Scale, Scope, and the International Expansion Strategies of Multiproduct Firms*

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Abstract

A growing literature seeks to understand how the characteristics of firms shape the manner in which they serve foreign markets. We consider an environment in which multiproduct firms can sell their products in multiple countries from multiple product locations. We show that there are strong empirical regularities in the expansion strategies of U.S. multinational firms and that simple extensions of standard models do not explain these regularities. We augment these models by introducing a framework in which managerial expertise is a scarce input that has to be allocated to particular products and production locations and show that the standard model, so amended, is consistent with the data. We then use the model to analyze the productivity effect of changes in international frictions both within and across firms.

1 Introduction

The world’s largest firms are incredibly complex organizations that sprawl across industries and countries. For instance, according to its annual report, Dupont operated production facilities in over 70 countries and produced a wide range of goods such as food, motor vehicle parts, electronics, plastics, construction materials, and industrial chemicals. In organizing the global activities of their firms, management must make a wide range of interrelated decisions: Which goods should they produce? Where should their focus lie? Where should it produce each good and

*The statistical analysis of firm-level data on U.S. multinational corporations reported in this study was conducted at U.S. Bureau of Economic Analysis under arrangements that maintained legal confidentiality requirements. Views expressed are those of the author and do not necessarily reflect those of the Bureau of Economic Analysis.
for which markets? Standard models within the international trade literature typically deal with only a subset of a firm’s activities and may miss important interactions between decisions.

This paper begins with a descriptive empirical analysis that establishes new facts from a confidential firm-level dataset for U.S. multinationals. This dataset allows us to link the domestic and export activity of the U.S. parent firms of U.S. multinationals to the activity of their foreign affiliates. We show that many large U.S. firms sell to unaffiliated customers in a given foreign market simultaneously through exports from the United States and through locally based affiliates. We interpret this fact as evidence that these firms opt to export a subset of their products and produce a different subset abroad. Further, we show that the breakdown of these sales by exports versus multinational production can be predicted by looking at the domestic operations of the parent firms. While parent firms with large sales in the U.S. markets have both larger export and foreign affiliate sales, the ratio of exports to affiliate sales rises in the domestic market share of parent firm. We also show that parent firms that concentrate the bulk of their domestic activities in a few product categories tend to expand abroad using foreign affiliates rather than exports. These results suggest a need to understand the interaction between a parent firm’s choice of the number of products to manage and the choice between mode (exports versus multinational production) that will be used to deliver these products to foreign customers.

To understand the forces at work in the data, we introduce a simple model in which firms produce multiple products for multiple countries in multiple locations. As in Bernard, Redding, and Schott (2011) firms may produce goods in a continuum of industries. We extend this setting to allow firms to tradeoff local production in foreign countries for export from the home country. As in Brainard (1993, 1997), Horstmann and Markusen (1992), and Helpman, Melitz, and Yeaple (2004) firms face a proximity-concentration tradeoff in choosing between these two modes of serving foreign markets.

We model firm heterogeneity in a very different way than standard treatments in the tradition of Melitz (2003). Production efficiency for any particular good requires managerial expertise, which is in fixed supply within the firm. Increasing productivity of some goods (or equivalently in our framework, raising quality) comes at the expense of productivity improvements in other goods. Firms are heterogeneous in two features of managerial expertise. First, management teams vary in their endowments of expertise. Firms with greater expertise have an absolute advantage producing in all goods for all countries. Second, manage-
ment teams differ in the relative efficiency with which they can deliver expertise to foreign affiliates, which creates a **comparative advantage** in producing at home or abroad.

Our model has interesting implications for the geographic structure of production. Firms with larger endowments of managerial expertise have higher aggregate sales in domestic markets, higher exports, and higher foreign affiliate sales. It turns out, however, that variation in absolute endowments of managerial expertise only govern the absolute size of a firm’s global operations and cannot predict the relative importance of exports versus affiliate sales. This is inconsistent with the empirical facts as demonstrated in the empirical section of the paper. Hence, a model based strictly on absolute advantage (i.e. strictly higher levels of aggregate productivity) cannot explain the composition of international commerce within the firm.

We show that it is the second source of firm heterogeneity in our model that can create the empirical link between small, concentrated parents and highly multinational operations. As one would expect, firms that have a comparative advantage in applying managerial expertise abroad tend to sell to foreign customers relatively more through a foreign affiliate rather than through exporting. However, because producing in multiple locations consumes managerial expertise, there is less managerial expertise that can be used to manage marginal product lines in the home market, leading the parent firm to be narrower and smaller than otherwise. In this sense, a comparative advantage in foreign operations reduces the absolute quantity of managerial resources deployed in the home market.

Finally, we use the model to analyze the impact of trade and MP frictions on the allocation of resources within the firm. We show that a reduction in trade costs induces the export oriented firms to increase the productivity of their export goods at the expense of other goods in their portfolio. Less export-oriented firms increase the productivity of all of their product lines. A reduction in the fixed cost of investing abroad leads the most MP-oriented firms to expand the range of goods produced abroad which ultimately leads to a reduction in managerial resources available for any given plant. Less MP-oriented firms shrink their product portfolio which has the implication of increasing the productivity of their remaining products.

This paper contributes to a broad range of the literature in International Economics. Most distinctively, it blends the elements of the literature on multiproduct firms with elements of the literature on multinational firms. Its treatment of multiproduct firms as having a heterogeneous portfolio of productivities across products makes it similar
to Bernard, Redding, and Schott (2011) and Arkolakis and Muendler (2011). In its focus on the span of control across product lines within the firm, the paper is similar to Nocke and Yeaple (2006) who treat product lines within the firm as symmetric. In a sense, the model presented in the paper blends elements of these two models. These papers, and all of the multiproduct firm papers in the literature of which we are aware, analyze an environment in which firms are not free to locate production overseas.

With respect to the treatment of multinational production, this paper belongs in the branch of the literature that focuses on a proximity-concentration framework that has been associated with Horstmann and Markusen (1992), Brainard (1993, 1997), and Helpman, Melitz and Yeaple (2004). In focusing on communication problems between managers located in one country and affiliates in another, the paper also has antecedents in Keller and Yeaple (2011).

The key contribution of this paper is to focus attention on the role of scarce managerial expertise within the firm and its affect on both product range and the expansion strategies of multinational firm. There is now a growing and vibrant literature on management practices and their effect on industry and country performance as typified by Bloom, Genakos, Sadun and Van Reenan (2012).

By focusing on scarce managerial expertise and multinational firms, our paper has some outward similarity to Burstein and Monge (2008), who also treat management as a rival factor within the firm but who do not address multiproduct firms and who analyze the use of scarce managerial resources in a vertical FDI framework. Outside of the international economics context, the idea that there is a fixed stock of expertise within the firm that can be applied across products can also be found in Matsusaka (2001) and Phillips and Maksimovic (2002) whose focus is on the understanding diversification in conglomerates. Finally, in separate work Agapitos and Yeaple (2012) analyze multiproduct, multinational

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1In the published version of the paper, the authors consider heterogenousity in demand levels across countries at the level of the individual variety. A special case of this model that would be consistent with our treatment is the case in which demand levels are not country specific.

2Other examples of multiproduct firms and trade, include Eckel and Neary (2009) who focus on flexible manufacturing and cannibalization effects. Dhingra (2010) is another example of a multiproduct firm framework in which trade liberalization has productivity effects within the firm. Baldwin and Ottaviano (2001) do in fact analyze multi-product multinationals in an oligopolistic environment where cannibalization places center stage.

3Similarly, our focus on scarce managerial resources is related to Antras, Garicano, and Rossi-Hansberg (2005), so like Burstein and Monge are focused on a more vertical style of offshoring in a single product environment.
firms in an environment in which the organization of international product of a firm is due to the interaction of firm characteristics when a firm’s products are close substitutes to one another.

The remainder of this paper is broken into three main sections. The next section describes a dimension of the data on multinational firms that has received less attention in the international trade literature: the firm-level composition of sales broken down by parent sales in the home market, parent exports by country, and affiliate sales by host country. We establish a number of interesting empirical regularities. Section three specifies and analyzes a simple extension of a popular multiproduct firm model in the trade literature to allow for multinational production and endogenous productivity through the allocation of scarce managerial resources. In section four, the equilibrium of the model is characterized with a focus on the mapping of a firm’s characteristics to its domestic and international operations. We show that a model in the tradition of Helpman et al (2004) naturally extended along the lines of Bernard et al (2011) cannot reproduce key elements of the facts presented in section two and show that introducing a span of control is critical to matching the qualitative features of the data. In section five, we present comparative statics results. Here our focus is on the effects of a reduction in trade and MP frictions affects the allocation of scarce managerial resources across product lines within the firm. We show that the nature of this reallocation depends on the firms’ inherent characteristics with some firms becoming more productive and others less so. The final section summarizes and concludes.

2 Features of Multiproduct Multinationals

In this section, we uncover several as yet unknown or underappreciated features of the international expansion strategies of multinational firms. First, we show that large multinational firms are very likely to both export to and engage in local production for unaffiliated customers in a given foreign market. The most natural interpretation of the phenomenon is that firms sell multiple products and individual products are sold exclusively by one mode or another. Second, we show that firms with large U.S. market shares and diverse product portfolios disproportionately sell in foreign markets via exports from the United States rather than local affiliates. The fact that smaller, highly focused firms are more likely to engage in multinational operation is not a prediction of the standard models. We argue that this feature of the data highlights the need to

\footnote{Rob and Vetters (2003) show that a firm might sell the same product through both modes when dynamic concerns are paramount.}
incorporate management capabilities into trade theory.

2.1 Data Description

Firm-level data of the international structure of U.S. multinationals’ operations come from the Bureau of Economic Analysis (BEA) surveys of U.S. Direct Investment Abroad, which are conducted for the purpose of producing aggregate statistics on direct investment activities for the general public.\(^5\) A U.S. multinational entity is the combination of a single U.S. legal entity that has made the direct investment, called the U.S. parent, and at least one foreign business enterprise, called the foreign affiliate. As a result of confidentiality assurances and penalties for non-compliance, the BEA believes that coverage in this survey is close to complete and the level of accuracy is high.

For reasons of data availability, we rely on the 1994 benchmark survey of the Bureau of Economic Analysis.\(^6\) We are interested in the manner in which U.S. firms serve unaffiliated customers in foreign markets; by exporting from the U.S. parent or by selling to these customers from a local affiliate. While every benchmark survey collected detailed firm-level data by country on the value of sales of the foreign affiliates to unaffiliated customers in their host country markets, firm-level data on parent firm exports to those countries is scarce. The last year that the BEA collected comprehensive data on the exports of parents to unaffiliated customers by destination country was 1994.\(^7\)

We also collect data on the scale and scope of the parent firms operations in the United States that are geared toward serving the U.S. market. We observe a parent firm’s sales to U.S. customers in the aggregate across all categories of goods, the number of three-digit manufacturing industries in which the parent is active, and the value of sales of each of these types of goods.\(^8\) From these data, we can infer a firm’s U.S. scope (the number of product categories), its scale (average U.S. sales per industrial category), and a Herfindahl index of the concentration

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\(^5\)U.S. direct investment abroad is defined as the direct or indirect ownership or control of a single U.S. legal entity of at least ten percent of the voting securities of an incorporated foreign business enterprise or the equivalent interest in an unincorporated foreign business enterprise.

\(^6\)For information on the survey, see the Methodology section of the data publication U.S. Direct Investment Abroad: 1994 Benchmark Survey, Final Results, which can be accessed on the BEA’s web site at http://www.bea.gov/scb/account_articles/international/usdia94.htm.

\(^7\)Data collection on this variable was gradually phased out after 1994. In 1999, the reporting threshold was raised substantially so that smaller parents need not report. After 1999, this part of the survey was eliminated.

\(^8\)In the 1994 benchmark survey, the BEA asked firms to report their top eight industries, so there are likely a number of firms for which this restriction binds.
of sales across product lines. We focus on U.S. firms whose main-line-of-business is manufacturing and count only the product classifications that correspond to manufacturing industries. The parent firm’s export data is aggregated across all product classifications but is disaggregated by final destination. Only exports in excess of $500,000 are reported. To these sales, we also include affiliate imports of goods classified for resale without further processing.

For each parent firm, we observe their network of foreign affiliates by country and industry. Our measure of affiliate activity by firm and country is the sales of manufacturing affiliates to unaffiliated customers in their host country. In cases in which the same firm owned more than one manufacturing affiliate, we aggregated over affiliates to create a single firm-country observation.

Descriptive statistics for our sample are reported in Table 1. There are 725 parent firms in our sample. These firms tend to be very large as shown in their average U.S. sales of $1.7 billion. Moreover, they tend to be active in more than one product classification. The average parent firm exports to 11 countries and owns affiliates in 3.4 countries, so exports are the more common mode in terms of destinations served. The volume of sales by mode is very different, as the average export by country is only $2.4 million while the average affiliate sales by country is $97 million. Note that both types of sales are highly skewed, which is why we consider a log-log specification below. Further, while firms tend to export to a larger number of countries than they engage in affiliate sales, affiliate sales in the aggregate account for 57% of total foreign sales, and when measured across countries 77 percent. The bulk of firms engage in both exports and affiliate sales to unaffiliated customers in at least one country. Firms that only engage in affiliate sales tend to have substantially smaller parent sales than firms that engage in both.

### 2.2 Empirical Analysis

Table 2 reports the results of our simple regression analysis. Columns 1, 2, and 3 correspond to dependent variables that are (1) the logarithm of a firm’s foreign affiliate to local customers, (2) the logarithm of the parent’s exports to unaffiliated local customers, and (3) the logarithm of the share of affiliate sales in total firm sales. The rows correspond to the explanatory variables. The first row corresponds to the logarithm of the parent firm’s sales in the United States. The remaining four rows correspond to standard gravity controls: log GDP, log GDP per capita, log distance from the United States, and an indicator variable for English as the official language. Columns 4, 5, and 6 show the results of replacing the country characteristics with country fixed effects.
We begin by discussing the results for the levels of sales by each mode. Looking across the first row, we see that larger parents sell larger quantities to any given foreign country whether through their foreign affiliates (columns 1 and 4) or by exporting from the United States (columns 2 and 5). The coefficients on parent sales in the United States are highly statistically significant using standard errors that have been corrected for heteroskedascity and for clustering at the firm level. Further, the results in columns 1 and 2 show that gravity fits quite well at the firm level for both affiliate sales and for parent exports, although there are differences in the relative sizes of the coefficient estimates. Note the larger number of observations for the export specification demonstrates that parents are more likely to export to any given location than they are to open an affiliate there.

In columns 3 and 6, the coefficient estimates associated with a dependent variable that is the ratio of affiliate sales to total firm sales in a given foreign market. The coefficient is negative and highly statistically significant, indicating that while larger parents have larger affiliate sales and larger export sales to a given market, larger parent firms rely relatively more heavily on export sales to a given market than on affiliate sales. As our interest is on the coefficients on parent firm variables, we henceforth focus exclusively on country fixed-effect regressions.

In Table 3, we further explore the relationship between parent firm local market behavior and the firm’s foreign market behavior. In column 1 parent sales are decomposed into scale (logarithm of U.S. sales per product) and scope (logarithm of number of products). The results indicate that both outcome variables for the parent firm predict the firm’s foreign expansion strategies: high scale or high scope is associated with larger exports relative to affiliate sales. In column 2, we add two additional parent firm characteristics, the logarithm of their R&D intensity and the logarithm of their capital to labor ratio. We find that although capital intensity does predict higher affiliate sales (perhaps because of a greater likelihood of internalization?) adding these additional parent firm characteristics only raises the coefficient on scope.

In column three of Table 3, we add a measure of concentration at the level of the parent firm: ie the sum of squared product category shares by firm. An increase in this variable, which represents a more concentrated firm, is associated with an increase in the relative importance of affiliate sales in the expansion strategies of firms. Note that adding this variable makes the coefficient on scope change sign and to become not statistically significant. In the last column of Table 3, we drop product scope and scale and add back our measure of parent sales in the United States. The coefficient estimates are consistent with our previous results: parents
that are smaller and more highly focused on a narrow product range tend to expand relatively more through foreign affiliates than larger and more highly diversified parents.

We have run a number of robustness checks through which the essential message of Tables 2 and 3 remains intact. These include Heckman selection specifications using World Bank measures of business costs as an exclusion restriction and fixed effects specifications for the main-line-of-business. We also consider Tobit specifications with an upper limit of zero to account for observations in which only affiliate sales are observed. The effects are to increase the absolute values of the estimated coefficients, but not their sign or statistical significance.

3 Model Assumptions

To allow for firms to produce and sell their product in multiple locations we consider a world in which there are two identical countries indexed by \( l \) and \( k \). The preferences of the representative consumer in each country are two tier over a continuum of goods that are each differentiated by variety. Preferences over these goods are Cobb-Douglas with equal budget shares:

\[
U = \int_0^1 \ln C(i) di.
\]  

(1)

Each industry is differentiated by variety with subutility function given by

\[
Q(i) = \left[ \int_{\omega \in \Omega_i} q(\omega) \frac{\sigma}{\sigma - 1} d\omega \right]^{\frac{\sigma - 1}{\sigma}},
\]

(2)

where \( \sigma > 1 \) is the elasticity of substitution across individual varieties. For expositional convenience, we assume that the elasticity of substitution is common across goods \( i \).

All goods are produced using exclusively labor, which we choose as the numeraire, and an input we will refer to as managerial expertise. There exists a continuum of ex ante identical entrepreneurs. When an entrepreneur incurs a fixed cost \( F^E \), she receives a bundle of characteristics from distributions known ex ante. First, she receives the blueprint to produce one variety of each type of good. Each blueprint implies a level of “fundamental” productivity \( Z \) independently drawn from a Pareto distribution \( G(Z) = 1 - Z^{-\kappa} \), where \( \kappa > 1 \). As all firms draw from the same distribution, there is no aggregate variation across firms due to this source of heterogeneity.\(^9\)

\(^9\)Nor is it necessary that this heterogeneity take the form of productivity heterogeneity as it could be modeled as different levels of demand.
The actual productivity with which the firm will produce a product of fundamental productivity $Z$ depends as well on the quantity of “managerial expertise” that is dedicated by the firm to the plant producing that variety. If $t_j(Z)$ is the quantity of this managerial input allocated to a plant at location $j$ that produces a variety of fundamental productivity $Z$ then the productivity of that plant is

$$\tilde{\varphi}(t_j(Z), Z) = Z t_j(Z)^{\tilde{\theta}}, \quad (3)$$

where $\tilde{\theta} \in (0, 1/(\sigma - 1))$ is a measure of the span of control of managerial expertise by product. By location $j \in \{d, f\}$, domestic and foreign is relative to the country that the firm entered. If a firm entered in $l$ is producing in $k = l$ then $j = d$ whereas if $k \neq l$ then $j = f$.

When a firm enters it also receives a random draw that defines its managerial type, which is the source of intrinsic aggregate heterogeneity across firms. This types includes the firm’s stock of managerial expertise, $T$, and the efficiencies of this expertise in its home market ($d$) and in the other market ($f$) given by $(\lambda_d, \lambda_f)$. These characteristics are drawn from a joint probability distribution $H$ with density $h$.

Equation (3) shows that productivity is endogenous and depends on the amount of managerial expertise that the entrepreneur allocates to its production. In allocating time to the various goods produced by the firm, the manager must respect the constraint that she can allocate no more than $T$ units of her managerial expertise to all goods that are produced:

$$T \geq \int_0^{\infty} \sum_j \lambda_j t_j(Z) dG(Z), \quad (4)$$

where $t_j(Z)$ is the expertise allocated to a plant of productivity $Z$ that is located in country $j \in \{d, f\}$. Note the role played by the parameter $\lambda_j$. We assume that $\lambda_f = \lambda \geq \lambda_d = 1$ so that a firm has greater difficulty coordinating production in remote locations. We normalize the communication cost in domestic markets to unity because it can be safely subsumed into absolute managerial expertise $T$ and assume that the support for $\lambda$ is on $[1, \overline{\lambda}]$.\textsuperscript{10} Aside from the idiosyncratic productivity differences across goods within a firm, all of the heterogeneity across firms has been neatly bundled into the expertise allocation constraint across firms as shown in equation (4).

\textsuperscript{10}Since Hymer (1976), it is standard in the literature on multinational firms to think of foreign producers at a disadvantage due to the fact that they are remote from their home country. This “liability of foreignness” is what varies across firms depending on their managerial characteristics.
An active firm must decide where to produce each good. If a firm produces a particular variety at home, it must pay a fixed cost $F$. If it chooses to export that good to the foreign country, it must pay a variable iceberg-type trade cost $\tau \geq 1$. We will use in our exposition below the transformed, “freeness” of trade parameter $\rho \equiv \tau^{1-\sigma} < 1$. In addition, the firm must pay a fixed cost $F^x$ to find a distributor for its product. Finally, the firm might choose to produce abroad to serve the local market from a local affiliate. As is standard in the proximity-concentration literature, by opening the affiliate, the producer avoids trade costs $\tau$ but must pay a fixed cost $F^m$. To obtain an interior solution in which all modes will be observed by at least some firms, we make the following parameter restriction:\textsuperscript{11}

$$F < \frac{F^x}{(1 + \rho)^{1/\sigma} - 1} < F^m. \tag{5}$$

The timing of the model proceeds as follows. First, firms draw their type $(T, \lambda)$ and their good-specific productivities. Second, firms decide which goods to produce, which markets to serve, and where to locate production for each good and market (mode choice). Third, firms choose how much time to allocate to production of each good at each location. Finally, firms compete in monopolistically competitive fashion in each market.

In summary, the model extends the simplest version of Bernard, Redding, and Schott (2011) to a setting in which firms face a proximity-concentration tradeoff between exports and multinational production. While firms are heterogeneous in their productivity in various good categories, this heterogeneity is entirely idiosyncratic and not the origin of aggregate productivity differences across firms. Instead, productivity of individual products depend on the characteristics of management that enter the resource constraint (4). Note that because all fixed costs (aside from the entry fixed cost) are product specific. This will have the implication that once a firm has entered, it will not exit because it can sell at least some goods in all markets. We have purposely chosen to eliminate such selection effects across firms to focus on selection within firms.\textsuperscript{12}

\textsuperscript{11}In the special case that $\theta = 0$ this is equivalent to the parameter restriction made in Helpman et al (2004).

\textsuperscript{12}Such effects could be easily added by having fixed costs of serving various markets that occur at the firm level. As these selection effects are very well understood we leave this extension to the interested reader.
4 Results

In this section we characterize the equilibrium choices of heterogeneous firms as a function of their types \((T, \lambda)\). We solve the model backwards by first deriving sales and profits for each good and production location taking as given the allocation of managerial expertise to each product and the production location of that product vis-a-vis the final market served. Then, we solve for the optimal allocation of managerial expertise within a given firm across goods as a function of that good’s productivity \(Z\) and its production location (domestic or foreign). Next, we solve for the production location of each good. Finally, we impose a free entry condition.

Throughout the exposition of this section, our derivation is entirely from the perspective of a firm from the country \(l\) which has its mirror image in country \(k \neq l\).

4.1 Profits and Sales Volumes

The preference system given by (1) and (2) combined with the symmetry of the model implies the following demand function for any given good in either country:

\[
q = \frac{E}{P} \left( \frac{p}{P} \right)^{-\sigma},
\]

(6)

where \(p\) is the price charged in the market, \(E\) is aggregate expenditure across all goods and \(P\) is the price index for each industry given by

\[
P_l^{1-\sigma} = \int_{\omega \in \Omega_l} p(\omega)^{1-\sigma} d\omega,
\]

where \(\Omega_l\) is the set of goods available for sales in country \(l\). As is well known, profit maximizing firms facing iso-elastic demand (6) optimally charge a price that is a constant mark-up over marginal cost. For a good of fundamental productivity \(Z\) we have

\[
p(Z) = \frac{\sigma}{\sigma - 1} C(Z)
\]

(7)

where

\[
C(Z) = \begin{cases} \frac{1}{\varphi_j(Z) \lambda} & \text{if the good is produced in } j \text{ and sold in } j \\ \frac{1}{\varphi_j(Z) \tau} & \text{if the good is produced in } j \text{ and exported} \end{cases}
\]

(8)

Note that a firm will never produce in a foreign market for sale in the domestic market because doing so would require it to incur both communication costs \(\lambda\) and shipping costs \(\tau\) which is never sensible given
the symmetry of the two countries. Note also that we have normalized the wage in the two identical countries to unity.

Given the optimal pricing formula, and the cost function, we can derive the profits that accrue to any particular mode of serving global markets. For instance, a firm that sells a particular good $Z$ only its domestic market faces only the domestic fixed cost $F$ and faces no costs of shipping a good or communicating across borders. Hence, the profit associated with a purely domestic mode of operation for a product line with specific productivity $Z$ of a type $(T, \lambda)$ can be written

$$\pi^D(t_d, Z) = AZ^{\sigma-1}(t_d)^\theta - F,$$

where we have defined $\theta = \tilde{\theta}(\sigma - 1) < 1$, and $A$ is the markup adjusted demand level in each country given by

$$A \equiv \frac{1}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} EP^{\sigma-1}.$$

The firm may also choose to serve foreign markets in addition to its domestic market. If it chooses to export its product, it must incur fixed costs $F$ and $F^x$ and it must also incur variable costs $\tau$ on its sales to foreign customers. The resulting profit associated with exporting is thus

$$\pi^X(t_d, Z) = (1 + \rho)AZ^{\sigma-1}(t_d)^\theta - F - F^x,$$

where we have used $\rho \equiv \tau^{1-\sigma}$ to simplify notation. Note that a firm that exports benefits from consolidating the production of that variety in one location, which has the advantage of exploiting economies of scale in the provision of managerial expertise.

A firm that decides to engage in horizontal FDI, avoids all trade costs, but is now exposed to communication costs associated with its parent. Further, the firm must incur the higher fixed costs associated with international production given by $F^m$. The resulting profit is

$$\pi^M(t_d, t_f, Z) = AZ^{\sigma-1}((t_d)^\theta + (t_f)^\theta) - F - F^m.$$

This expression makes clear that a significant cost to multinational production vis-a-vis the export mode is the need to use managerial expertise for both the domestic and foreign operation. It also suggests immediately, that the foreign plants of a firm are likely to be at productivity disadvantage relative to their home country counterparts due to communication cost. This implication that overseas plants are less productive than domestic plants is consistent with some empirical work (see Keller.
and Yeaple, 2011). Given the profits associated with each mode in equations (9)-(11), the aggregate profits of a multiproduct, multinational firm can be written

\[
\pi(T, \lambda) = \int_1^\infty \max(0, \pi^D(t_d(Z), Z), \pi^X(t_d(Z), Z), \pi^M(t_d(Z), t_f(Z), Z))dG(Z).
\]

Sales of any good by a given mode can be computed using equations (6)-(8). We now turn to the optimal allocation of managerial expertise across goods and locations conditional on the location decisions of the firm for each variety.

### 4.2 Allocation of Managerial Expertise across Products

Suppose that a firm has made production location decisions by allocating product lines into three sets, \(\Phi_D\), \(\Phi_X\), and \(\Phi_M\), which are the product-specific productivities \(Z\) of goods allocated to pure domestic sales, export, and multiplant production, respectively. Goods whose \(Z\) is not an element of the union of these sets are not produced by the firm. Using the profit functions by mode (9)-(11), the aggregate profit function (12), and the resource constraint (4), the first-order conditions for the optimal choice of the amount of managerial expertise to allocate across various goods then imply

\[
t_j(Z; \lambda, T) = \begin{cases} 
\frac{T}{B} Z^{\frac{\sigma-1}{\tau-\sigma}} & \text{if } j = d \text{ and } Z \in \Phi_D \cup \Phi_M \\
\frac{T}{B} (1 + \rho)^{\frac{1}{\tau-\sigma}} Z^{\frac{\sigma-1}{\tau-\sigma}} & \text{if } j = d \text{ and } Z \in \Phi_X \\
\frac{T}{B} \lambda^{-\frac{1}{\tau-\sigma}} Z^{\frac{\sigma-1}{\tau-\sigma}} & \text{if } j = f \text{ and } Z \in \Phi_M 
\end{cases}
\]

where

\[
B = \int_{Z \in \Phi_D} Z^{\frac{\sigma-1}{\tau-\sigma}} dG(Z) + \int_{Z \in \Phi_X} (1 + \rho)^{\frac{1}{\tau-\sigma}} Z^{\frac{\sigma-1}{\tau-\sigma}} dG(Z)
\]

\[
+ \int_{Z \in \Phi_M} Z^{\frac{\sigma-1}{\tau-\sigma}} dG(Z)
\]

measures the total burden of the firm’s production network on its stock of managerial expertise. Equations (13) and (14) illustrate some of the tradeoffs facing firms. First, everything else equal, a firm that expands the number of goods that it manages will have less managerial resources to spend on each good that it produces and so will tend to be less productive (higher \(B\)). Second, the managerial resource allocation decision
of firms will magnify differences in the initial productivities across products produced within the firm. Third, a firm that reallocates a product from an export mode to a multinational production mode will spend less managerial resources on the foreign affiliate than it did in the exporting plant in the home country but may spend more managerial time on the product line in total because it needs to support two, rather than one, plant.

Given the optimal allocation of managerial resources across products, we can now rewrite the profits by mode from the system (9)-(11) as

\[
\begin{align*}
\pi^D(Z; \lambda, T) &= AT^\theta B^{-\theta} Z^{\frac{1}{1-\theta}} - F, \\
\pi^X(Z; \lambda, T) &= AT^\theta B^{-\theta} Z^{\frac{1}{1-\theta}} (1 + \rho)^{\frac{1}{1-\theta}} - F - F^x, \\
\pi^M(Z; \lambda, T) &= AT^\theta B^{-\theta} Z^{\frac{1}{1-\theta}} \left(1 + \lambda^{-\frac{\theta}{1-\theta}}\right) - F - F^m.
\end{align*}
\]

We now focus our attention to the manner in which firms assign various goods in their portfolio to various modes of serving global markets.

### 4.3 Allocation of Goods to Modes

Profits per mode are given by the equations in (15). Given this formulation, we can use the logic in Helpman et al. (2004) to assign products to modes. From the assumptions (5), it follows from the expressions in (15) that there exist cutoffs \(z_D\) and \(z_X > z_D\) such that for \(Z < z_D\) goods are not produced, goods \(Z > z_D\) will be sold in at least the domestic market, and goods \(Z > z_X\) will be sold in both markets. For firms for which the managerial efficiency costs abroad are sufficiently low, ie.

\[\lambda^{-\frac{\theta}{1-\theta}} > \Delta,\]

where \(\Delta \equiv (1 + \rho)^{\frac{1}{1-\theta}} - 1\) there will be at least some products for which multinational production is optimal. We henceforth assume that the support of distribution of firm inefficiencies \(\lambda\) is such that this condition is meant. Specifically, we assume that the upper bound of the support of international inefficiency satisfies \(\bar{\lambda}^{-\frac{\theta}{1-\theta}} > \Delta.\)

It follows immediately that there exists an additional cutoff, \(z_M > z_X\), such that goods \(Z > z_M\) will be produced (and sold) in both countries.

Given the existence of the three cutoffs \(z_D < z_X < z_M\) that define the sets \(\Phi_D\), \(\Phi_X\), and \(\Phi_M\), we may use equations (15), (14) to rewrite (12) as

\[\text{Relaxing this assumption is straightforward and will result in firms that are large and productive in their home market but that choose not to own any foreign affiliates.}\]
\[ \pi = AT^\theta B(z_D, z_X, z_M)^{1-\theta} - (1 - G(z_D))F - (1 - G(z_M))F^m - (G(z_m) - G(z_x))F^x \]  

where the function \( B \) is now written

\[ B(z_D, z_X, z_M) = \int_{z_D}^{z_X} Z^{\frac{\theta - 1}{1-\theta}}dG(Z) + (1 + \rho)^{\frac{1}{1-\theta}} \int_{z_X}^{z_M} Z^{\frac{\theta - 1}{1-\theta}}dG(Z) + \left(1 + \lambda^{-\frac{\theta}{1-\theta}}\right) \int_{z_M}^{\infty} Z^{\frac{\theta - 1}{1-\theta}}dG(Z). \]  

The first order conditions for profit maximization associated with (16) imply the following expressions for the three cutoffs:

\[ z_D = \left((1 - \theta)AT^\theta B(z_D, z_X, z_M)^{-\theta} \frac{1}{F}\right)^{\frac{1-\theta}{\sigma - 1}}, \]  
\[ z_X = \left((1 - \theta)AT^\theta B(z_D, z_X, z_M)^{-\theta} \frac{\Delta}{F^I}\right)^{-\frac{1-\theta}{\sigma - 1}}, \]  
\[ z_M = \left((1 - \theta)AT^\theta B(z_D, z_X, z_M)^{-\theta} \frac{\lambda^{-\frac{\theta}{1-\theta}} - \Delta}{F^I}\right)^{\frac{1-\theta}{\sigma - 1}}. \]

where \( F^I \equiv F^m - F^x \). To complete the characterization of a firm’s choices, we integrate (17) using the Pareto distribution and substitute the cutoffs (18), (19), and (20) to obtain:

\[ B(T, \lambda) = \left(\frac{a}{a - 1} \frac{\left((1 - \theta)AT^\theta\right)^{\alpha - 1}}{\Theta(\lambda)}\right)^{\frac{1}{1-\theta + \alpha \theta}}. \]

where \( a \equiv \kappa(1 - \theta)/(\sigma - 1) > 1 \) is a bundle of parameters and

\[ \Theta(\lambda) \equiv F^{1-a} + \Delta^a (F^x)^{1-a} + \left(\lambda^{-\frac{\theta}{1-\theta}} - \Delta\right)^a (F^I)^{1-a} \]

is an index of the various costs facing a firm that varies with firms’ foreign managerial inefficiency, \( \lambda \). As managerial foreign inefficiency \( \lambda \) rises, \( \Theta(\lambda) \) falls. By substituting (21) into the cutoff equations (18)-(20), we obtain reduced form expressions for each cutoff. We will use these expressions in the following section to analyze the cross-firm structure of international production.
4.4 The Firm-Level Structure of Production

In this section, we map the firm decisions into measures of firm-level aggregates that correspond to objects that we measure in the empirical section of the paper. Most derivations and the proof of proposition 3 can be found in the appendix. We begin by analyzing the level of firm sales by destination and production location of a firm of type \((T, \lambda)\). The level of sales by the parent firm in the domestic market for a good of productivity \(Z\) is given by

\[
S^D(T, \lambda, Z) = p(Z)q_d(Z) = AZ^\theta t_d(Z),
\]

where \(t_d(Z)\) is the solution to (13). Aggregate sales are then found by integrating over the range of these sales, we find

\[
S^D(T, \lambda) = \sigma \tilde{A} \left( \frac{T}{\Theta(\lambda)} \right)^{\frac{\theta \alpha}{1-\alpha+\theta}} \left[ F^{1-\alpha} + \left( (1 + \rho)^{\frac{1}{1-\theta}} - 1 \right) \left( \left( \frac{\Delta}{F^x} \right)^{a-1} - \left( \frac{\lambda^\theta - \frac{\theta}{1-\theta}}{F^I} \right)^{a-1} \right) \right],
\]

where

\[
\tilde{A} \equiv (A)\frac{a(1 - \theta)^{a-1}}{a - 1},
\]

is an alternative measure of demand. When managerial expertise is important (i.e. \(\theta > 0\)), costs that affect international markets have an indirect effect on the level of sales in the domestic market because of the intra-firm resource allocation effect.

Similarly, parent firm export sales are given by

\[
S^X(T, \lambda) = \sigma \tilde{A} \left( \frac{T}{\Theta(\lambda)} \right)^{\frac{\theta \alpha}{1-\alpha+\theta}} \rho (1 + \rho)^{\frac{\theta}{1-\theta}} \times \left[ \left( \frac{\Delta}{F^x} \right)^{a-1} - \left( \frac{\lambda^\theta - \frac{\theta}{1-\theta}}{F^I} \right)^{a-1} \right],
\]

and the sales of foreign affiliates are given by

\[
S^M(T, \lambda) = \sigma \tilde{A} \left( \frac{T}{\Theta(\lambda)} \right)^{\frac{\theta \alpha}{1-\alpha+\theta}} \lambda^\theta \left( \frac{\lambda^\theta - \frac{\theta}{1-\theta}}{F^I} \right)^{a-1}.
\]

The following proposition follows directly from inspection of expressions (23), (25), and (26).

**Proposition 1 (Absolute Advantage)** An increase in a firm’s endowment of managerial expertise, \(T\), increases the firm’s domestic sales, export sales, and local affiliate sales.
Firms that have higher levels of managerial expertise allocate more of this expertise to all modes of international commerce. In a sense, it is as if they have higher “core productivity” as in Bernard, Redding and Schott (2011). For any given good $Z$, an increase in core productivity raises the sales of that good and makes it more likely that the good will be produced by the firm. This result is consistent with the empirical fact in section 2: parent firms with larger domestic sales have a larger value of aggregate export sales and multinational sales by foreign market.

We now decompose a firm’s parent sales into its **scale** (local sales per product or its intensive margin) and its **scope** (number of products managed or its extensive margin) and explore the manner in which a firm’s type is revealed by these observable characteristics. We begin with the parent firm’s scope. A parent firm’s **scope** (number of products produced) is measured in the model as the share of product categories above the domestic production cutoff: $N(T, \lambda) = (z_D)^{-\alpha}$. Using (18) and (21), we find that

$$N(T, \lambda) = F_{a-a}^{\alpha} \left[ (1 - \theta)AT^\theta \left( \frac{\alpha}{a - 1} \Theta(\lambda) \right)^{-\theta} \right]^\frac{a}{1+\theta(a-1)}. \quad (27)$$

As shown in the appendix, a parent firm’s domestic **scale** is

$$\frac{S^D(T, \lambda)}{N(T, \lambda)} = \frac{\sigma}{1 - \theta} \frac{a}{a - 1} \left[ F^+ \left( (1 + \rho)^{1+\eta} - 1 \right) F^{\alpha} \left( \left( \frac{\Delta}{F^X} \right)^{a-1} - \left( \frac{\lambda^{1+\eta}}{F^X} - \Delta \right)^{a-1} \right) \right]. \quad (28)$$

Inspection of (27) and (28) establishes the following proposition that summarizes the relationship between a firm’s type and the parent scale and scope.

**Proposition 2 (Scale and scope)** A firm’s **scale** is independent of $T$ and increasing in $\lambda$. A firm’s **scope** is increasing in both $T$ and $\lambda$.

The proposition shows us how to perceive variation in a firm’s type from its scale and scope. A parent firm’s scale, an observable firm characteristic, is driven entirely by variation in its comparative advantage managing production in the home country. The intuition for this result is as follows. An increase in $T$ leads firms to add more managerial resources to its existing portfolio and to expand into weaker products, which shows up as an increase in scope. As these products have smaller sales than the average product sold previously, the Pareto parameterization of product-level productivity requires that the within-firm extensive
and intensive margins cancel out (as in Chaney, 2008) leaving average sales per product unchanged.

Now consider the effect of an increase in $\lambda$. This shift induces the firm to allocate managerial expertise away from expertise-intensive multinational operations toward domestic production. The resulting impact on the firm’s domestic operations is similar to that of an increase in $T$. There is, however, an additional effect. As the share of foreign sales shifts toward exports, more managerial resources are allocated to this end. Because the plants that produce for export also produce for the domestic market, the productivity of these export plants rises, lowering the cost of selling in the domestic market and raising the average size of domestic operations.

We now turn our attention from firms’ absolute advantages to firms’ comparative advantages. We start the discussion by focusing our attention on the composition of a firm’s foreign sales as represented by the ratio of the firm’s aggregate export to its local affiliate sales, which obtain by dividing (25) by (26):

$$
\frac{S_X(T, \lambda)}{S_M(T, \lambda)} = \rho (1 + \rho)^{1/1 - \sigma} \lambda^{1/1 - \sigma} \left[ \left( \frac{F^I}{F^E} \frac{\Delta}{\Delta - \frac{\sigma}{\lambda^{1 - \sigma}} - \Delta} \right)^{\frac{k}{\sigma - 1}} - 1 \right], 
$$

(29)

where $\tilde{k} \equiv k(1 - \theta)$. This expression should be very familiar to readers of Helpman, Melitz, and Yeaple (2004). In the special case in which managerial time is unnecessary for production ($\theta = 0$) this expression simplifies to the industry-level expression found in Helpman et al (2004). This shows how the Bernard, Redding and Schott (2011) framework, naturally extended as in Helpman, Melitz and Yeaple (2004), delivers within firm expansion strategies that are identical both within-firms and across industries!

In general, equation (29) shows that a firm’s endowment of managerial expertise, $T$, has no impact on $S_X(T, \lambda)/S_M(T, \lambda)$. Given the Pareto parameterization, an increase in $T$ causes the cutoffs $Z_M$ and $Z_X$ to shift down with the implication that both types of sales rise by exactly the same proportion. As variation in $T$ plays the role of core productivity in Bernard et al (2011), it follows that this type of mechanism does not determine the composition of commerce at the firm level: simple selection driven models do not deliver variation in the within-firm composition of commerce that we demonstrated exists in section 2.

One might propose that firms differ in the the degree of productivity dispersion across industry, i.e. that lower $k$ might be a feature of particular types of firms. If so, a model without scarce managerial resources within the firm is unnecessary to explain the facts in the empirics section.
While this does indeed generate the proposed effect on $S^X/S^M$ as can be seen from (29), the following proposition shows that in a model without scarce managerial resources, increased dispersion within the firm is not consistent with the data (proof is in appendix).

**Proposition 3** Suppose that $\theta = 0$ so that managerial expertise is irrelevant. An increase in within-firm heterogeneity (lower $\bar{\kappa}$) increases the multinational sales relative to exports and increases both the domestic sales of the same firm and the number of varieties sold in the domestic market.

Intuitively, increasing within-firm productivity dispersion increases the mass of activity above an given cutoff value. This is precisely the point made in Helpman et al (2004). Hence, as productivity becomes more dispersed within the firm, more of the productivity draws exceed the domestic productivity cutoff raising both domestic sales and the number of products sold in the domestic market. As this runs counter to the facts demonstrated in section 2, we turn to the comparative advantage mechanism in our model.

A quick glance at equation (29) confirms that comparative advantage within the firm, across markets plays the role one would expect. The greater the difficulty that a firm has in providing managerial inputs to its affiliate, the higher the export to multinational production ratio will be. The next proposition, which follows from inspection of equations (27), (28), and (29), shows that the managerial expertise mechanism in the model is consistent with the empirics shown in section 2.

**Proposition 4** Holding fixed firms’ absolute managerial expertise $T$, firms that have difficulty communicating abroad (high $\lambda$) sell a wider range of goods in their domestic market (scope), have larger sales per product in the domestic market (scale) and tend to serve foreign markets through exports rather than affiliate sales.

The mechanism giving rise to these effects is intuitive. When a firm chooses not to open many foreign plants because it is relatively costly to do so, it frees up managerial resources that can be used to raise the productivity of marginal domestic plants. As a result, parent firms expand domestically by increasing both their product scale and their product scope. Hence, the model can generate the facts described in section 2 both in terms of absolute levels of sales across modes (Proposition 1), and in terms of the relationship between domestic levels and the relative mode choice by firms in international markets (Proposition 4).
4.5 Free Entry

Using the results from the previous sections, we can finally compute the profits of a firm of type \((T, \lambda)\):

\[
\pi(T, \lambda) = \frac{1 - \theta + a\theta}{a} \tilde{A}T^{\frac{\theta}{1+\sigma}} \left[ \Theta(\lambda) \right]^{\frac{1-\theta}{1+\sigma}}, \tag{30}
\]

where \(\tilde{A}\) is given by (24). Let the joint distribution of firm types be given by \(H(T, \lambda)\) with joint probability density \(h(T, \lambda)\) the free entry condition can then be written

\[
\int \int \pi(T, \lambda)h(T, \lambda)dTd\lambda - F_E = 0 \tag{31}
\]

5 Trade Liberalization and Intra-Firm Productivity

In this section, we consider a few simple comparative static exercises to illustrate the workings of the model. First, we consider the effect of a reduction in trade frictions between countries, which corresponds to an increase in \(\rho\). Second, we consider a decrease in the fixed cost of opening a foreign affiliate \(F_I\). Our focus is on the novel aspect of this model that a change in the economic environment alters the within-firm allocation of scarce managerial expertise. All proofs are in the appendix.

The intra-firm productivity effects work through the mechanisms in (13) and (14). For instance, a reduction in trade cost (increase in \(\rho\)) has a direct impact on the allocation of time within a firm by causing a reallocation of managerial resources toward exported goods. Because managerial resources are in limited supply within the firm there is also an impact on all goods that works through changes in \(B\). Allocating more resources to exported goods necessarily reduces the amount of resources available to all other goods. Further, to the extent that the change in the external environment causes firms to switch modes for individual products (changes in the cutoffs \(z_D, z_X, z_M\)) this too will have an impact on the resources available to any individual good.

We begin by showing that the model shares in common with standard models of heterogeneous multiproduct firms that a reduction in international friction leads to a rationalization effect within the firm.

**Proposition 5** An increase in the freeness of trade, \(\rho\), or a reduction in the fixed cost of investing abroad, \(F_I\), reduces the product range of all firms.

A reduction in an international friction has a push and pull effect on the resource allocation within the firm. First, as international frictions
get less intense, the free entry condition requires that the mark-up adjusted demand level $A$ must fall, and this discourages the production of marginal goods within the firm. The same reduction in trade frictions will also encourage a reallocation of managerial resources away from goods that produced exclusively for the domestic market toward goods that are sold in the foreign market.

We now focus our attention on the within firm productivity effects of a reduction in trade costs $\tau$ (a rise in trade freeness $\rho$). We summarize the effects of trade and multinational production liberalization through its effect on the productivity of individual goods within the firm defined as $\varphi(Z) = Zt(Z)^{\bar{\theta}}$. We begin with the following intermediate result:

**Lemma 1:** An increase in trade freeness, $\rho$, causes at least some products produced by a firm to switch from not being exported to being exported.

At least some of the goods that were previously produced in both the domestic and foreign market will have their production rationalized to being exported from a domestic plant. It is also possible that some products will that were previously sold exclusively in the domestic market will also begin to be exported. What happens to the productivity of plants that have switched from not exporting? The following proposition establishes this productivity effect.

**Proposition 6** The productivity of a plant that switches from not exporting its product to exporting its product after an increase in trade freeness rises.

Products that switch from not being exported to being exported must see their productivity rise both relative to other goods in the firm’s portfolio and in absolute terms. This is consistent with the empirical results of Lileeva and Treffer (2009), who show that Canadian plants that switch from domestic only to export become more productive. What is novel about our approach is that the effect is not due to increase R&D but to the fact that managers reallocate managerial expertise within the firm. Once a firm consolidates the production of a good in one location for two markets the plant producing that good receives a higher proportion of the managerial resources available to the firm. If the push of managerial resources out of other activities such as marginally productive goods sold exclusively in the domestic market is sufficiently strong, then the productivity of all goods must rise.

We now turn our attention to the productivity at the firm level rather than at the level of the individual product line. The following proposition
considers the effect of a reduction in variable trade costs, or an increase in the freeness of trade.

**Proposition 7** There exists a cutoff level of domestic comparative advantage, \( \tilde{\lambda} < \tilde{\lambda} \) such that for all firms with \( \lambda < \tilde{\lambda} \) the productivity of all goods produced by the firm increases while for \( \lambda > \tilde{\lambda} \) the productivity of all non-exported goods falls.

The absolute productivity effects at the firm level of an increase in trade freeness depends on the firm’s initial orientation toward the foreign market. If the firm has a high comparative cost advantage producing in the domestic market for export (i.e. \( \lambda > \tilde{\lambda} \)), then many of its managerial resources will be allocated toward exporting already. In such a case the pull of the export market leads to such a large reallocation of managerial resources out of non-exported goods that the productivity of non-exported goods ultimately falls. If the firm exports relatively few products, then the fall in the mark-up adjusted demand level \( A \) brought about by the increase in trade freeness drives managerial resources out of marginal plants, and this release of managerial resources dominates the pull of resources toward exported products. The net result is an increase in the productivity of remaining plants.

The asymmetric effect of a change in the international environment across firms also appears when we consider the effect of a reduction in the fixed cost of engaging in multinational production as the following proposition makes clear.

**Proposition 8** Consider a reduction in the fixed cost of international operations, \( F^I \). There exists a \( \tilde{\lambda} < \tilde{\lambda} \) such that for firms of type \( \lambda < \tilde{\lambda} \) the productivity of all goods decreases and for firms of type \( \lambda > \tilde{\lambda} \) the productivity of all goods increases.

Unlike an increase in the freeness of trade, a reduction in the fixed cost of international operations has no direct effect on the allocation of managerial time across goods. Two indirect effects are at work. First, there is a tendency for firms to substitute on the margin multinational production for exports. While less managerial resources are allocated to any one plant for a switching good, collectively the two plants require more managerial resources than a single plant. This tends to take managerial resources away from other goods (dispersion effect). Second, as noted above, the marginal domestic plant closes as the mark-up adjusted demand level falls making more managerial resources available for remaining plants (consolidation effect). Firms with a strong comparative advantage producing in their domestic market (\( \lambda > \tilde{\lambda} \)) will see their productivity increase as the consolidation effect outweighs the dispersion effect while the opposite is true for the remaining firms.
6 Conclusion

The key feature of multiproduct firms is that they internalize the effects of decisions directed toward one set of goods on the outcomes of another set of goods. The types of effects that are internalized can either be on the product market side (e.g., cannibalization effects) or on the production side. The focus of this paper has been squarely on the latter. We have shown that when managerial expertise is a scarce resource in the firm (as in Lucas 1977 and Rosen 1982), that the decision of how many goods to produce, where to produce them (export versus FDI), and for which markets to produce become inter-related.

Several important insights emerge from our analysis. First, we show that considering the internalization effects of managerial time provides insight into the standard “proximity-concentration” model. An important benefit to consolidating production in a single location is that managerial resources are conserved, which allows the firm to produce the same set of goods more efficiently while also producing a wider range of goods.

Second, we have derived a new set of facts on the behavior of large multiproduct firms that both export and engage in MP in foreign markets and demonstrated that standard proximity-concentration models naively adapted to a multiproduct setting heterogeneity cannot explain these facts. Comparative advantage in domestic versus foreign management across firms combined with an internal resource constraint within the firm is consistent with these facts, however.

Third, we have shown how an internal managerial resource constraint leads to within-firm productivity effects that differ substantially from those of purely selection driven models. Changes in the international trading environment affect the way that firms allocate scarce managerial resources across products with the implication that some firms will appear to become more productive as they narrow their product range and concentrate production in fewer locations, while other firms will appear to become less productive as they expand their product range and allocate more resources to foreign production.

There are several natural extensions to the model. First, by adding additional countries one can generate export platform multinational production that provides a firm with the benefit of conserving scarce managerial expertise relative to replicating production in many locations. Second, by adding idiosyncratic differences in demand across countries and products, it becomes possible to generate a number of new outcomes such as the same firm exporting “both ways” between two countries. Third, if individual goods within a firm’s product portfolio receive a productivity shock that leads the firm to introduce a previously dor-
mant product, then the firm may drop a previously produced product line (or vice versa). This means that the model can be used to explain the product churn that has been documented in the literature.

Finally, an area that we have not explored in this paper, but which would be a fruitful subject of further analysis would be to consider how the allocation of scarce managerial time within the firm could be combined with contractual frictions as in Antras (2003) and Antras and Helpman (2004). In the latter, in any given industry there is an efficient contracting configuration that all firms would choose to adopt but for fixed costs associated with that mode. In the setting considered here, another benefit of outsourcing that could conserve on managerial expertise but which might come at the cost of a reduction in incentives. Such an extension would help to bring a wider array of international commerce into a single framework.

References


7 Appendix

7.1 Derivation of Aggregate Sales

7.1.1 Domestic Sales

As noted in the text, the definition of domestic sales is

\[ S_D(T, \lambda) = \int_{z_D}^{z_M} Z_{\xi-D}^{\frac{\alpha}{\alpha-\theta}} dG(Z) + (1 + \rho)^{\frac{\alpha}{\alpha-\theta}} \int_{z_M}^{\infty} Z_{\xi-D}^{\frac{\alpha}{\alpha-\theta}} dG(Z) \]

Integrating using the Pareto distribution, we obtain

\[ S_D(T, \lambda) = \sigma AT^\theta B^{-\theta} \left[ \left( z_D \right)^{\frac{\alpha}{\alpha-\theta} - 1} \left( (1 + \rho)^{\frac{\alpha}{\alpha-\theta}} - 1 \right) \left( z_X \right)^{\frac{\alpha}{\alpha-\theta} - \kappa} - \left( z_M \right)^{\frac{\alpha}{\alpha-\theta} - \kappa} \right] \]

Now substituting for the cutoffs using (18), (19), and (20), and defining \( a = \kappa / (\sigma - 1) \), we obtain

Substituting for the cutoffs we obtain

\[ S_D(T, \lambda) = \frac{\sigma F}{1 - \theta} (z_D)^{-\kappa} \frac{a}{a - 1} \left[ 1 + \left( (1 + \rho)^{\frac{\theta}{1-\theta}} - 1 \right) \left( \frac{F^{x}}{F} \right)^{1-a} - \left( \frac{F^{I}}{F(\lambda^{-\frac{\theta}{1-\theta}} - \Delta)} \right)^{1-a} \right] \]

Reorganizing this expression yields parent firm scale, which is equation (28) in the text. Finally, substituting for \( z_D \) using (18) and substituting out \( B \) using (21), we obtain (23).

7.1.2 Export Sales

The definition of export sales is \( S_X(T, \lambda) = \sigma A \rho \int_{z_X}^{z_M} Z_{\xi-D}^{\frac{\sigma}{\alpha-1} t_d(Z)^\theta} dG(Z) \), where \( t(Z) \) was derived in (13). Substituting for the allocation of managerial time we obtain

\[ S_X(T, \lambda) = \sigma AT^\theta B^{-\theta} \rho \left[ (1 + \rho)^{\frac{\theta}{1-\theta}} \int_{z_X}^{z_M} Z_{\xi-D}^{\frac{\alpha}{\alpha-\theta}} dG(Z) \right] . \]

Integrating using the Pareto distribution yields

\[ S_X(T, \lambda) = \frac{a\sigma \rho}{a - 1} AT^\theta \left( \frac{T}{B(\Phi, \lambda)} \right)^{\frac{\theta}{1-\theta}} \left( (1 + \rho)^{\frac{\theta}{1-\theta}} \left( z_X \right)^{\frac{\alpha}{\alpha-\theta} - \kappa} - \left( z_M \right)^{\frac{\alpha}{\alpha-\theta} - \kappa} \right) . \]

Applying the appropriate cutoff conditions (19) and (20), we obtain

\[ S_X(T, \lambda) = \frac{a\sigma (1 - \theta)^{\alpha-1} \rho}{a - 1} (AT^\theta B^{-\theta})^a (1 + \rho)^{\frac{\theta}{1-\theta}} \left[ \left( \frac{F^{x}}{\Delta} \right)^{1-a} - \left( \frac{F^{I}}{\lambda^{-\frac{\theta}{1-\theta}} - \Delta} \right)^{1-a} \right] \]

Finally, substituting for \( B \) using (21), we obtain (25).
7.1.3 Local Affiliate Sales

The definition of aggregate multinational sales is

\[ S^M(T, \lambda) = \sigma A \int_{z_M}^{\infty} Z^{\sigma-1} \left( \frac{t_f(Z)}{\lambda} \right)^\theta dG(Z) \]

where \( t(Z) \) was derived in (13). Following the same steps as above we obtain

\[ S^M(T, \lambda) = \sigma (AT^\theta) \frac{\Psi}{\Theta(\lambda)} \left( \frac{\lambda - \theta}{F^\theta - \delta} \right)^{a-1} \]

7.2 Proof of Proposition 3

Setting \( \theta = 0 \), the expression for domestic sales (23) simplifies to

\[ S^D(a) = \frac{\sigma a}{a-1} (A)^a F^{1-a} \]

where we have written sales as a function of \( a \) because we have put all firm heterogeneity into this variable. Note that a decrease in \( a \) is associated with an increase in dispersion across goods within an industry. Taking the logarithms of this expression we obtain

\[ \log S^D(a) = \log \sigma + \log(a) - \log(a-1) + a \log(A) + (1-a) \log F \]

Now differentiating, we obtain

\[ \frac{dS^D(a)}{da} = -\frac{1}{a(a-1)} - \log \left( \frac{F}{A} \right) \]

Because \( z_D = (\frac{F}{A})^{\frac{1}{\sigma-1}} \geq 1 \) we have \( dS^D(a)/da < 0 \). So an increase in dispersion must raise local sales.

7.3 Free Entry Condition Derivation

From (16) profits are defined as

\[ \pi = AT^\theta B^{1-\theta} - (1 - G(z_D))F - (1 - G(z_M))F^m - (G(z_m) - G(z_x))F^x \]

Substitute for the Pareto Distribution to obtain

\[ \pi = AT^\theta B^{1-\theta} - (z_D)^{-\kappa}F - (z_X)^{-\kappa}F^x - (z_M)^{-\kappa}F^I \]

Substitute for the cutoffs using (18)-(20) to obtain

\[ \pi = AT^\theta B^{1-\theta} - \left( (1 - \theta)AT^\theta B^{-\theta} \right)^a \Theta(\lambda) \]
Using the definition of $B$, solving for $\Theta(\lambda)$, and substituting the resultant expression, this can be simplified to

$$\pi = \frac{1 + \theta(a - 1)}{a} A T^\theta B^{1 - \theta}.$$

Now, substitute for $B$ and simplify to obtain

$$\pi = \frac{1 + \theta(a - 1)}{a} \tilde{A} T^\frac{\theta}{1 - \theta + a \theta} \Theta(\lambda)^{\frac{1 - \theta}{1 - \theta + a \theta}}.$$

Combining these (16) and (31), we obtain

$$\frac{1 + \theta(a - 1)}{a} \tilde{A} \int \int T^\frac{\theta}{1 - \theta + a \theta} \left[ \Theta(\lambda) \right]^{\frac{1 - \theta}{1 - \theta + a \theta}} h(T, \lambda) dT d\lambda - F^E = 0.$$

As our model delivers no extensive margin across firms by abstracting from on-going corporate fixed costs, this parameter never changes in any comparative statics.

### 7.4 Proof of Proposition 5

The least productive good produced by a firm is given by equation (18). Let a prime denote the value of a variable after a reduction in an international friction. We have

$$z_D' = \left( \frac{A}{A'} \right) (B')^\theta \frac{1}{1 - \theta + a \theta} - \frac{\theta}{1 - \theta + a \theta}.$$

Neither trade or MP friction enters this expression, so all the effects work through the endogenous variables $AB^{-\theta}$. Using the definition of $B$ given by (21) we find

$$z_D' = \left( \frac{A}{A'} \right) \tilde{A} \left( \Theta(\lambda)' \right)^\frac{1}{1 - \theta + a \theta} - \frac{\theta}{1 - \theta + a \theta}.$$

Note that if this variable falls with a reduction in international frictions, then the cutoff rises as we now show. Let primed variables be the values after a trade or MP liberalization. It is immediate from our parameter restrictions that $\Theta(\lambda)' > \Theta(\lambda)$ for all firms. Using the free entry condition (31), we obtain

$$A^{\frac{1}{1 - \theta + a \theta}} = \left( \frac{\tilde{A}}{\tilde{A}'} \right) \frac{F^E}{\left( \frac{a(1 - \theta)^{a - 1}}{a - 1} \right)^{\frac{1}{1 - \theta + a \theta}} \int \int T^\frac{\theta}{1 - \theta + a \theta} \left[ \Theta(l) \right]^{\frac{1 - \theta}{1 - \theta + a \theta}} h(T, l) dT dl}.$$

29
Hence, we have

\[
\left( \frac{A'}{A} \right)^{\frac{1}{1-\theta+\sigma}} = \left( \frac{\int \int T^{\frac{0}{1-\theta+\sigma}} [\Theta(l)]^{\frac{1-\theta}{1-\theta+\sigma}} h(T, l) dT dl}{\int \int T^{\frac{0}{1-\theta+\sigma}} [\Theta(l)]^{\frac{1-\theta}{1-\theta+\sigma}} h(T, l) dT dl} \right)^{\frac{1}{\alpha}} < 1
\]

So either type of liberalization lowers the mark-up adjusted demand level. Thus, we have

\[
z_M' = \frac{z'_D}{z_D} = \left( \frac{\int \int T^{\frac{0}{1-\theta+\sigma}} [\Theta(l)]^{\frac{1-\theta}{1-\theta+\sigma}} h(T, l) dT dl}{\int \int T^{\frac{0}{1-\theta+\sigma}} [\Theta(l)]^{\frac{1-\theta}{1-\theta+\sigma}} h(T, l) dT dl} \right)^{\frac{1-\theta}{\sigma-1}} > 1
\]

The cutoff for operating a good for all firms must rise.

### 7.5 Proof of Lemma 1

Consider the ratio of cutoffs \(z_M/z_D\). Using the cutoff definitions (18) and (20), we have

\[
z_M = \frac{1}{F} \left( \frac{1}{\lambda^{\frac{\theta}{1-\theta+\sigma}} - \Delta} \right)^{\frac{1-\theta}{\sigma-1}}
\]

An increase in trade freeness raises \(\Delta\) directly, and by proposition 5 raises \(z_D\) thus at least some goods that were previous produced in both locations (and thus not exported) become exported.

### 7.6 Proof of Proposition 6

For a small change in the freeness of trade there are two types of goods that could switch from not being exported to being exported. First there are goods that were not sold abroad at all (\(Z\) near \(z_X\)). Second, there are goods that were sold abroad through a multinational affiliate (see Lemma 1) whose production is rationalized with an increase in the freeness of trade (\(Z\) near \(z_M\)). Let a prime indicate the value of a variable after a change, the change in managerial resources allocated to any plant that only served the domestic market before the rise in trade freeness is

\[
\frac{t(Z')}{t(Z)} = \frac{B}{B'} (1 + \rho')^{\frac{1}{\theta}}.
\]

There are two possibilities. First, \(B\) falls. In this case, the time allocated to all remaining plants must rise including those that are exported. Second, \(B\) might rise so that \(B/B' < 1\). In this case, equations (13) shows that all goods that are not exported must have fewer managerial resources allocated to them. The change in \(t(Z)\) for an incumbent
exporter is
\[
\frac{t(Z')}{t(Z)} = \frac{B}{B'} \left( \frac{1 + \rho'}{1 + \rho} \right)^{\frac{1}{1+\rho}},
\]
which is strictly less than for a good that has switched from non-exporting
to exporting. Hence, as some goods must have more managerial re-
resources allocated so that resource allocation constraint binds, it must be
that the productivity of switchers rises.

7.7 Proof of Proposition 7

As made clear by (13) if an economic shock lowers the managerial time
burden \(B\) then the productivity of all incumbent products must increase.
We start by differentiating (21) with respect to \(\rho\) to obtain
\[
\frac{dB}{B \partial \rho} = \frac{1}{1 + \theta(a - 1)} \left[(a - 1) \frac{dA}{Ad\rho} + \eta(\lambda)\right].
\]
where
\[
\eta(\lambda) \equiv \frac{\partial \Theta(\lambda)}{\partial \rho} \Theta(\lambda) = \frac{\alpha}{1 - \theta} (1 + \rho)^{\frac{\theta}{1+\rho}} \left(\frac{F_x}{\Delta} \right)^{1-a} - \left(\frac{F_I}{\lambda^{\frac{a}{1-\theta}} - \Delta} \right)^{1-a} > 0.
\]
Next, totally differentiate the zero profit condition (31) to obtain
\[
\frac{dA}{Ad\rho} = -\frac{1 - \theta}{a} \int \int w(T, \lambda') \eta(\lambda') d\lambda' dT < 0
\]
where
\[
w(T, \lambda) = \frac{T^{1-\theta} \Theta(\lambda')^{1-\theta} h(T, \lambda')}{\int T^{1-\theta} \Theta(\lambda'')^{1-\theta} h(T, \lambda'') dT d\lambda''}.
\]
A reduction in trade costs must increase entry and so make the mark-up
adjusted demand level \(A\) fall. The magnitude of the decrease in \(A\) is
proportional to a weighted average of percent change in the cost index
\(\Theta(\lambda)\) across firms. Combining expressions, we obtain
\[
\frac{dB}{Bd\rho} = \frac{1}{1 + \theta(a - 1)} \left[\eta(\lambda) - \frac{(a - 1)(1 - \theta)}{a} \int \int w(T, \lambda') \eta(\lambda') d\lambda' dT\right]
\]
From this expression we note that \(\frac{(a - 1)(1 - \theta)}{a} < 1\) so that there must be
some firms for whom \(dB/d\rho > 0\). Note also that \(\partial \eta(\lambda)/\partial \lambda > 0\). This
means that there must be a \(\hat{\lambda} < \bar{\lambda}\) such that for all \(\lambda > \hat{\lambda}, dB/d\rho > 0\) and
for \(\lambda < \bar{\lambda}\), \(dB/d\rho < 0\). It may be that \(\bar{\lambda} < 1\) in which case \(dB/d\rho > 0\)
for all firms. For the firms with \(\lambda\) above this cutoff, the productivity of
non-export goods will fall.
7.8 Proof of Proposition 8

As made clear by (13) if an economic shock lowers the managerial time burden $B$ then the productivity of all incumbent products must increase. We start by differentiating (21) with respect to $F^I$ to obtain

$$
\frac{dB}{BdF^I} = \frac{1}{1 + \theta(a - 1)} \left[ (a - 1) \frac{dA}{AdF^I} + \eta(\lambda) \right],
$$

where

$$
\eta(\lambda) \equiv \frac{\partial \Theta(\lambda)}{\partial F^I} \frac{1}{\Theta(\lambda)} = \frac{(1 - a) \left( \lambda^{-\frac{a}{F^I}} \Delta \right)^a}{F^{1-a} + \Delta a \left( F^x \right)^{1-a} + \left( \lambda^{-\frac{a}{F^I}} - \Delta \right)^a (F^I)^{1-a}} < 0.
$$

Note that the inequality follows from the fact that $a > 1$. Next, totally differentiate the zero profit condition (31) to obtain

$$
\frac{dA}{AdF^I} = -\frac{1 - \theta}{a} \int \int w(T, \lambda') \eta(\lambda') d\lambda' dT > 0
$$

where

$$
w(T, \lambda) = \frac{T^{-\frac{a}{v+\theta}} [\Theta(\lambda')]^{-\frac{1-\theta}{v+\theta}} h(T, \lambda')}{\int \int T^{-\frac{a}{v+\theta}} [\Theta(\lambda'')]^{-\frac{1-\theta}{v+\theta}} h(T, \lambda'') dT d\lambda''}.
$$

An increase in $F^I$ must reduce $A$, the mark-up adjusted demand level, and the magnitude of this fall is proportional to a weighted average of percent change in the cost index $\Theta(\lambda)$ across firms. Combining expressions, we obtain

$$
\frac{dB}{BdF^I} = \frac{1}{1 + \theta(a - 1)} \left[ \eta(\lambda) - \frac{(a - 1) (1 - \theta)}{a} \int \int w(T, \lambda') \eta(\lambda') d\lambda' dT \right]
$$

From this expression we note two things. First, $\frac{(a - 1)(1 - \theta)}{a} < 1$ and so there must exist some firms for whom an increase in $F^I$ lowers $B$. Second, note that $\partial \eta(\lambda)/\partial \lambda > 0$. It then follows that there must exist a $\tilde{\lambda} < \bar{\lambda}$ such that for all $\lambda > \tilde{\lambda}$, $dB/dF^I > 0$ and for $\lambda < \tilde{\lambda}$, $dB/dF^I < 0$, and it may be that $\tilde{\lambda} < 1$ in which case $dB/dF^I > 0$ for all firms. Hence, a reduction in $F^I$ will make $B$ fall for $\lambda > \tilde{\lambda}$, increasing productivity of all goods, and increase $B$ for $\lambda < \tilde{\lambda}$, lowering the productivity of all goods.
Table 1: Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Number of Observations</th>
<th>Average</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parent US Sales</td>
<td>725</td>
<td>$1.7 billion</td>
<td>$620 million</td>
</tr>
<tr>
<td>Parent Number of Product lines</td>
<td>725</td>
<td>2.7</td>
<td>2</td>
</tr>
<tr>
<td>Parent Concentration</td>
<td>725</td>
<td>0.71</td>
<td>0.29</td>
</tr>
<tr>
<td>Parent Exports</td>
<td>8,244</td>
<td>$2.4 million</td>
<td>$19 million</td>
</tr>
<tr>
<td>Affiliate Sales</td>
<td>2,579</td>
<td>$96.5 million</td>
<td>$443 million</td>
</tr>
<tr>
<td>Share of Affiliate Sales in Total Sales*</td>
<td>2,579</td>
<td>0.77</td>
<td>0.53</td>
</tr>
</tbody>
</table>

Notes: Data are levels. * indicates that average is conditional on Affiliate Sales observed.

Table 2: Foreign Market Activity and the Domestic Operations of Multinational Firms

<table>
<thead>
<tr>
<th></th>
<th>Local Affiliate Sales</th>
<th>Parent Exports</th>
<th>Share of Affiliate Sales in total</th>
<th>Local Affiliate Sales</th>
<th>Parent Exports</th>
<th>Share of Affiliate Sales in total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parent Sales</td>
<td>0.58 (0.03)</td>
<td>0.46 (0.03)</td>
<td>-0.07 (0.01)</td>
<td>0.57 (0.03)</td>
<td>0.47 (0.03)</td>
<td>-0.052 (0.01)</td>
</tr>
<tr>
<td>GDP</td>
<td>0.57 (0.04)</td>
<td>0.31 (0.02)</td>
<td>0.01 (0.01)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>GDPPC</td>
<td>0.08 (0.07)</td>
<td>0.18 (0.03)</td>
<td>-0.06 (0.02)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIST</td>
<td>-0.20 (0.03)</td>
<td>-0.37 (0.02)</td>
<td>0.08 (0.02)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LANG</td>
<td>0.13 (0.07)</td>
<td>0.19 (0.04)</td>
<td>0.03 (0.02)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed Effects by Country?</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>N</td>
<td>2,579</td>
<td>8,244</td>
<td>2,579</td>
<td>2,579</td>
<td>8,244</td>
<td>2,579</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.38</td>
<td>0.25</td>
<td>0.03</td>
<td>0.42</td>
<td>0.30</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Notes: All variables (except language) are in logarithms. Standard errors (shown in parentheses) are robust to heteroskedascity and clustered by firm. Column headings indicate the dependent variable.
Table 3: Scale versus Scope and the Within Firm Composition of Commerce

| Dependent Variable: logarithm of affiliate sales in exports plus affiliate sales |
|------------------|------------------|------------------|------------------|
| Scale            | -0.03 (0.01)     | -0.05 (0.02)     | -0.03 (0.01)     |
| Scope            | -0.12 (0.02)     | -0.12 (0.02)     | 0.04 (0.05)      |
| Parent Size      |                  |                  | -0.05 (0.01)     |
| Herfindahl       |                  | 0.24 (0.08)      | 0.15 (0.04)      |
| R&D intensity    | -0.01 (0.02)     | -0.02 (0.02)     | 0.01 (0.03)      |
| Capital Intensity| 0.05 (0.02)      | 0.03 (0.02)      | 0.01 (0.03)      |
| N                | 2586             | 2468             | 2468             | 2468             |
| R-sq             | 0.09             | 0.09             | 0.10             | 0.16             |

Notes. All specifications include country fixed effects. All variables in logarithms. All standard errors (shown in parentheses) are robust to heteroskedascity and clustered by firm.