Firm Size Distortions and the Productivity Distribution: Evidence from France[*]

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Abstract

A major empirical challenge in economics is to identify how regulations (such as firing costs) affect economic efficiency. Almost all countries have regulations that increase costs when firms cross different size thresholds. We show how these size-contingent regulations can be used to identify the equilibrium and welfare effects of regulation through combining a new model with the firm-level distributions of size and productivity. Our framework adapts the Lucas (1978) model to a world with size-contingent regulations and applies this to France where there are sharp increases in firing costs when firms employ 50 or more workers. Using administrative data on the population of firms 2002 through 2007, we show how this regulation has major effects on the distribution of firm size (a "broken power law") and productivity. We then econometrically recover the key parameters of the model in order to estimate the costs of regulation which appear to be non-trivial: our baseline estimates suggest an annual welfare cost of the regulation of around 0.6% of GDP.

Keywords: Firm size, productivity, labor regulation, power law

JEL Classification: L11, L51, J8, L25.6

1 Introduction

A recent literature has documented empirically how distortions that raise the cost of labor or capital affect aggregate productivity though misallocations of resources from more productive to less productive firms. As Restuccia and Rogerson (2008) argued[1] more efficient firms may have "too little" output or employment allocated to them due to various distortions in their economies. Hsieh and Klenow (2009) have argued that these misallocations go a long way towards explaining the gap in aggregate productivity between the US, China and India.[2] In this paper, we focus on understanding the impact and the size of one specific distortion on the French firm size distribution: regulations sharply increasing labor costs when firms reach 50 workers.

The idea that misallocations of resources may be partly behind the productivity gap is attractive in understanding the differences between the US and Europe. According to the European Commission (1996) the average production unit in the EU employed 23% less workers than in the US. As Figure 1 shows, there appear to be far fewer French firms which are able to grow to the same scale as the productive US firms. Figure 1 shows two interesting patterns.[3] First, there is a large bulge in the number of firms with employment just below 50 workers in France, but no such bulge in the distribution of American firms. Second, there is a much larger share of very large firms in the US - the US has many more firms with over 2,500 employees than does France. This paper focuses on the first of those patterns, although we plan to examine the absence of very large French firms in future work.

Labor legislation in France substantially increases firing costs when firms employ 50 or more workers. Specifically, such firms must formulate a "social plan," which is designed to facilitate reemployment, through training, etc. As a result, the costs of employing workers also rise (see Bertola and Bentolila, 1990) at that threshold. Figure 2 shows that indeed the legislation binds, so that there is a clear threshold effect at precisely 50 firms. This uses 2007 data, but the discontinuity in the size distribution at 50 workers is present in all years of our data (see below).

What are the implications for the firm size distribution, the firm productivity distribution and aggregate productivity from those distortions? Our approach relies on revealed preference and on positive sorting effects. Some firms that would have been larger without the regulation choose to remain below the legal threshold to

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[1] See also Parente and Prescott (2000), Bloom and Van Reenen (2010) and Petrin and Sivadasan (2010).

[2] In development economics many scholars have pointed to the "missing middle", i.e. a preponderance of very small firms in poorer countries compared to richer countries (see Banerjee and Duflo, 2005, or Jones, 2011). For example, in the late 1980s in India lareg firms were banned from producing about 800 product groups (Little et al, 1987). Many explanations have been put forward for this such as financial development, taxes, human capital, lack of competition in product markets, and social capital. One possibility, related to our approach, is size related labor regulations. Besley and Burgess (2004), for example, suggest that labor regulation is one of the reasons why the formal manufacturing sector is much smaller in some Indian states compared to others.

[3] Bartelsman et al (2009) examine misallocation using micro-data across many OECD countries and make a similar point. In particular, they find that the "Olley Pakes" (1996) covariance between size and productivity is much smaller in France (0.24 in their Table 1) and other European countries compared to the US (0.51 in their Table 1).

avoid these additional costs. In this paper we aim to identify these firms, calculate their counterfactual size, and use this observation to infer the cost of this legislation.

There has been extensive discussion of the importance of Employment Protection Legislation (EPL) for unemployment and more recently productivity (e.g. Layard and Nickell, 1999; European Commission, 2006). The OECD, World Bank and other agencies have developed various indices of the importance of these regulations based on examination of laws and (sometimes) surveys of managers. It is very hard, however, to see how these can be quantified as "adding up" the legal and regulatory provisions has a large arbitrary component. A contribution of our paper is to offer a methodology for quantifying the "tax equivalent" of a regulation, albeit in the context of a specific model.

There are different views on the underlying sources of heterogeneity in firm productivity. We follow Lucas (1978) in taking the stand that managerial talent is the primitive, and that the economy-wide observed resource distribution is, as Manne (1965) felicitously put it, "a solution to the problem: allocate productive factors over managers of different ability so as to maximize output." Managers make discrete decisions or solve problems (Garicano, 2000). Making better decisions, or solving problems that others cannot solve, raises everyone's marginal product. This means that, in equilibrium, better managers must be allocated more resources. In fact, absent decreasing returns to managerial talent, the best manager must be allocated all resources. Given limits to managerial time or attention, the better managers are allocated more workers and more capital to manage. This results in a "scale-of-operations" effect whereby differences in talent are amplified by the resources allocated.[4] Lucas (1978) first explored these effects in an equilibrium setting.[5]

Consequently better managers, those that are able to generate more productivity, should be allocated to (or equivalently, should choose) larger firm sizes. When managers are confronted with legislation that introduces a cost of acquiring a size that is beyond a certain threshold, they may choose to stay below the threshold and stay at an inefficiently small size. By studying the productivity of these marginal managers, we are able to estimate the cost of the legislation, the distortions in them, and thus the welfare cost of the legislation for the entire firm size distribution.[6]

We start by setting up a simple model of the allocation of a single factor, labor to firms in a world where

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[4]In a model of this kind, the source of decreasing returns are on the production size, and are linked to limits to managerial time. For our purposes here, as Hsieh and Klenow (2009) show, this source of decreasing returns is equivalent to having the decreasing returns come (as is more common in recent literature following Melitz, 2003) from the utility side.

[5]Such a scale of operations effects is at the heart of Rosen's (1982) theory of hierarchies, where efficiency units of labor controlled (and not just number of bodies) matter, and also in Garicano and Rossi-Hansberg (2006) where there is limited quantity-quality substitutability so that matching between workers and managers takes place. Empirically, this technology has been used to explain a wide-range of phenomena, for example the impact of scale of operations effects on CEO wages (Gabaix and Landier, 2008).

[6]Many empirical papers have shown that deregulation (e.g. Olley and Pakes, 1996), higher competition (e.g. Syverson, 2004) and trade liberalization (e.g. Pavcnik, 2002) have tended to improve reallocation by increasing the correlation between firm size and productivity.

there are decreasing returns to managerial talent. We use it to study the effect of a step change in labor costs after a particular size and show that there are four main effects:

1. Equilibrium wages fall as a result of the reduction in the demand for workers (i.e. some of the tax incidence falls on workers)

2. Firm size increases for all firms below the threshold as a result of the general equilibrium effect on wages

3. Firm size reduces to precisely the regulatory threshold for a set of firms that are not productive enough to justify incurring the regulatory costs

4. Firm size reduces proportionally for all firms that are productive enough to incur the additional cost of regulation.

We use the model to guide our estimation of the impact of these costs. The theory tells us there is a deviation from the 'correct' firm size distribution as a result of the regulation. That is, we expect to see a departure from the usual power law firm size distribution[7] as firms bunch up below the threshold (50 workers). Given factors such as measurement error, the observed empirical departure from the power law is not just at 49 workers but at a slightly smaller firm size. Similarly, there is not precisely zero mass to the left of the cut-off (at 50), but rather a "valley" were there are significantly fewer firms than we would expect from an unbroken power law. Then, at some point the distribution becomes again a power law, with a lower intercept. The jump in the power law identifies the size of the distortion.

Our results are consistent across specifications, and the cost of these regulations is quite precisely estimated. We find that these regulations operate mainly as a variable cost, and are equivalent to a 1 percentage point increase in wages across the distribution. There are thus potentially large misallocations of resources that follow from the implicit tax. Of course, it does not follow that the welfare loss is of this magnitude: society may value the increase in job security of workers in larger firms as sufficiently important to compensate such welfare losses (e.g. Lazear, 1990). Determining this, however, requires knowing the cost in terms of lost output, which is what our approach delivers. Furthermore, we do not identify any significant empirical "insurance" effects of labor regulation in our data when looking at wages.

We find that the welfare cost of these regulations is on the order of 0.6% of GDP, and that there are winners and losers. The biggest losers are workers, who suffer a 0.6% reduction in wages: the regulation limits firm size and reduces labor demand. Large firms attain a smaller size, athough the tax cost is somewhat compensated

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[7]See Axtell (2001), Sutton (1997) and Gabaix, (2009). There is a large literature on the size and productivity distribution of firms in macro, trade, finance and IO. Appropriately, the first major study in this area was by Gibrat (1931) who studied French industrial firms, the main focus of the empirical part of our paper.

by the wage reduction. Specifically, we find that the average large firm has a 2.41% lower profits than it would have without the regulation. The winners are small firms, who benefit from lower wages and do not have to incur the tax, which starts at 50 workers. We show that small firms have 2.8% higher profits than in the counterfactual world. Firms on the neighborhood of 50 workers benefit from the lower wages, but are costrained in size as they stay at 50. Overall, the effect is a net positive, and their profits are on average 2.15% higher than without the tax. We also show that there is no evidence that workers place a positive economic value on this regulation.

Overall, the labor protection regulations that we study and that affect firms over 50 workers, place a significant burden on the economy, by keeping firms below their optimal size and by reducing wages. Too many workers work for smaller firms, and too few for large firms, and these reduces the economy´s potential.

The most closely related paper to ours is Braguinsky, Branstetter and Regateiro (2011) who seek to explain the shift to the left in the Portuguese firm size distribution in the context of the Lucas model with labour regulations[8]. Their calibrations also show substantial effects of the regulations on lower aggregate productivity. A difference between our paper and theirs is that we exploit the sharp discontinuity at employment size 50 evident in the law and our data to identify the structural parameters of our model and the implicit cost of regulation, whereas there does not appear to be such a sharp break in their data on Portuguese firms[9].

The structure of the paper is as follows. Section 2 describes our theory and some extensions. Section 3 describes the institutional setting and data. Section 4 describes the empirical strategy we use to map the theory into the data. Section 5 contains the main results, which come in three parts. First we show that the main empirical predictions of the model in terms of the size and productivity distribution are consistent with the data. Secondly, we estimate the parameters of the structural model and use this to show that the costs of the regulation are non-trivial. Third, we show that there is considerable heterogeneity across sectors within manufacturing and outside manufacturing. We present various extensions and robustness tests in Section 6, such as looking at other margins of substitution into hours and skills to avoid the regulation before drawing some conclusions in the final section.

2 Theory

We aim to estimate the distortions in the productivity distribution and the reallocation effect that result from an implicit tax on firm size that starts at a particular threshold. Our strategy relies on analyzing the

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[8]On the quantitative theory side Guner et al (2006, 2008) also conside a Lucas model with size-contingent regulation. They calibrate this to uncover sizeable welfare losses. Unlike our paper and Barguinsky et al (2011), however, there is no econometric application.

[9]Braguinsky et al (2011) attribute this to the sheer multitude of size-related regulations in Portugal that makes it hard to identify any sharp cut-off in the size distribution.

choices of those firms that prefer to stay at a lower size in order to avoid the tax. Having done that, we will be able to estimate the general equilibrium effects of the tax through the changes in firm size.

We study regulatory effects on the firm size distribution and on the productivity distribution in the simplest possible version of Lucas' model. There is only one input in production, labor, and a single sector. The primitive of the model is the distribution $\phi(c)$ of 'managerial ability' α (which we will measure as Total Factor Productivity, TFP), with cdf $\Phi : R^+ \rightarrow [0,1]$. Ability is defined and measured by how much an agent can raise a team's output: a manager who has ability α and is allocated n workers produces $y = \alpha f(n)$. Larger teams produce more, $f' > 0$, but given e.g. limited managerial time, there are decreasing returns to the firm scale that a manager can manage, $f'' < 0$.

The key difference between our setting and the original Lucas model is that, in our application, there is a tax on firm size, which imposes a wedge between the wage the worker receives and the cost to the firm.[10] Since termination costs are generally denominated in years of salary, we assume this cost is a proportional increase in wage costs, taking the form of a labor tax. Moreover, this tax does not grow in a smooth way, but instead it begins hitting firms after they reach a given size N.

2.1   Individual Optimization

Let $\pi(\alpha)$ be the profits obtained by a manager with skill α when he manages a firm at the optimal size. These profits are then given by:

$$\pi(\alpha) = \max_n \alpha f(n) - \overline{k} - w\overline{\tau}n \begin{cases} \overline{\tau} = 1, \overline{k} = 0 \text{ if } n < N \\ \overline{\tau} = \tau, \ \overline{k} = k \text{ if } n \geq N \end{cases} \tag{1}$$

where w is the worker's wage, n is the number of workers, k is the fixed cost that must be incurred over threshold N, and τ is the tax, which only applies for firm over a minimum threshold of N (50) workers in our application).

Firm size at each side of the threshold is then determined by first order condition:

$$\alpha f'(n^*) - \overline{\tau}w = 0, \qquad \text{with } \tau = \overline{\tau} \ \text{ if } n \geq N \tag{2}$$

so that $n^* = f'^{-1}(\frac{\tau w}{\alpha})$. Note that $\partial n/\partial \alpha > 0$, $\partial n/\partial \tau < 0$ and $\partial n/\partial w < 0$.

The size constraint is reached at size N and managerial ability α_c (sub-script "c" for "constrained") is given by:

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[10]In our application the 'tax' involves an extra marginal cost and also a fixed cost component. However, previous studies of this problem, such as particularly Kramarz and Michaud (2003) show that the fixed cost component are second order relative to the marginal cost component.

$$\alpha_c = \frac{w}{f'(N-1)} \tag{3}$$

Firms can legally avoid being hit by the regulation by choosing to remain small. The cost of this avoidance is increasing in the talent (α) of the individual, and thus at a given ability level, given a choice between staying at $n = N - 1$ and avoiding the tax, managers choose to pay the tax. The ability of the "marginal manager" that is unconstrained (α_u) is defined by the indifference condition between remaining small and jumping to be a larger firm and paying the regulatory tax as:

$$\alpha_u f(N-1) - w(N-1) = \alpha_u f(n^*(\alpha_u)) - w\tau n^*(\alpha_u) - k \tag{4}$$

where $n^*(\alpha_u)$ is the optimal firm size for an agent of skill α_u We call this threshold α_u, where u denotes the boundary of the unconstrained firms, and the firm size $n^*(\alpha_u)$ is denoted n_u.

2.2 Equilibrium

The most skilled individuals choose to be manager-entrepreneurs, since they benefit from their higher ability in two ways. First, for a given firm size n, they earn more. Second, the most skilled individuals hire a larger team, $n(\alpha)$. We denote the ability threshold between managers and workers as $\alpha_{\min}$.

A competitive equilibrium is defined as follows:

Definition 1 *Given a distribution of managerial talent $\phi(\alpha)$, a per worker labor tax τ and a fixed cost k that binds all firms of size $n \geq N$, and a production function $\alpha f(n)$, a competitive equilibrium consists of:*

(i) a wage level w paid to all workers

(ii) an allocation $n(\alpha)$ that assigns a firm of size n to a particular manager of skill α

(iii) a triple of cutoffs$\{\alpha_{\min} \leq \alpha_c \leq \alpha_u\}$, such that $W = [0, \alpha_{\min}]$ is the set of workers, $M_1 = [\alpha_{\min}, \alpha_c]$ is the set of unconstrained, untaxed managers, $M_2 = [\alpha_c, \alpha_u]$ is the set of constrained, $n^* = N - 1$, but untaxed managers, and $M_2 = [\alpha_u, \infty]$ is the set of taxed managers such that:

(1) No agent wishes to change occupation from manager to worker or to change from unconstrained to constrained.

(2) The choice of $n(\alpha)$ for each manager α is optimal given their skills, taxes τ and wages w;

(3) Supply of labor equals demand for labor

Start with condition (1): an agent prefers to be a worker if $w > \alpha f(n) - wn$, or a manager if $w < \alpha f(n) - wn$, and thus we have:

$$\alpha_{\min} f(n) - wn = w \qquad (5)$$

Equilibrium condition (2), from the first order condition (2) implies that firm sizes are given by:

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\begin{aligned}
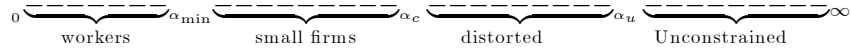n(\alpha) &= 0 & \text{if } \alpha < \alpha_{\min} & \qquad (6) \\
n(\alpha) &= f'^{-1}(\frac{w}{\alpha}) & \text{if } \alpha_{\min} < \alpha < \alpha_c & \qquad (7) \\
n(\alpha) &= N - 1 & \text{if } \alpha_c < \alpha < \alpha_u & \qquad (8) \\
n(\alpha) &= f'^{-1}(\frac{\tau w}{\alpha}) & \text{if } \alpha_u < \alpha < \infty & \qquad (9)
\end{aligned}
$$

Thus we have four categories of agents as the following figure shows:

Equilibrium partition of individuals into workers and firm types by managerial ability,α

Finally, from condition (3) in Definition 1, equilibrium requires that markets clear- that is the supply and demand of workers must be equalized. The supply of workers is $\Phi(\alpha_{\min})$, and the demand of workers by all available managers, $\int_{\alpha_{\min}}^{\infty} n(\alpha)d\Phi(\alpha)$, where $n(\alpha)$ is the continuous and piecewise differentiable function given as above. Thus:

$$\Phi(\alpha_{\min}) = \int_{\alpha_{\min}}^{\infty} n(\alpha)d\Phi(\alpha) \qquad (10)$$

Solving the model involves finding four parameters, the cutoff levels $\alpha_{\min}, \alpha_c, \alpha_u$, and the equilibrium wage w. For this we use the four equations (3), (4), (5) and (10).

The equilibrium is unique; the following proposition characterizes the comparative statics in the equilibrium:

Proposition 1 *The introduction of a tax/variable cost τ of hiring workers starting at firm size N has the following effects:*

1. *Reduces equilibrium wages as a result of the reduction in the demand for workers*

2. *Increases firm size for all firms below the threshold, $[\alpha_{\min}, \alpha_c]$, as a result of the general equilibrium effect that reduces wages*

3. *Reduces firm size to the threshold N for all firms that are constrained, that is those in $[\alpha_c, \alpha_u]$*

4. Reduces firm size for all firms that are taxed $[\alpha_c, \infty]$

Example. Consider a power law, $\phi(\alpha) = \frac{0.6}{\alpha^{1.6}}$ and returns to scale parameter of $\theta = 0.9$. Figure 4 shows the firm size distribution for a firm size cut-off at 50 employees, and an employment tax of 1%. As in the distribution in the data, there is a spike at 49 employees that breaks the power law. Figure 5 reports the productivity distribution α as a function of firm size n. It shows that we should expect a spike in the productivity distribution at the point in which the regulation starts to bind. Essentially the maximum bar of this graph is the most productive firm that is affected by the regulation. We can trace the firm size simply by moving horizontally to the right in the graph.

2.3 Empirical Implications

The econometric work that follows aims to use the theory as a guide to estimate the welfare losses that result from this regulation. As is well known, the firm size distribution generally follows a power law (see e.g. Axtell, 2001). Lucas (1978) shows that Gibrat's law implies that the returns to scale function must be $f(n) = n^\theta$, and that for it to be consistent with a power law, the managerial ability or productivity distribution must also be power, $\phi(\alpha) = c_\alpha \alpha^{-\beta_\alpha}$ with the constants $c_\alpha > 0$ and $\beta_\alpha > 0$. This is not a bad approximation: the distribution of TFP is somewhere between log normal and power, but the fit for a power distribution is good for a large fraction of the data.

In this case, from the first order conditions, firm sizes are given, for a given wage, by:

$$n^* = \left(\frac{\alpha\theta}{w\overline{\tau}}\right)^{1/(1-\theta)} \tag{11}$$

A firm below the tax threshold N chooses a firm size $n^* = f'^{-1}(\frac{w}{\alpha})$; while a firm above the threshold chooses $n^*_\tau = f'^{-1}(\frac{\tau w}{\alpha})$. In the empirical section we rely on this relationship between TFP and firm size n to estimate the counterfactual welfare if the labor regulation τ were to be removed:

$$n^*(\alpha) = \begin{cases} \left(\frac{\alpha\theta}{w}\right)^{1/(1-\theta)} = c_1.\alpha^{1/(1-\theta)} = n_1(\alpha) & \text{if} \quad \alpha_{\min} < \alpha < \alpha_c \\ N - 1 = n_1(\alpha_c) & \text{if} \quad \alpha_c \le \alpha < \alpha_u \\ \left(\frac{\alpha\theta}{w.\tau}\right)^{1/(1-\theta)} = \underbrace{c_1.\tau^{-1/(1-\theta)}}_{=c_2}.\alpha^{1/(1-\theta)} = \underbrace{n_1(\alpha).\tau^{-1/(1-\theta)}}_{=n_2(\alpha)} < n_1(\alpha) & \text{if} \quad \alpha_u \le \alpha \end{cases} \tag{12}$$

with $c_1 = \left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}}$ and $c_2 = c_1.\tau^{-\frac{1}{1-\theta}}$. In $[\alpha_{min}, \alpha_c]$ we find firms that are not directly affected by the distortion. The only impact of the regulation comes through the general equilibrium impact because of lower wages w. This induces some low-ability individuals to became small firms rather than remain as workers (i.e.

the regulatory distortion creates too many entrepreneurial small firms). At $[\alpha_c, \alpha_u]$ we find the constrained firms: firms that, given the choice between paying the labor cost τw^* and choosing their optimal size and paying w^* but staying at size $n < 50$, prefer to stay below 50. Once productivity exceeds a higher threshold α_u firms are sufficiently productive that they pay the tax in order to produce at a higher level.

Given our assumption above that the distribution of $\phi(\alpha)$ follows a power law, $\phi(\alpha) = c_\alpha . \alpha^{-\beta_\alpha}$ the distribution of firm sizes $\chi(n)$ is also power (apart from the threshold), since by the change of variable formula, $\chi(n) = \phi(\alpha(n)) * n^{-\theta} w \bar{\tau} \frac{(1-\theta)}{\theta}$ (omitting the threshold). The power law on n^* is then given by:

$$\chi^*(n) = \begin{cases} c_\alpha(1-\theta)c_1^{\beta-1}n^{-\beta} & \text{if} & n < N-1 = n_1(\alpha_c) \\ \int_{\alpha_c}^{\alpha_u} \phi(\alpha)\mathrm{d}\alpha = \delta & \text{if} & n = N-1 = n_1(\alpha_c) \\ 0 & \text{if} & N-1 < n < n_u = n_2(\alpha_u) \\ c_\alpha(1-\theta)c_2^{\beta-1}n^{-\beta} & \text{if} & n_2(\alpha_u) = n_u \leq n \end{cases} \qquad (13)$$

Where $\beta = \beta_\alpha . (1-\theta) + \theta$ and δ is the mass of firms whose size is distorted- these are firms that choose to stay below the firm size threshold, rather than getting to a large size and paying the additional labor cost, τ.

The adding up constraints on δ can be written more conveniently in the size (n) space rather than the ability space. After some straightforward manipulation, relegated to Appendix A, we can rewrite the pdf of n^* as:

$$\chi^*(n) = \begin{cases} \left(\frac{1-\theta}{\theta}\right)^{1-\beta} (\beta-1)n^{-\beta} & \text{if} & \frac{\theta}{1-\theta} < n < 49 = n_1(\alpha_c) \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta} \left(49^{1-\beta} - Tn_u^{1-\beta}\right) & \text{if} & n = 49 = n_1(\alpha_c) \\ 0 & \text{if} & 49 < n < n_u = n_2(\alpha_u) \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta} (\beta-1)Tn^{-\beta} & \text{if} & n_2(\alpha_u) = n_u \leq n \end{cases} \qquad (14)$$

where $T = \tau^{-\frac{\beta-1}{1-\theta}}$. The upper employment threshold, n_u, is unknown and must be estimated alongside β, the power law term, and T. Note that the scale parameter in the power law β is unaffected by the law; instead, in log-log space, the labor regulations generate a parallel shift in the firm size distribution measured by T. Thus the key empirical implication is that the tax can be recovered from the jump T in the power law.

In Section 4, we propose an empirical model in which we introduce an error term in the model so that we can take it to the data. Such empirical model must account for two departures in Figure 2 from the predictions in the theory

1. The departure from the power law does not start at N, but slightly earlier: there is a bump in the distribution starting at around 46 workers.

2. The region immediately to the right of N does not have zero density, but rather there are some firms with positive employment levels just to the right of the regulatory cut-off, N.

The model we propose to account for these departures features a measurement error. The justification for such mis-measurement is straightforward: the measurement of firm size that we have is not exactly the same one as the one used to determine whether a firm is subject to the regulation or not. From the perspective of the regulation, the relevant concept of employment is the number of workers at the precise date where the collective dismissal is announced. Our measure of firm size is the mandatory item that is reported in the firm's fiscal accounts - the arithmetic mean of the workforce at the end of the quarter of the fiscal year.[11]

3 Empirical Strategy

In this section we explain how we apply our theoretical framework to the data. First, we allow for some measurement error which is necessary to fit the employment data. Second, we discuss how identification and inference. Third, we show how we can make empirical welfare calculations.

3.1 Empirical Model

Recall that our starting point is the pdf of n^*, which is, according to the theory, given by equation (14).

Employment is measured with error so we assume that rather than observing $n^*(\alpha)$ we observe:

$$n(\alpha, \varepsilon) = n^*(\alpha)e^{\varepsilon}$$

Where the measurement error ε is unobservable. In the data we observe the distribution of n, and thus obtaining the likelihood function requires that we obtain the density function of n. The law of $n|\varepsilon$, has support on $[e^{\varepsilon}; +\infty]$. The conditional cumulative distribution function is given by (see Appendix A):

$$\mathbb{P}(x < n|\varepsilon) = \begin{cases} 0 & \text{if} & \ln(n) - \ln\left(\frac{\theta}{1-\theta}\right) < \varepsilon \\ 1 - \left(\frac{1-\theta}{\theta}\right)^{1-\beta} (ne^{-\varepsilon})^{1-\beta} & \text{if} & \ln(n) - \ln(49) < \varepsilon \leq \ln(n) - \ln\left(\frac{\theta}{1-\theta}\right) \\ 1 - \left(\frac{1-\theta}{\theta}\right)^{1-\beta} Tn_u^{1-\beta} & \text{if} & \ln(n) - \ln(n_u) < \varepsilon \leq \ln(n) - \ln(49) \\ 1 - \left(\frac{1-\theta}{\theta}\right)^{1-\beta} T\left(ne^{-\varepsilon}\right)^{1-\beta} & \text{if} & \varepsilon \leq \ln(n) - \ln(n_u) \end{cases}$$

Integrating over ε we can compute the unconditional CDF simply as:

$$\forall n > 0, \quad \mathbb{P}(x < n) = \int_{\mathbb{R}} \mathbb{P}(x < n|\varepsilon) \frac{1}{\sigma} \cdot \varphi\left(\frac{\varepsilon}{\sigma}\right) d\varepsilon.$$

In Appendix A we show that no further constraints on the parameters are required for this object to be a CDF:

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[11] Fiscal definition, article 208-III-3 du Code General des Impots.

Lemma 1 *Let ε be normally distributed with mean 0 and variance σ so that the measurement error is log normal. Then the function $\mathbb{P}(x < n)$ is a cumulative distribution function, that is strictly increasing in n, with $\lim_{n \longrightarrow 0} \mathbb{P} = 0$ and $\lim_{n \longrightarrow \infty} \mathbb{P} = 1$ for all feasible values of all parameters, σ, T, β, n_u.*

Thus taking the derivative of $\mathbb{P}$ formulated in this way we can obtain the density of the observed n. Given such a density, it is straightforward to estimate the parameters of the model by maximum likelihood. Specifically, the maximum likelihood estimation yields estimates of the parameters: $\widehat{\sigma}, \widehat{T}, \widehat{\beta}, \widehat{n}_u$.[12]

Figure 6 shows the difference between the pure model where employment was measured without error and the true model where there is measurement error. The solid (blue) line shows the firm size distribution under the pure model of Section 2 (same as Figure 4) whereas the hatched line shows the firm size distribution when we allow for measurement error. The smoothness of the hump around 50 will depend on the degree of measurement error - Figure 6 shows that if we increase the measurement error to $\sigma = 0.5$ instead of $\sigma = 0.15$ it is almost impossible to visually identify the effects of the regulation.

3.2 Identification and Inference

ML estimation over the size distribution allows us to obtain most of the parameters of interest. Intuitively, the slope of the line in Figure 6 (which is the same before and after the cut-off) identifies β, the the power law parameter. The composite parameter $T = \tau^{-\frac{\beta-1}{1-\theta}}$ which is a function of our key object of interest the implicit tax, τ, is identified from three related features of the data. First, the downward shift of the power law slope around 49 employees, the "intercept". Second, the hump of firms just before the regulatory threshold at 49 employees and third the "hole" in the size distribution between 49 employees and where the power law recovers at n_u. The larger is the implicit tax, the greater will be the size of the intercept, the hump of firms at the regulatory threshold and the hole in the firm size distribution. The fixed costs k/w, are identified from the indifference equation (4) of the marginal manager around the regulatory threshold and the measurement error from the size of the random deviations of size from the broken power law.

Given these estimates we still need an estimate for the returns to scale parameter θ to identify the key tax parameter,τ. There are several ways to obtain the scale. In principle, it can be obtained from the size distribution itself jointly with the other parameters (see Appendix A.2). This method relies on rather strong assumptions over the identity of the smallest firm from the indifference condition between being a worker and a manager. As discussed in Appendix A2 the data is not rich enough to estimate θ from the size distribution

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[12]In a previous version of the paper we generated OLS estimators of these parameters. However, Bauke (2007) and Howell (2002), both within the physics literature, have shown that least square methods may be unreliable. Gabaix and Ibragimov (2011) make the same point and propose a simple rank-based method with the robust approximations for for standard errors. Calculation of these is non-trivial, however. In Appendix B we show how to obtain OLS estimates of the parameters of interest developing a new methodology borrowed from the time series literature on structural breaks. These results suggest a larger implicit tax of the regulation.

alone, so we consider several alternatives in order to examine the empirical robustness of our estimates of τ. Our first approach is to calibrate θ from existing estimates. Since this is well recognized to be an important parameter in the macro reallocation literature there is a sizeable literature here. Basu and Fernald (1997) show a large number of estimates based on US data and suggest a value of 0.8 is reasonable. Atkeson and Kehoe (2005) using a version of the Lucas model with organizational capital use a $\theta = 0.85$. Most calibrations seem to take a value of around 0.8 (e.g. Guner et al, 2006, use a $\theta = 0.802$ for Japan. We also consider more extreme values of $\theta = 0.5$ and $\theta = 0.9$ in the results section.

A second approach is to use information from the production function. Since we have rich data on firms we can estimate production functions and from the sum of the coefficients on the factor inputs estimate returns to scale. Appendix C details how we do this using a variety of methods adapting Levinsohn and Petrin (2003), Olley and Pakes (1996) in addition to the more standard Solow residual approach. A final and related method is to use the relationship between size and TFP from equation (12) to back out an estimate of the returns to scale.

For each of these estimates of θ, we have an estimate of the implicit tax of regulation as:

$$\widehat{\tau} = \widehat{T}^{-\frac{1-\widehat{\theta}}{\widehat{\beta}-1}} \tag{15}$$

We obtain standard errors for the estimate of the tax using the delta method. To be precise, let $\widehat{\Theta} = (\widehat{\beta}, \widehat{T}, \widehat{\theta})'$ be the vector of asymptotically Gaussian estimates obtained in the previous steps. The asymptotic variance-covariance matrix of this vector can be written as:

$$V(\widehat{\Theta}) = \begin{pmatrix} V_{\widehat{\beta}} & Cov_{\widehat{\beta},\widehat{T}} & 0 \\ . & V_{\widehat{T}} & 0 \\ . & . & V_{\widehat{\theta}} \end{pmatrix}$$

Such that an estimator of the asymptotic variance of $\widehat{\tau}$ is:

$$V(\widehat{\tau}) = (\frac{\partial \tau}{\partial \beta}, \frac{\partial \tau}{\partial T}, \frac{\partial \tau}{\partial \theta}).V(\widehat{\Theta}).(\frac{\partial \tau}{\partial \beta}, \frac{\partial \tau}{\partial T}, \frac{\partial \tau}{\partial \theta})'$$

3.3   Welfare calculations

Total output of a given team, managed by manager of skill α and with size $n_\tau^*(\alpha)$ given by replacing the equilibrium wages w in $n^*(\alpha)$ as given by equation (12) is:

$$y(\alpha, N, \tau, k) = \alpha f(n_\tau^*(\alpha)) - \bar{k} - w\bar{\tau}n_\tau^*(\alpha) + wn_\tau^*(\alpha) \begin{cases} \bar{\tau} = 1, k = 0 \text{ if } n_\tau^*(\alpha) < N \\ \bar{\tau} = \tau, \bar{k} = k \text{ if } n^*(\alpha, \tau) \geq N \end{cases}$$

$$= \alpha f(n_\tau^*(\alpha)) - \bar{k} - (\tau - 1)wn_\tau^*(\alpha) \begin{cases} \bar{\tau} = 1, k = 0 \text{ if } n_\tau^*(\alpha) < N \\ \bar{\tau} = \tau, \bar{k} = k \text{ if } n^*(\alpha, \tau) \geq N \end{cases}$$

and thus total output is given by:

$$Y(N, k, \tau) = \int_{\alpha_{\min}}^{\alpha_c} \alpha f(n_\tau^*(\alpha)) d\Phi(\alpha) +$$
$$+ \int_{\alpha_c}^{\alpha_u} \alpha f(N - 1) d\Phi(\alpha) +$$
$$+ \int_{\alpha_c}^{\infty} \alpha f(n_\tau^*(\alpha)) - k - w^* n_\tau^*(\alpha)(\tau - 1)$$

And thus the welfare change is then given as follows:

$$\Delta Y = Y(N, k, \tau) - Y(N, 0, 1) = \int_{\alpha_{\min}}^{\alpha_{fb\min}} \alpha \left[f(n_\tau^*(\alpha)) \right] d\Phi(\alpha) + \int_{\alpha_{fb\min}}^{\alpha_c} \alpha \left[f(n_\tau^*(\alpha)) - f(n) \right] d\Phi(\alpha)$$
$$+ \int_{\alpha_c}^{\alpha_u} \alpha \left[f(N - 1) - f(n) \right] d\Phi(\alpha) +$$
$$+ \int_{\alpha_c}^{\infty} \alpha \left\{ [f(n_\tau^*(\alpha)) - f(n)] - k - w^* n_\tau^*(\alpha)(\tau - 1) \right\} d\Phi(\alpha) \qquad (16)$$

where n is the first best team size and $\alpha_{fb\min}$ is the first best cutoff between workers and managers.

The welfare losses are then the result of adding up three effects:

(1) There is an output gain given by the gain in size of small firms resulting from the general equilibrium effect on wages (wages go down) of the firing costs. This is the first row of equation (16).

(2) There is a first "local" output loss, that is the result of the firms that would have had optimal size but instead are constrained at $N - 1$ (49 workers). This is the second row of equation (16).

(3) The top row captures two positive effects on total output from the fall in the equilibrium wage arising from the regulation. First, there will be some additional firms since marginal workers are drawn into becoming entrepreneurs by cheaper labor. Second, the firms who are below the regulatory threshold (And not paying the tax) will be able to hire more workers as their wages are lower.

Once we have our estimates of all the model parameters, we use this calculation to obtain the output losses by obtaining the counterfactual firm size distribution in the world without tax and the total production loss in this economy due to the tax distortion.

4 Institutional Setting and Data

4.1 Institutions: The French Labor market and Employment Costs

France is renowned for having a highly regulated labor market (see Abowd and Kramarz, 2003; Kramarz and Michaud, 2010). What is less well known is that most of these laws only bind on a firm when it reaches a particular employment size threshold. By far the most important size threshold is when a firm hits fifty employees - at this point of number of labour market regulations bind regarding the firm's ability to adjust its labor. Although there are some regulations that bind when a firm (or less often, a plant) reaches a lower threshold such as 10, 20 or 25 employees, 50 is generally agreed by labour lawyers and business people to be the critical threshold when costs rise significantly (see Appendix D)[13].

Perhaps the most important of these is a set of regulations introduced under a major piece of legislation in 1989. This required firms with 50 or more employees to formulate a "social plan" before laying off 10 or more workers (a "collective termination"). This social plan must place a limit on the total number of terminations, lay out plans to facilitate reemployment of terminated workers and will typically insist on an extensive retraining program. Union representatives or personnel delegates and the departmental director of the Ministry of Labor must also be informed of the plan. Two public meetings of the works council ("comité d'entreprise") must be organized with an interval between the meetings of 2–4 weeks depending upon the number of terminations proposed. The works council may require the firm to hire a consulting accountant (at the company's expense) to help the council with its analysis. During this period, the Ministry of Labor must be continuously informed of the proceedings, the plan, and the names of the proposed terminated workers. In addition to these firing costs in the 1989 law, there are some other pieces of regulation that bite at size 50 (see Appendix D).

How important are such provisions for firms? It is hard to know directly, as the opportunity cost of managerial time involved in preparing for such eventualities may be very great. Our framework is designed to recover the costs of such regulations. We treat such firing costs as an increase in the cost of labour. Firms face future shocks which will require them to adjust labor. Firms facing such a firing cost will effectively face a much higher cost in the eventuality that they face a negative shock. This affects the decision to hire and is (in expected value terms) very much like a labor tax. Since our analysis is mainly cross sectional we will model the firing cost as a labour tax.

There are other laws affecting French firms, so in one sense we are estimating a lower bound to the cost of regulation. But we are alert to the problem that some of the data is also affected by other laws which may

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[13]http://www.travail-emploi-sante.gouv.fr/informations-pratiques,89/fiches-pratiques,91/licenciement,121/le-plan-de-sauvegarde-de-l-emploi,1107.html

also have a size-related threshold. Discussions with the labor ministry, lawyers, unions and business people confirm that the threshold of 50 is the most important one in France, so it makes sense to begin our analysis here.

4.2   Data

Our main dataset is administrative data covering the universe of French firms between 2002 and 2007. These hold about 2.2m observations per year. Our main results are on the approximately 200,000 firms active in manufacturing industries (NACE2 class 15 to 35) as productivity is easier to measure. We also look at all the other main private non-manufacturing sectors in extensions of the baseline results. The data are the (mandatory) fiscal returns of all French firms ("FICUS") and are the appropriate level for analysis as it is on this administrative unit ("entreprise") that the main laws pertain to. In addition to accurate information on employment (average number of workers in last quarter of the fiscal year), FICUS contains balance sheet information on labor, capital, investment, wage bills, materials, four digit industry affiliation, zipcode, etc. that are important in estimating productivity. More details of the dataset are given in the Appendix.

We take several approaches to estimating productivity. Our baseline results use the Levinsohn and Petrin (2003) version of the Olley and Pakes (1996) method of using a control function approach to deal with unobserved productivity shocks and selection when estimating production functions. Because we have a panel of firms we can implement this and estimate the production function coefficients. The details of these regressions are reported in Appendix C. There are several issues with this approach (see Ackerberg et al, 2007) to estimating production functions so we also estimate TFP using a variety of other methods (see Appendix C for details).

5   Results

5.1   Qualitative analysis of the data

Before moving to the econometrics we first examine some qualitative features of the data to see whether they are consistent with our model. Many commentators have expressed skepticism about the quantitative importance of employment regulations as it is sometimes hard to observe any clear change in the size distribution around important legal thresholds[14], so we first focus on this issue. Figure 7 presents the empirical distribution of firm size around the cut-off of 50 employees for two datasets. The dataset we use (FICUS),

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[14] For example, Schivardi and Torrini (2008) and Boeri and Jimeno (2005) on Italian data, Braguinsky et al (2011) on Portuguese data or Abidoye et al (2010) on Sri Lankan data. The authors find that there is slower growth just under the threshold consistent with the regulation slowing growth, but they find relatively little effect on the cross-sectional distribution. This may be because of the multitude of regulations, variable enforcement or measurement error in the employment data (see sub-section 2.3).

the fiscal files of the French tax administration, is the population dataset of the universe of French firms that forms the basis of our econometric work. Panel 7.1 in Figure 7 is the same as Figure 2 except now on 2002 data instead of 2007 data. The qualitative features are clearly stable across years: just as in Figure 2 there is a sharp discontinuity in size precisely at 50 employees which is strong non-parametric evidence for the importance of the regulation. There are just over 400 firms with exactly 49 employees and then only about 130 with 50 employees. Importantly, the distribution which declines from 31 employees flattens after about 44 employees, just before the stacking up at 49 employees then dropping off a sharp cliff when size hits 50. The top right hand side of Figure 7 shows this in log-log space clearly indicating the evidence of a "broken power law".

The next panel of Figure 7 compares FICUS with another dataset, DADS (Déclaration Annuelle de Données Sociales), that is also frequently typically used by labor economists. DADS is a worker-level dataset containing information on occupation (see Figure 7.2), wages and demographics. In Panel 7.2 we aggregate employment up to the appropriate level for each FICUS firm. This enables us to investigate different measures of employment such as employment dated on 31st December. The discrete jump at 50 shows up here almost as clearly as the FICUS data. The bottom panels of Figure 7 uses Full-Time Equivalents which shows less of a jump than the straight count of employees in the previous panels (the main labor laws relate to the number of workers rather than full-Time Equivalents, so this is expected). Figure 7 illustrates the importance of good data - one of the reasons that other studies have not identified such a clear discontinuity around the regulatory threshold is that they may have been using data with even greater measurement error than our own and not using the full population (as we do).

Figure 8 shows the firm size distribution over a larger range between 1 and 1,000 employees. Overall, firm size seems to approximate a power law in the employment size distribution prior to the bulge around 50. After 50, there is a sharp fall in the number of firms and the line more flat than expected before resuming what looks like another power law. Broadly, outside a "distorted" region around 50 employees, one could describe this pattern a "broken power law" with the break at 50.[15] The finding of the power-law for firm size in France is similar to that for many other countries and has been noted by other authors (e.g. Giovanni et al, 2010; Giovanni and Levchenko, 2010), but the finding of the break in the law precisely around the main labor market regulation has not (yet) been documented in the academic literature, except in Ceci-Renaud and Chevalier (2011). As is well known the power law fits rather less well for the very small firms. Additionally, there does appear to be some break in the power law at firm size 10. This corresponds to the size thresholds from other pieces of labor and accounting regulations (see Appendix D). In order to avoid conflating these

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[15]See Howell (2002) for examples of how to estimate these types of distributions. More generally see Bauke (2007) for ways of consistently estimating power laws.

issues we focus our analysis on firms with 10 or more employees. We can generalize the methods used here to other breaks in the Power Law which we will exploit in future versions.

The distribution of TFP is presented in Figure 9. This shows that the mean level of TFP is higher in each size class of firm which is what we would expect from the model. Our basic model, following Lucas, has the implication that more talented managers leverage their ability over a greater number of workers.

We cut the same data in a slightly different way in Figure 10 plotting the mean TFP levels by firm size. Panel A does this for firms between 5 and 100 employees whereas Panel B extends the threshold out to firms with up to 1000 employees. In all panels productivity appears to rise monotonically with size, although there is more heteroskedacity for the larger firms as we would expect because there are fewer firms in each bin. The relationship between TFP and size is broadly log-linear. What is particularly interesting for our purposes, however, is the "bulge" in productivity just before the 50 employee threshold. We mark these points in red. This looks consistent with our model where some of the more productive firms who would have been just over 50 employees in the counterfactual world, choose to be below 50 employees to avoid the cost of the regulation. Firms just below the cut-off are a mixture of firms who would have had a similar employment level without the implicit tax and those firms whose size is distorted by the size-related regulation.

We exploit the relationship between size and TFP to identify the θ, returns to scale parameter in some of the empirical estimates.

5.2   Econometric Implementation of the model

5.2.1   Main results

The key parameters are estimated from the size distribution of firms using the ML procedure described above. We begin in Table 1 with a set of baseline results using calibrated values of θ for the entire sample of French manufacturing firms 2002-2007[16]. Given the large sample sizes the results are very precise.

We begin by using a calibrated value of $\theta = 0.8$ from Basu and Fernald (1997) and Guner et al (2006, 2008) in column (1). The slope of the power law is about 1.8 and highly significant. The upper employment threshold is estimated to be about an employment level of 58 and we obtain a standard deviation of the measurement error of just over 0.10, which suggests significant, but not major amounts of mismeasured employment. Turning to the estimates of the tax equivalent costs of the regulation, we obtain $T = 0.948$ which is determined in part by the implicit variable labor tax which we estimate to be $\tau = 1.013$ and highly significant. This implies that the regulation increases variable costs by 1.3 percentage points which is a

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[16]We use a sample of firms with between 10 and 1,000 employees correcting our estimates for censoring at the lower and upper known thresholds. We do this because there are other regulations that bite at 10 employees. The results are robust to reasonable extensions around these thresholds.

moderately large and important effect. By contrast the fixed cost component of the regulation is insignificant at the 5% level, incorrectly signed and small in magnitude (for a 100 worker firm the implies estimates are less than 0.5% of the wage bill).[17]

Figure 11 shows the data and the fit of the model using the estimated parameters. Although not perfect, we seem to do a reasonable job at mimicking the size distribution even around the regulatory threshold when allowing for measurement error.

Column (2) of Table 1 considers an alternative calibration of $\theta = 0.85$ from Atkeson and Kehoe (2005). The results are very stable, although as expected the estimate of the marginal tax falls from 1.3% to 1%. This is because the importance of the distortions of the tax depend on returns to scale. When returns to scale are close to unity the most efficient firms have a very large share of output, so it only takes a small distortionary tax to have a large effect on the size distribution. As decreasing returns set in it takes a much larger estimate of τ to rationalize any given distorted distribution (the "intercept", "hump" and "hole" in firm size). Column (3) considers $\theta = 0.5$ and column (4) a $\theta = 0.9$. The first case is formally equivalent to a minimum firm size of unity which is empirically consistent with the data and implies a large tax of 3.3 percentage points. The second case reduces the implicit tax to 0.4 percentage points.

Table 2 uses estimates of the returns to scale parameter directly estimated from a production function (see Table A1 and Appendix C). The first column just reproduces our baseline results from column (1) of Table 1. Column (2) contains the results using the estimations from the production function giving a value of $\theta = 0.855$ that is highly significant. Other parameter estimates remain stable and we obtain an estimate of $\tau = 1.010$ only slightly lower than the baseline case of $\tau = 1.013$. Column (3) uses the TFP estimates from the production function to estimate the TFP-size relationship as in Figure 10. We recover an estimate of θ from the slope of this relationship which is $\theta = 0.799$ and re-estimate all the parameters. This generates a generally stable values with an estimate of $\tau = 1.013$, very close to the baseline results.

5.3 Industry Heterogeneity

Holding the parameters constant across industries is an attempt to focus on the macro-economic consequences of the regulation. But there is nothing in our approach we requires we do this. Consequently we have investigated various ways of allowing the coefficients to vary across industries. We begin with simply splitting the industries into "high tech" and "low tech" following OECD definitions (these are based on R&D intensity). The estimates of parameters are given in columns (4) and (5) of Table 2 and the analogous production function

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[17]If we constrain the model to set the fixed costs to zero, the results are stable, but the implied value of the variable tax falls to 1.004 with a standard error of 0.001. This does suggest that the unconstrained model may have some desirable features.

estimates are in the last two columns of Table A1. There does appear to be significant heterogeneity with the estimated implicit tax insignificantly different from zero in the high tech sectors and bearing more heavily in low-tech sectors (1.3%).

Next, in Table 3, we examine the other main sectors outside manufacturing. We use calibrated values of $\theta = 0.80$ and $\theta = 0.85$. The first two columns repeat the baseline estimates using these values. We estimate the models for the other four large sectors of the French economy outside manufacturing in the rest of the table - Transport (columns (3) and (4)), Construction (columns (5) and (6)), wholesale and distribution Trade (columns (7) and (8)) and Business Services (columns (9) and (10)). The implicit tax seems to be more important in both Transport (2.5%) and Construction (2.0%) than it was in manufacturing (1.3%). In business services, by contrast, the regulation seems to be estimated to be insignificantly different from zero.

Finally, we estimated the production functions separately by three digit sectors and used the full ML technique with estimated production functions as in column (2) of Table 2. This allows the scale (θ) and all other parameters to be freely estimated. Some of the industries have insufficient number of firms to do this estimation but we are still able to do this for a large number. The results are in Figure A1 which again demonstrates a substantial degree of heterogeneity with some sectors with estimates of the implied tax from near zero to over 50%.

5.4 Estimates GDP and welfare loss

Our model (see the explanation in the Appendix) allows us to calculate the impact of the tax on te firm size distribution and from it, on output. As we have seen, the slope of the power law does not change as a result of the tax. The impact of the tax is a paralell move upwards of the firm size distribution at sizes $n < 49$, a spike at $n = 49$, and a parallel move down for $n > 58$. Thus very simply the counterfactual firm size is a power law with the exponent β we calculated in our analysis, and $\tau = 1$. The position of the intercept is pinned down by the labor market condition, which requires that the number of workers employed in total remain constant, and by the minimum firm size which is pinned down by the returns to scale parameter and stays constant.

Conservatively, our estimates only reach firms of up to 500 workers. Since we are estimating the costs of this legislation off the distortions it imposes on the medium size firms, plainly our estimates are less trustworthy as we increase the size substantially. In this sense our estimates are a lower bound of the true estimates.

Figure 12 presents the resuts of such exercise graphically. The two upper charts (Panels A and A Zoom) simply compare the FSD and the counterfactual FSD. The right hand side panel expands on the left one by focusing only on firm sizes around the threshold and cutting the spike at 49 to do this. Indeed, as the theory

led us to expect, the counterfactual FSD appears in between the two empirical distributions, the one that characterizes the distribution up to 49 workers, and the one from 59 onwards. The lower two charts (panels B and B zoom) show the output levels for each level of TFP or skill. They show that output losses for firms below 49 workers are tiny, as firm size is almost unchanged. A large triangle of loss appears for those managers whose ability would leave them to choose sizes between 49 and 58. Finally, a significant loss can be observed for all firms over that size. Table 4 next quantifies these gains and losses in terms of their profit and wage consequences.

Table 4 and Figure 13 compare the world with the labor employment protection legislation and the world without such legislation, in terms of output. We note first that managers prefer to stay below the threshold if their optimal size with tax is 58 or less. Some of the welfare cost of these regulations fall on the workers, who expereince a significant drop in their wages, of the order of 0.7% of GDP. This is a result of the tax reducing the firm size of all large firms. Large firms attain a smaller size, athough the tax cost is somewhat compensated by the wage reduction: the tax is around 1.13%, and the reduction in wages of around .69%, so firms that choose sizes over 50 workers incur labor costs that are .6% higher than absent the regulation. Thus small firms increase by around 3.3%, and large firms reduce in size by around 3%.

As a result, large firm have 2.41% lower profits than it would have without the regulation. The winners are small firms, who benefit from lower wages and do not have to incur the tax, which starts at 50 workers. We show that small firms have 2.8% higher profits than in the counterfactual world. Firms on the neighborhood of 50 workers benefit from the lower wages, but are costrained in size as they stay at 50. Overall, the effect is a net positive, and their profits are on average 2.15% higher than without the tax. We also show that there is no evidence that workers place a positive economic value on this regulation.(16).

Of course, all of the above results focus GDP losses. There may be some utility, gains to workers of these regulations, if workers value the EPL as a flow of services and insurance they receive. In workers value this amanity then, as Lazear (1990) argues, firms can respond to employment protection regulation in a "Coasian" manner by contracting around the regulation. Since workers enjoy greater insurance from negative shocks through the increased firing costs they should be prepared to accept lower wages as a compensating differential. With sufficiently flexible and enforceable contracts, the cost to the firm of being above 50 may not matter. Contracts may be very incomplete of course, causing downward nominal wage rigidity especially in France where there are strong unions and high minimum wages. Low trust between employers and employees and credit constraints for workers could also limit such contractual flexibility.

We examine this empirically by looking at wages around the threshold (see Figure 14). If there are such insurance benefits we would expect wages to be lower after the threshold of 50 employees. As expected

the wage is upward sloping, but there does not appear to be a significant fall in wages after the regulatory threshold. Therefore we do not think that firms are able to completely "contract around" the regulation in this manner to offset the costs of the implicit labor tax.

6   Extensions and Robustness

In this section we consider several extensions to our framework and robustness tests of the results.

6.1   Changing the organizational structure of corporations

An obvious way in which a business group could respond to the regulation is by splitting itself into smaller subsidiaries. Thus a firm which wished to grow to 50 employees could split itself into two 25 employee firms controlled by the group CEO. There are costs to such a strategy - the firm will have to file separate fiscal and legal accounts, demonstrate that the affiliates are operating autonomously and suffer from greater problems of loss of control. The authorities are well aware of such strategies of large firms pretending to be small in order to avoid regulation and there are hefty fines and prison terms for executives seeking to do this.

Nevertheless, one way to check for this issue is to split the sample into those firms that are stand alone businesses and those that are part of larger groups. If groups could simply split themselves into smaller subsidiaries when they crossed the threshold of 50 employees we should expect to see no discontinuity in group size around the threshold. Figure 15 splits the size distribution into subsidiaries which are standalone and those which are part of larger groups. For the latter we aggregate employment to the group level. We can see a clear discontinuity around 50 employees for the group size (as well as the standalone firms). This suggests that corporate restructuring does not full undue the regulation[18].

6.2   Other margins of adjustment to the regulation

The simplest version of the model focuses on the decision over firm size based on employment. However, there are many other possible margins of adjustments that firms could take to avoid the regulation. This can be allowed for in the model by re-writing output as $y = \alpha[h(n,x)]^\theta$ instead of $y = \alpha n^\theta$ where x are the other factors of production such as hours or human capital. If there was perfect substitutability between labor and these other factors then the firm could avoid the size-distortion we have discussed. More realistically when there is imperfect substitution the firm can mitigate some of the costs of the regulation through substitution. Of course, having to sub-optimally adjust into other factors of production is going to generate some welfare loss by itself.

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[18]A more extreme reaction of the firm would be to engage in franchising. This has some further costs as the CEO no longer has claims over the residual profits of the franchisee and loses much control. In any case, franchising is rare in manufacturing.

The first way that the firm could adjust is by making its workforce work harder rather than expand the number of employees. We find clear evidence that firms respond in this way in Figure 16 as the number of annual hours increases just before the threshold of 50 employees. This is a combination of firms making workers do more overtime hours and substituting towards full time workers and away from part-timers. A second method would be by substituting across workers of different occupational types, such as employing workers who are of higher quality with more human capital. Figure 17 shows that change in the share of the main three skill groups in French firms across firm size (managers - the most skilled group, manual workers - the least skilled group and clerical workers - the main middle group). Panel A shows the share of managers (excluding the CEO). This share seems to rise with firm size, but there is a clear change in the pattern around the threshold with firms choosing to increase their proportion of managers just after the regulatory threshold. Panel B shows almost a mirror image for manual workers - firms seem to reduce their reliance on less skilled workers around the threshold. The middle group of workers in Panel C is relatively unaffected (the smaller residual groups look broadly like Panel C). this indicates a pattern whereby instead of expanding the quantity of workers a as it nears the threshold, firms will increase the quality of employees by substituting away from low skilled manuals to more skilled managers. This enables them to increase output without necessarily increasing employment and paying the extra regulatory cost.

We also looked at other possible margins of adjustment. We see an increase in capital intensity around the threshold and a greater use of outsourced workers[19], which is also consistent with firms trying to avoid the regulatory costs.

Since we observe all these margins we are able to take account of them in our estimation of the production function. They should therefore not in principle bias our estimates of TFP. If the firm was able to perfectly substitute into these other types of activity the regulation would have little welfare effect as the firm would be able adjust around the regulation by smoothly adjusting into other factors.

In summary, firms do appear to be adjusting to the regulation around the threshold, in particular by attempting to increase hours and skills rather than raw labor when they get close to 50 employees. This is reassuring as it suggests that firm size is not just being misreported to avoid the regulation - firms are genuinely changing their activities in a theoretically expected direction.

6.3   Growth Analysis near the threshold

In a dynamic sense, the knowledge that going over 50 employees will generate a large increase in costs should affect the growth behavior of firms near the threshold. Since firms are hit by idiosyncratic shocks (e.g.

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[19]These outsourced workers will enter as an intermediate input and are therefore not included in total employment.

less quits than usual), they may inadvertently get caught in the high regulation regime. Thus there is a strong disincentive to grow for firms which are approaching the threshold size. We can examine this issue by looking at growth dynamics at different size thresholds. Indeed we find that the growth process appears to be distorted near the threshold with firms just below the threshold significantly less likely to grow and firms just above the threshold significantly more likely to shrink.

More subtly, the difficulty of growing beyond a threshold may affect firms dynamic decisions. For example, if firms learn about their TFP through growing then the tax on size will reduce the learning process and cause a dynamic further. Also, if firms are concerned for stochastic reasons they may cross the threshold they may cut their size back even further below 49 in order to be well away from the threshold.

7   Conclusions

How costly is labor market regulation? This is a long-debated subject in policy circles and economics. We have tried to shed light on this issue by introducing a structural methodology that combines a simple theoretical general equilibrium approach based on the well known Lucas (1978) model of the size and productivity distribution of firms. We introduce size-specific regulations into this model, exploiting the fact that in most countries labor regulation only bites when firms cross specific size thresholds. We show how such a model generates predictions about the changes in the size and productivity distribution and moreover, can be used to generate an estimate of the implicit tax of the regulation. Intuitively, firms will optimally choose to remain small to avoid the regulation, so the size distribution becomes distorted with "too many" firms just below the size threshold and "too few" firms just above it. Furthermore, the distribution of productivity is also distorted: some of those firms just below the cut-off are "too productive" as they have been prevented from growing to their optimal size by the regulation. We show how the regulation creates welfare losses by (i) allocating too little employment to more productive firms who choose to be just below the regulatory threshold, (ii) allocating too little employment to more productive firms because they bear the implicit labor tax (whereas small firms do not) and (iii) through reducing equilibrium wages (due to some tax incidence falling on workers) this encourages too many individuals to become small entrepreneurs rather than working as employees for more productive entrepreneurs.

We implement this model on the universe of firms in the French private economy. France has onerous labor laws which bite when a firm has 50 employees, so is ideally suited to our framework. We find that the qualitative predictions of the model fit very well: (i) there is a sharp fall off in the firm size distribution precisely at 50 employees resembling a "broken power law" and (ii) there is a bulge in productivity just to the left of the size threshold. Having good employment measures over the population of firms helps a lot.

We then estimate the key parameters of the theoretical model from the firm size distribution. Our approach delivers quite a stable and robust cost of the employment regulation which seems to place an additional cost on labor of around 1% of the wage, but with considerable variation across industries. We show that we expect this cost to translate into significant, output losses, in our baseline specification of around 2.7%.

This is just the start of our research program. We need to do a lot more testing of the results and extensions to the greater institutional complexity of the labor market. We believe that our approach is a simple, powerful and potentially fruitful way to tackle the vexed problem of the impact of regulation on modern economies. Size-contingent regulations are ubiquitous and our methodology can be used for other regulations, other parts of the size distribution, other industries[20] and other countries.

---

[20] For example, the retail sector has a large number of size-contingent regulations with "big boxes" being actively discouraged in many countries and US cities (e.g. Bertrand and Kramarz, 2002, or Baily and Solow, 2001).

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A   Omitted Proofs

A.1   Adding up constraint on δ

How do we derived equation (14) from equation (13)? The firm size distribution is given by the broken power law in equation (13):

$$\chi^*(n) = \begin{cases} c_\alpha.(1-\theta).c_1^{\beta-1}.n^{-\beta} & \text{if} & \underbrace{n(\alpha_{min})}_{=\theta/(1-\theta)} < n < 49 = n_1(\alpha_c) \\ \int_{\alpha_c}^{\alpha_u} \xi(\alpha)\mathrm{d}\alpha = \delta & \text{if} & n = 49 = n_1(\alpha_c) \\ 0 & \text{if} & 49 < n < n_u = n_2(\alpha_u) \\ c_\alpha.(1-\theta).c_2^{\beta-1}.n^{-\beta} & \text{if} & n_2(\alpha_u) = n_u \leq n \end{cases}$$

We have *two* additional restrictions:

- The constraints on δ can be re-written more conveniently in terms of regimes 1 and 2 as (the two are equivalent up to a variable change):

$$
\begin{aligned}
\delta &= \int_{n_1(\alpha_c)}^{n_1(\alpha_u)} c_\alpha.(1-\theta).c_1^{\beta-1}.n^{-\beta}\mathrm{d}n = \int_{n_1(\alpha_c)=49}^{\tau^{\frac{1}{1-\theta}}.n_2(\alpha_u)=\tau^{\frac{1}{1-\theta}}.n_u} c_\alpha.(1-\theta).c_1^{\beta-1}.n^{-\beta}\mathrm{d}n \\
&= \int_{n_2(\alpha_c)}^{n_2(\alpha_u)} c_\alpha.(1-\theta).c_2^{\beta-1}.n^{-\beta}\mathrm{d}n = \int_{\tau^{-\frac{1}{1-\theta}}.n_1(\alpha_c)=49.\tau^{-\frac{1}{1-\theta}}}^{n_2(\alpha_u)=n_u} c_\alpha.(1-\theta).c_1^{\beta-1}.\tau^{-\frac{\beta-1}{1-\theta}}.n^{-\beta}\mathrm{d}n \\
&= \underbrace{c_\alpha.\frac{1-\theta}{\beta-1}.c_1^{\beta-1}}_{=C}.\left(49^{1-\beta} - \underbrace{\tau^{-\frac{\beta-1}{1-\theta}}}_{=T}.n_u^{1-\beta} \right)
\end{aligned}
$$

- This is a pdf, so this adds up to 1 (with support on $[\theta/(1-\theta); +\infty[$):

$$
\begin{aligned}
\delta &= 1 - c_\alpha.(1-\theta).c_1^{\beta-1}.\frac{49^{1-\beta} - \left(\frac{\theta}{1-\theta}\right)^{1-\beta}}{1-\beta} - c_\alpha.(1-\theta).c_2^{\beta-1}.\frac{-n_u^{1-\beta}}{1-\beta} \\
&= 1 - c_\alpha.\frac{1-\theta}{\beta-1}.c_1^{\beta-1}.\left(\left(\frac{\theta}{1-\theta}\right)^{1-\beta} - 49^{1-\beta} + \tau^{-\frac{\beta-1}{1-\theta}}.n_u^{1-\beta} \right)
\end{aligned}
$$

Taken together, these relations imply:

$$\delta = C.\left(49^{1-\beta} - T.n_u^{1-\beta} \right) = 1 - C.\left[\left(\frac{\theta}{1-\theta}\right)^{1-\beta} - \left(49^{1-\beta} - T.n_u^{1-\beta} \right) \right]$$

Therefore:

$$C = \left(\frac{1-\theta}{\theta}\right)^{1-\beta} \geqslant 1 \tag{17}$$

$$\chi^*(n) = \begin{cases} \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.(\beta-1).n^{-\beta} & \text{if} & \frac{\theta}{1-\theta} < n < 49 = n_1(\alpha_c) \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.\left(49^{1-\beta} - T.n_u^{1-\beta} \right) & \text{if} & n = 49 = n_1(\alpha_c) \\ 0 & \text{if} & 49 < n < n_u = n_2(\alpha_u) \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.(\beta-1).T.n^{-\beta} & \text{if} & n_2(\alpha_u) = n_u \leq n \end{cases}$$

A.2 Proof of Lemma 1

When employment is measured with error, we can only observe the following quantity:

$$n(\alpha, \varepsilon) = n^*(\alpha).e^\varepsilon$$

We can then write the conditional CDF of this variable denoted by x below:

$$\mathbb{P}(x < n|\varepsilon) = \begin{cases} 0 & \text{if} & n.e^{-\epsilon} < \frac{\theta}{1-\theta} \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.(\beta-1).\int_{\frac{\theta}{1-\theta}}^{n.e^{-\epsilon}} x^{-\beta}\mathrm{d}x & \text{if} & \frac{\theta}{1-\theta} \le n.e^{-\epsilon} < 49 \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.(\beta-1).\underbrace{\int_{\frac{\theta}{1-\theta}}^{49} x^{-\beta}\mathrm{d}x}_{\left(\frac{\theta}{1-\theta}\right)^{1-\beta}-49^{1-\beta}} + \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.\left(49^{1-\beta}-T.n_u^{1-\beta}\right) & \text{if} & 49 \le n.e^{-\epsilon} < n_u \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.\left(\left(\frac{\theta}{1-\theta}\right)^{1-\beta}-T.n_u^{1-\beta}\right) + \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.(\beta-1).T.\int_{n_u}^{n.e^{-\epsilon}} x^{-\beta}\mathrm{d}x & \text{if} & n_u \le n.e^{-\epsilon} \end{cases}$$

$$= \begin{cases} 0 & \text{if} & \ln(n) - \ln\left(\frac{\theta}{1-\theta}\right) < \varepsilon \\ 1 - \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.\left(n.e^{-\varepsilon}\right)^{1-\beta} & \text{if} & \ln(n) - \ln(49) < \varepsilon \le \ln(n) - \ln\left(\frac{\theta}{1-\theta}\right) \\ 1 - \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.T.n_u^{1-\beta} & \text{if} & \ln(n) - \ln(n_u) < \varepsilon \le \ln(n) - \ln(49) \\ 1 - \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.T.\left(n.e^{-\varepsilon}\right)^{1-\beta} & \text{if} & \varepsilon \le \ln(n) - \ln(n_u) \end{cases}$$

This time we can compute the unconditional probability as:

$$\begin{aligned} \forall n > 0, \quad \mathbb{P}(x < n) &= \int_{\mathbb{R}} \mathbb{P}(x < n|\varepsilon)\frac{1}{\sigma}.\varphi\left(\frac{\varepsilon}{\sigma}\right)\mathrm{d}\varepsilon \\ &= \int_{\ln(n)-\ln(49)}^{\ln(n)-\ln\left(\frac{\theta}{1-\theta}\right)} \left[1 - C.n^{1-\beta}.e^{\varepsilon.(\beta-1)}\right]\frac{1}{\sigma}.\varphi\left(\frac{\varepsilon}{\sigma}\right)\mathrm{d}\varepsilon + \int_{\ln(n)-\ln(n_u)}^{\ln(n)-\ln(49)} \left[1 - C.T.n_u^{1-\beta}\right]\frac{1}{\sigma}.\varphi\left(\frac{\varepsilon}{\sigma}\right)\mathrm{d}\varepsilon \\ &\quad + \int_{-\infty}^{\ln(n)-\ln(n_u)} \left[1 - C.T.n^{1-\beta}.e^{\varepsilon.(\beta-1)}\right]\frac{1}{\sigma}.\varphi\left(\frac{\varepsilon}{\sigma}\right)\mathrm{d}\varepsilon \\ &= \underbrace{\Phi\left(\frac{\ln(n) - \frac{\ln(C)}{\beta-1}}{\sigma}\right)}_{=A(n)} - \underbrace{C.T.n_u^{1-\beta}.\left[\Phi\left(\frac{\ln(n)-\ln(49)}{\sigma}\right) - \Phi\left(\frac{\ln(n)-\ln(n_u)}{\sigma}\right)\right]}_{=B(n)} \\ &\quad \underbrace{- C.n^{1-\beta}.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.\left[\Phi\left(\frac{\ln(n) - \frac{\ln(C)}{\beta-1}}{\sigma} - \sigma.(\beta-1)\right) - \Phi\left(\frac{\ln(n)-\ln(49)}{\sigma} - \sigma.(\beta-1)\right)\right]}_{=C(n)} \\ &\quad \underbrace{- C.T.n^{1-\beta}.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.\Phi\left(\frac{\ln(n)-\ln(n_u)}{\sigma} - \sigma.(\beta-1)\right)}_{=D(n)} \end{aligned}$$

In fact there is no additional constraint in the parameters, because we can show that this function is strictly increasing (straightforward from the way we constructed it), with limits 0 in 0 and 1 in $+\infty$:

$$A(n) \underset{n\to+\infty}{\longrightarrow} 1 \qquad\qquad A(n) \underset{n\to 0}{\longrightarrow} 0$$

$$B(n) \underset{n\to+\infty}{\longrightarrow} Cst \times (1-1) = 0 \qquad\qquad B(n) \underset{n\to 0}{\longrightarrow} Cst \times (0-0) = 0$$

$$C(n) \underset{n\to+\infty}{\longrightarrow} 0 \times (1-1) = 0 \qquad\qquad C(n) \underset{x\to 0}{\longrightarrow} +\infty \times (0-0) = 0 \ (*)$$

$$D(n) \underset{n\to+\infty}{\longrightarrow} 0 \times 1 = 0 \qquad\qquad D(n) \underset{n\to 0}{\longrightarrow} +\infty \times 0 = 0 \ (*)$$

To solve the two problematic cases, marked with $(*)$, let us consider $F(n)$ defined as:

$$
\begin{aligned}
F(n) \quad &= \quad n^{1-\beta}.\Phi\left(\frac{\ln(n)}{\sigma}+F\right) = \frac{\Phi\left(\frac{\ln(n)}{\sigma}+F\right)}{n^{\beta-1}} \\[2mm]
\text{(L'Hôpital's rule)} \quad &\underset{n\to 0}{\sim} \quad \frac{\frac{1}{n.\sigma}\varphi\left(\frac{\ln(n)}{\sigma}+F\right)}{(\beta-1).n^{\beta-2}} \\[2mm]
&\underset{n\to 0}{\sim} \quad \frac{1}{\sigma.\sqrt{2\pi}.(\beta-1)}.e^{-\frac{F^2}{2}}.\underbrace{n^{1-\beta}.e^{-\frac{1}{2}.\frac{(\ln n)^2}{\sigma^2}}.e^{-\frac{1}{2}.F.\frac{\ln(n)}{\sigma}}}_{e^{\ln(n).\left(1-\beta-\frac{1}{2}.\frac{F}{\sigma}-\frac{\ln(n)}{2.\sigma^2}\right)}} \\[2mm]
&\underset{n\to 0}{\longrightarrow} \quad 0
\end{aligned}
$$

Last, the density that corresponds to the CDF is given by:

$$
\begin{aligned}
\chi(n) \quad = \quad & \frac{1}{\sigma.n}.\varphi\left(\frac{\ln(n)-\frac{\ln(C)}{\beta-1}}{\sigma}\right) - \frac{1}{\sigma.n}.C.T.n_u^{1-\beta}\left[\varphi\left(\frac{\ln(n)-\ln(49)}{\sigma}\right) - \varphi\left(\frac{\ln(n)-\ln(n_u)}{\sigma}\right)\right] \\[2mm]
& -C.n^{-\beta}.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.(1-\beta).\left(\Phi\left(\frac{\ln(n)-\frac{\ln(C)}{\beta-1}}{\sigma}-\sigma.(\beta-1)\right) - \Phi\left(\frac{\ln(n)-\ln(49)}{\sigma}-\sigma.(\beta-1)\right)\right) \\[2mm]
& -C.n^{-\beta}.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.\frac{1}{\sigma}.\left(\varphi\left(\frac{\ln(n)-\frac{\ln(C)}{\beta-1}}{\sigma}-\sigma.(\beta-1)\right) - \varphi\left(\frac{\ln(n)-\ln(49)}{\sigma}-\sigma.(\beta-1)\right)\right) \\[2mm]
& -C.n^{-\beta}.T.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.\left[(1-\beta).\Phi\left(\frac{\ln(n)-\ln(n_u)}{\sigma}-\sigma.(\beta-1)\right) + \frac{1}{\sigma}.\varphi\left(\frac{\ln(n)-\ln(n_u)}{\sigma}-\sigma.(\beta-1)\right)\right] \quad (18)
\end{aligned}
$$

We use standard ML techniques to estimate the parameters in equation (18).

Note that we obtain an estimate of C from this procedure from which we can, in principle recover an estimate of the coefficient θ from equation (17). This is unlikely to be a powerful way of identifying the scale parameter, however. We found empirically that the likelihood was very flat when trying to estimate θ in this way, suggesting it was not well identified. Instead, as discussed in the main text we generate estimates of θ from three alternative routes (i) calibration, (ii) estimates from the production function and (iii) using the TFP-size relationship. We use this when estimating equation (18) together with equation (17). For example, a calibrated $\theta = 0.5$ implies that C = 1. When we use methods (ii) and (iii) and estimate θ using the productivity distribution, we take into account the variance around the estimation of θ in calculating the

correct variance-covariance matrix.

B   Least squares estimation of broken Power Law

We discuss here an alternative to our MLE approach. Taking as our starting point the power law for firm sizes, we can proceed as follows:

$$\ln \chi(n) = \ln k - \beta \ln n + \delta(D_{n>n_u}) + \sum_{i=n_c}^{n_u} d_i \tag{19}$$

where $D_{n>n_u}$ is a dummy variable that turns on to 1 for firms above the threshold n_u and is zero otherwise), but we have added d_i dummies that pick up the average number of firms in the distorted size categories, i.e. between the upper (n_u) and lower (n_c) employment thresholds. Equation (19) is estimated subject to the constraint $\sum_{i=n_c}^{n_u} d_i = 0$.

Following Axtell (2001), we can estimate equation (19) through OLS[21], conditional on the 'structural breaks' at n_c and n_u.To find these structural break points, we follow Bai (1997) and Bai and Perron (1998) in their study of structural breaks in time series models. In our context, their result implies that for each partition $\{\{1, ... n_c\}, \{n_c ... n_u\}, \{n_u, ...\}\}$, one obtains the OLS estimators of $\{k, \beta, \delta_1, \delta_2\}$ subject to constraint $\sum_{i=n_c}^{n_u} d_i = 0$.[22] Letting the sum of squared errors generated by each of these partitions be $SSE(n_u, n_c)$, our estimates of the 'break points', n_u and n_c are:

$$(\widehat{n}_u, \widehat{n}_c) = \arg \min_{n_u, n_c} SSE(n_u, n_c) \tag{20}$$

Bai and Perron (1998) show that, for a wide range of error specifications (including heteroskedastic like in our case) the break points are consistently estimated, and converge at rate $\acute{N}$,where $\acute{N}$ is the maximum firm size as long as $n_u - n_c > \varepsilon \acute{N}$, and $n_c < n_u$, (the break points are asymptotically distinct) which is true in our framework since we know $n_c < N < n_u$.

Armed with these parameter estimates we can the proceed to estimate τ using the results above. One intuitive way of seeing the procedure is as follows. Fix the lower employment threshold (say 43) and estimate the power law (conservatively) only on the part of the employment distribution below this and on the upper

---

[21] See Gabaix and Ibragimo (2008) for improvements in the OLS procedure using ranks, which is preferred for small samples and for the upper part of the distribution (not the middle, our focus).

[22] Perron and Qu (2006) show that the framework can accomodate linear restrictions on the parameter; and that the consistency and rate of convergence results hold and the limiting distribution is unaffected. However, our constraint is non-linear and no results exist on whether the results hold.

part of the size distribution that is undistorted (say under 43 and over 100).[23] This procedure generates a mass of firms (entrepreneurs) displaced to the "bulge" in the distribution between n_c and N (i.e. 43 and 50) as shown in Figure 9. These firms are drawn from between N and n_u, and since we know the counterfactual slope of the power paw over this region, we can reallocate these firms so as to minimize the deviation from this counterfactual power law. n_u is estimated as the maximum employment bin which is attained in this procedure.

Rather than fixing n_c, the Bai and Perron (1998) procedure estimates this efficiently by minimizing a sum of squares criterion along with the other parameters in the model as in equation (20).

This procedure gives us all the parameters necessary to estimate the implicit cost of the regulation which we calculate is equivalent to a labor tax of around 26% ($\tau = 1.26$).

C   Using information from the productivity distribution

C.1   Incorporating TFP into the estimation method

We can do much better if we have direct information on the TFP Distribution. Estimation is a challenge here (see next sub-section), but let us initially assume we have reliable on TFP. First, recall from equation (12) the relationship between firm size and TFP:

$$
n^*(\alpha) = \begin{cases} \left(\frac{\alpha\theta}{w}\right)^{1/(1-\theta)} & \text{if} & \alpha < \alpha_c \\ N - 1 & \text{if} & \alpha_c \leq \alpha < \alpha_u \\ \left(\frac{\alpha\theta}{w.\tau}\right)^{1/(1-\theta)} & \text{if} & \alpha_u \leq \alpha \end{cases}
$$

The empirical model adds a stochastic error term to this to obtain:

$$
n^*(\alpha) = \begin{cases} \left(\frac{\alpha\theta}{w}\right)^{1/(1-\theta)} e^\varepsilon & \text{if} & \alpha < \alpha_c \\ (N - 1)e^\varepsilon & \text{if} & \alpha_c \leq \alpha < \alpha_u \\ \left(\frac{\alpha\theta}{w.\tau}\right)^{1/(1-\theta)} e^\varepsilon & \text{if} & \alpha_u \leq \alpha \end{cases}
$$

Or

$$
\begin{aligned}
\ln n_1 &= \frac{1}{1-\theta}\ln\alpha + \frac{1}{1-\theta}\ln\left(\frac{\theta}{w}\right) + \varepsilon \\
\ln n_2 &= \ln(N-1) + \varepsilon \\
\ln n_3 &= \frac{1}{1-\theta}\ln\alpha + \frac{1}{1-\theta}\ln\tau + \frac{1}{1-\theta}\ln\left(\frac{\theta}{w}\right) + \varepsilon
\end{aligned}
$$

---

[23] We could in principle use all firms as small as one employee and up the largest firm in the economy. In practice the Power Law tends to be violated at these extremes of the distribution in all countries (e.g. Axtell (2001), so we follow that standard approach of trimming the upper and lower tails. We show that nothing is sensitive to these exact maximum and minimum employment thresholds as can be seen from the various figures.

Combining these together:

$$\ln n = \ln n_1 I_{\{\alpha < \alpha_c\}} + \ln n_2 I_{\{\alpha_c \leq \alpha < \alpha_u\}} + \ln n_3 I_{\{\alpha_u \leq \alpha\}} \qquad (21)$$

where I is an indicator function for a particular regime. If we have a measure of firm-specific α, TFP, then we can estimate equation (21). This is one way to obtain an estimate of θ that is needed to calculate the implicit tax of regulation. Alternatively, we can estimate θ directly as the returns to scale parameter directly from a production function. We show the results from both methods in Table 2.

C.2   Estimation of TFP

There is no one settled way of best estimating TFP on firm level data and there are many approaches suggested in the literature. Fortunately, at least at the micro-level, different methods tend to produce results where the correlation of TFP estimated by different methods is usually high (see Syverson, 2010).

In the baseline result we follow the method of Levinsohn and Petrin (2003) who propose extending the Olley and Pakes (1996) control function method to allow for endogeneity and selection. Olley and Pakes proposed inverting the investment rule to control for the unobserved productivity shock (observed to firm but unobserved to econometrician) that affects the firm's decision over hiring (and whether to stay in business). Because of the problem of zero investment regimes (common especially among smaller firms that we use in our dataset) Levinsohn and Petrin (2003) recommended using materials as an alternative proxy variable that (almost) always takes an observed positive value.

We use this estimator to estimate firm-level production functions on French panel data 2002-2007 (using the unbalanced panel) by each of the four-digit manufacturing industries in our dataset. We also did the same for the retail sector and the business services sector. The production functions take the form (in each industry):

$$\ln y_{it} = \beta_n \ln n_{it} + \beta_k \ln k_{it} + \beta_m \ln m_{it} + \omega_{it} + \tau_t + \eta_{it} \qquad (22)$$

where y = output, n = labour, k = capital, m = materials, ω is the unobserved productivity shock, τ_t is a set of time dummies and η is the idiosyncratic error of firm i in year t. From estimating the parameters of the production function we can then recover our estimate of the persistent component of TFP. Note that TFP is always normalized within industry and year.

There are of course many problems with these estimation techniques. For example, Ackerberg et al (2006) focus on the problem of exact multicollinearity of the variable factors conditional on the quasi-fixed factors

given the assumption that input prices are assumed to be common across firms. Ackerberg et al (2007) suggest various solutions to this issue.

We consider alternative ways to estimate TFP including the more standard Solow approach. Here we assume that we can estimate the factor coefficients in equation (22) by using the observed factor shares in revenues. We do this assuming constant returns to scale, so $\beta_n = \frac{wn}{py}$; $\beta_m = \frac{cm}{py}$ and $\beta_k = 1 - \frac{wn}{py} - \frac{cm}{py}$ where c = the price of materials. We used the four digit industry factor shares averaged over our sample period for the baseline but also experimented with some firm-specific (time invariant) factor shares. As usual these alternative measures led to similar results.

A problem with both of these methods is that we do not observe firm-specific prices so the estimates of TFP as we only control for four digit industry prices. Consequently, the results we obtain could be regarded as only revenue-based TFPR instead of quantity-based TFPQ (see Hsieh and Klenow, 2009). TFPQ is closer to what we want to theoretically obtain as our estimate of α. In practice, there is a high correlation between these two measures as shown by Foster et al (2008) who have actual data on plant level input and output prices. So it is unclear whether this would make too much of a practical difference to our results.

An alternative approach would be to follow de Loecker (2010) and put more structure on the product market. For example, assuming that the product market is monopolistically competitive enables the econometrician in principle to estimate the elasticity of demand and correct for the mark-up implicit in TFPR to obtain TFPQ. We will pursue this in future work.

D   More Details of some Size-Relation Labor Market Regulations in France

The main bite of labor (and some accounting) regulations comes when the firm reaches 50 employees. But there are also some other size-related thresholds at other levels. The main other ones comes at 10-11 employees. For this reason we generally trim the analysis below 12 employees to mitigate nay bias induced in estimation from these other thresholds. For more details on French regulation see inter alia Abowd and Kramarz (2003) and Kramarz and Michaud (2010).

D.1   Labor Regulations

From fifty employees:

- Obligation to use a complex redundancy plan with oversight, approval and monitoring from Ministry of Labor in case of a collective redundancy for 9 or more employees (threshold based on total employment

at time of redundancy). See text.

- Appointing a shop steward if demanded by workers (threshold exceeded for 12 consecutive months during the last three years);

- Obligation to establish a committee on health, safety and working conditions (HSC) and train its members (threshold exceeded for 12 months during the last three years)

- Obligation to establish a profit sharing (threshold exceeded for six months during the accounting year within one year after the year end to reach an agreement);

- Obligation to establish a staff committee with business meeting at least every two months (plant level: threshold exceeded for 12 months during the last three years)

From twenty-five employees:

- Duty to supply a refectory if requested by all employees;

- Electoral colleges for electing representatives. Increased number of delegates from 26 employees.

From twenty employees:

- Contribution to the National Fund for Housing Assistance;

- Increase the contribution rate for continuing vocational training of 1.05% to 1.60%

- Compensatory rest of 50% for mandatory overtime beyond 41 hours per week

From eleven employees:

- Allowance of at least six months salary if terminated without cause or serious;

- Obligation to conduct the election of staff representatives(threshold exceeded for 12 consecutive months over the last three years).

- **From ten employees:**

- Monthly payment of social security contributions, instead of a quarterly payment (according to the actual last day of previous quarter);

- Obligation for payment of transport subsidies (Article L. 2333-64 of the General Code local authorities);

- Increase the contribution rate for continuing vocational training of 0.55% to 1.05% (threshold exceeded on average 12 months).

D.2   Accounting rules

The additional requirements depending on the number of employees of enterprises, but also limits on turnover and total assets are as follows:

From fifty employees:

- loss of the possibility of a simplified presentation of Schedule 2 to the accounts (also if the balance sheet total exceeds 2 million or if the CA exceeds 4 million);

- requirement for LLCs, the CNS, limited partnerships and legal persons of private law to designate an auditor (also if the balance sheet total exceeds 1.55 million euros or if the CA is more than 3.1 million euros, applicable rules of the current year).

From ten employees:

- loss of the possibility of a simplified balance sheet and income statement (also if the CA exceeds 534 000 euro or if the balance sheet total exceeds 267 000 euro, applicable rule in case of exceeding the threshold for two consecutive years).

Table 1: Parameter estimates (calibrating returns to scale, θ)

Method	(1) Unconstrained (θ calibrated from Basu and Fernald, 1997)	(2) Unconstrained (θ calibrated from Atkeson and Kehoe, 2005)	(3) Unconstrained (θ calibrated at 0.5)	(4) Unconstrained (θ calibrated at 0.9)
θ , scale parameter	0.8	0.85	0.5	0.9
β , power law	1.822	1.822	1.822	1.829
	(0.059)	(0.059)	(0.059)	(0.057)
$T = \tau^{\frac{1-\beta}{1-\theta}}$	0.948	0.948	0.948	0.965
	(0.018)	(0.018)	(0.018)	(0.015)
n_u, upper employment threshold	57.898	57.898	57.898	52.562
	(0.024)	(0.024)	(0.024)	(0.002)
σ, variance of measurement error	0.104	0.104	0.104	0.036
	(0.025)	(0.025)	(0.025)	(0.008)
τ , implicit tax, variable cost	1.013	1.010	1.033	1.004
	(0.005)	(0.004)	(0.013)	(0.002)
k/w, implicit tax, fixed cost	-0.496	-0.372	-1.243	-0.201
	(0.257)	(0.192)	(0.653)	(0.099)
Mean (Median) # of employees	55.7 (23)	55.7 (23)	55.7 (23)	55.7 (23)
Observations	238,701	238,701	238,701	238,701
Firms	57,008	57,008	57,008	57,008
Ln Likelihood	-1065936	-1065936	-1065936	-1066165

Notes: Parameters estimated by ML with standard errors below in parentheses (clustered at the four digit level). Estimation is on unbalanced panel 2002-2007 of population of French manufacturing firms with 10 to 1,000 employees. These estimates of the implicit tax are based on different estimates of θ ; the methods are indicated in the different columns.

Table 2: Parameter estimates (exploiting information from the Production Function to estimate returns to scale, θ)

| Method	(1) Baseline (column (1) of Table 1)	(2) Using Production Function estimates	(3) TFP/Size relationship	(4) High-Tech Sectors (Using Production Function estimates)	(5) Low-Tech Sectors (Using Production Function estimates)
θ, scale parameter	0.8	0.855	0.799	0.882	0.848
		(0.013)	(0.025)	(0.012)	(0.016)
β, power law	1.822	1.822	1.822	1.625	1.864
	(0.059)	(0.052)	(0.055)	(0.063)	(0.063)
$T = \tau^{\frac{1-\beta}{1-\theta}}$	0.948	0.948	0.948	0.997	0.929
	(0.018)	(0.020)	(0.023)	(0.012)	(0.032)
n_u, upper employment threshold	57.898	57.899	57.898	50.000	58.328
	(0.024)	(1.133)	(1.342)	(2.474)	(1.603)
σ, variance of measurement error	0.104	0.104	0.104	0.000	0.114
	(0.025)	(0.020)	(0.024)	(0.029)	(0.034)
τ, implicit tax, variable cost	1.013	1.010	1.013	1.001	1.013
	(0.005)	(0.004)	(0.005)	(0.003)	(0.007)
k/w, implicit tax, fixed cost	-0.496	-0.359	-0.498	-0.026	-0.514
	(0.257)	(0.177)	(0.219)	(0.129)	(0.304)
Mean (Median) # of employees	55.7 (23)	55.7 (23)	55.7 (23)	78.3 (29)	51.3 (23)
Observations	238,701	238,701	238,701	38,713	199,988
Firms	57,008	57,008	57,008	9,099	48,139
Ln Likelihood	-1065936.	-	-	-	-

Notes: Parameters estimated by ML with standard errors below in parentheses (clustered at the four digit level). Estimation is on unbalanced panel 2002-2007 of population of French manufacturing firms with 10 to 1,000 employees. These estimates of the implicit tax are based on different estimates of θ; the methods are indicated in the different columns. Standard errors are calculated using bootstrap in columns (2) to (5). "Using TFP-Size relationship" calculates $\theta = 1 - \frac{\partial \ln \alpha}{\partial \ln n}$ where $\frac{\partial \ln \alpha}{\partial \ln n}$ $\partial \ln n / \partial \ln \alpha$ is calculated from the coefficient of a regression of ln(TFP) on ln(employment) on firms with 10 to 45 workers. "Using the production function" calculates θ as the sum of the coefficients on the factor inputs (see Table A1). "High tech" sectors are based on R&D intensity as defined by the OECD.

Table 3: Variation in estimates across different sectors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Method	Baseline		Calibrated		Calibrated		Calibrated		Calibrated	
	Manufacturing industries		Transport		Construction		Trade		Business services	
θ	0.8	0.85	0.8	0.85	0.8	0.85	0.8	0.85	0.8	0.85
β, power law	1.822	1.822	1.878	1.878	2.372	2.372	2.128	2.128	2.001	2.001
	(0.059)	(0.059)	(0.098)	(0.099)	(0.147)	(0.147)	(0.100)	(0.100)	(0.087)	(0.087)
$T = \tau^{\frac{1-\beta}{1-\theta}}$	0.948	0.948	0.898	0.989	0.871	0.872	0.885	0.885	0.984	0.984
	(0.018)	(0.018)	(0.029)	(0.029)	(0.016)	(0.016)	(0.022)	(0.021)	(0.020)	(0.020)
n_u, upper	57.898	57.898	55.312	55.318	57.874	57.873	57.151	57.151	58.254	58.256
employment	(0.024)	(0.024)	(0.016)	(0.016)	(0.013)	(0.013)	(0.015)	(0.015)	(0.018)	(0.018)
threshold										
σ,measurement	0.104	0.104	0.060	0.060	0.089	0.089	0.084	0.084	0.106	0.106
error	(0.025)	(0.025)	(0.023)	(0.023)	(0.037)	(0.037)	(0.050)	(0.050)	(0.024)	(0.024)
τ, implicit	1.013	1.010	1.025	1.019	1.020	1.015	1.022	1.016	1.003	1.003
variable tax	(0.005)	(0.004)	(0.010)	(0.007)	(0.004)	(0.003)	(0.006)	(0.004)	(0.004)	(0.024)
k/w, implicit	-0.496	-0.372	-1.137	-0.850	-0.842	-0.631	-0.951	-0.711	0.003	0.002
tax, fixed cost	(0.257)	(0.192)	(0.492)	(0.367)	(0.179)	(0.133)	(0.288)	(0.215)	(0.188)	(0.150)
Mean	55.7 (23)	55.7 (23)	48.2	48.2	29.3 (17)	29.3	36.8 (19)	36.8 (19)	45.3 (20)	45.3 (20)
(Median) # of			(23)	(23)		(17)				
employees										
Observations	238,701	238,701	70,479	70,479	159,440	159,440	255,812	255,812	205,835	205,835
Firms	57,008	57,008	14,487	14,487	41,768	41,768	66,848	66,848	61,906	61,906

Notes: Parameters estimated by ML with standard errors below in parentheses (clustered at the four digit level). Estimation is on unbalanced panel 2002-2007 of firms with 10 to 1,000 employees. These estimates of the implicit tax are based on different estimates of θ; the methods are indicated in the different columns.

Table 4. Consequences of the 50+ Regulation

Variable	Impact (world with tax-world without)
Size of smallest firm that is unaffected by threshold	59
Change in labor costs (wages reduction), Small firms (below 49)	-0.69%
Change in labor costs (tax increase minus wage reduction), large firms (over 50)	0.61%
Excess entry by small firms (percent increase in number of firms)	2.87%
Increase in size of small firms	3.3%
Increase in size of large firms	-3.0%
Annual Welfare Loss	-0.57%
Winners and losers (losers in italics)	
Change in wages for workers who remain workers	*-0.69%*
Average gain by entering entrepreneurs of small firms	1.06%
Average profit gain by small unconstrained firms	2.82%
Average profit gain by firms constrained at 49	2.15%
Change in profits of Large Firms (over 58)	*- 2.41%*

Notes: Using the estimates of the Baseline Specification (see Table 2), this Table compares the world with and without tax. See Appendix X for details of the calculation of the counterfactual.

Figure 1: The Firm size distribution in the US and France
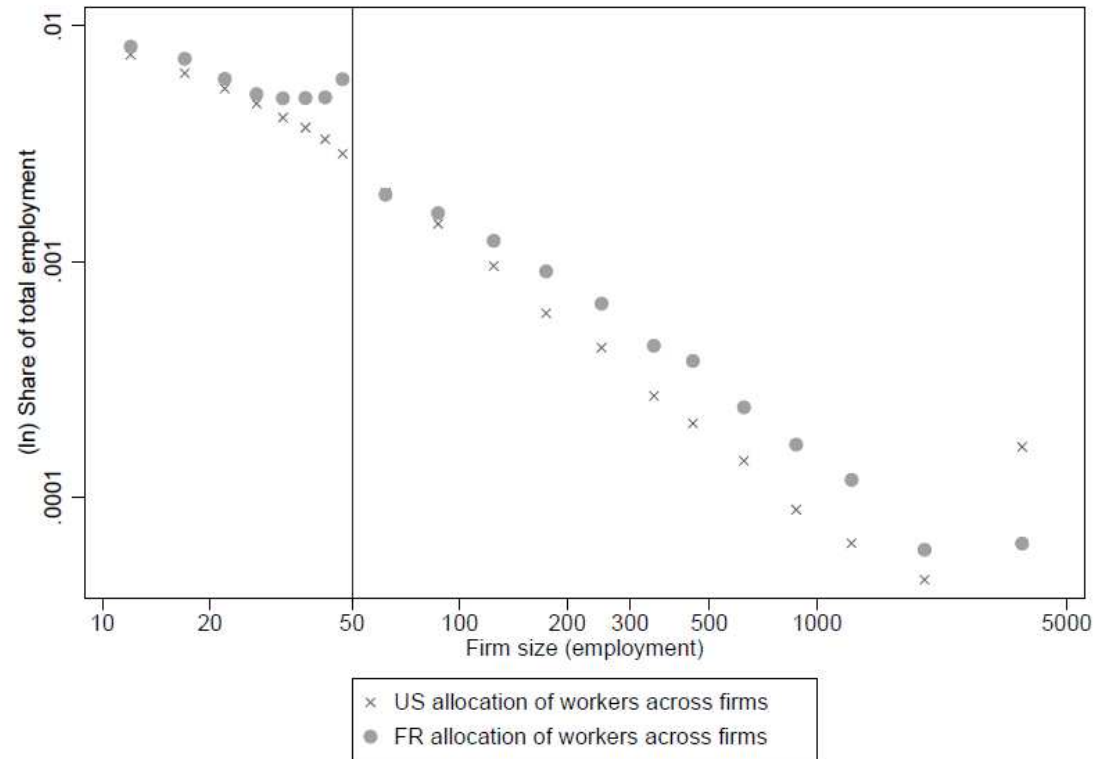
Source: FICUS for France and Census for the US. Population databases of all firms.

Notes: This is the distribution of firms (not plants). Authors' calculations

Figure 2: Number of Firms by employment size in France
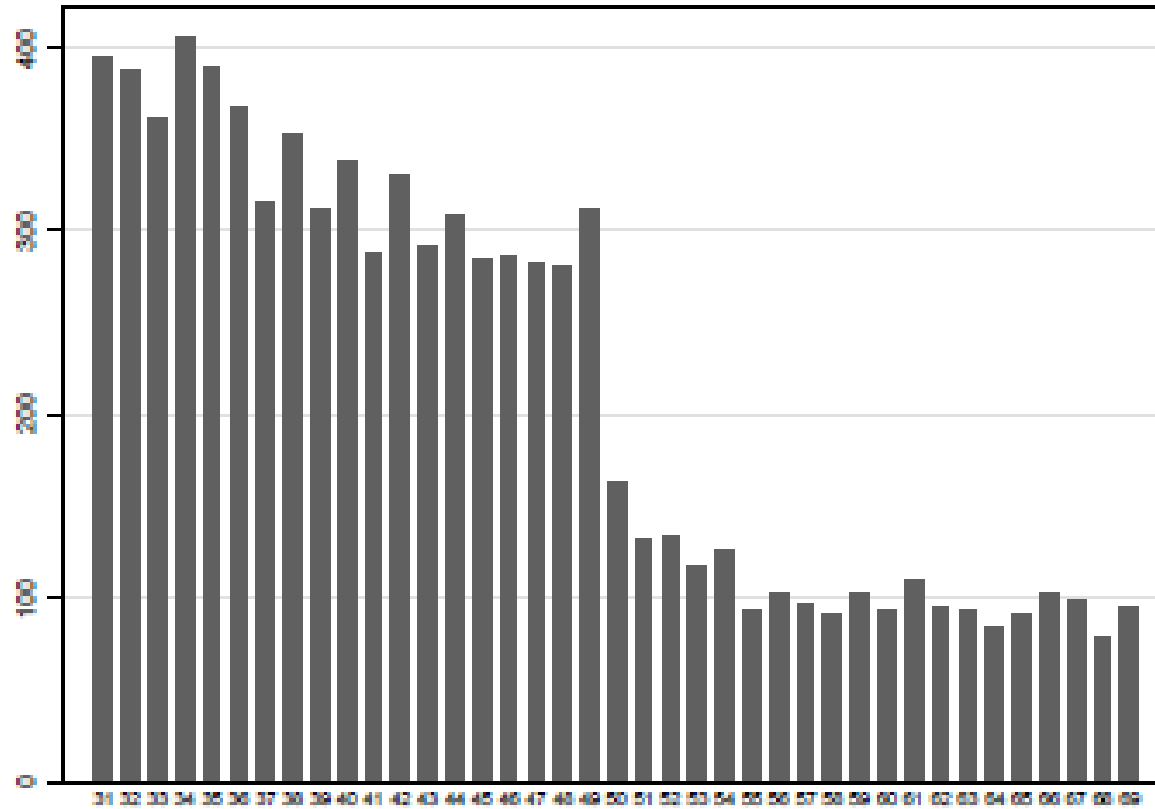
Source: FICUS

Notes: This is the population of manufacturing firms in France with between 31 and 69 employees. This plots the number of firms in each exact size category (i.e. raw data, no binning). There is a clear drop when the employment regulation begins for firms with 50 or more employees.

Figure 3: Definitions of regimes in terms of managerial ability
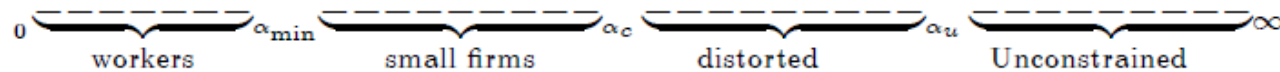
Notes: This figure shows the definitions of different regimes in our model. Individuals with managerial ability below α_min choose to be workers rather than managers. Individuals with ability between α_{min} and α_c are "small firms" who (conditional on the equilibrium wage, which is lower under regulation) do not change their optimal size. Between α_c and α_u are individuals who are affected by the regulatory constraint and choose their firm size to be smaller than they otherwise would have been - we call these individuals/firms who are in a "distorted" regime. Individuals with ability above α_u are choosing to pay the tax rather than keep themselves small.

Figure 4: Theoretical Firm size distribution with regulatory constraint

Firm Size Distribution
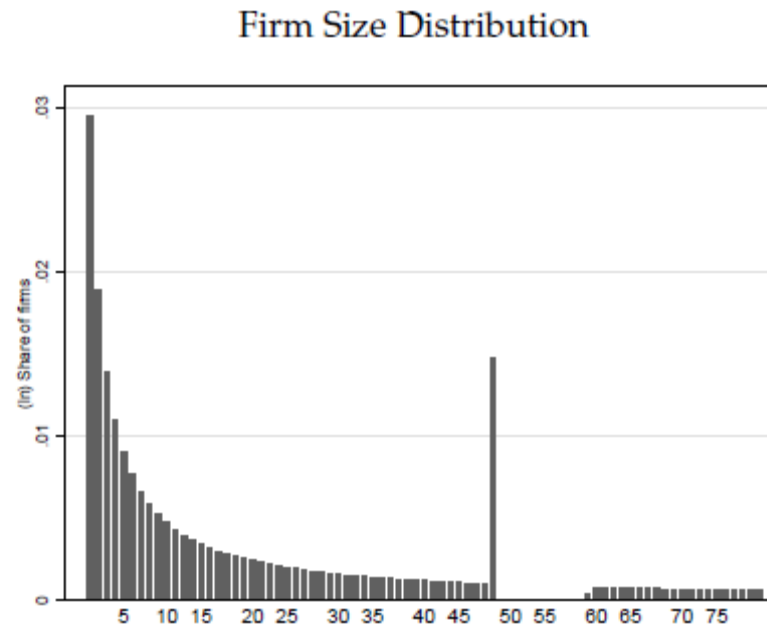
Notes: This figure shows the theoretical firm size distribution with exponentially increasing bins. The tallest bar represents the point at which the size constraint bins. Parameters: $\beta_\alpha = 1.6$, $\tau = 1.01$, $n_u = 60$, $\theta = 0.9$, $\beta = 1.06$.

Figure 5: Theoretical Relationship between TFP (managerial talent) and firm size

TFP/Size Relation
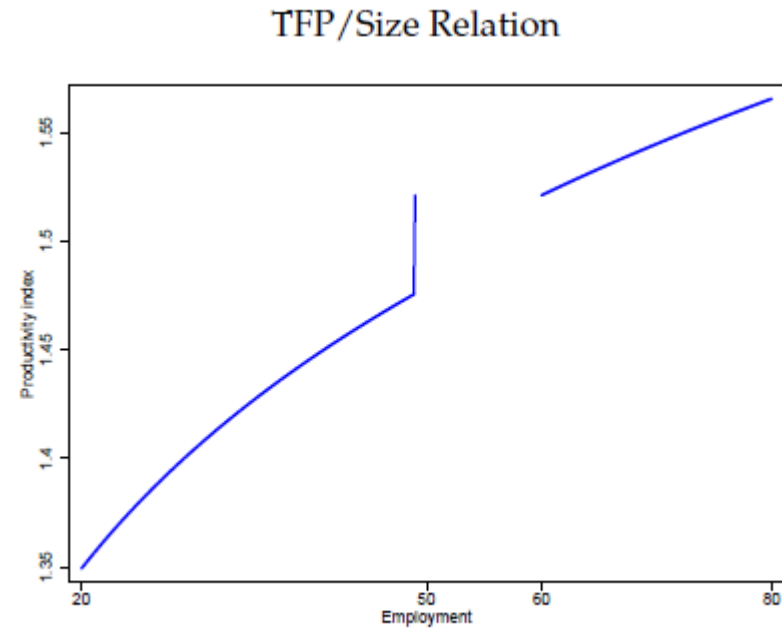
Notes: This figure shows the theoretical relationship between TFP and firm size. There is a mass of firms at employment size=50 where the regulatory constraint binds. Parameters: $\beta_\alpha = 1.6$, $\tau = 1.01$, $n_u = 60$, $\theta = 0.9$, $\beta = 1.06$.

Figure 6: The Theoretical Firm Size Distribution when employment is measured with error
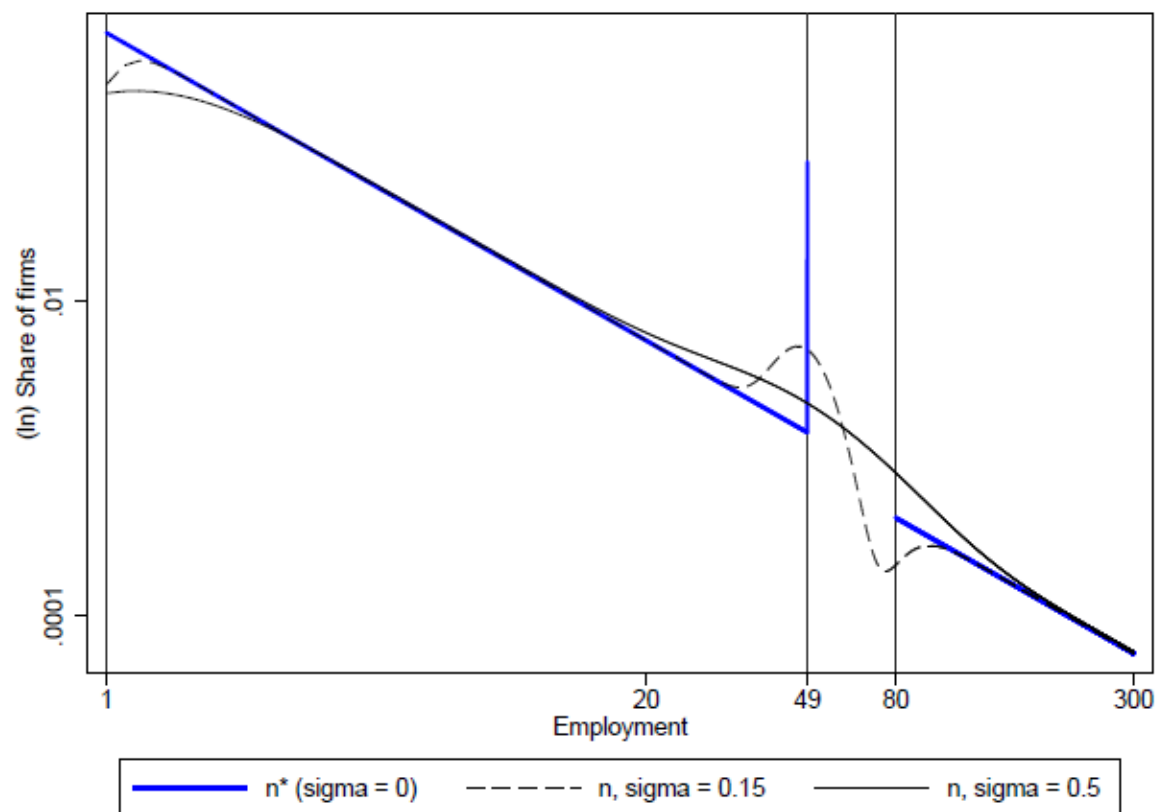
Note: The solid (blue) line shows the theoretical firm size distribution (broken power law), n*. The dashed line shows the new firm size distribution when we extend the model, to allow employment size to be measured with error with $\sigma = 0.15$. The solid dark line increases the measurement error to $\sigma = 0.5$

Figure 7: The effect on the measured firm size distribution using Alternative Datasets and definitions of employment

Bar plot   *Log-log plot*

FICUS (2002): Fiscal source
Arithmetic average of quarterly head counts.
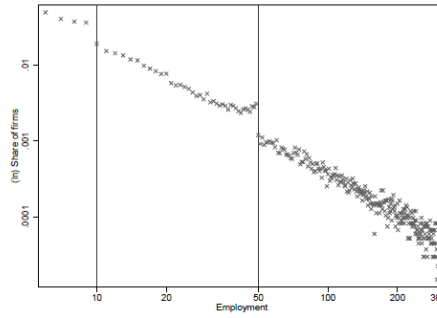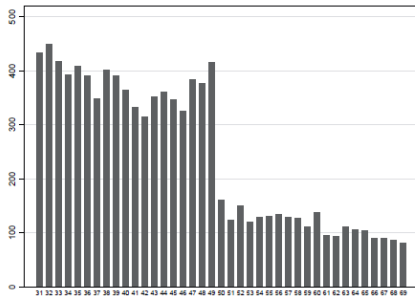
Bar plot   *Log-log plot*

DADS (2002): Payroll tax reporting to social administration
"Declared" workers on Dec. 31st: cross-sectional "fractional" count, i.e. taking account of part-timers.
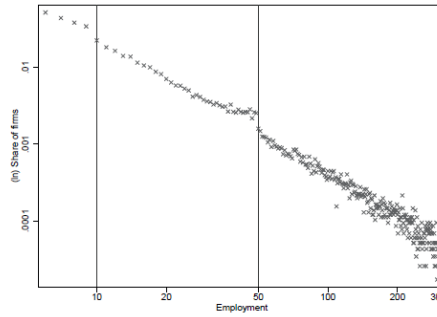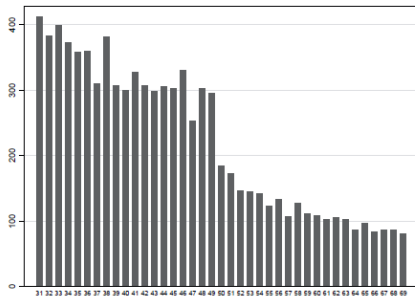
DADS (2002): Payroll tax reporting to social administration
"Full-time equivalent", computed by the French statistical institute as $\min\left\{\dfrac{\text{hours}}{P^{75}\left(H_{\text{Ind., Size}}\right)}, 1\right\}$
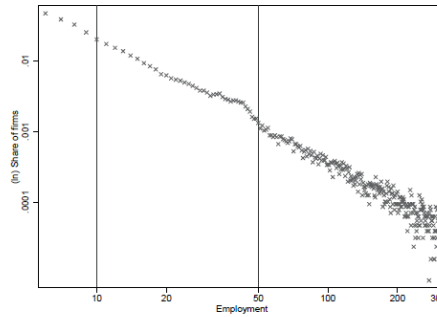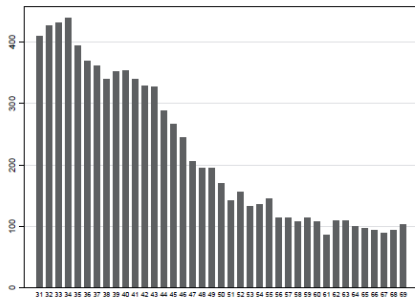
Figure 8: Share of Firms by employment size
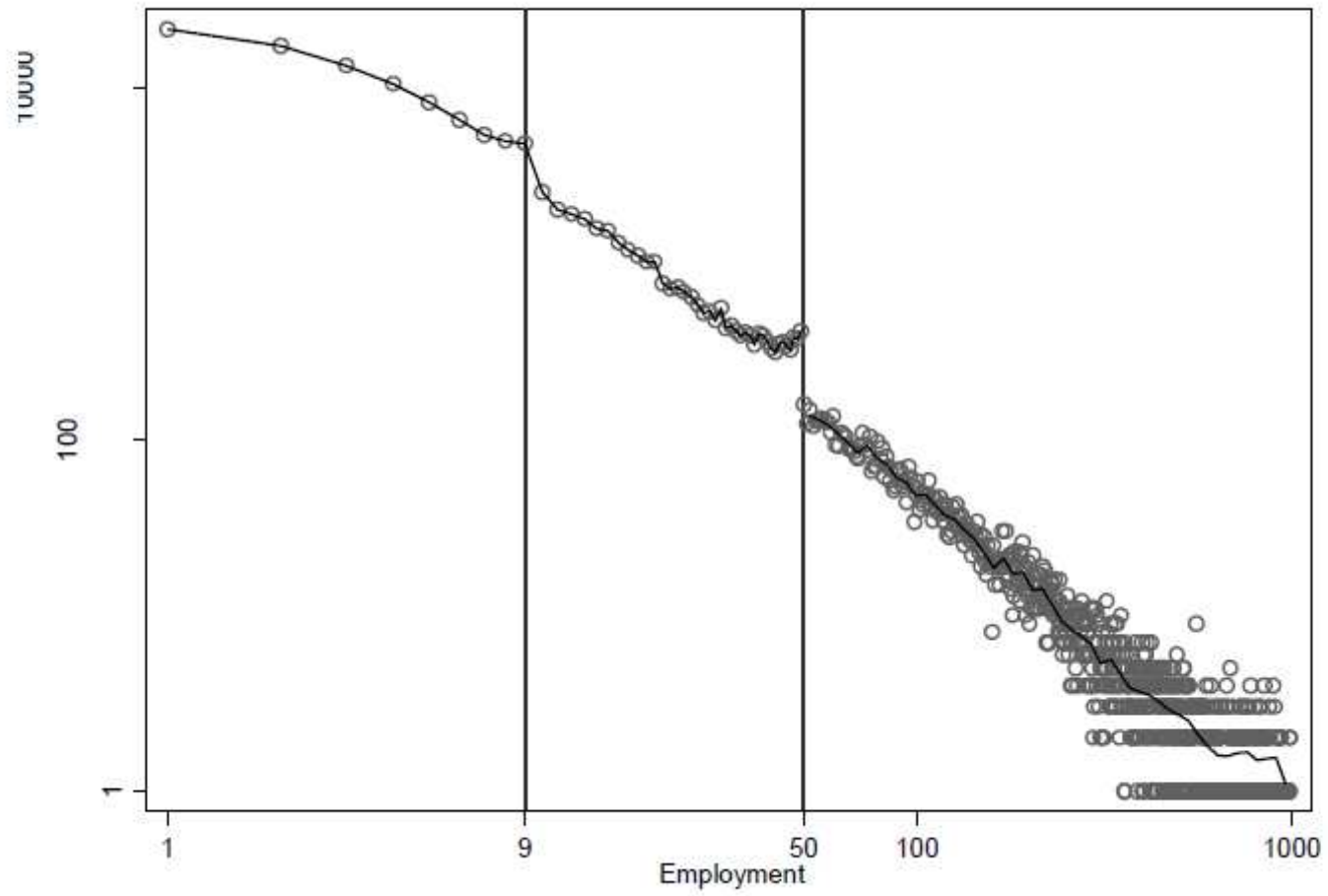
Figure 9: TFP Distribution by Firm Size
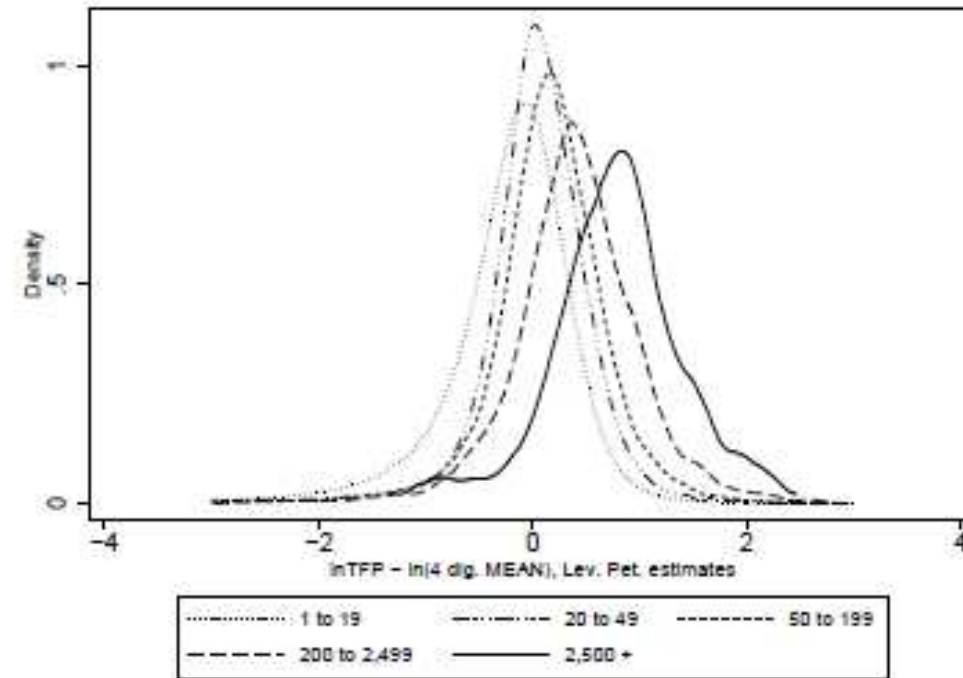
Notes: This figure plots the (kernel density smoothed) distribution of TFP (estimated by the Levinsohn Petrin method) across four size classes. TFP is relative to the four digit industry by year average.

Figure 10: TFP Distribution around the regulatory threshold of 50 employees

Panel A: Short Employment span
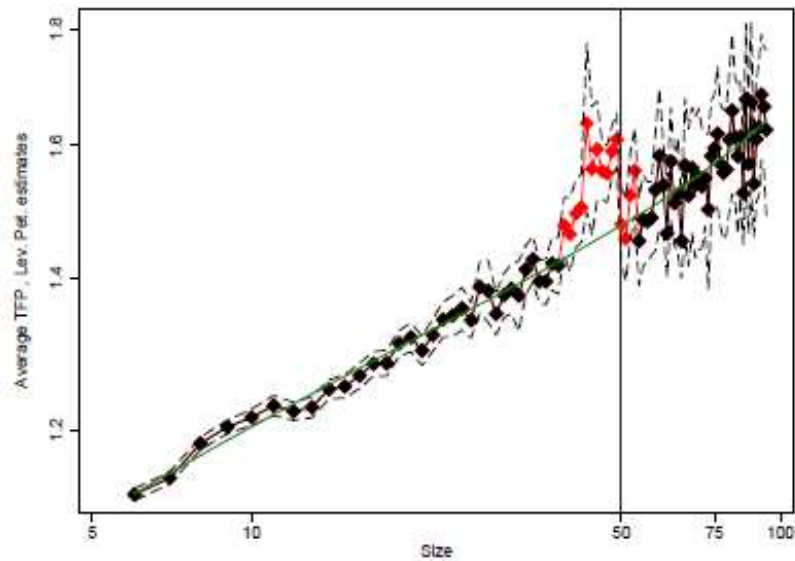
Panel B: Longer Employment span
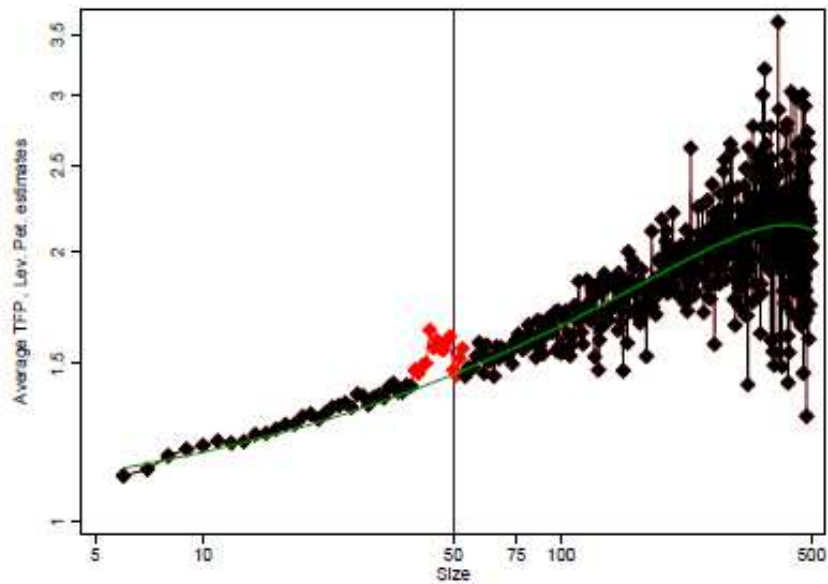
Notes: This figures plots the mean level of TFP by firm employment size using an upper support of 100 (Panel A) or 500 (Panel B). A fourth order polynomial is displayed in both panels using only data from the "undistorted" points (shown in red).

Figure 11: Firm Size Distribution and Broken Power Law: Data and Fit of Model
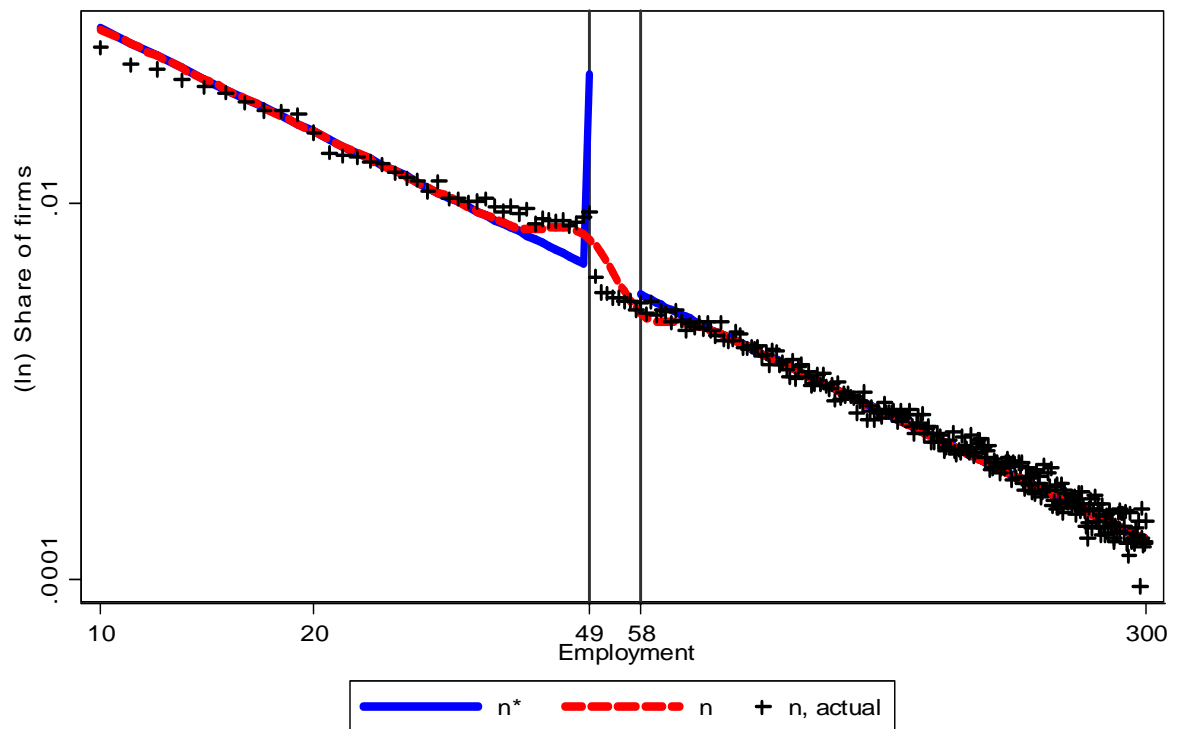
Notes: This shows the difference between the fit of the model (dashed red line) which allows for measurement error with the actual data. We also include the "pure" theoretical predictions (in blue).

.**Figure 12: Counterfactual size and output (in red)**
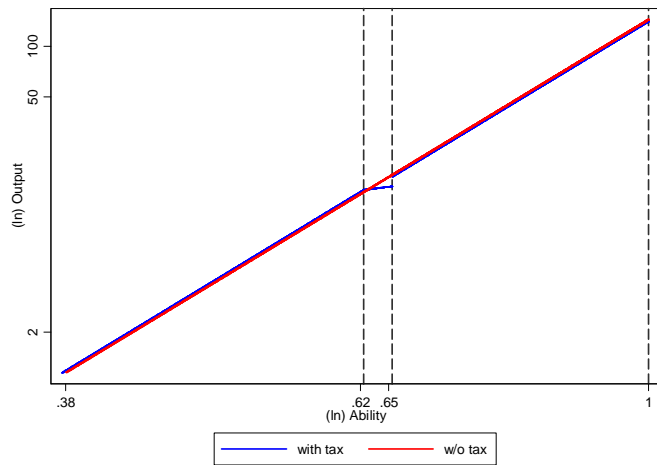
Panel A: Firm size distribution
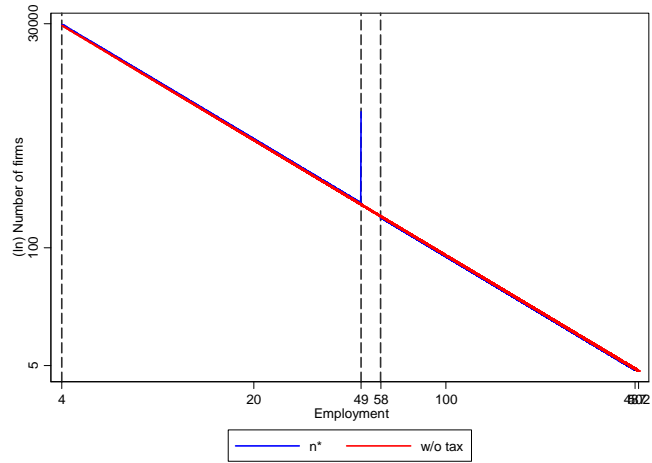
Panel A ZOOM: Firm size distribution, size around regulation
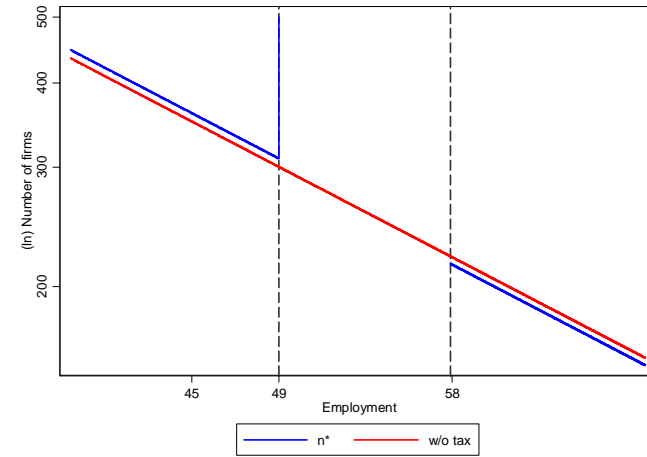
Both panels use the structural parameters of the odel to recover the distribution of firm sizes that would be observed in the counterfactual (no EPL) world.

Panel B: Output as a function of TFP, all sizes
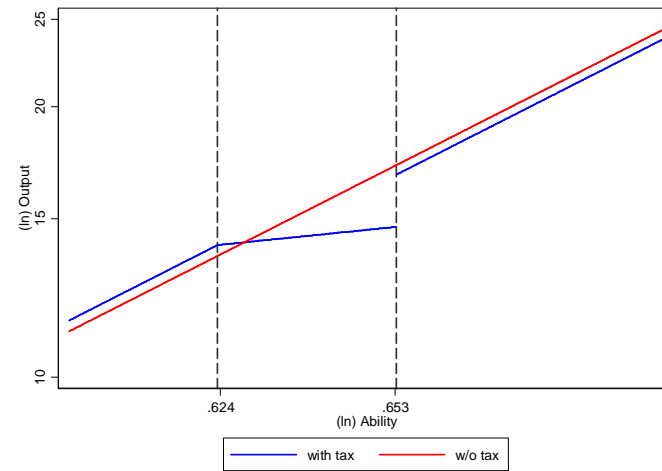

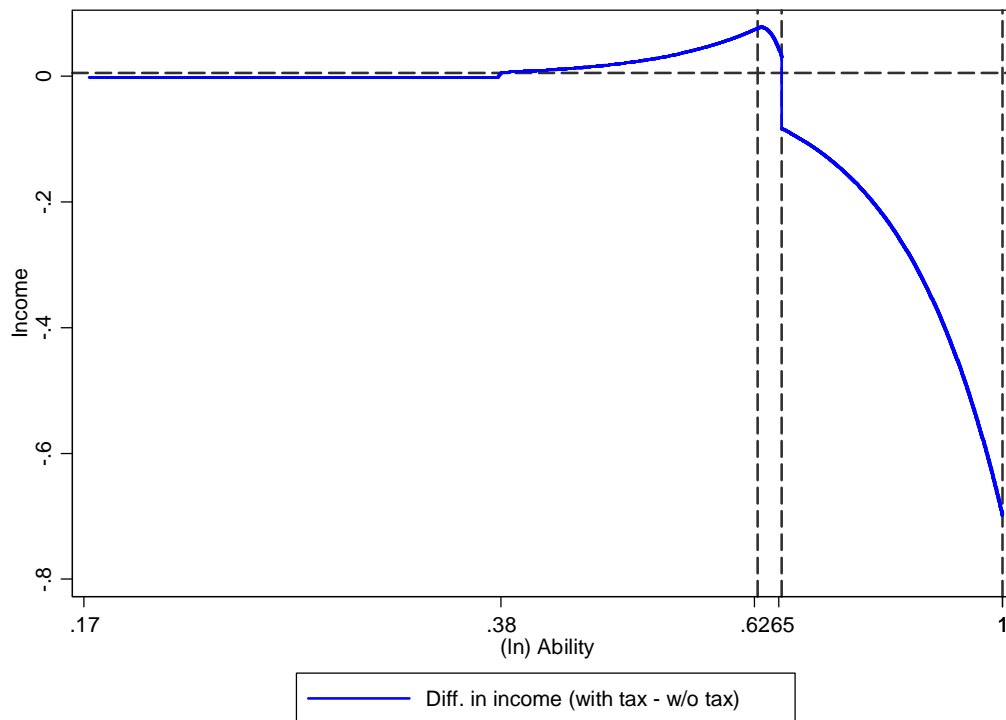**Panel B ZOOM: Output as a function of TFP**

Figure 13: Welfare effect of regulation

Notes: This figure shows the impact on welfare of the regulations discussed in the paper. It is built on the Baseline case, and reflects the same analysis as Table 4. It shows that workers lose due to lower wages, except for those who become managers. Midsize firms benefit from the lower wage costs. Large firms lose, the largest losses being to the most productive firms.
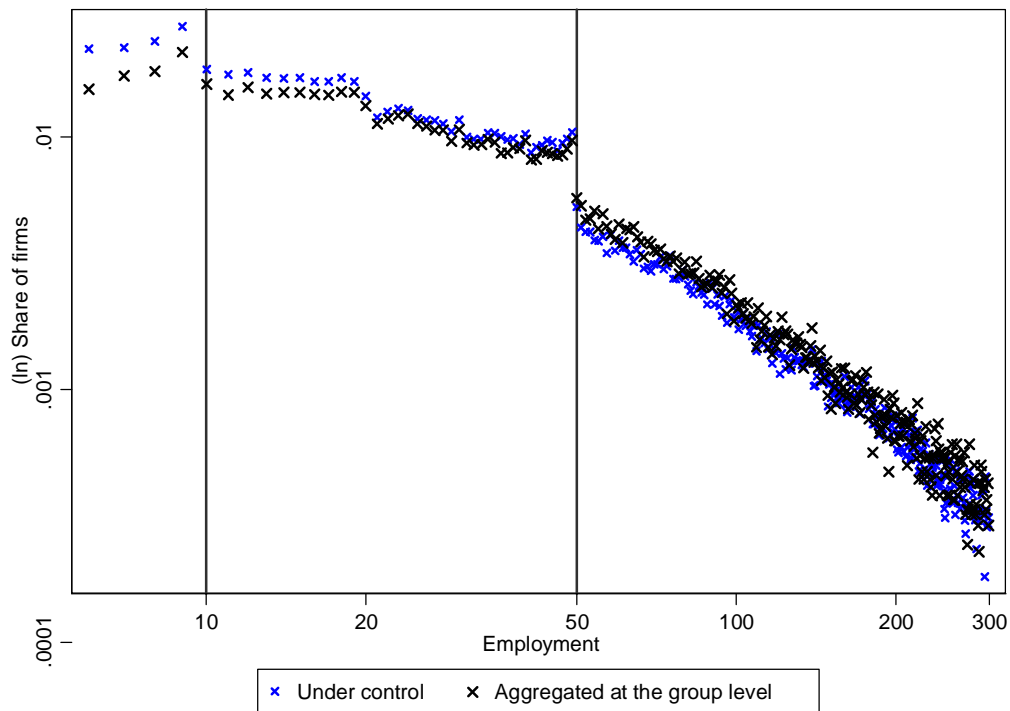
Output lost by firms   Total Output lost
larger than 58

Figure 14: Welfare (Wages): Workers do not appear to be accepting significantly lower wages in return for "insurance" of employment protection
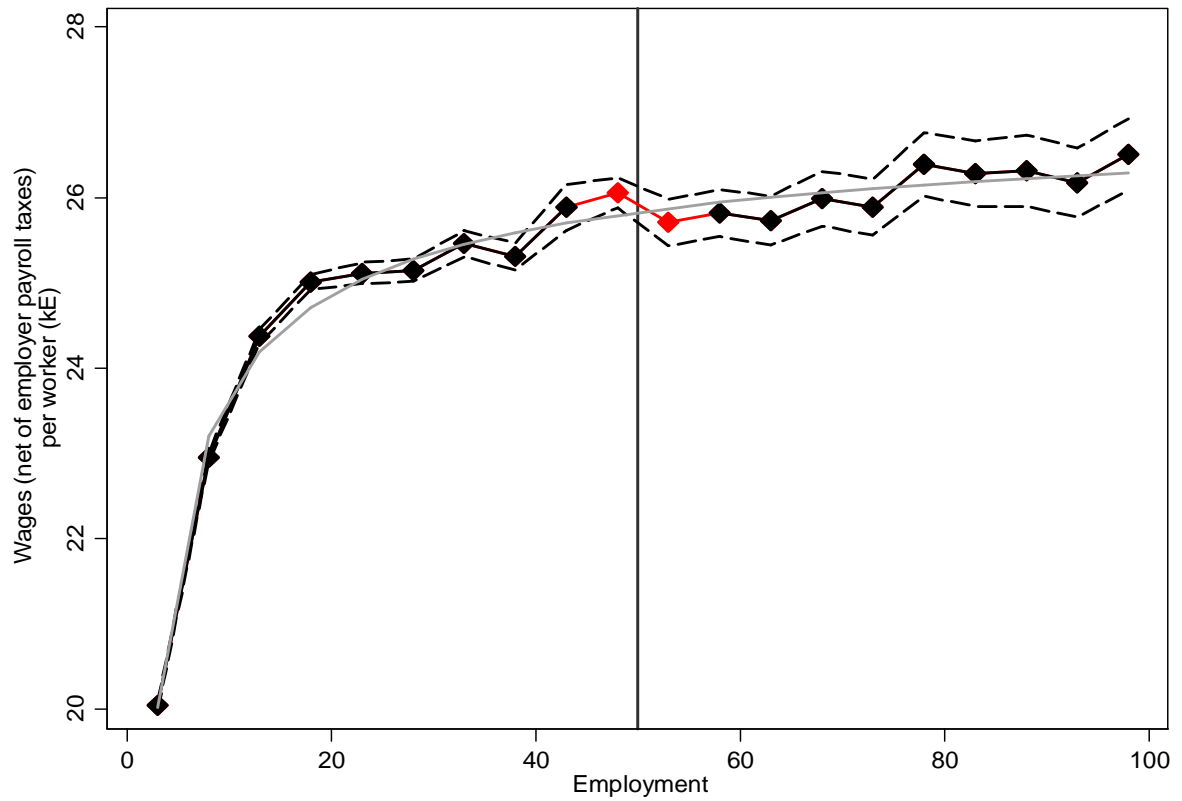

Notes: Wages is the nominal wage (net of payroll tax) by employer size. 95% confidence intervals shown.

Figure 15: Corporate Restructuring in Response to the Regulation?

Notes: "Under Control" are firms who are subsidiaries of larger business groups (blue crosses). Black crosses reports employment aggregated to the business group level. This shows that the discontinuity at 50 is not simply due to subsidiaries of larger business groups as we can see the relationship even at the group level.
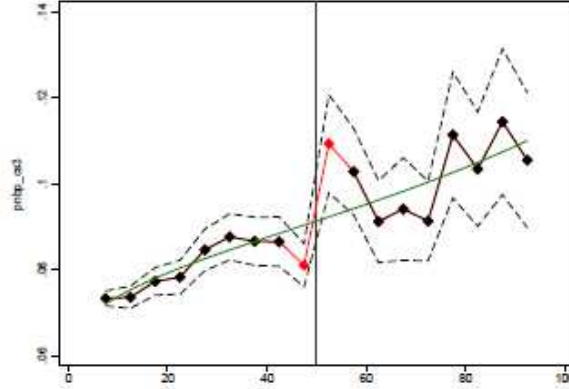
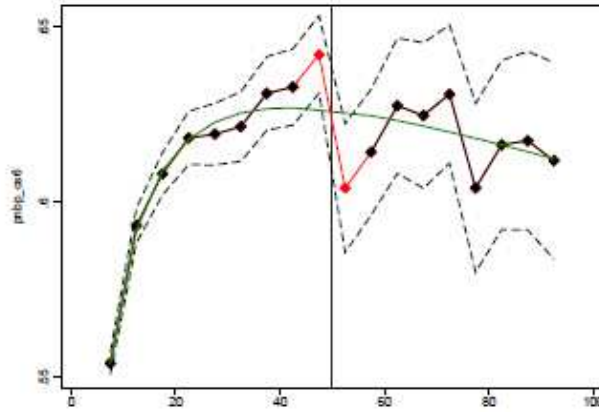# Figure 16 Adjustment in the hours margin around the threshold (annual hours per worker)
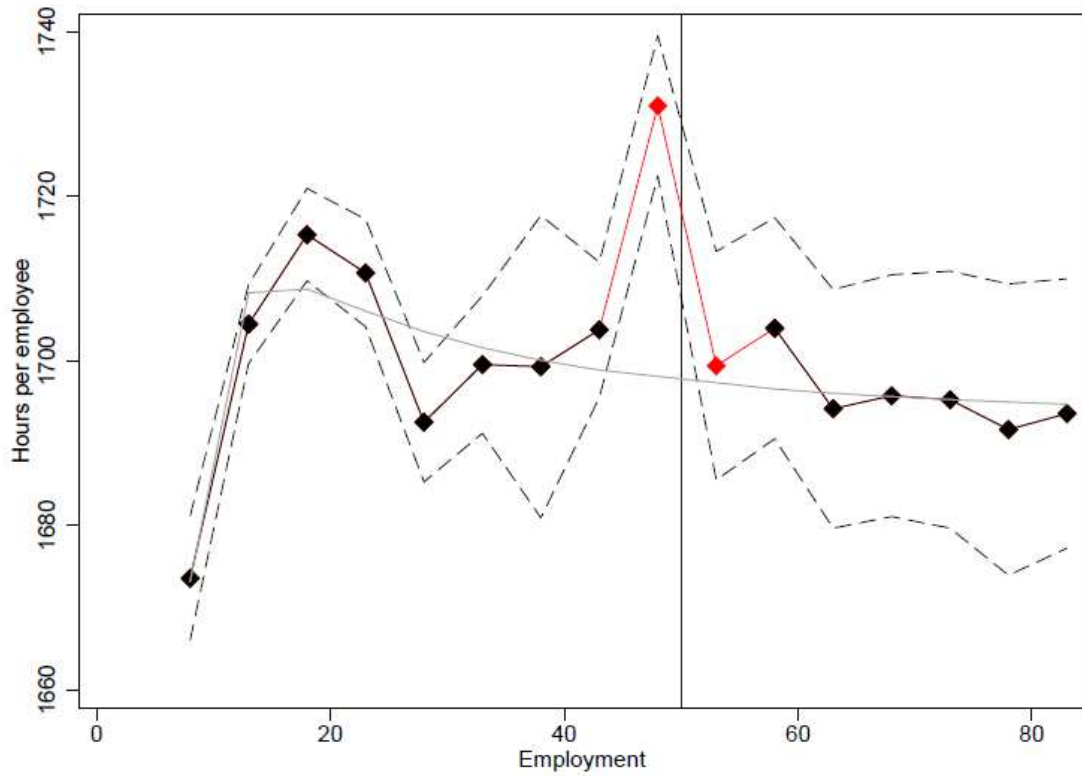
Notes: Annual average hours per worker - combined FICUS and DADs data. 95% confidence intervals shown.

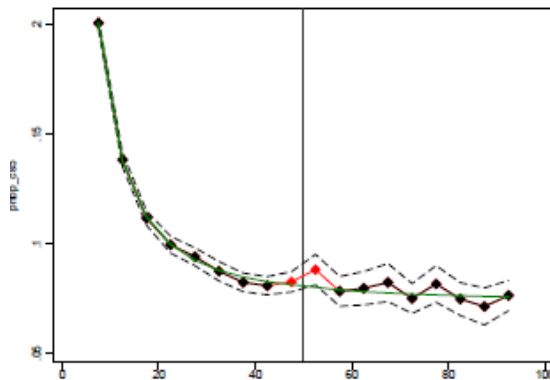# Figure 17: Adjustment in Types of Labor

A. Share of Managers

B. Share of manual workers

C. Share of Clerical Workers

Notes: This looks at the share of the main three occupational groups using combined FICUS and DADs data. Managers and professionals are the most skilled group and manual workers are the least skilled group (clerical workers are in between). 95% confidence intervals shown.

Appendices

Table A1: Production Function Estimation

	(1) Baseline	(2) High Tech Sectors	(3) Low Tech Sectors
Labor	0.739	0.756	0.735
	(0.004)	(0.011)	(0.004)
Capital	0.116	0.126	0.113
	(0.004)	(0.012)	(0.003)
Observations	219,938	35,233	184,705
Firms	53,127	8,410	44,931

Notes: Parameters estimated by Levinsohn-Petrin (2003) method. Estimation on unbalanced panel 2002-2007 of population of French manufacturing firms with 10 to 1,000 employees. "High tech" sectors are based on R&D intensity as defined by the OECD.

Figure A1: Heterogeneity of Results by three digit sector in manufacturing

Notes: These are the results from industry-specific estimation on the same lines as column (2) of Table 2