Firm-to-Firm Trade:

Imports, Exports, and the Labor Market

Jonathan Eaton,[1] Samuel Kortum,[2] Francis Kramarz,[3] Raul Sampognaro[4]

June 2013

[1]The Pennsylvania State University, jxe22@psu.edu

[2]Yale University, samuel.kortum@yale.edu

[3]CREST(ENSAE), kramarz@ensae.fr

[4]CREST, SciencesPo

1 Introduction

We build a model of firm-to-firm trade to capture the role of intermediate inputs in production and international transactions. At the firm level, the model generates predictions for imports, exports, and the share of labor in production broadly consistent with observations on French manufacturers. At the aggregate level, firm-to-firm trade determines bilateral trade shares as well as labor's share of output in each country.

Without intermediates, the model reduces to a version of Melitz (2003), Chaney (2008), and Eaton, Kortum, and Kramarz (2011). The model of firm-to-firm trade is complementary to the recent work of Oberfield (2013). Firm production combines the output of a number of tasks with a firm-specific efficiency level. Each task can be performed by the firm's employees or by an intermediate input purchased by the firm. The intermediates available to a firm are determined by a matching process, with the firm replacing its own workers to perform a particular task if a cheap enough intermediate is available. The distribution of prices for intermediates is itself determined by the distribution of costs of the other firms which produce them.

Our work relates to several strands in the literature. Recent papers looking at exports or imports and labor markets include Hummels, Jørgensen, Munch, and Xiang (2011), Felbermayr, Prat, and Shmerer (2008), Egger and Kreickemeier (2009), Helpman, Itskhoki, and Redding (2010), Caliendo and Rossi-Hansberg (2012). In addition to Oberfield (2013), other theories of networks or input-output interactions include Lucas (2010), Acemoglu and Autor (2011), Chaney (2013), and Acemoglu and Carvalho (2012). Quantitative work on exports, imports, and labor markets includes Irarrazabal, Moxnes, and Ulltveit-Moe (2010), Klein, Moser, and Urban (2010), Frias, Kaplan, and Verhoogen (2010), Kramarz (2009), Caliendo, Monte, and Rossi-Hansberg (2013), and Helpman, Itskhoki, Muendler, and Redding (2013). Recent work on firm-level imports includes Halpern, Koren, and Szeidl (2011) as well as Blaum, Lelarge, and Peters (2013).

2 A Model of Firm-to-Firm Trade

Consider $i = 1, 2, ..., N$ countries, each endowed with labor L_i. Countries manufacture differentiated goods by combining labor and manufactured intermediates. These goods themselves may be consumed directly or used as intermediates in the production of other goods. These goods are traded, subject to iceberg cost: to deliver a unit to country n, country i must ship $d_{ni} \geq 1$ units.

We first focus on firms in a particular country i. A firm produces a unique good j, with firm-specific efficiency $z_i(j)$, by combining the output of K tasks in a Cobb-Douglas constant-returns-to-scale production function. Each task k can be performed by the firm's own workers, at wage $w_{k,i}$, or by purchasing an intermediate produced by another firm, at price $p_{k,i}$. The firm will choose to do the task in house with its own workers if $w_{k,i} \leq p_{k,i}$ or choose to buy the intermediate if $w_{k,i} > p_{k,i}$. We assume that the measure of firms in i with efficiency above z is:

$$\mu_i^z(z) = T_i z^{-\theta}.$$

In addition to firm efficiency, the intermediate goods available to the firm is also heterogenous. The prices of these intermediates depends on the set of suppliers with whom the firm has made contact. The total unit cost of the firm thus depends on the vector of intermediate prices it faces, $\boldsymbol{p} = (p_{1,i}, p_{2,i}, ..., p_{K,i})'$, one for each task. Given $\boldsymbol{p}$, the firm's unit cost of delivering to country n is:

$$c_{ni} = \frac{d_{ni} b_i(\boldsymbol{p})}{z},$$

where

$$b_i(\boldsymbol{p}) = \prod_{k=1}^{K} \min \{w_{k,i}, p_{k,i}\}^{\beta_k},$$

and

$$\sum_{k=1}^{K} \beta_k = 1.$$

If all firms in i faced the same vector of intermediate prices $\boldsymbol{p}$, the measure that could serve market n at a cost below c would be:

$$\mu_i^z(d_{ni} b_i(\boldsymbol{p})/c) = T_i \left[d_{ni} b_i(\boldsymbol{p})\right]^{-\theta} c^{\theta}. \tag{1}$$

In fact, they will face different intermediate prices. To solve the model, we need to integrate this measure over the distribution of intermediate prices faced by firms in country i. We now derive this distribution.

2.1 Distribution of Intermediate Prices

For each task k a firm encounters $h_{k,i}$ price quotes from different suppliers (domestic or foreign) of the the intermediate, where $h_{k,i}$ is distributed Poisson with parameter $\lambda_{k,i}$ and each price is drawn from a distribution $F_i(p)$, independent across k and across draws for each k. Each seller present in a market is equally likely to encounter any buyer, regardless, for instance, of its origin or cost.

The firm would choose only the supplier with the lowest quote. Conditioning on the number of quotes $h_{k,i}$, the distribution of the minimum quote is

$$\Pr\left[p_{k,i} \leq p | h_{k,i}\right] = 1 - \left[1 - F_i(p)\right]^{h_{k,i}}.$$

The unconditional distribution is:

$$\begin{aligned} G_{k,i}(p) &= \sum_{h_{k,i}=0}^{\infty} \frac{e^{-\lambda_{k,i}} \left(\lambda_{k,i}\right)^{h_{k,i}}}{h_{k,i}!} \left(1 - \left[1 - F_i(p)\right]^{h_{k,i}}\right) \\ &= 1 - \left(e^{-\lambda_{k,i} F_i(p)} \sum_{h_{k,i}=0}^{\infty} \frac{e^{-\lambda_{k,u}\left[1 - F_u(p)\right]} \left(\lambda_{k,i}\left[1 - F_n(p)\right]\right)^{h_{k,i}}}{h_{k,i}!}\right) \\ &= 1 - e^{-\lambda_{k,i} F_i(p)}. \end{aligned}$$

As we derive below, the individual quotes are drawn from the distribution:

$$F_i(p) = \left(\frac{p}{\bar{c}_i}\right)^{\theta}, \tag{2}$$

for $p \leq \bar{c}_i$, where $\bar{c}_i$ is the highest possible price in country i for any task k, also derived below.

Substituting into the distribution of the minimum price above gives:

$$G_{k,i}(p) = 1 - e^{-\nu_{k,i} p^{\theta}}, \tag{3}$$

where

$$\nu_{k,i} = \lambda_{k,i} \bar{c}_i^{-\theta}$$

Thus the minimum intermediate price that the individual firm sees for a particular task (3) is distributed Weibull with location parameter $\nu_{k,i}$, with a higher value meaning that a lower price is more likely, and shape parameter θ, with a higher value meaning that prices are less disbursed.

Having derived the distribution of the minimum available price of an intermediate we now turn to the distribution of costs.

2.2 Distribution of Costs

We can integrate (1) over (3) to obtain an expression for the measure of firms from i that can deliver to market n at a unit cost below c:

$$\begin{aligned}\mu_{ni}(c) &= \int_0^\infty ... \int_0^\infty \mu_i^z(d_{ni}b_i(\boldsymbol{p})/c)dG_{1,i}(p_1)...dG_{K,i}(p_K) \\ &= T_i d_{ni}^{-\theta} c^\theta \prod_{k=1}^{K} \left(\int_0^\infty \min\left\{w_{k,i}, p_k\right\}^{-\theta\beta_k} dG_{k,i}(p_k) \right), \end{aligned} \tag{4}$$

where we have used the fact that the price draws are independent across tasks.

A complication in solving this integral is determining the relevant support of the intermediate goods prices. If $w_{k,i} > \bar{c}_i$ then any available intermediate would always displace workers, so any price up to the upper bound $\bar{c}_i$ is relevant. If $w_{k,i} \leq \bar{c}_i$ then prices above $w_{k,i}$ would always be rejected, so that $w_{k,i}$ is the relevant upper bound. We will impose restrictions that ensure:

$$\bar{c}_i \geq \max_k \left\{w_{k,i}\right\}, \tag{5}$$

to avoid uninteresting taxonomy. This inequality implies that there are active firms in the economy whose unit costs are so high that they make no intermediate sales, since it would be cheaper for any buyer to produce in house rather than to use intermediates produced by such firms. This inequality also implies that a firm take advantage of an intermediate to perform task k with probability:

$$G_{k,i}(w_{k,i}) = 1 - \exp\left[-\nu_{k,i}(w_{k,i})^\theta\right].$$

Note that the term $\nu_{k,i}(w_{k,i})^{\theta}$ is equal to the arrival rate of quotes $\lambda_{k,i}$ times the probability $(w_{k,i}/\overline{c}_i)^{\theta}$, from (2), that a quote is below the wage $w_{k,i}$.

Since w_{ki} is the relevant upper bound on the intermediate goods prices that a buyer would consider, we can write:

$$\begin{aligned}
\int_0^{\infty} \min\{w,p\}^{-\theta\beta} dG(p) &= [1-G(w)]\,(w)^{-\theta\beta} + \int_0^{w} p^{-\theta\beta} dG(p) \\
&= e^{-\nu w^{\theta}} w^{-\theta\beta} + \int_0^{w} p^{-\theta\beta}\theta\nu p^{\theta-1} e^{-\nu p^{\theta}} dp \\
&= w^{-\theta\beta}\left[e^{-\nu w^{\theta}} + \left(\nu w^{\theta}\right)^{\beta} \int_0^{\nu w^{\theta}} y^{-\beta} e^{-y} dy\right] \\
&= w^{-\theta\beta}\left[e^{-\nu w^{\theta}} + \left(\nu w^{\theta}\right)^{\beta} \gamma\left(1-\beta,\nu w^{\theta}\right)\right] \qquad (6)
\end{aligned}$$

where

$$\gamma(1-\beta,\nu w^{\theta}) = \int_0^{\nu w^{\theta}} y^{-\beta} e^{-y} dy$$

is the incomplete gamma function.

Using (6), we can solve (4):

$$\mu_{ni}(c) = T_i \Xi_i \left(\overline{w}_i d_{ni}\right)^{-\theta} c^{\theta},$$

where

$$\Xi_i = \prod_{k=1}^{K}\left[e^{-\nu_{k,i}(w_{k,i})^{\theta}} + \left[\nu_{k,i}(w_{k,i})^{\theta}\right]^{\beta_k}\gamma(1-\beta_k,\nu_{k,i}(w_{k,i})^{\theta})\right],$$

and

$$\overline{w}_i = \prod_{k=1}^{K_s}\left(w_{k,i}\right)^{\beta_k}$$

is the cost of a bundle of labor inputs if no intermediates are available.

Aggregating across all sources of supply, the measure of firms that can deliver a good to market n at a cost below c is:

$$\begin{aligned}\mu_n(c) &= \sum_{i=1}^{N} \mu_{ni}(c) \\ &= \Upsilon_n c^{\theta}\end{aligned}$$

where:

$$\Upsilon_n = \sum_i T_i \Xi_i \left(\overline{w}_i d_{ni}\right)^{-\theta}. \tag{7}$$

Since the cutoff entry cost in country n (derived in the next section) is $\overline{c}_n$, the measure of active firms in n is $\mu_n(\overline{c}_n)$.

We assume that intermediates are available at the marginal cost of production.[1] Hence the distribution of price quotes for intermediates in a market corresponds to the distribution of unit costs for active firms. This distribution is:

$$F_n(p) = \frac{\mu_n(p)}{\mu_n(\overline{c}_n)} = \left(\frac{p}{\overline{c}_n}\right)^{\theta},$$

consistent with the form asserted in (2).

2.3 Firm Entry

While different goods can be perfect substitutes as an intermediate for a firm in performing a task, and are sold at unit cost, final buyers perceive the outputs of different firms as differentiated, and pay a markup over unit cost.

[1] Two justifications for this assumption are: (i) that buyers have all the bargaining power in the relationhip between buyers and sellers of intermediates or (ii) that bargaining between them is efficient, so that sellers make their goods available at marginal cost. In the first case (but not necessarily the second) buyers glean all the surplus from the exchange. In the second case sellers might gain a share of the surplus through, say, a lump sum transfer.

In particular, consumers of final goods have Dixit-Stiglitz preferences, with an elasticity of substitution:

$$1 < \sigma < \theta + 1.$$

Given total consumption spending X_n^C, consumers in country n will spend

$$x_n^C = \left(\frac{p^C}{P_n^C}\right)^{-(\sigma-1)} X_n^C$$

on a particular good whose final price is p^C, where P_n^C is the price index for consumption goods.

If a firm's unit cost of producing a good (including transport) is c, it will charge the monopoly price to final consumers of $p^C = \bar{m}c$, where

$$\bar{m} = \frac{\sigma}{\sigma - 1}.$$

Hence its sales in country n are:

$$x_n^C(c) = \left(\frac{\bar{m}c}{P_n^C}\right)^{-(\sigma-1)} X_n^C. \tag{8}$$

For simplicity, we assume that firms earn no profit from intermediate sales. Hence, in deciding whether or not to enter market n, a firm compares its variable profit $x_n^C(c)/\sigma$ with the cost of entry. We allow the entry cost E_n to vary by destination but not by source. Faced with this entry cost, a firm with unit cost c in delivering to n will be active in the market if $c \leq \bar{c}_n$, where this upper bound cost threshold satisfies

$$x_n^C(\bar{c}_n) = \sigma E_n,$$

so that

$$\bar{c}_n = \frac{P_n}{\bar{m}} \left(\frac{\sigma E_n}{X_n^C}\right)^{-1/(\sigma-1)}, \tag{9}$$

which applies to any potential producer regardless of its location i.[2]

2.4 Trade Shares and Price Indices

Country i's share in n's demand for final goods is:

$$\pi_{ni} = \frac{\mu_{ni}(\overline{c}_n)}{\mu_n(\overline{c}_n)} = \frac{\Xi_i T_i \left(\overline{w}_i d_{ni}\right)^{-\theta}}{\Upsilon_n}.$$

Finally, the price index P_n for final goods, given parameters and wages, solves:

$$\begin{aligned}\left(\frac{P_n}{\bar{m}}\right)^{1-\sigma} &= \sum_{i=1}^{N} \int_0^{\overline{c}_n} c^{-(\sigma-1)} d\mu_{ni}(c) \\ &= \sum_{i=1}^{N} \left[\frac{\theta}{\theta-(\sigma-1)} \Xi_i T_i \left(\overline{w}_i d_{ni}\right)^{-\theta} \left(\frac{P_n}{\bar{m}}\right)^{\theta-(\sigma-1)} \left(\frac{\sigma E_n}{X_n^C}\right)^{1-\theta/(\sigma-1)}\right],\end{aligned}$$

where the second equality follows from (9). Rearranging:

$$P_n = \bar{m}\left[\frac{\theta}{\theta-(\sigma-1)} \left(\frac{\sigma E_n}{X_n^C}\right)^{1-\theta/(\sigma-1)} \Upsilon_n\right]^{-1/\theta}, \tag{10}$$

which can be solved given parameters, wages, and market size. The result can be substituted back into (9) to solve for the entry threshold:

$$\overline{c}_n = \left(\frac{\theta}{\theta-(\sigma-1)} \Upsilon_n \frac{\sigma E_n}{X_n^C}\right)^{-1/\theta}.$$

It is now clear that assumption (5) is satisfied for large enough values of E_n/X_n^C.

The measure of entrants is thus:

$$\mu_n(\overline{c}_n) = \frac{\theta-(\sigma-1)}{\theta} \frac{X_n^C}{\sigma E_n} \tag{11}$$

[2] Our restriction (5) ensures that, if we allowed any distribution of the buyer's surplus to the seller of an intermediate, there would be no consequence for entry. The reason is that, in order to be of interest to the buyer of an intermediate, the seller would need to have a unit cost strictly below the entry threshold.

The new elements here are the terms Ξ_i, which capture the efficiency gains from outsourcing either domestically or abroad. Suppose $\lambda_{k,i} = 0$ (and hence $\nu_{k,i} = 0$) for all k and i, so that no intermediates are available and a firm will use its own workers for task k with probability 1. As a result, $\Xi_i = 1$ for all i and hence $\Upsilon_n = \sum_{i=1}^{N} T_i \left(\overline{w}_i d_{ni}\right)^{-\theta}$ for all n, as in EKK (2011).

2.5 Intermediate Sales

Given a firm's cost c, its final sales in a market i are deterministic, as given by (8). Its intermediate sales depend on its number of buyers and on how much each one buys, both of which are random. We denote its expected intermediate sales as $x_i^I(c)$, equal to its expected number of customers times its expected sales per customer.

How many buyers in country i will a seller with unit cost c there have? There are two components, the number of encounters that she has with potential buyers, and the probability of making a sale conditional on an encounter.

For meetings to balance, the number of potential buyers for input k that a seller encounters is distributed Poisson with parameter $\lambda_{k,i}\pi_{ii}$, since there are π_{ii} buyers (since they must be producing in i) for each seller (which is any firm selling in i).

Given that a seller has matched with a potential buyer, what is the probability that she makes a sale? First, her unit cost c has to lie below the wage $w_{k,i}$. We denote that event by the indicator function $\delta(c \leq w_{k,i})$. Second, c has to be less than the unit costs of all of the competitors. That probability is simply:

$$1 - G_{k,i}(c) = \exp\left(-\nu_{k,i} c^{\theta}\right) = \exp\left[-\lambda_{k,i}\left(c/\overline{c}_i\right)^{\theta}\right]$$

Hence the number of buyers of input k in country i for a seller with cost c is distributed

Poisson with parameter:

$$\Lambda_{k,i}(c) = \pi_{ii}\lambda_{k,i}\exp\left[-\lambda_{k,i}\left(c/\overline{c}_i\right)^{\theta}\right]\delta\left[c \leq w_{k,i}\right].$$

Not surprisingly, this parameter, which is the expected number of customers, falls with c. Note how the effect of $\lambda_{k,i}$ is nonmonotonic. At $\lambda_{k,i} = 0$, meeting a intermediate customer is impossible, so the number of customers is zero. Increasing $\lambda_{k,i}$ from zero initially increases the expected number of customers. But at some point, further increases in $\lambda_{k,i}$ mean that a seller is more likely to be beat out by lower cost rivals, reducing the expected number of customers per firm.

We now turn to intermediate sales per buyer in country i, which depend on the buyer's final and intermediate sales in each destination country n.

Consider sales of input k to a buyer with unit production cost c'. The buyer enters country n if $c'd_{ni} \leq \overline{c}_n$, in which case its final sales there are $x_n^C(c'd_{ni})$ and its expected intermediate sales are $x_n^I(c'd_{ni})$.

Integrating over the distribution of costs c' for active buyers in i, we get an expression for expected intermediate sales of input k for a seller in i with unit cost c:

$$x_{k,i}^I(c) = \Lambda_{k,i}(c)\beta_k \sum_{n=1}^{N} \int_0^{\overline{c}_n/d_{ni}} \left(\frac{x_n^C(c'd_{ni})}{\overline{m}} + x_n^I\left(c'd_{ni}\right)\right)\theta\overline{c}_i^{-\theta}\left(c'\right)^{\theta-1}dc'.$$

Changing the variable of integration from c' to $c'' = d_{ni}c'$ yields:

$$x_{k,i}^I(c) = \Lambda_{k,i}(c)\beta_k \sum_{n=1}^{N} \int_0^{\overline{c}_n} \left(\frac{x_n^C(c'')}{\overline{m}} + x_n^I\left(c''\right)\right)\left(d_{ni}\overline{c}_i\right)^{-\theta}\theta\left(c''\right)^{\theta-1}dc''.$$

which simplifies to

$$x_{k,i}^I(c) = \Lambda_{k,i}(c)\beta_k \sum_{n=1}^{N} \left(\frac{\overline{c}_n}{\overline{c}_i d_{ni}}\right)^{\theta}\left(\frac{x_n^C}{\overline{m}} + x_n^I\right),$$

where:

$$x_n^I = \int_0^{\bar{c}_n} x_n^I\left(c'\right)\theta\bar{c}_n^{-\theta}\left(c'\right)^{\theta-1}dc'.$$

Aggregating across sales of the different inputs:

$$x_i^I(c) = \sum_{k=1}^{K} x_{k,i}^I(c) = \sum_{k=1}^{K}\pi_{ii}\lambda_{k,i}\beta_k \exp\left[-\lambda_{k,i}(c/\bar{c}_i)^\theta\right]\delta\left[c \leq w_{k,i}\right]\sum_{n=1}^{N}\left(\frac{\bar{c}_n}{\bar{c}_i d_{ni}}\right)^\theta\left(\frac{x_n^C}{\overline{m}} + x_n^I\right)$$

Finally, integrating over the distribution of costs for firms selling in i:

$$\begin{aligned}
x_i^I &= \int_0^{\bar{c}_i} x_i^I(c)\theta\bar{c}_i^{-\theta}c^{\theta-1}dc \\
&= \sum_{k=1}^{K}\pi_{ii}\lambda_{k,i}\beta_k\sum_{n=1}^{N}\left(\frac{\bar{c}_n}{\bar{c}_i d_{ni}}\right)^\theta\left(\frac{x_n^C}{\overline{m}} + x_n^I\right)\int_0^{\bar{c}_i}\exp\left[-\lambda_{k,i}(c/\bar{c}_i)^\theta\right]\delta\left[c \leq w_{k,i}\right]\theta\bar{c}_i^{-\theta}c^{\theta-1}dc \\
&= \sum_{k=1}^{K}\pi_{ii}\beta_k\sum_{n=1}^{N}\left(\frac{\bar{c}_n}{\bar{c}_i d_{ni}}\right)^\theta\left(\frac{x_n^C}{\overline{m}} + x_n^I\right)\int_0^{w_{k,i}} dG_{k,i}(c) \\
&= \pi_{ii}\sum_{n=1}^{N}\left(\frac{\bar{c}_n}{\bar{c}_i d_{ni}}\right)^\theta\left(\frac{x_n^C}{\overline{m}} + x_n^I\right)\left(1-\beta_i^L\right), \qquad (12)
\end{aligned}$$

where:

$$\beta_i^L = \sum_{k=1}^{K}\beta_k \exp\left[-\lambda_{k,i}\left(w_{k,i}/\bar{c}_i\right)^\theta\right]. \qquad (13)$$

The term β_i^L is the total share of labor in manufacturing production costs. Note how it declines as the wage rises.

2.6 Aggregate Manufacturing Inputs

Having solved for the expected intermediate sales of a typical firm selling in i we now turn to total spending on intermediates and on manufacturing labor there.

Total intermediate sales in country i, X_i^I is the product of expected intermediate sales per firm x_i^I, given in (12), and the measure of firms selling in country i, $\mu_i(\bar{c}_i)$:

$$X_i^I = \Upsilon_i\bar{c}_i^\theta x_i^I.$$

Substituting in (13):

$$
\begin{aligned}
X_i^I &= \Upsilon_i \pi_{ii} \left(1-\beta_i^L\right) \sum_{n=1}^{N} \left(\frac{\overline{c}_n}{d_{ni}}\right)^{\theta} \left(\frac{x_n^C}{\overline{m}} + x_n^I\right) \\
&= \left(1-\beta_i^L\right) \sum_{n=1}^{N} \Upsilon_i \pi_{ii} \left(d_{ni}\right)^{-\theta} \frac{1}{\Upsilon_n} \left(\frac{X_n^C}{\overline{m}} + X_n^I\right) \\
&= \left(1-\beta_i^L\right) \sum_{n=1}^{N} \pi_{ni} \left(\frac{X_n^C}{\overline{m}} + X_n^I\right).
\end{aligned}
$$

Writing the set of equations as a system lets us solve for intermediate demand as a function of final demand:

$$
\begin{aligned}
\boldsymbol{X^I} &= \frac{\sigma-1}{\sigma} \left[\boldsymbol{I_N} - (\boldsymbol{I_N} - \boldsymbol{\beta}^L)\boldsymbol{\pi}'\right]^{-1} (\boldsymbol{I_N} - \boldsymbol{\beta}^L)\boldsymbol{\pi}'\boldsymbol{X^C} \\
&= \frac{\sigma-1}{\sigma} (\boldsymbol{I_N} - \boldsymbol{\beta}^L) \left[\boldsymbol{I_N} - \boldsymbol{\pi}' (\boldsymbol{I_N} - \boldsymbol{\beta}^L)\right]^{-1} \boldsymbol{\pi}'\boldsymbol{X^C} \qquad (14)
\end{aligned}
$$

where $\boldsymbol{X^I} = (X_1^I, X_2^I, ..., X_N^I)'$, $\boldsymbol{X^C} = (X_1^C, X_2^C, ..., X_N^C)'$, $\boldsymbol{\beta}^L$ is a diagonal matrix with β_i^L on the diagonal, $\boldsymbol{\pi}$ is a matrix with representative element π_{ni}, and $\boldsymbol{I_N}$ is an N-dimensional identity matrix.[3]

[3] The second equality in (14) follows from:

$$
\begin{aligned}
\left[\boldsymbol{I_N} - (\boldsymbol{I_N} - \boldsymbol{\beta}^L)\boldsymbol{\pi}'\right]^{-1} (\boldsymbol{I_N} - \boldsymbol{\beta}^L) &= \left[(\boldsymbol{I_N} - \boldsymbol{\beta}^L)^{-1} \left(\boldsymbol{I_N} - (\boldsymbol{I_N} - \boldsymbol{\beta}^L)\boldsymbol{\pi}'\right)\right]^{-1} \\
&= \left[(\boldsymbol{I_N} - \boldsymbol{\beta}^L)^{-1} - \boldsymbol{\pi}'\right]^{-1} \\
&= \left[\left(\boldsymbol{I_N} - \boldsymbol{\pi}'(\boldsymbol{I_N} - \boldsymbol{\beta}^L)\right)(\boldsymbol{I_N} - \boldsymbol{\beta}^L)^{-1}\right]^{-1} \\
&= (\boldsymbol{I_N} - \boldsymbol{\beta}^L)\left[\boldsymbol{I_N} - \boldsymbol{\pi}'(\boldsymbol{I_N} - \boldsymbol{\beta}^L)\right]^{-1}.
\end{aligned}
$$

This second formulation is immediate if we start with the system of equation for total variable costs V:

$$
V_i = \sum_{n=1}^{N} \pi_{ni} \left[\frac{\sigma-1}{\sigma} X_n^C + (1-\beta_n^L) V_n\right],
$$

which can be expressed in matrix notation as

$$
\boldsymbol{V} = \frac{\sigma-1}{\sigma} (\boldsymbol{I_N} - \boldsymbol{\beta}^L) \left[\boldsymbol{I_N} - \boldsymbol{\pi}' (\boldsymbol{I_N} - \boldsymbol{\beta}^L)\right]^{-1} \boldsymbol{\pi}'\boldsymbol{X^C}.
$$

This expression gives us intermediate absorption of manufactures around the world as a function of the final demand for manufactures around the world. Similarly, spending on manufacturing labor H_i^M is:

$$\boldsymbol{H}^M = \frac{\sigma-1}{\sigma}\boldsymbol{\beta}^L\left[\boldsymbol{I_N} - \boldsymbol{\pi}'\left(\boldsymbol{I_N} - \boldsymbol{\beta}^L\right)\right]^{-1}\boldsymbol{\pi}'\boldsymbol{X^C} \tag{15}$$

Now we turn to the determination of final demand and overall equilibrium.

2.7 Manufacturing Profits

We can also relate profits earned in each source country i to consumption of manufactures around the world. We begin by considering profits generated in each destination country n without regard to who has claims to them. Variable profits generated in n are simply X_n^C/σ from which we need to subtract entry costs.

Using the expression (11) for the measure of entrants, total fixed costs from entry in market n are:

$$E_n\mu_n(\bar{c}_n) = \frac{\theta-(\sigma-1)}{\theta}\frac{X_n^C}{\sigma}. \tag{16}$$

so that total net profits generated are:

$$\frac{X_n^C}{\sigma} - \frac{\theta-(\sigma-1)}{\theta}\frac{X_n^C}{\sigma} = \frac{(\sigma-1)X_n^C}{\sigma\theta}.$$

Since a fraction π_{ni} of those net profits flow to country i, we have

$$\Pi_i = \sum_{n=1}^{N}\pi_{ni}\frac{(\sigma-1)X_n^C}{\sigma\theta}.$$

which we can write in matrix form as:

$$\boldsymbol{\Pi} = \frac{(\sigma-1)}{\sigma\theta}\boldsymbol{\pi}'\boldsymbol{X^C} \tag{17}$$

where $\Pi = (\Pi_1, \Pi_2, ..., \Pi_N)'$.

2.8 The Specification of the Search Function

We assume that in each destination n the parameter of the Poisson arrival of price quotes is proportional to the measure of active sellers:

$$\lambda_{k,n} = \bar{\lambda}_{k,n} \mu_n (\overline{c}_n) = \bar{\lambda}_{k,n} \Upsilon_n \overline{c}_n^{\theta},$$

and hence

$$\nu_{k,n} = \bar{\lambda}_{k,n} \Upsilon_n,$$

where the parameter $\bar{\lambda}_{k,n}$ determines the rate of contacts for task k in country n.[4]

We can solve for the vector of Υ_n from the system of equations:

$$\Upsilon_n = \sum_{i=1}^{N} T_i \left(\overline{w}_i d_{ni}\right)^{-\theta} \prod_{k=1}^{K} \left[e^{-\Upsilon_i \bar{\lambda}_{k,i} (w_{k,i})^{\theta}} + \left(\Upsilon_i \bar{\lambda}_{k,i} (w_{k,i})^{\theta} \right)^{\beta_k} \gamma(1 - \beta_k, \Upsilon_i \bar{\lambda}_{k,i} (w_{k,i})^{\theta}) \right], \quad (18)$$

for $n = 1, ..., N$. The appendix describes an iterative procedure to solve this system. To show that a unique finite solution exists, we assume that in each country, there exists at least one task k, with $\beta_k > 0$, such that $\bar{\lambda}_{k,n} = 0$.

2.9 General Equilibrium

Consumers spend a share α of their income on manufactures. The rest is spend on nonmanufactures, which are produced with only labor. Hence spending on manufactures in country n is simply a fraction α of final aggregate spending X^A:

$$X_n^C = \alpha X_n^A,$$

[4] This specification avoids the implication of treating $\lambda_{k,n}$ as a parameter that finding a low price quote is less likely in an economy with more sellers.

where aggregate spending is equal to the sum of labor income, profit income Π_n, and the overall deficit D_n. Labor income is generated in three sectors: manufacturing H_n^M, nonmanufacturing H_n^N, and in fixed cost activities H_n^F.

Labor income in manufacturing is given in (15) while profits are given by (17). Labor income in nonmanufacturing is:

$$H_n^N = (1-\alpha)X_n^A - D_n^N$$

where D_n^N is the deficit in nonmanufactures. We assume that local labor is the only component of fixed costs, so that H_n^F is given by (16).

Hence we can write:

$$X_n^A = H_n^M + H_n^N + H_n^F + \Pi_n + D_n$$

where D_n is the overall deficit, which we treat as exogenous.

To complete the specification of the model, we assume constant labor requirements to perform fixed-cost services a_n^F and to produce nonmanufactures a_n^N. Thus, the cost of entry into market n is

$$E_n = a_n^F w_n^F,$$

and, if country n produces nonmanufactures in equilibrium, the price of nonmanufactures there is

$$P_n^N = a_n^N w_n^N,$$

where w_n^s is the wage for labor in sector $s \in \{M, F, N\}$. We now turn to the determination of these wages.

Aggregating over tasks, total manufacturing production labor income in country i is:

$$w_i^M L_i^M = \sum_{k=1}^{K} w_{k,i} L_{k,i},$$

where

$$L_i^M = \sum_{k=1}^{K} L_{k,i}.$$

If labor is mobile between tasks then $w_{k,i} = w_i^M$, while if it is immobile:

$$w_i^M = \sum_{k=1}^{K} \frac{L_{k,i}}{L_i^M} w_{k,i},$$

with $L_{k,i}$ fixed. Employment by task (in the case of labor mobility) or the task specific wage (in the case of immobile labor) is determined by the share equations:

$$\frac{w_{k,i} L_{k,i}}{w_i^M L_i^M} = \frac{\beta_k \exp\left[-\lambda_{k,i}(w_{k,i}/\overline{c}_i)^\theta\right]}{\sum_{k'=1}^{K} \beta_{k'} \exp\left[-\lambda_{k',i}\left(w_{k',i}/\overline{c}_i\right)^\theta\right]}.$$

We can proceed for now without taking a stand on whether labor is mobile across tasks.

The aggregate labor force is in fixed supply, L_i. Labor can be employed in manufacturing production, fixed-cost activities, or nonmanufacturing:

$$L_i = L_i^M + L_i^F + L_i^N.$$

We consider only settings in which equilibrium employment in nonmanufacturing is strictly positive in each country.

Labor income in sector $s \in \{M, F, N\}$ is thus

$$H_n^s = w_i^s L_i^s.$$

Aggregate income (GDP) is:

$$Y_i^A = X_i^A - D_i = w_i^M L_i^M + w_i^F L_i^F + w_i^N L_i^N + \Pi_i.$$

We now turn to simulations of the model in which we make particular assumptions about sectoral mobility of labor and tradability of nonmanufactures.

3 Simulations

We simulate the model to gauge the effect of lower trade barriers on the wages of workers who differ in their exposure to outsourcing. So far we limit ourselves to a symmetric two country case (and hence drop country subscripts in what follows).

We assume that nonmanufactures are not traded and that trade in manufactures is balanced $D = 0$. We resolve the indeterminacy of the wage level by normalizing world labor income to 1.

We consider three types of labor: nonmanufacturing, skilled, and unskilled. Nonmanufacturing labor produces nonmanufactures and also performs fixed-cost services (thus corresponding to $L^N + L^F$ in the notation above). This type of labor is perfect mobile between these two activities. Manufacturing labor L^M is segmented into skilled and unskilled workers, with no mobility between these skill levels. We assume that the tasks performed by skilled workers are less likely to be outsourced than the tasks performed by unskilled workers.

To keep things as stark as possible, we consider only $K = 3$ tasks. The first $k = 1$ is not subject to outsourcing, and is carried out by a nonmanufactured intermediate (hence w_1 is proportional to the nonmanufacturing wage). The second $k = 2$ is the skilled task (hence w_2 is the skilled wage) and the third $k = 3$ is the unskilled task (hence w_3 is the unskilled wage).

The parameter settings are listed Table 1. A few of the values require some explanation. The share of manufactures in preferences is 60 percent, far larger than the share of manufactures in GDP. The reason is that a major use of nonmanufactures (in the model) is as an input to manufactures. We set the share of nonmanufactured intermediates to 40 percent of manufacturing production costs, as can be seen near the bottom of the table. This large share captures, in a crude way, the fact that almost no manufacturing firms (at least in the French data) have an intermediate share of production costs below 40 percent. Finally, note that skilled and unskilled workers are treated symmetrically (with the same labor supply and task share) except that the outsourcing parameter is two orders of magnitude higher for the unskilled, as can be seen at the bottom of the table.

The results are shown in Table 2. Start with the first column, in which the iceberg cost of $d = 4$ leaves the countries nearly in autarky (the import share is 0.002, which rounds 0.00 in the table). Even with little access to imported intermediates, 70 percent of firms outsource unskilled tasks to other domestic firms. Since only 13 percent of firms outsource skilled tasks, labor market equilibrium demands a lower wage for the unskilled (a skill premium of nearly 3). Due to manufacturing's heavy use of nonmanufactured intermediates, manufacturing value added is only 31 percent of GDP, which contrasts with the 60 percent share of manufactures in consumption).

Moving to the right along the rows of Table 2 illustrates the effect of globalization. (The last column assumes frictionless trade, with an import share of 50 percent.) All the outcomes in the table are strictly monotonic in trade costs (rounding makes some inequalities appear weak). The first two rows show that globalization leads to a declining cost threshold for entrants (tougher competition), with only a slight reduction in the number of active firms in each market. Lower cost intermediates, both imported and domestic, make outsourcing a more attractive option. The fraction of skilled tasks outsourced rises from 13 to 27 percent (70 to 78 percent for unskilled). The skill premium rises by 13 percent.

The real wages of the unskilled workers actually decline with globablization. While these workers benefit from lower prices of consumption goods, lower prices of intermediates put downward pressure on their wages. Skilled workers face a low enough threat from outsourcing that their real wage rises. Since the nonmanufacturing workers are totally insulated from outsourcing, they obtain the greatest benefit from globalization. Their high wage feeds into high price for nonmanufactured consumption goods in the face of globalization, contributing to the real wage decline for unskilled workers. Aggregating over the winners and losers, globalization raises overall welfare by 14 percent.

In addition to the distributional effects of globalization, the model also captures the way in which globalization hollows out the manufacturing sector. This hollowing out is associated with only a small decline in the share of manufacturing value added in GDP, however. The striking feature is instead the share of manufacturing value added in manufacturing gross production, which fall from 44 percent to 38 percent. Thus hollowing out is associated with a rise in total manufacturing gross output rather than a fall in value added.

References

Acemoglu, Daron and David Autor (2011) "Skills, Tasks and Technologies: Implications for Employment and Earnings," chapter 12 in David Card and Orley Ashenfelter editors, *Handbook of Labor Economics*, volume 4, Elsevier.

Acemoglu, Daron and Vasco Carvalho (2012) "The Network Origins of Aggregate Fluctuations," *Econometrica,* 80: 1977-2016.

Arkolakis, Konstantinos (2010) "Market Penetration Costs and the New Consumers Margin in International Trade," *Journal of Political Economy*, 118: 1151-1199.

Bernard, Andrew B., Jonathan Eaton, J. Bradford Jensen, and Samuel Kortum (2003) "Plants and Productivity in International Trade," *American Economic Review,* 93: 1268-1290.

Bernard, Andrew B. and J. Bradford Jensen, (1995) "Exporters, Jobs, and Wages in US Manufacturing: 1976-87," *Brookings Paper on Economic Activity: Microeconomics,* 67-112.

Binmore, Kenneth, Ariel Rubinstein, Asher Wolinsky (1986) "The Nash Bargaining Solution in Economic Modelling," *Rand Journal of Economics*, 17: 176-188.

Biscourp, Pierre and Francis Kramarz (2007) "Employment, Skill Structure, and International Trade: Firm-Level Evidence for France," *Journal of International Economics,* 72: 22-51.

Blaum, Joaquin, Claire Lelarge, and Michael Peters (2013) "Firm Heterogeneity and Import Behavior: Evidence from French Firms," working paper, Yale University.

Cahuc, Pierre and André Zylberberg (2004) *Labor Economics.* Cambridge, MA: MIT Press.

Caliendo, Lorenzo and Esteban Rossi-Hansberg (2012) "The Impact of Trade on Organization and Productivity," forthcoming *Quarterly Journal of Economics.*

Caliendo, Lorenzo, Ferdinando Monte, and Esteban Rossi-Hansberg (2012) "The Anatomy of French Production Hierarchies," working paper, Yale School of Management.

Chaney, Thomas (2008) "Distorted Gravity: Heterogeneous Firms, Market Structure, and the Geography of International Trade," *American Economic Review,* 98: 1707-1721.

Chaney, Thomas (2013) "The Network Structure of International Trade," *NBER Working Paper* No. 16753.

Eaton, Jonathan, Samuel Kortum, and Francis Kramarz (2004) "Dissecting Trade: Firms, Industries, and Export Destinations," *American Economic Review, Papers and Proceedings*, 94: 150-154. (Data sources and tables available in, *NBER Working Paper #10344.*)

Eaton, Jonathan, Samuel Kortum, and Francis Kramarz (2011) "An Anatomy of International Trade: Evidence from French Firms," *Econometrica*, 79(5), 1453-1498.

Egger, Harmut and Udo Kreickemeier (2009) "Firm Heterogeneity and the Labor Market Effects of Trade Liberalisation," *International Economic Review*, 50: 187-216.

Felbermayer, Gabriel, Julien Prat, and Hans-Jörg Schmerer (2008) "Globalization and Labor Market Outcomes: Bargaining, Search Frictions, and Firm Heterogeneity," IZA discussion paper no. 3363.

Frías, Judith A., David S. Kaplan, and Eric A. Verhoogen (2009) "Exports and Wage Premia: Evidence from Mexican Employer-Employee Data," working paper, Columbia University.

Garetto, Stefania (2013) "Input Sourcing and Multinational Production," *American Economic Journal: Macroeconomics*, 5: 118-151.

Gouriéroux, Christian and Alain Monfort (1994) *Simulation Based Econometric Methods*, CORE Lecture Series, Université catholique de Louvain.

Halpern, Laszlo, Miklos Koren, and Adam Szeidl (2011) "Imported Inputs and Productivity,"' working paper, Central European University.

Helpman, Elhanan, Marc J. Melitz, and Stephen R. Yeaple (2004) "Exports vs. FDI with Heterogeneous Firms," *American Economic Review,* 94: 300-316.

Helpman, Elhanan, Oleg Itskhoki, and Stephen Redding (2010) "Inequality and Unemployment in a Global Economy," *Econometrica,* 78: 1239-1283.

Helpman, Elhanan, Oleg Itskhoki, Marc Muendler, and Stephen Redding (2013) "Trade and Inequality: From Theory to Estimation," *NBER Working Paper* No. 17991.

Hummels, David, Rasmus Jørgensen, Jakob R. Munch, and Chong Xiang (2011) "The Wage

Effects of Offshoring: Evidence from Danish Matched Worker-Firm Data," *NBER Working Paper* No. 17496.

Irarrazabal, Alfonso, Andreas Moxnes, and Karen Helene Ulltveit-Moe (2010) "Heterogeneous Firms or Heterogeneous Workers? Implication for Exporter Premia and the Gains from Trade," working paper, University of Oslo.

Klein, Michael W., Christoph Moser, and Dieter M. Urban (2010) "The Skill Structure of the Export Wage Premium: Evidence from German Manufacturing," working paper, Tufts University.

Kramarz, Francis (2009) "Offshoring, Wages, and Employment: Evidence form Data Matching Imports, Firms, and Workers," working paper, INSEE-CREST.

Krugman, Paul R. (1980) "Scale Economies, Product Differentiation, and the Patterns of Trade," *American Economic Review,* 70: 950-959.

Lucas, Robert E (2009) "Ideas and Growth," *Economica* 76: 1-19.

McDonald, Ian M. and Robert M. Solow (1981) "Wage Bargaining and Employment," *American Economic Review,* 71: 896-908.

Melitz, Marc J. (2003) "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica,* 71: 1695-1725.

Mortensen, Dale (2003), *Wage Dispersion: Why are Similar People Paid Differently*, Cambridge: MIT Press.

Oberfield, Ezra (2013) “Business Networks, Production Chains, and Productivity: A Theory of Input-Output Architecture,” Federal Reserve Bank of Chicago.

Postel-Vinay, Fabien and Jean Marc Robin (2002) “Equilibrium Wage Dispersion with Worker and Employer Heterogeneity.” *Econometrica*, 70(6), 2295-2350.

Yeaple, Stephen R. (2005) “A Simple Model of Firm Heterogeneity, International Trade, and Wages,” *Journal of International Economics*, 65: 1-20.

A Appendix: Computing Υ

Here we discuss an algorithm for computing Υ.

A.1 Simplest Case

We start with the simplest case of $K = 1$ and $N = 1$ so that Υ is the fixed point

$$\Upsilon = f(\Upsilon),$$

of the function f defined as:

$$f(x) = A\left[e^{-xu_1} + (xu_1)^{\beta_1}\gamma(1-\beta_1, xu_1)\right],$$

where

$$A = T\overline{w}^{-\theta},$$

$$u_1 = \bar{\lambda}_1 w_1^{\theta},$$

and

$$\gamma(1-\beta_1, xu_1) = \int_0^{xu_1} y^{-\beta_1}e^{-y}dy$$

The derivative is

$$f'(x) = A\frac{\beta_1}{x}(xu_1)^{\beta_1}\gamma(1-\beta_1, xu_1) > 0.$$

Assuming $\beta_1 < 1$, for large x we have

$$\lim_{x\to\infty}\frac{f(x)}{x} = \frac{A(xu_1)^{\beta_1}\gamma(1-\beta_1)}{x} = Au_1^{\beta_1}\gamma(1-\beta_1)x^{\beta_1-1} = 0.$$

Furthermore, note that $f(0) = A > 0$. Thus, for $\beta_1 < 1$ we know there is a strictly positive fixed point.

If we can show that $f'(x) < 1$ when evaluated at a fixed point $x = \Upsilon$, then we will know that the fixed point is unique. Evaluating the derivative at the fixed point we get:

$$f'(\Upsilon) = \beta_1 \frac{\Upsilon - Ae^{-\Upsilon w_1^\theta}}{\Upsilon} < 1,$$

since $Ae^{-\Upsilon w_1^\theta} > 0$ and $\beta_1 \leq 1$.

We can compute the fixed point by iterating on:

$$x_n = f(x_{n-1}),$$

starting with $x_0 = 0$. Our approximation is justified, since:

$$\Upsilon = \lim_{n \to \infty} x_n.$$

This result also give us the comparative statics. We have that Υ is increasing in A (higher technology or lower average wage) and in u_1 (higher arival of price quotes or higher wage in this task).

We can get a crude approximation $\overline{\Upsilon}$ for Υ by setting the exponentially declining term e^{-xu_1} to zero and replacing the incomplete by the complete gamma function. Thus

$$\overline{\Upsilon} = A \left(\overline{\Upsilon} u_1\right)^{\beta_1} \gamma(1 - \beta_1)$$

so that

$$\overline{\Upsilon} = \left(A u_1^{\beta_1} \gamma(1 - \beta_1)\right)^{1/(1-\beta_1)}.$$

A.2 Multiple Tasks

We now consider an arbitary $K \geq 1$, showing that the results of the simplest case all go through under a particular restriction on the β_k's. In the case of multiple tasks, Υ is the fixed point

$$\Upsilon = f(\Upsilon),$$

of the function f defined as:

$$f(x) = A \prod_{k=1}^{K} \left[e^{-xu_k} + (xu_k)^{\beta_k} \gamma(1 - \beta_k, xu_k) \right],$$

where

$$A = T\overline{w}^{-\theta}$$

and

$$u_k = \bar{\lambda}_k w_k^{\theta},$$

Suppose that $\bar{\lambda}_k = 0$ and hence $u_k = 0$ for some task for which $\beta_k > 0$. Without loss of generality, let this protected task be $k = K$. In this case, we can simply drop this task from the definition of $f(x)$. More generally, suppose $u_k = 0$ for all $k > K^*$. We can then work with:

$$f(x) = A \prod_{k=1}^{K^*} \left[e^{-xu_k} + (xu_k)^{\beta_k} \gamma(1 - \beta_k, xu_k) \right]. \tag{19}$$

The point of this formulation is that:

$$\sum_{k=1}^{K^*} \beta_k < \sum_{k=1}^{K} \beta_k = 1,$$

which is a restriction that we will need to impose. In what follows, we will work directly with (19), imposing this restriction.

The derivative of (19) is:

$$
\begin{aligned}
f'(x) &= A\sum_{k=1}^{K^*}\beta_k u_k\,(xu_k)^{\beta_k-1}\,\gamma(1-\beta_k,xu_k)\prod_{j\neq k}\left[e^{-xu_j}+(xu_j)^{\beta_j}\,\gamma(1-\beta_j,xu_j)\right]\\
&= A\sum_{k=1}^{K^*}\frac{\beta_k}{x}\,(xu_k)^{\beta_k}\,\gamma(1-\beta_k,xu_k)\prod_{j\neq k}\left[e^{-xu_j}+(xu_j)^{\beta_j}\,\gamma(1-\beta_j,xu_j)\right],
\end{aligned}
$$

which is positive. For large x we have

$$
\lim_{x\to\infty}\frac{f(x)}{x} = A\prod_{k=1}^{K^*}\frac{(xu_k)^{\beta_k}\,\gamma(1-\beta_k)}{x} = Ax^{-\left[1-\sum_{k=1}^{K^*}\beta_k\right]}\prod_{k=1}^{K^*}u_k^{\beta_k}\gamma(1-\beta_k) = 0.
$$

As in the simplest case $f(0) = A > 0$. Thus, we know there is a strictly positive fixed point.

Evaluating (19) at the fixed point $x = \Upsilon$, we have:

$$
\frac{\beta_k}{\Upsilon}\,(\Upsilon u_k)^{\beta_k}\,\gamma(1-\beta_k,\Upsilon u_k) = \frac{\beta_k}{\Upsilon}\left(\frac{\Upsilon}{A\prod_{j\neq k}\left[e^{-\Upsilon u_j}+(\Upsilon u_j)^{\beta_j}\,\gamma(1-\beta_j,\Upsilon u_j)\right]} - e^{-\Upsilon u_k}\right).
$$

This expression allows us to write the derivative, evaluated at the fixed point, as:

$$
\begin{aligned}
f'(\Upsilon) &= A\sum_{k=1}^{K^*}\frac{\beta_k}{\Upsilon}\left(\frac{\Upsilon}{A\prod_{j\neq k}\left[e^{-\Upsilon u_j}+(\Upsilon u_j)^{\beta_j}\,\gamma(1-\beta_j,\Upsilon u_j)\right]} - e^{-\Upsilon u_k}\right)\prod_{j\neq k}\left[e^{-\Upsilon u_j}+(\Upsilon u_j)^{\beta_j}\,\gamma(1-\beta_j,\Upsilon u_j)\right]\\
&= \sum_{k=1}^{K^*}\beta_k\left(1-A\frac{e^{-\Upsilon u_k}}{\Upsilon}\prod_{j\neq k}\left[e^{-\Upsilon u_j}+(\Upsilon u_j)^{\beta_j}\,\gamma(1-\beta_j,\Upsilon u_j)\right]\right). \qquad (2
\end{aligned}
$$

The term in round brackets is positive and less than one since:

$$
A\frac{e^{-\Upsilon u_k}}{\Upsilon}\prod_{j\neq k}\left[e^{-\Upsilon u_j}+(\Upsilon u_j)^{\beta_j}\,\gamma(1-\beta_j,\Upsilon u_j)\right] = \frac{e^{-\Upsilon u_k}}{\left[e^{-\Upsilon u_k}+(\Upsilon u_k)^{\beta_k}\,\gamma(1-\beta_k,\Upsilon u_k)\right]} < 1.
$$

Thus (20) is simply a sum of terms, each one of which is less than β_k. Since $\sum_{k=1}^{K^*}\beta_k < 1$, we get the result that $f'(\Upsilon) < 1$. It follows that there is a unique fixed point, which we can compute using an iterative procedure.

The simple approximation $\overline{\Upsilon}$ of the fixed point now satisfies:

$$\overline{\Upsilon} = A \left[\prod_{k=1}^{K^*} u_k^{\beta_k} \gamma(1-\beta_k) \right] \overline{\Upsilon}^{\sum_{k=1}^{K^*} \beta_k}$$

so that

$$\overline{\Upsilon} = \left(A \prod_{k=1}^{K^*} u_k^{\beta_k} \gamma(1-\beta_k) \right)^{1/\left(1-\sum_{k=1}^{K^*} \beta_k\right)} .$$

A.3 Multiple Countries

We now generalize the multiple-task case to allow for an arbitrary number of countries, $N \geq 1$.

Similar to the one-country case, we assume:

$$\sum_{k=1}^{K^*} \beta_k < \sum_{k=1}^{K} \beta_k = 1,$$

where, in each country i, $\bar{\lambda}_{k,i} = 0$ for all $k > K^*$.

With multiple countries, $\boldsymbol{\Upsilon}$ is an N by 1 vector satisfying the fixed point

$$\boldsymbol{\Upsilon} = f(\boldsymbol{\Upsilon}),$$

which the n'th element is:

$$f_n(\boldsymbol{x}) = \sum_{i=1}^{N} A_{ni} \prod_{k=1}^{K^*} \left[e^{-x_i u_{k,i}} + (x_i u_{k,i})^{\beta_k} \gamma(1-\beta_k, x_i u_{k,i}) \right],$$

where

$$A_{ni} = T_i \left(\overline{w} d_{ni} \right)^{-\theta}$$

and

$$u_{k,i} = \bar{\lambda}_{k,i} w_{k,i}^{\theta}.$$

Consider the approximation to the fixed point, with each element $\overline{\Upsilon}$ the same:

$$\overline{\Upsilon} = \left(\max_n \left\{ \sum_{i=1}^{N} A_{ni} \prod_{k=1}^{K^*} u_{k,i}^{\beta_k} \gamma(1-\beta_k) \right\} \right)^{1/\left(1-\sum_{k=1}^{K^*} \beta_k\right)}$$

[*** To be completed ***]

B Appendix: Derivation of Intermediate Sales

Here we go into details about our derivation of the expected intermediate sales of a seller of type s with unit cost c. Having found a buyer of type s' seeking to replace its k'th input, the seller will replace labor paid a wage w_k. The buyer's partial cost (that is, for its tasks other than k) is $b_{-k,s'} = (w_0)^{\beta_{0,s'}} \prod_{k' \neq k} (\min\{w_{k'}, p'_k\})^{\beta_{k',s'}}$. A buyer for input k of type s' has efficiency Z which is distributed:

$$\Pr[Z \leq z] = 1 - \left(\frac{z}{\underline{z}_k(\overline{c}_{s'}, c)}\right)^{-\theta}.$$

Here $\underline{z}_k(\overline{c}_{s'}, c)$ is the lowest efficiency that any active buyer of type s' buying input k from a seller with unit cost c can have, given that the cost cutoff for the buyer is $\overline{c}_{s'}$. Thus:

$$\underline{z}_k(\overline{c}_{s'}, c) = \frac{b_{-k,s'} c^{\beta_{k,s'}}}{\overline{c}_{s'}}.$$

The buyer's demand for the intermediate depends both on its final and its own intermediate sales. Given the buyer's unit cost c' its sales are just as in Melitz. But the buyer's intermediate sales depend on its encounters with buyers. We define the expected intermediate sales of a buyer of type s' with unit cost c' as $x^M_{s'}(c')$.

Using $x^M_{s'}(c')$ we can then write an expression for the expected intermediate sales of a seller with unit cost c to buyers of type s' for input k, conditioning on b_{-k}, as:

$$x^M_{k,s'}(c) = \Lambda_{k,s'}(c)\beta_{k.s'} \int_{\underline{z}_k(\overline{c}_{s'},c)}^{\infty} \left[\frac{1}{\overline{m}}\frac{X}{P^{1-\sigma}}\left(\frac{\overline{m} b_{-k,s'} c^{\beta_{k,s'}}}{z'}\right)^{1-\sigma} + x^M_{s'}\left(\frac{b_{-k,s'} c^{\beta_{k,s'}}}{z'}\right)\right] \theta \left[\underline{z}_k(\overline{c}_{s'}, c)\right]^{\theta} (z')^{-\theta-1}\, dz'. \tag{21}$$

We implement the change of variable:

$$c' = \frac{b_{-k,s'} c^{\beta_{k,s'}}}{z'}$$

so that:

$$z' = \frac{b_{-k,s'} c^{\beta_{k,s'}}}{c'} \text{ and therefore } dz' = -\frac{b_{-k,s'} c^{\beta_{k,s'}}}{\left(c'\right)^2} dc'$$

to get:

$$\begin{aligned}
x^M_{k,s'}(c) &= \Lambda_{k,s'}(c)\beta_{k,s'} \int_0^{\bar{c}_{s'}} \left[\overline{m}^{-\sigma} \frac{X}{P^{1-\sigma}} \left(c'\right)^{1-\sigma} + x^M_{s'}\left(c'\right)\right] \theta \left(\frac{b_{-k,s'} c^{\beta_{k,s'}}}{\bar{c}_{s'}}\right)^{\theta} \left(\frac{b_{-k,s'} c^{\beta_{k,s'}}}{c'}\right)^{-\theta-1} \frac{b_{-k,s'} c^{\beta_{k,s'}}}{\left(c'\right)^2} dc' \\
&= \Lambda_{k,s'}(c)\beta_{k,s'} \int_0^{\bar{c}_{s'}} \left[\overline{m}^{-\sigma} \frac{X}{P^{1-\sigma}} \left(c'\right)^{1-\sigma} + x^M_{s'}\left(c'\right)\right] \theta \bar{c}_{s'}^{-\theta} \left(c'\right)^{\theta-1} dc' \\
&= \Lambda_{k,s'}(c)\beta_{k,s'} \left[\overline{m}^{-\sigma} \frac{\theta}{\theta - (\sigma - 1)} \frac{X}{P^{1-\sigma}} \left(\bar{c}_{s'}\right)^{1-\sigma} + x^M_{s'}\right],
\end{aligned}$$

Table 1: Parameter Settings for Counterfactual

Parameter	symbol	value
Entry cost labor requirement	aF	0.0001
Nonmanufacturing labor requirement	aN	1
Manufactures share in preferences	alpha	0.6
Elasticity of substitution in preferences	sigma	4
Pareto parameter	theta	4.5
Technology level	T	1
Labor force (per country)	L	0.5
Labor by type (fractions of labor force):		
Nonmanufacturing		0.8
Mfg. skilled		0.1
Mfg. unskilled		0.1
Task shares:	beta	
Nonmanufacturing intermediates		0.4
Skilled		0.3
Unskilled		0.3
Outsourcing parameters:	lambda_bar	
Nonmanufacturing intermediates		0
Skilled		0.01
Unskilled		10

Table 2: Counterfactual Results

	Level of iceberg trade cost				
	d=4	d=2	d=1.5	d=1.1	d=1
Measure of entrants	289.81	289.36	288.16	283.66	280.90
Cost threshold for entry	3.38	3.33	3.21	2.81	2.61
Mfg. value added share :					
Share of GDP	0.31	0.31	0.31	0.29	0.29
Share of mfg. gross production	0.44	0.44	0.43	0.40	0.38
Fraction of tasks outsourced:					
Skilled	0.13	0.14	0.16	0.23	0.27
Unskilled	0.70	0.71	0.71	0.76	0.78
Labor share of mfg. variable cost:					
Skilled	0.26	0.26	0.25	0.23	0.22
Unskilled	0.09	0.09	0.08	0.07	0.07
Import share	0.00	0.04	0.14	0.39	0.50
Wage:					
Nonmanufacturing	0.96	0.96	0.96	0.98	0.99
Skilled	1.73	1.73	1.71	1.65	1.60
Unskilled	0.60	0.59	0.58	0.52	0.49
Skill premium (skilled/unskilled)	2.89	2.92	2.98	3.17	3.27
Real wage:					
Nonmanufacturing	1.82	1.84	1.88	2.05	2.15
Skilled	3.29	3.30	3.34	3.44	3.48
Unskilled	1.14	1.13	1.12	1.08	1.06
Welfare (real per capita consumption)	2.11	2.12	2.17	2.32	2.41

1. Total payments to labor around the world are normalized to 1.

2. Nonmanufacturing labor provides fixed cost services.