{v^*_k}{v^*} \) goes to zero for all \( p^* \) in the support, while \( \frac{h_{k^*}}{h} \) approaches \( A > 0 \), therefore

\[
\frac{\partial}{\partial k^*} \left[ v^* \theta (p^*, k^*) h(p^*, k^*) \right] < 0.
\]

We summarize this analysis with

**Proposition 6.** Suppose shocks are importer-specific. If there is sufficient income risk aversion and the support of \( \lambda \) is sufficiently small, then the optimal MPA increases investment in the export sector.

Broadly interpreted, this proposition suggests that under the condition that generates an uncertainty-reducing motive for a TA, namely a strong degree of income-risk aversion, the agreement leads to higher investment in the export sector, provided the underlying uncertainty in the environment is small enough. We also note that the same result would hold if we replaced the condition that \( \theta \) is sufficiently negative with the alternative condition that the export supply elasticity \( \varepsilon^*_x \) is constant (or sufficiently close to constant), as can be easily verified given the argument made above.

While the model suggests a simple set of sufficient conditions under which an uncertainty-reducing MPA has the intuitive effect on capital allocation, we emphasize that the opposite effect (lower TOT uncertainty leading to less investment in the export sector) is also possible when the support of the shock is not small or if shocks are common.

\[29\] It is worth noting that with Cobb-Douglas preferences \( \Omega^* \) would be increasing in \( k^* \) regardless of the amount of uncertainty.
4.2. Trade effects of the MPA

We now examine the impact of the optimal MPA on the expected trade volume and on uncertainty about trade flows. We focus again on the case of importer-specific shocks.

Recall first from the analysis of the static model that, if the MPA reduces TOT uncertainty, then expected trade increases if the export supply elasticity \( e_x^* (\pi^*) \) is decreasing in \( \pi^* \). Moreover, in the “neutral” case where \( e_x^* \) is constant, an MPA that decreases TOT uncertainty (does not affect expected trade and) decreases uncertainty in trade volume, i.e. \( \ln x^* (\pi^N) \) is a MPS of \( \ln x^* (\pi^*) \). When \( e_x^* \) is not constant, on the other hand, it is hard to make statements on the impact of an MPA on the uncertainty of trade flows.

We can now extend this analysis to the presence of ex-ante investment. Trade volume can be written as a function of TOT and ex-post capital level, thus the MPA increases expected log trade if the following is positive

\[
\int \ln x^* (\pi^*, k^*_{MPA}) \, dF^k_{MPA}(\pi^*) - \int \ln x^* (\pi^*, k^*_{N}) \, dF^k_{N}(\pi^*)
\]

where we add the superscript \( k \) to the distributions, \( F^k_{N} \) and \( F^k_{MPA} \) to avoid confusion with the distributions without investment. The first term in the expression above is analogous to the one in the static model that we discussed previously, so it depends on whether \( e_x^* (\pi^*, k^*_{N}) \equiv \frac{\partial \ln x^* (\pi^*, k^*_{N})}{\partial \pi^*} \) is increasing or decreasing in \( \pi^* \). The second term captures the expected growth in exports due to the change in investment. If \( k^* \) increases, this effect will be positive if the support of the shock is sufficiently small and the economy is not completely specialized. To see this, note that \( \frac{\partial x^* (\pi^*, k^*)}{\partial k^*} = \frac{\partial (q^* - \pi^*)}{\partial k^*} = \frac{\partial q^*}{\partial k^*} - \frac{\partial \pi^*}{\partial k^*} \cdot \frac{\partial R^*}{\partial k^*} \), where \( \frac{\partial R^*}{\partial k^*} \) is the ex-post differential in the rate of return to capital across sectors. This differential is zero in expectation under risk neutrality, while it can differ from zero with risk aversion, but if the shock has small support it is close to zero at the optimal ex-ante allocation. Thus if the support of \( \lambda \) is sufficiently small then \( \frac{\partial q^*}{\partial k^*} > 0 \), provided that \( \frac{\partial q^*}{\partial k^*} > 0 \), which is the case if the economy is not completely specialized.

Summarizing, in the case of importer-specific shocks considered here, if risk aversion is sufficiently strong and uncertainty is sufficiently small, the optimal MPA reduces uncertainty in TOT and increases investment in the export sector. Moreover, under these conditions, expected trade increases provided the export supply elasticity does not increase too rapidly with TOT.

5. Uncertainty and trade agreements between large countries

The case we analyzed thus far is useful for two reasons. First, it disentangles the “mean” and “uncertainty” motives for a small country to join a TA, and second, it provides the basic tools
for the analysis of a setting with two-way policy externalities. In what follows, we examine such a setting in the context of the reduced-form framework of section 2 without ex-ante investments.

We represent the reduced-form payoff functions as $G(t, t^*, \lambda)$ and $G^*(t^*, t, \lambda^*)$. The central difference relative to the small-large country setting is the presence of two policy dimensions. For tractability, we assume that countries are mirror-image symmetric. And we continue to assume a single dimension of uncertainty, or $\lambda^* = \lambda$; the interpretation is that there is a global shock that affects symmetrically the two countries, or equivalently, two domestic shocks that are perfectly correlated.

Given symmetry, we define the common payoff given a common tariff $t$ as $\tilde{G}(t, \lambda) \equiv G(t, t, \lambda)$. We make the following assumptions:

(i) Single crossing properties: $\tilde{G}_{t\lambda} > 0$, $G_{t\lambda} > 0$ (and by symmetry, $G^*_{t^*\lambda} > 0$);

(ii) Concavity: $\tilde{G}$ concave in $t$, $G$ concave in $t$ (and by symmetry, $G^*$ concave in $t^*$);

(iii) Stability of reaction functions: $|G_{tt}| > G^*_{tt^*}$ (and analogously for Foreign).

Given that countries are symmetric, the Nash equilibrium tariffs are symmetric, and implicitly defined by the following FOC:

$$G_t(t^N, t^N, \lambda) = 0.$$  

This condition yields the Nash equilibrium tariff schedule $t^N(\lambda)$. Given our assumptions, $t^N(\lambda)$ is increasing, as can be verified by implicitly differentiating the FOC:

$$\frac{dt^N}{d\lambda} = \frac{G_{t\lambda}^N}{-(G_{tt}^N + G^*_{tt^*})} > 0$$

where the numerator is positive by the single crossing property and the denominator is positive by the stability assumption.

Given the symmetry of the problem, it is reasonable to focus on the optimal symmetric MPA, which is given by:

$$t^{MPA}(\lambda) = \arg \max_{t(\lambda)} E\tilde{G}(t(\lambda), \lambda) \text{ s.t. } Et(\lambda) = Et^N(\lambda). \quad (5.1)$$

We can write the Lagrangian for this problem as

$$L = \int [\tilde{G}(t, \lambda) + \psi t^N(\lambda) - \psi t(\lambda)] dF_\lambda(\lambda)$$

Maximizing this Lagrangian pointwise yields the FOCs

$$\tilde{G}_t(t(\lambda), \lambda) = \psi \text{ for all } \lambda$$

$$Et(\lambda) = Et^N(\lambda)$$

We can prove the following:

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31 Given the concavity of the payoff functions, we conjecture that the global maximum is indeed symmetric, so that there is no loss of generality in focusing on a symmetric MPA.
Proposition 7. If \((G_{tt}^N + G_{tt}^s) \cdot \frac{dN}{d\lambda} + G_{t\lambda}^s < 0 (> 0)\) for all \(\lambda\) then there is an uncertainty reducing (increasing) motive for a TA. If \((G_{tt}^N + G_{tt}^N) \cdot \frac{dN}{d\lambda} + G_{t\lambda}^N = 0\) for all \(\lambda\) then there is no uncertainty motive for a TA.

Proof:
Focus on the case \((G_{tt}^N + G_{tt}^N) \frac{dN}{d\lambda} + G_{t\lambda}^N < 0\), or equivalently \(\frac{d}{d\lambda} G_{t}^N(t^N(\lambda), t^N(\lambda), \lambda) < 0\). The key is to prove the analog of Lemma 1, namely that \(t^{MPA}(\lambda)\) intersects \(t^N(\lambda)\) once and from above.

We argue by contradiction. Suppose \(t^{MPA}(\lambda)\) intersects \(t^N(\lambda)\) at some point \(\hat{\lambda}\) from below. Consider two values of \(\lambda\) on the opposite sides of this intersection, \(\lambda_1 < \hat{\lambda} < \lambda_2\), such that \(t^{MPA}(\lambda_1) > t^N(\lambda_1)\) and \(t^{MPA}(\lambda_2) > t^N(\lambda_2)\).

Recalling that \(G_t(t^N(\lambda), t^N(\lambda), \lambda) = 0\) and \(\frac{d}{d\lambda} G_t^N(t^N(\lambda), t^N(\lambda), \lambda) < 0\) for all \(\lambda\), then

\[G_t(t^N(\lambda_2), \lambda_2) = G_t(t^N(\lambda_2), t^N(\lambda_2), \lambda_2) < G_t^N(t^N(\lambda_1), t^N(\lambda_1), \lambda_1) = G_t(t^N(\lambda_1), \lambda_1)\]

These inequalities and the concavity of \(\tilde{G}\) in \(t\) imply

\[\tilde{G}_t(t^{MPA}(\lambda_2), \lambda_2) < \tilde{G}_t(t^N(\lambda_2), \lambda_2) < \tilde{G}_t(t^N(\lambda_1), \lambda_1) < \tilde{G}_t(t^{MPA}(\lambda_1), \lambda_1)\]

This contradicts the FOC, which requires that \(\tilde{G}_t(t^{MPA}(\lambda), \lambda)\) be equalized across states.

Having proved the analog of Lemma 1, the claim of the proposition follows immediately: just observe that the assumed single crossing properties imply \(t^N(\lambda)\) and \(t^{MPA}(\lambda)\) are increasing, and apply a similar argument to that in the proof of Proposition 1. QED

We can now contrast the result of Proposition 7 with the corresponding result for the small-large country setting. The general condition for a policy uncertainty reducing motive, \(\frac{d}{d\lambda} G_t^s < 0\), is similar to the small-large setting, but in the large-large country setting this expression includes an additional term, namely \(G_{tt}^N\). We label this the “strategic interaction” effect, which is positive if tariffs are strategic complements and negative if they are strategic substitutes. Thus an interesting new insight that emerges is that the strategic-interaction effect works in favor of the uncertainty reducing motive if tariffs are strategic substitutes, and vice-versa if tariffs are strategic complements. Whether tariffs are strategic substitutes or complements depends on the specifics of the trade structure (see for example Syropoulos, 2002), so the direction of this effect is ultimately an empirical question.

Note also that while the other terms are similar, they will reflect additional effects. In particular, the attitude towards foreign policy risk \(G_{tt}^N\) and the policy-externality-shifting effect \(G_{t\lambda}^N\) will now include tariff-revenue and pass-through elasticity effects that were absent in the small-large country setting.

One important remark concerns the policy-externality-shifting effect \(G_{t\lambda}^s\). As we explained in a previous section, this term effectively disappears if the shocks are importer-specific. But with two large symmetric countries, the shock will always directly affect both countries and hence this term cannot disappear. This implies that even if countries are TOT-risk neutral there will generically be an uncertainty motive for a TA.

Finally, we can show that the expressions derived in section 2.3, which approximate the gains from managing policy risk and policy mean relative to the noncooperative outcome (and
the relative importance of the two motives) are still valid as stated in this symmetric-country setting.

6. Conclusion

In this paper we initiate a formal examination of the nexus between trade agreements and economic/political uncertainty. We disentangle the uncertainty-managing motive for a trade agreement from its more standard mean motive, explore under what conditions there is an uncertainty-reducing motive for a TA, and analyze the determinants of the relative importance of the two motives.

We highlight a "puzzle" that arises in a standard trade model with income-risk neutrality, namely that there tends to be an uncertainty-increasing role for a TA. This motivates us to introduce income-risk aversion in the model, and in this case we find that the uncertainty-managing motive for a TA is determined by interesting trade-offs, which in turn can be linked to observable (or estimable) quantities.

A key message of our analysis is that the origin of the shocks (country-specific or global) and their type (shocks to economic fundamentals or to political preferences) matters a great deal for determining the direction and strength of the uncertainty-managing motive for a TA.

Finally, we examine how the uncertainty-managing component of a TA affects ex-ante investment and trade volume, highlighting conditions under which a reduction in policy uncertainty leads to more investment in the export sector and more expected trade.

There are several potentially interesting avenues for future research. We think it would be interesting to examine the role of uncertainty-reducing trade agreements in settings where the underlying reason for the agreement is not the classic TOT externality: in particular, one might consider settings in which agreements are motivated by the governments’ need for domestic commitment, or by the presence of non-TOT international externalities. Also, a challenging but potentially fruitful direction of research would be to enrich the model with a view of exploring its implications empirically: this would probably require allowing for imperfectly correlated shocks across countries, as well as multiple countries and multiple goods.

References


7. Appendix

Proof of Corollary 1:

We start with part (ii). Proposition 1 states that the optimal MPA implies a mean-preserving compression of $t$ relative to $t^N$ if $\frac{\partial}{\partial \lambda} G^*_t(t^N(\lambda), \lambda) < 0$ for all $\lambda$, which in the setting under consideration is equivalent to $-\frac{\partial}{\partial \lambda} G^*_t(\pi^*(t^N(\lambda), \lambda)) < 0$. So we obtain $-\frac{\partial}{\partial \lambda} G^*_t(\pi^*(t^N(\lambda), \lambda)) = -G^*_t \cdot \frac{d\pi^*(t^N(\lambda), \lambda)}{d\lambda}$, and the claim follows. For part (i) note that we can apply Lemma 1 to $\pi^*$ instead of $t$ if we think of Home as choosing $\pi^*$ instead of $t$. In this case we only need to look at the sign of $G^*_N$, since Foreign’s objective is $G^*(\pi^*(t, \lambda))$. The assumption that $\frac{\partial^2 G}{\partial t^2}$ has the same sign for all $(\pi^*, \lambda)$ ensures that the slopes of $\pi^N(\lambda)$ and $\pi^MPS(\lambda)$ have the same sign, and we can therefore apply the same argument as in the proof of proposition 1.

QED

Proof of Proposition 5:

We combine the proof of lemma 1 and proposition 1 and extend to the case with investment. The mean constraint and the continuity of $t$ ensure the existence of at least one intersection. Consider one such intersection $\lambda$, so that $t^{MPS}(\lambda) = t^N(\lambda)$. By the FOC, $G^*_t(t^N(\lambda), \lambda, k^{MPS}) = \psi$. Since $G_t(t^N(\lambda), \lambda) = 0$ this implies $G^*_t(t^N(\lambda), \lambda, k^{MPS}) = \psi$. Now if $\frac{\partial}{\partial \lambda} G^*_t(t^N(\lambda), \lambda, k^{MPS}) = 0$ then $G^*_t(t^N(\lambda), \lambda, k^{MPS}) = \psi$ for all $\lambda$, which in turn implies $G^*_t(t^N(\lambda), \lambda, k^{MPS}) = \psi$ for all $\lambda$. Therefore the schedule $t^N(\lambda)$ satisfies the FOC, hence $t^{MPS}(\lambda) = t^N(\lambda)$ for all $\lambda$ and $k^{MPS} = k^N$.

We next prove part (a). Again, $t^{MPS}(\lambda)$ and $t^N(\lambda)$ must intersect at least once. We now argue that if $\frac{\partial}{\partial \lambda} G^*_t(t^N(\lambda), \lambda, k^{MPS}) < 0$ for all $\lambda$ then $t^{MPS}(\lambda)$ can only intersect $t^N(\lambda)$ from above. This, together with continuity, will also ensure the uniqueness of the intersection.

We argue by contradiction. Suppose $t^{MPS}(\lambda)$ intersects $t^N(\lambda)$ at some point $\hat{\lambda}$ from below. Consider two values of $\lambda$ on the opposite sides of this intersection, $\lambda_1 < \hat{\lambda} < \lambda_2$, such that $t^{MPS}(\lambda_1) < t^N(\lambda_1)$ and $t^{MPS}(\lambda_2) > t^N(\lambda_2)$.

Recalling that $G_t(t^N(\lambda), \lambda) = 0$ for all $k^*$ and assuming $\frac{\partial}{\partial \lambda} G^*_t(t^N(\lambda), \lambda, k^{MPS}) < 0$ for all $\lambda$ then

$$G^*_t(t^N(\lambda_1), \lambda_1, k^{MPS}) = G^*_t(t^N(\lambda_2), \lambda_2, k^{MPS})$$

These inequalities and the concavity of $G^*_t$ in $t$ imply

$$G^*_t(t^{MPS}(\lambda_2), \lambda_2, k^{MPS}) < G^*_t(t^N(\lambda_2), \lambda_2, k^{MPS})$$

$$< G^*_t(t^N(\lambda_1), \lambda_1, k^{MPS}) < G^*_t(t^{MPS}(\lambda_1), \lambda_1, k^{MPS})$$

QED.