International Shocks and Domestic Prices: How Large Are Strategic Complementarities?*

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Abstract

How do international shocks affect domestic prices? In this paper, we estimate the effect of exchange rate changes on the prices that firms charge in their domestic market. These prices can be affected directly through the marginal cost channel for firms that import their intermediate inputs and indirectly through the markup channel as firms respond to changes in their competitors’ prices. The contribution of this paper is to examine, theoretically and empirically, the impact of exchange rate shocks on domestic prices, isolating the role of both the marginal cost and the markup channels, while taking explicit account that all prices in the economy are set simultaneously. We find that strategic complementarities play an important role in transmitting international shocks into domestic prices. We show that about a half of exchange rate movements is transmitted into the average domestic prices, with the marginal cost and the markup channels playing nearly equal roles. Firm heterogeneity plays a central role in this transmission mechanism, with small firms reacting mostly through the marginal cost channel and large firms adjusting more through the markup channel. Large firms exhibit substantially stronger strategic complementarities than small firms. Lastly, we provide a calibrated model of variable markups able to match a number of salient features of our data, and then use it to undertake a number of counterfactuals.

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1 Introduction

How do international shocks affect domestic prices? Although this question is at the heart of international economics the answers have remained unclear. In this paper, we estimate the effect of exchange rate changes on the prices that firms charge in their domestic market. These prices can be affected directly through the marginal cost channel for firms that import their intermediate inputs and indirectly through the markup channel as firms respond to changes in their competitors’ prices. The contribution of this paper is to examine, theoretically and empirically, the impact of exchange rate shocks on domestic prices, isolating the role of both the marginal cost and markup channels, while taking explicit account that all prices in the economy are set simultaneously. We find that strategic complementarities, as well as firm heterogeneity, play an important role in transmitting international shocks into prices of domestic goods.

There have been two major challenges that have made it difficult for researchers to address this question. The first challenge is being able to find data that allows you to measure firm-product-level domestic prices, marginal costs and markups. We use highly disaggregated Belgium firm-product level data combined with trade data to proxy for all these variables. One distinguishing feature of this data set is the availability of information on imported inputs at the firm level, making it possible to measure the component of marginal costs that is most directly affected by exchange rate movements. In addition, we proxy for the markup channel by constructing a comprehensive competitors price index at the industry level. With the matched industrial and trade data at the firm-product level, we can include all components of the competitors’ price index - price changes of both domestic and foreign competitors. There are few data sets that provide all these necessary ingredients.

To estimate the size of the strategic complementarities, we need to take into account that all prices in the economy are set simultaneously. We use instrumental variables estimation, instrumenting for the competitor’s price index with their marginal costs of importing inputs and sector-level exchange rates.

The second challenge is how to map the firm-product level results into movements in the aggregate price indexes. An important policy question in international macroeconomics is how much exchange rate movements matter for inflation. We develop a general equilibrium model, where we can construct counterfactuals using the firm-level results as inputs into the calibration.

Our results show that half of exchange rate movements are transmitted into domestic prices. On average, the contributions of the marginal cost and markup channels to price changes are roughly equal. We show that firm heterogeneity matters for these results. Small firms react to shocks relatively more through the marginal cost channel and large firms adjust more through the markup channel. Large firms have much stronger strategic complementarities than small firms.

There is a dearth of evidence on the effects of exchange rates on domestic prices and its components. Previous studies have either relied on aggregate industry level data (Goldberg and Campa 2010, Auer and Schoenle 2013). The more disaggregated empirical studies that use product-level data (Cao, Dong, and Tomlin 2012, Pennings 2012) have not been able to match the product level price movements with firm characteristics. These studies have generally found very low pass-through rates into domes-
tic prices. In contrast, we show that aggregate pass-through is much higher than previous studies have found, once you weight the observations by domestic sales of the firms. Without data on firm characteristics, you cannot distinguish between the marginal cost channel or strategic complementarities. The lack of data on domestic product prices at the firm-level matched with international data has meant that researchers have turned to looking at pass-through into import or export prices. Gopinath and Itskhoki (2011) regressed import prices on trade-weighted sector level exchange rates to try to identify the strategic complementarity channel, but the sector-level exchange rate can affect firm’s prices via its marginal costs if it imports its intermediate inputs, and/or through the markup channel if firms have variable markups and respond to changes in their competitors price index. Amiti, Konings, and Itskhoki (2014) decomposed the pass-through of exchange rates to export prices into the markup and marginal cost channels, but in that study the industry price index was held constant and the focus was on export prices. In this paper, we explicitly account for changes in the industry price index to estimate the size of the firms’ strategic complementarities in setting domestic prices. With firm-level data matched to trade data, we can estimate the relative magnitude of these channels.

Another closely related literature is the research on the pass-through of tariff liberalization on domestic prices. Although this literature is limited to studying developing countries because import tariffs have been low in developed countries for a long time, the mechanisms for which tariffs affect prices are analogous to exchange rate shocks. De Loecker, Goldberg, Khandelwal, and Pavcnik (2012) find that falls in output tariffs have procompetitive effects in India and Edmond, Midrigan, and Xu (2012) find procompetitive effects in Taiwan. Both of these studies have detailed firm-product domestic level data but neither has matched import data, thus making it difficult to measure the marginal cost component that is most directly affected by the international shock. As such, each has taken a distinct path in identifying markups, with De Loecker, Goldberg, Khandelwal, and Pavcnik (2012) using production function estimation to back out markups but not imposing any demand or market structure, which prevents them from performing any counterfactuals. In contrast, Edmond, Midrigan, and Xu (2012) rely on the structure of the Atkeson and Burstein (2008) model to back out the distribution of markups. Our paper falls in the middle of these two, where we rely more on the actual data to infer markup distributions and our industry equilibrium model enables us to perform counterfactuals, and thus make statements about the aggregate effects of international shocks on domestic prices.

The rest of the paper is organized as follows. In section 2, we set out the accounting framework to guide our empirical analysis. Section 3 describes the data and presents the empirical results. Section 4 develops the general equilibrium model and performs counterfactuals. Section 5 concludes.
2 Accounting Framework

Consider a general accounting framework following Gopinath, Itskhoki, and Rigobon (2010) and Burstein and Gopinath (2012). The log price set by a firm-product \( i \) in period \( t \) is the sum of the marginal cost and the markup:

\[
p_{it} = \mu_{it} + mc_{it}.
\]

Denote the elasticity of markup with respect to own price to be \( \Gamma_{it} \) and with respect to competitor price index to be \( \Gamma_{-i,t} \). Then for small price changes the following approximation is accurate:

\[
\Delta p_{it} = \frac{1}{1 + \Gamma_{it}} \Delta mc_{it} + \frac{\Gamma_{-i,t}}{1 + \Gamma_{it}} \Delta P_{-i,t} + \varepsilon_{it},
\]

where \( \Delta \) denotes the time change, \( P_{-i,t} \) is the log of competitor price index, and \( \varepsilon_{it} \) is the change in markup unrelated to the changes in prices. The competitor price index is approximated as:

\[
\Delta P_{-i,t} = \frac{1}{1 - \omega_{i,t}} \sum_{j \neq i} \omega_{j,t} \Delta p_{j,t},
\]

where \( \omega_{j,t} \) is the expenditure share on product \( j \). All the analysis is done within industry.

In a wide class of models analyzed below, the markup elasticity is symmetric for both own and competitor price, but may vary with the characteristics of the firm or the product:

\[
\Gamma_{it} = \Gamma_{-i,t} \equiv \Gamma(z_{it}),
\]

where \( z_{it} \) is a vector of firm-product characteristics which may include firm size, market share, proxies for quality, etc.

Equation (1) can be estimated with data on firm prices and marginal costs, but it requires instrumentation the competitor prices, as prices of all firms are set simultaneously as an outcome of a game in which the optimal price depends on the actions of the competitors. We carry out such estimation first for the unrestricted case in which the coefficients on the own marginal costs and competitor prices without constraining the coefficients to sum to one, and then test whether they do. We then introduce additional interaction terms to allow for \( z_{it} \) variables, to characterize the heterogeneity in firms’ responses.

[OI: discuss heterogeneity results here from one of my early notes...]

3
3 Empirical Evidence

3.1 Data Description

A rare feature of our data set is the availability of highly disaggregated information on values and quantities of all products produced by manufacturing firms in Belgium, which enables us to construct domestic unit values - the main variable of interest. It is the same type of data that is more commonly available for firm’s exports. This firm-product level production data (PRODCOM) is from the National Bank of Belgium, collected by Statistic Belgium. Firms in the Belgium manufacturing sector report production values and quantities for all their products, defined at the PC 8-digit (1,700 products). The survey covers all Belgium firms with a minimum of 10 employees, which covers at least 90% of production in each NACE 4-digit (that is, the first 4 digits of the PC 8-digit code). Firms are required to report total values and quantities but are not required to report the breakdown between domestic sales and exports. Therefore, to get a measure of domestic values and quantities we merge on the export data from customs and subtract total export values and quantities from total production values and quantities sold.

The import and export data are also from the National Bank of Belgium. These data are reported at the firm level by destination and source country for each product classified at the 8-digit combined nomenclature (CN) in values and quantities, with around 10,000 distinct products. The first 6-digits of the CN codes correspond to the World Harmonized System (HS). These data are easily merged with the PRODCOM data using a unique firm identifier; however, the product matching between the two data sets was more complicated (and described in the data appendix).

A third data set, on firm characteristics, comes from the Belgian Business Registry. These data are used to construct measures of total costs and total factor productivity. They are available on an annual frequency at the firm level. Each firm reports their main economic activity within a 5-digit NACE industry, but there is no individual firm-product level data available from this data set.

We combine these three data sets for the period 1995 to 2008 to construct the key variables for our analysis. Let $V$, $Q$, and $p$ denote values, quantities, and prices (unit values), respectively, while log changes in these variables are denoted by $\Delta V$, $\Delta Q$, $\Delta p$, respectively. Each of these variables are indexed by firms ($f$), products ($h$) for CN product codes and $g$ for PC product codes, and years ($t$). Every good (whether denoted by an $h$ or a $g$) belongs to one industry $N$. Imports, exports, domestic production, and total production are denoted by the superscripts $m$, $x$, $d$, and $p$.

**Domestic Prices**  The main variable of interest is the price of the domestically sold goods. We proxy for this using the log change in the domestic unit value, denoted $\Delta p_{fgt}^d$. The domestic unit value is the ratio of production sold domestically to quantity sold domestically. In order to get at the domestic portion of total production, we need to net out the firm’s exports:

$$p_{fgt}^d = \frac{V_{fgt}^p - V_{fgt}^x}{Q_{fgt}^p - Q_{fgt}^x} = \frac{V_{fgt}^d}{Q_{fgt}^d}$$ (3)


One complication in constructing domestic unit values is the issue of carry-along-trade (see Bernard et al), which arises when firms export products that they do not themselves produce. To address this issue we drop all observations for which exports of a firm in period $t$ are greater than 95% of production sold in terms of value and quantity (dropping 11% of the observations and 15% of the production value, which amounts to a much lower share of domestic value sold since most of this production is exported). We clean the data by dropping the observations for which the year-to-year log change in domestic unit values is greater than 200% or less than minus 66%. The other key variables of interest in our analysis are the firm’s marginal cost and the competition variables.

**Marginal Cost** Changes in a firm’s marginal cost can arise from changes in the price of imported and domestic intermediate inputs. We have detailed information on a firm’s imported inputs, however the data sets only include total expenditure on domestic inputs without any information on individual domestic input prices or quantities. Given this limitation, we need to infer the firm’s overall marginal cost, which we construct as follows:

$$
\Delta MC^d_f = (\Delta TVC_f - \Delta Q_f)
$$

with $\Delta Q_f = \Delta Revenue_f - \Delta P_f$ and total variable costs is the sum of the total material cost and the total wage bill. This is likely to be a noisy measure of a firm’s marginal cost so we also utilize the foreign component of a firm’s marginal cost, defined as follows:

$$
\Delta MC^m_f = \frac{V_m}{TVC_f} \Delta P^m_f = s_f \left( \sum_{h,k} w_{fhk} \Delta p_{fhk}^m \right)
$$

where $\Delta MC^m_f$ is a weighted average of the change in the log unit value of the firm’s imports, at the CN 8-digit level from all source countries $k$, and the weights are the average of $t$ and $t-1$ import shares. This is multiplied by the share of imports, $V_f^m$, in total variable cost, $TVC_f$. We drop any change in import unit values greater than 200% and less than 66%. In our baseline case, we assume all imports are intermediate inputs. However, we also experiment with an alternative definition of intermediate imported inputs where we allow for the possibility that some imports by PROCOM sample firms may be final goods. Here, we define an import to be a final good for a firm if it also reports positive production of that good. To illustrate, suppose a firm imports cocoa and chocolate, and it also produces chocolate. In that case we would classify the imported cocoa as an intermediate input and the imported chocolate as a final good, and hence only the imported cocoa would enter in the marginal cost variable.

**Competition Variables** When selling goods in the Belgium market, Belgium firms in the PRODCOM sample face competition from other Belgium firms in the PRODCOM sample that produce their goods in Belgium and sell in the domestic market as well as from Belgium firms not in the PRODCOM sample who import their goods and sell in them in the Belgium market. To capture these two different sources of competition, we construct competitor price indexes for each at the industry level. The import price competition variable faced by each firm-product in industry $N$ is the weighted average log change in
the import price of goods imported by its competitors, $f'$:

$$\Delta P^m_{fg} = \sum_{f' \in N} \frac{V^m_{fg}}{\text{market}_N} \left( \sum_{f' \neq f, g \in N} w^m_{f'g} \Delta p^m_{f'g} \right)$$

(6)

Only the imports categorized as final goods enter in the construction of this variable i.e. any imports that are not included in the construction of the marginal costs. We also split this variable into two components, separating euro and noneuro countries. The euro grouping comprise a time-invariant group, which includes all euro countries countries except Slovenia and Slovakia because of their volatile exchange rates.

Similarly, the domestic price competition variable for each firm-product in industry $N$ is constructed as the weighted average log change in the domestic price of goods sold by its competitors:

$$\Delta P^d_{fg} = \sum_{f' \in N} \frac{V^d_{fg}}{\text{market}_N} \left( \sum_{f' \neq f, g \in N} w^d_{f'g} \Delta p^d_{f'g} \right)$$

(7)

where

$$\text{market}_N = \sum_{f' \in N} \sum_{g \in N} (V^p_{f'g} - V^x_{f'g} + V^m_{f'g})$$

(8)

The market size variable comprises all domestic sales by Belgium firms in the PROCOM survey with positive domestic sales in industry $N$ plus imports and minus exports by Belgium firms in the customs data that are not in the PRODCOM sample. An overall competitors price index is constructed as the weighted average of the foreign and domestic indexes.

A firm $f'$s market share of product $g$ in industry $N$ sold in Belgium is defined as (omitting the $t$ subscript):

$$\text{market}_N = \sum_{f' \in N} \sum_{g \in N} (V^p_{f'g} - V^x_{f'g} + V^m_{f'g})$$

(9)

We define an industry at the NACE 4-digit level (around 175 industries). We chose this level of aggregation in order to avoid huge market shares arising solely due to narrowly defined industries. Our results are robust to more disaggregated industries at the 5-digit and 6-digit levels.

3.2 Empirical Results

To estimate the effect of exchange rate changes on firm-product level prices, we first project price changes and its components on industry level exchange rates $\Delta e_s$. In column 1 of table 1, we regress $\Delta p_{fg}$ on $\Delta e_s$ without weighting the observations and find a small positive coefficient that is barely significant at 0.1. But when we weight the observations by the firm’s domestic sales of that product in column 2, the coefficient increases in magnitude to 0.5 and in significance, so a 10% euro depreciation against its trading partners is associated with a 5% rise in domestic prices. All of the subsequent estimation at the firm-level are weighted by firm-product-level domestic sales in order to give more
Table 1: Exchange Rate Projections

<table>
<thead>
<tr>
<th>Dep. var.:</th>
<th>$\Delta p_{fg}$</th>
<th>$\Delta p_{fg}$</th>
<th>$\Delta mc_f$</th>
<th>$\Delta mc_f^*$</th>
<th>$\Delta P_s$</th>
<th>$\Delta P_s^F$</th>
<th>$\Delta P_s^D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta e_s$</td>
<td>0.108*</td>
<td>0.525***</td>
<td>0.463**</td>
<td>0.265***</td>
<td>0.484***</td>
<td>0.631***</td>
<td>0.322***</td>
</tr>
<tr>
<td></td>
<td>(0.058)</td>
<td>(0.186)</td>
<td>(0.219)</td>
<td>(0.064)</td>
<td>(0.063)</td>
<td>(0.061)</td>
<td>(0.087)</td>
</tr>
<tr>
<td># obs.</td>
<td>63,882</td>
<td>63,882</td>
<td>63,882</td>
<td>63,882</td>
<td>1,714</td>
<td>1,714</td>
<td>1,714</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.000</td>
<td>value</td>
<td>value</td>
<td>0.014</td>
<td>0.033</td>
<td>0.058</td>
<td>0.007</td>
</tr>
<tr>
<td>Weighted</td>
<td>none</td>
<td>value</td>
<td>value</td>
<td>nobs</td>
<td>nobs</td>
<td>nobs</td>
<td></td>
</tr>
</tbody>
</table>

Regressions do not include fixed effects. $\Delta e_s$ is the (log change in) sector import-weighted exchange rate. $\Delta p_{fg}$ is the (log change in) firm-product price. $\Delta mc_f$ is the (log change in) firm marginal cost. $\Delta mc_f^*$ is the (log change in the) imported component of the firm marginal cost. $\Delta P_s$ is the (log change in) the dectoral price index. $\Delta P_s^F$ ($\Delta P_s^D$) is the (log change in) the sectoral price index of imported (domestic) goods.

1Note that errors in all the firm-level regressions are clustered at the industry level to take into account that the exchange rate is an industry-level measure.

weight to larger sales where unit values are likely to be measured with more precision. In order to get a sense of which channel the exchange rate affects domestic prices, we project each component of the the change in firms’ domestic prices on the change in its sector level trade-weighted exchange rate. In column 3, the dependent variable is the firm’s change in its overall marginal cost $\Delta mc_f$, and the coefficient on $\Delta e_s$ is also positive and close to 0.5. The firm’s marginal cost is likely to be affected by exchange rates through its imports of intermediate inputs. In the next column we project the foreign component of a firm’s marginal cost $\Delta mc_f^*$ on exchange rates, and we see that the coefficient is also positive and estimated with a much smaller standard error. Next, we turn to the markup channel. One way to assess the role of strategic complementarities is to regress sector-level price indices on sector-level exchange rates, as in columns 5 to 7. These regressions are weighted by the number of firm-product-level observations that were used to construct each sector-level price index. In column 5, where the dependent variable is the sector-level overall price index $\Delta P_s$, the coefficient is 0.5, and when we break up this price index into the foreign $\Delta P_s^F$ and domestic $\Delta P_s^D$ component in columns 6 and 7, we see that the magnitude is much higher for the foreign price index and lower for the domestic price index, as expected. The size of the coefficient on exchange rates for the foreign price index is twice the size of that for the domestic price index. Nonetheless, we find even the domestic component of the price index significantly responsive to the exchange rate movements. These numbers will prove useful when we turn to interpreting the economic significance of exchange rate shocks into domestic prices.

Another way to assess the importance of the markup channel is to restrict the sample to the set of firms that source all of their intermediate inputs domestically or from within the Euro area in order to switch off the marginal cost channel. However, one might argue that this subsample of firms’ marginal costs may still be indirectly affected by exchange rates as the domestic price of intermediate inputs responds to competing imports. To check for this, we regress the firm’s change in its overall marginal cost on the sector level exchange rates for the sub-sample of firms that do not import anything from
outside the euro area. We see from column 1 of Table 2 that the coefficient is close to zero. Being reassured that this indirect mechanism is not significant, we regress firm-product-level prices on the sector-level exchange rate for this subsample. The result in column 2 indicates a sizeable role for strategic complementarities, with a coefficient on the exchange rate equal to 0.35. As suggested by theory (see Section 4), that this effect should be stronger for firms with larger market shares. Even though many of these firms will be smaller than those that import their inputs from outside the euro area, we can still check for this heterogeneity in a couple of ways. In column 3, we interact the exchange rate with a \( \text{Large}_f \) dummy equal to one for firms with average market shares of greater than 1%, and we see that this interaction term is big and statistically significant. Alternatively, we can see this heterogeneity by interacting the exchange rate with the market share measure itself (\( S_f \)). Again we find a significant and large coefficient on the interaction term, suggesting that large market share firms do indeed respond more strongly to competitors’ prices.

Drawing on the accounting framework developed in section 2, we estimate the change in firm-product prices on marginal cost and competitors price index, as in equation 1. The results are reported in Table 3. First, we estimate equation 1 using OLS, without industry effects in column 1 and with industry fixed effects in column 2. The coefficient on the competitors price index and the firm’s marginal cost are of similar magnitude, indicating equal contributions to price changes from marginal cost shocks and changes in competitors’ prices. Notably, the sum of the two coefficients is less than one whereas we would have expected it to be equal to one. This may be due to measurement error in the construction of marginal cost and/or the endogeneity of the competitors’ price index. The advantage of our proxy for marginal cost is that it encompasses all of the components of marginal costs, but the disadvantage is that it is measured with a lot of noise. We can instrument for the firm’s marginal cost using the foreign component of its marginal cost, which is that part of overall marginal cost that is most likely affected by exchange rate movements and it is more precisely measured than the other components of the firm’s

### Table 2: Non-importers

<table>
<thead>
<tr>
<th>Dep. var.:</th>
<th>( \Delta mcf )</th>
<th>( \Delta p_{fg} )</th>
<th>( \Delta p_{fg} )</th>
<th>( \Delta p_{fg} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>( \Delta e_s )</td>
<td>0.064 (0.129)</td>
<td>0.350*** (0.154)</td>
<td>0.026 (0.138)</td>
<td>0.232* (0.138)</td>
</tr>
<tr>
<td>( \Delta e_s \times \text{Large}_f )</td>
<td>0.651** (0.327)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta e_s \times S_f )</td>
<td></td>
<td>4.361** (1.929)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

# obs. | 22762 | 22762 | 22762 | 22762 |
Adj. \( R^2 \) | -0.000 | 0.001 | 0.002 | 0.001 |
Weighted | value | value | value | value |

Sample of firms that do not import from outside the EU. Notation as in Table 1. In column 3, \( \text{Large}_f \) is defined as average marketshare at the firm-product level with 4-digit industry > 1%. In column 4, the market share \( S_f \) interaction is linear.
Table 3: Domestic Prices, Marginal Costs, and Competitors’ Price Index

<table>
<thead>
<tr>
<th>Dep. var.:</th>
<th>$\Delta p_{fg}$ (1)</th>
<th>$\Delta p_{fg}$ (2)</th>
<th>$\Delta p_{fg}$ (3)</th>
<th>$\Delta p_{fg}$ (4)</th>
<th>$\Delta p_{fg}$ (5)</th>
<th>$\Delta p_{fg}$ (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta mc_f$</td>
<td>0.346*** (0.041)</td>
<td>0.345*** (0.042)</td>
<td>0.701*** (0.166)</td>
<td>0.704*** (0.162)</td>
<td>0.561*** (0.179)</td>
<td>0.530*** (0.192)</td>
</tr>
<tr>
<td>$\Delta P_{-f,s}$</td>
<td>0.477*** (0.081)</td>
<td>0.380*** (0.091)</td>
<td>0.355* (0.197)</td>
<td>0.337* (0.198)</td>
<td>0.549** (0.242)</td>
<td>0.574** (0.265)</td>
</tr>
<tr>
<td># obs.</td>
<td>63882</td>
<td>63882</td>
<td>63882</td>
<td>63882</td>
<td>47850</td>
<td>63399</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.181</td>
<td>0.193</td>
<td>0.064</td>
<td>0.035</td>
<td>0.146</td>
<td>0.119</td>
</tr>
<tr>
<td>Industry F.E.s</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Instrumental Vars</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Weak Instrument Test (F-stat)</td>
<td>159.44</td>
<td>4.6</td>
<td>3.97</td>
<td>1.77</td>
<td>0.1</td>
<td>1.04*** (0.098)</td>
</tr>
<tr>
<td>Hansen-J Test $\chi^2$ p-value</td>
<td>4.6</td>
<td>3.97</td>
<td>1.77</td>
<td>0.1</td>
<td>1.04*** (0.098)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: all regressions include year fixed effects. Standard errors are clustered at the industry level. Notation as in Table 1. $\Delta P_{-f,s}$ denotes the firm’s competitors price index. Instrument set: $\Delta e_s$, $\Delta MC_{-f,s}$ (firm’s competitors marginal cost), $\Delta mc_f$, $(1 - \phi)\Delta w_f$ (change in firm wages). The IV regressions pass the weak instrument test with F-stats over 50. Column 6 uses a stricter definition of inputs, excluding any import in a 4-digit industry that the firm produces.

Marginal cost, as we saw in Table 1. We also include the firm’s average wage cost in the instrument set. Another potential problem with these estimations is the endogeneity of the competitors’ price index. All prices are set simultaneously, so we are unable to infer a causal relationship. To address the endogeneity of the price index, we instrument using the weighted average of the competitors’ marginal cost of imported inputs and the sector-level exchange rate. From column 3, we see that the coefficient on the firm’s marginal cost increases in size and the coefficients now sum to one. The results are almost identical with industry fixed effects in column 4. From now on, all estimations will include industry fixed effects. If we constrain the coefficients to sum to one, the coefficient on the firm’s marginal cost is unaffected, equal to 0.7 (not reported).

We check the robustness of these results in subsequent columns. In column 5, we limit the sample to each firm’s main industry to deal with the multi-product firm issue. This avoids the problem of attributing responses to final goods prices to the marginal cost channel. In column 6, we adopt a more narrow measure of intermediate imports, by only including the firm’s imports outside any 4-digit industry that the firm has any sales. We see that the coefficients on both the firm’s marginal cost and competitors price index are around 0.5 and we cannot reject that the sum of the two coefficients equals one. The first-stage regressions are reported in the appendix.

According to the theory, we should see a lot of heterogeneity in how firm’s respond to international
Table 4: Firm Heterogeneity

<table>
<thead>
<tr>
<th>Dep. var.</th>
<th>$\Delta p_{fg}$ (1)</th>
<th>$\Delta p_{fg}$ (2)</th>
<th>$\Delta p_{fg}$ (3)</th>
<th>$\Delta p_{fg}$ (4)</th>
<th>$\Delta p_{fg}$ (5)</th>
<th>$\Delta p_{fg}$ (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta mc_f$</td>
<td>0.864*** (0.143)</td>
<td>0.930*** (0.193)</td>
<td>0.870*** (0.147)</td>
<td>0.955*** (0.200)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta mc_f \times Large_f$</td>
<td>0.533** (0.260)</td>
<td>-0.357 (0.346)</td>
<td>0.533** (0.261)</td>
<td>-0.388 (0.351)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta P_{-f,s}$</td>
<td>0.157 (0.173)</td>
<td>0.142 (0.230)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta P_{-f,s} \times Large_f$</td>
<td>0.486* (0.286)</td>
<td>0.305 (0.386)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

# obs. 48939 14993 63932 48939 14993 63932
Adj. $R^2$ 0.037 0.072 0.038 0.030 0.048 0.013

All regressions have industry fixed effects and year fixed effects. Standard errors are clustered at the industry level. Observations are weighted with domestic sales. Notation as in Tables 1–3. Large$_f$ is the dummy for employment>100. Instrument set: $\Delta e_s$, $\Delta MC^*_{-f,s}$, $\Delta mc_f$, $(1-\phi)\Delta w_f$.

shocks. We explore this in Table 4, where we allow the coefficients on the marginal cost and competitors price index to vary with the firm’s size, which we measure in terms of the firm’s employment and its market share. In Table 4, we define a large firm as any firm that has more than 100 workers on average over the sample period. In columns 1 and 2, we estimate the sample of small and large firms separately. From column 1, we see that small firms have a larger coefficient on their marginal cost, equal to 0.9, and a small insignificant coefficient of 0.2 on the competitors price. However, large firms have roughly equal coefficients of around 0.5 on both marginal cost and competitors’ price index. In column 3, we use the full sample of firms and include an interactive large term and we find a similar pattern. These three regressions are repeated in columns 4 to 6, now constraining the sum of the two coefficients on marginal cost and competitors’ prices to equal one. Again, we see that the coefficient on the $\Delta mc_f$ variable is 0.9 for small firms and 0.5 for large firms. Using the firm’s market share (averaged over time) of a product within a 4-digit industry produces with various cutoffs similar results, e.g. observations in the top twenty percentile within a 4-digit industry, top ten percentile, a 3% market share cutoff, and a 2% market share cutoff (see appendix for these results).
4 Model of Variable Markups

Monopolistic competition under CES demand yields constant markups. In this section we relax both assumptions, allowing for both general non-CES homothetic demand and oligopolistic competition. Our model nests both Kimball (1995) and Dixit and Stiglitz (1977) with large firms (as in Krugman 1987, Atkeson and Burstein 2008).

Consider the following aggregator for the sectoral consumption $C$:

$$\frac{1}{N} \sum_{i \in \Omega} A_i \Upsilon \left( \frac{N C_i}{\xi_i C} \right) = 1,$$  \hspace{1cm} (10)

where $\Omega$ is the set of products $i$ in the sector with $N = |\Omega|$ denoting the number of goods, and $C_i$ is the consumption of product $i$; $A_i$ and $\xi_i$ denote the two shifters (a quality parameter and a demand parameter, respectively, as will become clear later); $\Upsilon(\cdot)$ is the demand function such that $\Upsilon(\cdot) > 0$, $\Upsilon'(\cdot) > 0$, $\Upsilon''(\cdot) < 0$ and $\Upsilon(1) = 1$.

There are two important limiting cases that we consider. First, in the limiting case of $N \to \infty$, the demand aggregator becomes:

$$\frac{1}{|\Omega|} \int_{i \in \Omega} A_i \Upsilon \left( \frac{|\Omega| C_i}{\xi_i C} \right) \, di = 1,$$  \hspace{1cm} (11)

where now $|\Omega|$ is the mass of products in the sector. This limiting case corresponds to the Kimball (1995) demand model, as used for example in Klenow and Willis (2006) and Gopinath and Itskhoki (2010).

The second limiting case obtains when the demand aggregator becomes a power function, $\Upsilon(z) = z^{(\sigma-1)/\sigma}$, which corresponds to the conventional CES aggregator which we can rewrite as:

$$C = \left[ N^{-1/\sigma} \sum_{i \in \Omega} \left( A_i \xi_i^{\sigma-1/\sigma} C_i^{\sigma-1} \right)^{\sigma/\sigma-1} \right]^{\sigma-1/\sigma},$$  \hspace{1cm} (12)

which for finite $N$ corresponds to the demand structure in the pricing-to-market papers of Krugman (1987) and Atkeson and Burstein (2008) and for infinite $N$ is the standard monopolistic competition model of Dixit and Stiglitz (1977), later used in Krugman (1980) and much of the macro and international literature.

Consumers allocate expenditure $E$ to the purchase of products in the sector, and we assume that $E = \alpha P^{1-\eta}$, where $P$ is the sectoral price index and $\eta$ is the elasticity of substitution across sectors. This assumption corresponds to the case of the CES aggregator of sectoral outputs, when each sector is too small to affect economy-wide price index. Formally, we write the sectoral expenditure (budget) constraint as:

$$\sum_{i \in \Omega} P_i C_i = E.$$  \hspace{1cm} (13)

Given prices $\{P_i\}_{i \in \Omega}$ of all products in the sector and expenditure $E$, consumers allocate consumption
\{C_i\} optimally across products within sectors to maximize the consumption index \(C\):

\[
\max_{\{C_i\}_{i \in \Omega}} \left\{ C \mid \text{s.t. (10) and (13)} \right\}.
\]  

(14)

The first-order optimality condition for this problem defines consumer demand (see appendix for derivation), and is given by

\[
C_i = \frac{\xi_i C}{N} \cdot \psi \left( \frac{x_i}{\gamma_i} \right), \quad \text{where} \quad x_i \equiv \frac{P_i/\gamma_i}{P/D}.
\]  

(15)

In this expression, \(\gamma_i \equiv A_i/\xi_i\) is the quality parameter and \(\psi(\cdot) \equiv \Upsilon^{-1}(\cdot)\) is the demand curve, while \(\xi_i C/N\) is the normalized demand shifter, where \(C\) is sectoral consumption. \(P\) is the ideal price index such that \(C = E/P\) and \(D\) is an additional auxiliary variable determined in industry equilibrium that is needed to characterize demand outside the CES case.\(^2\) Note that an increase in \(\gamma_i\) directly reduces the effective price for the good in the eyes of the consumers, which corresponds to a shift along the demand curve. At the same time, an increase in \(\xi_i\) (holding \(\gamma_i\) constant), shifts out the demand curve holding the effective price unchanged. This is why we refer to \(\xi_i\) as the demand shifter, and \(\gamma_i\) as the quality parameter.

We show in the appendix that \(P\) and \(D\) are defined by:\(^3\)

\[
\frac{1}{N} \sum_{i \in \Omega} A_i \Upsilon \left( \psi \left( \frac{P_i/\gamma_i}{P/D} \right) \right) = 1,
\]  

(16)

\[
\frac{1}{N} \sum_{i \in \Omega} \xi_i P_i \frac{P}{P/D} \psi \left( \frac{P_i/\gamma_i}{P/D} \right) = 1.
\]  

(17)

Equation (16) ensures that (10) is satisfied given the demand (15), i.e. that \(C\) is indeed attained given the consumption allocation \(\{C_i\}\). Equation (17) ensures that the expenditure constraint (13) is satisfied given the allocation (15). Note that condition (17) simply states that the sum of market shares in the

\(^2\)Note that the ideal price index \(P\) exists since the demand defined by (10) is homothetic, i.e. a proportional increase in \(E\) holding all \(\{P_i\}\) constant results in a proportional expansion in \(C\) and in all \(\{C_i\}\) holding their ratios constant; \(1/P\) equals the Lagrange multiplier for the maximization problem in (14) subject to the expenditure constraint (13).

\(^3\)In the limiting case of CES, we have \(\Upsilon(z) = z^{\frac{\sigma-1}{\sigma}}\), and hence \(\Upsilon'(z) = \frac{\sigma-1}{\sigma} z^{-1/\sigma}\) and \(\psi(x) = \left( \frac{\sigma}{\sigma-1} x^{1-\sigma} \right)\). Substituting this into (16)–(17) and taking their ratio immediately pins down the value of \(D\). We have, \(D \equiv (\sigma-1)/\sigma\) and is independent of \(\{P_i\}\) and other parameters, and hence this auxiliary variable is indeed redundant in the CES case. Given this \(D\), the price index can be recovered from either condition in its usual form:

\[
P = \left[ \frac{1}{N} \sum_{j \in \Omega} (A_j^\sigma \xi_j^{1-\sigma}) P_j^{1-\sigma} \right]^{1/\sigma}.\]

The case of CES is a knife-edge case in which the demand system can be described with only the price index \(P\), which summarizes all information contained in micro-level prices needed to describe aggregate allocation. More generally, the second auxiliary variable \(D\) is needed to characterize the aggregate effects of micro-level heterogeneity. As will become clear later, \((P,D)\) are sufficient statistics to describe the relevant moments of the price distribution, which at the first-order approximation could be thought of as measures of the average price and the dispersion of prices.
sector equals one, with the market share given by

\[ s_i \equiv \frac{P_i C_i}{PC} = \frac{\xi_i P_i}{NP} \psi\left(\frac{P_i/\gamma_i}{P/D}\right), \quad (18) \]

where we substituted in for \( C_i \) from the demand equation (15). In addition, we introduce the demand elasticity as a characteristic of the slope of the demand curve \( \psi(\cdot) \):

\[ \sigma_i \equiv \sigma(x_i) = -\frac{d \log \psi(x_i)}{d \log x_i}, \quad (19) \]

where \( x_i \) is the effective price of the firm as defined in (15). Outside the CES case, the demand elasticity is non-constant and is a function of the effective price of the firm. We further show in the appendix the following results for the effects of changes in the individual firm prices on aggregate variables \( P \) and \( D \):

\[
\begin{align*}
\text{d} \log P &= \sum_{i \in \Omega} s_i \, \text{d} \log P_i, \\
\text{d} \log \frac{P}{D} &= \sum_{i \in \Omega} \frac{s_i \sigma_i}{\sum_{j \in \Omega} s_j \sigma_j} \, \text{d} \log P_i.
\end{align*}
\]

Given this, we can calculate the full elasticity of demand, which takes into account the effects of \( P_i \) on \( P \) and \( D \). Substituting \( C = E/P = \alpha P^{-\eta} \) into (15), we have:

\[
\Sigma_i \equiv -\frac{d \log C_i}{d \log P_i} = \eta s_i + \sigma_i \left(1 - \frac{s_i \sigma_i}{\sum_{j \in \Omega} s_j \sigma_j}\right), \quad (20)
\]

where \( \sigma_i \) is given in (19). With this demand elasticity, the firm profit maximization problem under constant returns to scale production, \( \Pi_i = \max_{P_i} [P_i - MC_i] C_i \), yields the following expression for the optimal price:

\[ P_i = M_i MC_i, \quad M_i \equiv \frac{\Sigma_i}{\Sigma_i - 1}. \]

The two analytically tractable cases are: (1) monopolistic competition with \( s_i \to 0 \) for all \( i \in \Omega \), and (2) CES demand with \( \sigma_i \equiv \sigma \) for all \( i \). Indeed in those two cases, the formula in (20) simplifies considerably: \( \Sigma_i = \sigma_i \) in the former and \( \Sigma_i = \eta s_i + \sigma(1 - s_i) \) in the latter. The latter case corresponds to Atkeson and Burstein (2008) and has been studied in Amiti, Konings, and Itskhoki (2014), where we showed that the markup elasticity is symmetric:

\[ \Gamma_i \equiv -\frac{\partial \log M_i}{\partial \log P_i} = \frac{d \log M_i}{d \log P} = \frac{(\rho - 1)(\rho - \eta)s_i}{\Sigma_i(\Sigma_i - 1)}, \]

13
and is increasing in the market share $s_i$. Therefore, for that case we can write:

$$d \log P_i = \frac{1}{1 + \Gamma_i} d \log MC_i + \frac{\Gamma_i}{1 + \Gamma_i} d \log P + \epsilon_i.$$  

In the case of monopolistic competition under non-CES demand, the markup elasticity is somewhat different, and can be written as:

$$d \log P_i = \frac{1}{1 + \Gamma_i} d \log MC_i + \frac{\Gamma_i}{1 + \Gamma_i} d \log P + \epsilon_i,$$

where $\Gamma_i$ is defined in the same way, but now does not depend on $s_i$, but rather depends on the relative effective price of the firm $x_i$, as we discuss further below. Also note that $d \log (P/D)$ is different from $d \log P$, and $d \log D$ is not necessarily orthogonal with $d \log P$. Nonetheless, if variation in $P_i$ is dominated by firm-idiosyncratic shocks, then $d \log D$ would indeed be close to orthogonal to $d \log P$, as we show numerically in the following section.

The more general case with both non-CES demand and oligopolistic competition is analytically intractable, and we analyze it numerically in the next section.

Before turning to a more special case of the Kimball demand, we discuss briefly some of its general properties. First, Kimball demand is homothetic and separable in the sense that the cross-partial elasticities are symmetric for all varieties (as is also the case for the most common parameterization of the translog demand, see Feenstra ??). Second, Kimball demand nests CES as a special case. Third, Kimball demand (given in (15)) for variety $i$ in general depends on the own price of the variety $P_i$ and only the two moments of the price distribution $\{P_i\}$—the two auxiliary variables $P$ and $D$, defined in (16)–(17). These auxiliary variables summarize all relevant information contained in the distribution of prices $\{P_i\}$ and, roughly speaking, capture the mean and the variance of this distribution, as we illustrate below. In the limiting case of the CES, the ideal price index $P$ is the unique sufficient statistic for demand, while $D = (1 - 1/\sigma)$ is constant in this case and does not depend on the distribution of prices.

### 4.1 Klenow-Willis aggregator

For our quantitative analysis, we adopt a tractable specification of the Kimball aggregator introduced by Klenow and Willis (2006). Specifically, the demand curve in this case is given by:

$$\psi(x_i) = \left[1 - \bar{\epsilon} \log \left(\frac{\sigma}{\sigma - 1} x_i\right)\right]^{\bar{\sigma}/\bar{\epsilon}},$$  

\[\text{(21)}\]

\[\text{4An alternative expression is}\]

$$d \log P_i = \frac{1}{1 + \Gamma_i} d \log MC_i + \frac{\Gamma_i}{1 + \Gamma_i} d \log P_{-i} + \epsilon_i,$$

where $\Gamma_i' \equiv (1 - s_i)\Gamma_i$ and $P_{-i}$ is the competitor price index such that $P = \left[(\xi_i \gamma_{\sigma}^* \right) P_i^{1-\sigma} + (1 - \xi_i \gamma_{\sigma}^*) P_{-i}^{1-\sigma}]^{1/(1-\sigma)}$.

\[\text{5These two auxiliary variables corresponds to the the Lagrange multipliers in the consumer optimization, corresponding to constraints (13) and (10) respectively (see the appendix).}\]
where \( x_i \) is the effective price of the firm, as defined in (15). The two demand parameters \( \bar{\sigma} > 1 \) and \( \bar{\varepsilon} \geq 0 \) control respectively the elasticity of demand and the elasticity of markup for a representative firm. In the limiting case of \( \bar{\varepsilon} = 0 \), the demand in (21) converges to a constant elasticity demand curve with \( \sigma = \bar{\sigma} \). The appendix provides a closed-form expression for \( \Upsilon(\cdot) \), which gives rise to the demand curve in equation (21).

For concreteness, we specialize to the case of the monopolistic competition \( (N \rightarrow \infty \) and \( s_i \rightarrow 0 \) for all \( i \in \Omega \)), and briefly discuss the cross-sectional properties of this demand. The demand elasticity and super-elasticity functions are given by:

\[
\begin{align*}
\sigma_i &\equiv \sigma(x_i) = - \frac{\partial \log \psi(x_i)}{\partial \log P_i} = \frac{\bar{\sigma}}{1 - \bar{\varepsilon} \log \left( \frac{\sigma}{\sigma-1} x_i \right)}, \\
\varepsilon_i &\equiv \varepsilon(x_i) = \frac{\partial \log \sigma(x_i)}{\partial \log x_i} = \frac{\bar{\varepsilon}}{1 - \bar{\varepsilon} \log \left( \frac{\sigma}{\sigma-1} x_i \right)}.
\end{align*}
\]

Under this demand, the optimal markup is given by:

\[
\mathcal{M}_i \equiv \frac{\sigma(x_i)}{\sigma(x_i) - 1} = \frac{\bar{\sigma}}{1 + \frac{\varepsilon}{\bar{\sigma} - 1} \log \left( \frac{\sigma}{\sigma-1} x_i \right)},
\]

and therefore the elasticity of markup is:

\[
\Gamma_i \equiv \Gamma(x_i) = - \frac{\partial \log \mathcal{M}_i}{\partial \log P_i} = \frac{\varepsilon(x_i)}{\sigma(x_i) - 1} = \frac{\frac{\varepsilon}{\bar{\sigma} - 1}}{1 + \frac{\varepsilon}{\bar{\sigma} - 1} \log \left( \frac{\sigma}{\sigma-1} x_i \right)}.
\]

Therefore, both markups \( \mathcal{M}_i \) and markup elasticity \( \Gamma_i \) are decreasing in the effective relative price \( x_i \), and hence the idiosyncratic pass-through rate \( \Psi_i \equiv 1/(1 + \Gamma_i) \) is increasing in \( x_i \).

The Klenow-Willis demand with \( \bar{\varepsilon} > 0 \) has a few notable properties, whereas the limit of \( \bar{\varepsilon} \rightarrow 0 \) correspond to the CES demand. First, it is log-concave (as can be immediately observed from (21)), while the CES limit is log-linear. Second, in contrast to the CES limit, it has a choke-off price defined by \( \psi(\hat{x}) = 0 \) and equal to \( \hat{x} = \frac{\bar{\sigma}-1}{\bar{\sigma}} e^{1/\bar{\varepsilon}} \). Third, there is a least price below which the elasticity demand is below one (and hence inconsistent with profit maximization), as defined by \( \sigma(\bar{x}) = 1 \) and given by \( \bar{x} = \frac{\bar{\sigma}-1}{\bar{\sigma}} e^{-(\bar{\sigma}-1)/\bar{\varepsilon}} < 1 \). Note that at this price the markup becomes infinite, \( \mathcal{M}(\bar{x}) = \infty \), and therefore in equilibrium this price can be charged only by firms with zero marginal costs, and in the absence of such firms, every firm charges an effective price strictly above \( \bar{x} \). Lastly, the idiosyncratic pass-through \( \Psi(x_i) \) varies from zero for the firm with a least price \( \bar{x} \) to a maximum of \( \bar{\Psi} = \frac{1}{1 + \varepsilon/\bar{\sigma}} \) for the firm with the choke-off price \( \hat{x} \). We illustrate these properties in Figure 1 in the appendix.

Finally, we discuss the properties of the industry equilibrium. Note that the price of each firm can be written as \( P_i = \mathcal{M}(x_i) MC_i \), where \( x_i = \frac{P_i/\gamma_i}{P/P_D} \) is the effective relative price of the firm, and \( P \) and \( D \) are the solution to (16)–(17). This defines a joint fixed point problem for the aggregate variables \( P \)

---

6Note that with this demand, the elasticity of elasticity with respect to quantity is constant: \( d \log \sigma_i / d \log C_0 = \bar{\varepsilon}/\bar{\sigma} \). Furthermore, the markup elasticity \( \Gamma_i \) is proportional to the level of markup \( \mathcal{M}_i \) (we introduce both below): \( \Gamma_i/\mathcal{M}_i = \bar{\varepsilon}/\bar{\sigma} \).
and $D$, as well as for the individual prices $\{P_i\}$. The firm fixed point problem has an implicit closed form solution given by:

$$P_i = P \cdot W\left(\exp\left(\frac{\bar{\sigma} MC_i}{P}\right)\right), \quad \text{where} \quad P = \frac{\bar{\sigma} - 1}{\bar{\sigma}} e^{-\frac{\bar{\sigma} - 1}{\bar{\sigma}}} \cdot \frac{P}{D}$$  \hspace{1cm} (26)

is the least price (corresponding to $z$), and $W(\cdot)$ is the Lambert W function, defined as the solution to $W(z)e^{W(z)} = z$.

There exists no closed-form solution for $P$ and $D$ in general. We provide the implicit equations defining $P$ and $D$—the counterparts of (16)–(17)—for the case of Klenow-Willis demand in the appendix. Here we discuss a special tractable case with $\bar{\sigma} = \bar{\epsilon} > 1$ and $\xi_i = A_i \equiv 1$ for illustration purposes, while the appendix offers derivations and general expressions. When $\bar{\sigma} = \bar{\epsilon}$, the utility aggregator has a simple closed form given by $\Upsilon(z_i) = 1 + (\sigma - 1)(1 - \exp\{(1 - z_i)/\bar{\sigma}\})$. Using this expression, we can simplify and manipulate the sector equilibrium conditions (16)–(17) to yield the following results:

$$P = \bar{P} \cdot [1 - \bar{\sigma}T],$$  \hspace{1cm} (27)

$$D = \frac{\bar{\sigma} - 1}{\bar{\sigma}} \frac{P}{\bar{P}} = \frac{\bar{\sigma} - 1}{\bar{\sigma}} (1 - \bar{\sigma}T),$$  \hspace{1cm} (28)

where $\bar{P} \equiv \frac{1}{|\Omega|} \int_{i \in \Omega} P_i \, di$ is the average price and $T \equiv \frac{1}{|\Omega|} \int_{i \in \Omega} \frac{P_i}{\bar{P}} \log \frac{P_i}{\bar{P}} \, di$ is the Theil index of price dispersion in the industry. Therefore, the mean and dispersion (measured by the Theil index) of prices form a sufficient statistic for the industry equilibrium, as they allow to recover both $P$ and $D$. The ideal price index $P$ equals the average price in the industry adjusted for the dispersion of prices; given the average price $\bar{P}$, the ideal price index is lower the larger is the dispersion of prices $T$ and/or the larger is the elasticity of substitution parameter $\bar{\sigma}$. The second auxiliary variable $D$ measures the departure of the price index from the average price, and hence is decreasing in the dispersion of prices. This example illustrates the role of the two auxiliary variables $P$ and $D$, and while it corresponds to a very special case of the model, it provides more general insights about the types of the moments of the price distribution, which shift the demand schedules.

## 5 Exchange Rates, Markups and Prices in a Calibrated Model

In this section we provide a numerical analysis of the model of variable markups described above under certain specific function form and parametric assumptions. We focus on an industry equilibrium, where the costs of the firms follow exogenous processes, which we describe below. We analyze the joint price setting by different firms that are subject to idiosyncratic cost shocks, as well as an aggregate exchange rate shock which affects them with heterogeneous intensities. The main question that we ask is what dimensions of the within industry equilibrium affect the aggregate pass-through of the exchange rate into the industry price level.
5.1 Firms and costs

A representative industry is populated by \( N \) domestic and \( N^* \) foreign firm-products, which we denote with \( i \). Firm \( i \) has the following marginal cost in period \( t \):

\[
MC_{it} = \frac{W_t}{\Omega_{it}} \cdot \left( \frac{V^*_t \cdot E_t}{W_t} \right)^{\varphi_i},
\]

where \( W_t \) is the cost index of domestic inputs, \( V^*_t \) is the cost index of the foreign inputs in the foreign currency, \( E_t \) is the nominal exchange rate (unit of domestic currency for one unit of foreign currency), \( \varphi_i \) is the firm’s exposure to foreign inputs, and \( \Omega_{it} \) is the firm’s productivity. We assume that the firm’s exposure to foreign inputs is constant over time. This assumption is justified in the data as we look at rather short time span of about 10 years. We calibrate the joint distribution of \( \{\varphi_i, \Omega_{it}\} \) using the data on import intensity of the firms and market shares of the firms.

We assume that \( \{W_t, V^*_t, E_t\} \) follow exogenous processes. In particular, we let the nominal exchange rate follow a random walk in logs:

\[
e_t = e_{t-1} + \sigma_e u_t,
\]

where \( e_t \equiv \log E_t, u_t \sim iid \mathcal{N}(0, 1) \), and \( \sigma_e \) is the standard deviation of the log change in the exchange rate. The initial value of the exchange rate is to one, that is \( e_0 = 0 \). For simplicity, we normalize \( W_t \equiv V^*_t \equiv 1 \). These assumptions reflect the partial equilibrium nature of our exercise. We assume that foreign and domestic firms are symmetric in all respects but their draws of \( \varphi_i \)—the exposure to foreign inputs. Specifically, the foreign firms have a greater exposure to foreign inputs, as we detail further below in the discussion of our calibration. The other difference between foreign and domestic firms is in their respective numbers \( N \) and \( N^* \), which reflects the selection forces, as modeled in detail in Eaton, Kortum, and Sotelo (2012).

We further assume that the productivity process \( \Omega_{it} \) follows a random growth process:

\[
\omega_{it} = \mu + \omega_{i,t-1} + \sigma_\omega v_{it},
\]

where \( \omega_{it} \equiv \log \Omega_{it}, \mu \) is the drift, \( v_{it} \sim iid \mathcal{N}(0, 1) \), and \( \sigma_\omega \) is the standard deviation of the innovation to log productivity. Additionally, we impose a reflecting barrier at \( \omega \), in which case the productivity process becomes:

\[
\omega_{it} = \begin{cases} 
\mu + \omega_{i,t-1} + \sigma_\omega v_{it}, & \text{if } \omega > \omega, \\
\omega - (\mu + \omega_{i,t-1} + \sigma_\omega v_{it}), & \text{otherwise}.
\end{cases}
\]

That is, the process follows (30) as long as it stays above lower bound \( \omega \), and otherwise it reflects from the lower bound by the amount (30) was going to undershoot \( \omega \) without the reflection. The initial productivities are drawn from a Pareto distribution, \( \Omega_{i0} \sim iid \text{Pareto}(k, e\omega) \), where \( k \) is the shape parameter and \( \omega \) is the lower bound for \( \omega_{i0} = \log \Omega_{i0} \). That is, the cumulative distribution function for \( \Omega_{i0} \) is given by \( G_0(\Omega) = 1 - (\Omega/e\omega)^{-k} \) for \( \Omega \geq e\omega \). When \( \mu = -k\sigma_\omega^2 / 2 < 0 \), the cross-sectional distribution of \( \Omega_{it} \) stays unchanged at \( G_0(\cdot) \), as discussed e.g. in Gabaix (2009).
This specifies the distribution of costs for the firms in each period \( t \), \( \{MC_{it}\} \). Given the costs, we calculate the equilibrium prices \( \{P_{it}\} \) according to (26) and find the equilibrium price index \( P_t \) and the auxiliary variable \( D_t \) according to (16)–(17).\(^7\) We also calculate the market shares \( \{s_{it}\} \) according to (18). Note that for simplicity we ignore the heterogeneity in the demand and quality parameters and set \( \xi_i = A_i \equiv 1 \). We then calculate the measured log change in the price index according to:

\[
\Delta \log P_t = \sum_{i=1}^{N+N^*} \frac{s_{it} + s_{i,t-1}}{2} \left( \log P_{it} - \log P_{i,t-1} \right),
\]

as we did in the data. We calculate similar price indexes for domestic and foreign subsets of firms. Additionally, we calculate the change in the competitor price index for each firm as:

\[
\Delta \log P_{-i,t} = \sum_{j \neq i} \frac{s_{jt}^i + s_{j,t-1}^i}{2} \left( \log P_{jt} - \log P_{j,t-1} \right),
\]

where \( s_{jt}^i \equiv s_{jt} / (1 - s_{it}) \) so that \( \sum_{j \neq i} s_{jt}^i = 1 \).

5.2 Parameterization and calibration

In order to calibrate the model, we focus on a typical 4–digit sector in the Belgian economy. We set the number of domestic firm-products to \( N = 60 \), which slightly above the median (of 57) and below the mean (of 79) across 146 sectors. Interestingly, the incidence of multiple products sold by a single firm within a 4–digit sector is not large, and thus we ignore for now the issue of the strategic price setting by multi-product firms, to which we return in the end of the paper. Similarly, for now we adopt the approximation that all firms in an industry are small enough not to be able to affect significantly the sectoral price index, and we analyze the alternative case in the end of this section.

We do not observe directly the number of foreign firm-products, yet we observe the total sales of foreign products. Following the insights from Eaton, Kortum, and Sotelo (2012), we calibrate the number of foreign firms \( N^* \) to correspond to the overall market share of foreign sold goods in sectoral sales. Specifically, we set \( N^* = 40 \), corresponding to a 40% market share of imported goods, slightly above the average (of 38%) across the sectors. Table ?? in the appendix summarizes the distribution of number of firms, foreign market shares and other variables across the sectors of the Belgian economy.

[OI: we might want to consider Poisson draws with means 60 and 40 to be more in line with EKS.]

We set the elasticity of substitution across the 4–digit sectors to \( \eta = 1 \), as is conventional in the literature following Atkeson and Burstein (2008) (see, for example, Edmond, Midrigan, and Xu 2012), and we also experiment with an arguably more realistic number of \( \eta = 2 \). This number does not play a role in our calibration until the end of this section when we consider large firms that can affect the sectoral price index. The other two demand parameters that we calibrate are the elasticity of demand within sector \( \bar{\sigma} \) and the super-elasticity of demand \( \bar{\varepsilon} \). We set \( \bar{\sigma} = 4 \), which is standard in the literature

\[^7\text{For now we assume that } N \text{ and } N^* \text{ are large enough, and the maximum market shares are small enough, so that it is an accurate assumption that } \Sigma_i \approx \sigma_i \text{ (see (20)).}\]
Table 5: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of domestic firms</td>
<td>(N = 60)</td>
<td>Data</td>
</tr>
<tr>
<td>Number of foreign firms</td>
<td>(N^* = 40)</td>
<td>Import share of sales</td>
</tr>
<tr>
<td>Elasticity across sectors</td>
<td>(\eta = 1, 2)</td>
<td>EMX (2014)</td>
</tr>
<tr>
<td>Elasticity of demand</td>
<td>(\bar{\sigma} = 4)</td>
<td>BW (2006)</td>
</tr>
<tr>
<td>Super-elasticity of demand</td>
<td>(\bar{\varepsilon} = 1.33)</td>
<td>Firm-level pass-through</td>
</tr>
<tr>
<td>Pareto shape parameter</td>
<td>(k = 0.75)</td>
<td>Size distribution of firms</td>
</tr>
<tr>
<td>Firm productivity st.dev.</td>
<td>(\sigma_\omega = 0.15)</td>
<td>corr(s_{it}, \Delta s_{it})</td>
</tr>
<tr>
<td>Firm productivity drift</td>
<td>(\mu = -k\sigma_\omega^2/2)</td>
<td>Distribution stationarity</td>
</tr>
<tr>
<td>Exchange rate st.dev.</td>
<td>(\sigma_e = 0.1)</td>
<td>Data on trade-weighted ER</td>
</tr>
</tbody>
</table>

Table 6 (see, e.g. Broda and Weinstein 2006), and we set \(\bar{\varepsilon} = 1.75\), which allows us to match the level and variation in pass-through within industries. Recall from our discussion above that this parametrization implies the maximum pass-through of idiosyncratic shocks equal to \(\hat{\Psi} = \frac{\bar{\sigma}}{\bar{\varepsilon} + \bar{\sigma}} = 70\%\).

In the initial productivity distribution \(G_0(\Omega)\), we set the lower bound to one, which correspond to \(\omega = 0\). We set the shape parameter to \(k = 0.75\), which is much lower the numbers conventionally used in practice (e.g., \(k \approx \sigma - 1 = 3\), as in Eaton, Kortum, and Sotelo 2012), but reflects that we work with a non-CES demand, which is log-concave. This very fat-tailed productivity distribution together with our demand assumptions allows us to match the size distribution of firms, and in particular the Herfindahl index among the domestic firms (see the appendix).

For the dynamic process, we set the drift of idiosyncratic productivity \(\mu\) to keep the distribution of firm productivities stationary, and we calibrate the standard deviation of productivity innovations \(\sigma_\omega\) to match the joint distribution of markets shares and changes in markets shares... For the exchange rate process, we set \(\sigma_e = 0.1\) corresponding to the annual standard deviation of the trade-weighted Euro exchange rate.

Finally, we match the overall import intensity of the sector, as well as the distribution of import intensity across Belgian firms. [OI: We need a detailed discussion of the calibration of \(\{\phi_i\}\) here...]

We set the import intensity across foreign firms to a common number equal to \(\varphi^F = 0.7\) to match the aggregate pass-through of the exchange into the price index of foreign products.

5.3 Simulation results

Using the calibrated model, we simulate a panel of firm-product prices across 20 industries and 11 times periods, roughly corresponding to the structure of our dataset. Given the calibrated exogenous marginal cost process in (29), we use the model to solve for the (Bertrand-Nash) equilibrium of the simultaneous price setting game. In addition to firm market shares and prices, we calculate the evolution of sectoral price indexes according to (31), the way it is done by statistical agencies. Provided this simulated panel dataset, we run the same regression specifications as in Tables 1–4.

In Table 6 we report the results from two regression specification. In the first row, we report the
Table 6: Sectoral pass-through regressions

<table>
<thead>
<tr>
<th></th>
<th>All firms</th>
<th>Domestic firms</th>
<th>Foreign firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Sector-level</td>
<td>0.345</td>
<td>0.342</td>
<td>0.068</td>
</tr>
<tr>
<td>Firm-level pooled</td>
<td>0.351</td>
<td>0.355</td>
<td>0.092</td>
</tr>
</tbody>
</table>

sector-level specification in which we run the log change in the price index $\Delta \log P_t$ calculated according to (31), as well as a similarly constructed index of the change in the log marginal cost of all firms ($\Delta \log MC_t$), on the change in the log exchange rate $\Delta \log E_t$. We do it for the full sample of all firms, as well as for the subsamples of domestic and foreign firms separately. Columns (2), (4), (6) of Table 6 correspond to columns (5), (7) and (6) of Table 1: the model sectoral pass-through rates are 0.34, 0.21 and 0.53 for all, domestic and foreign respectively, in parallel with 0.48, 0.32 and 0.63 pass-through estimated in our Belgian dataset. The marginal cost regressions for the domestic and foreign firms recover closely the respective average exposures to foreign inputs, $\tilde{\varphi}^H = 0.1$ and $\varphi^F = 0.7$ respectively, as well as for the sample of all firms, equal to $S^H\tilde{\varphi}^H + S^F\varphi^F = 0.6 \cdot 0.1 + 0.4 \cdot 0.7 = 0.34$, where $S^F = (1 - S^H)$ is the sectoral sales share of foreign firms. Note the similarity in the sectoral-level coefficients for marginal costs and prices for the sample of all firms (both equal to 34%), reflecting that at the aggregate there is a complete pass-through. At the same time, the price of domestic firms move substantially more that the marginal costs (21% versus 7%), reflecting the role of strategic complementarities in competition with foreign firms (which, in their turn, pass-through only 53% of the exchange rate movement while their costs move by 70%). This implies that an exchange rate devaluation results in an increase in markup by domestic firms and a reduction in markups by foreign firms, which nearly offset each other.

The second row of Table 6 report the results from the pooled firm-product-level regressions of changes in log marginal costs ($\Delta \log MC_{it}$) and prices ($\Delta \log P_{it}$) on the change in the log exchange rate ($\Delta \log E_t$). The results closely replicate those form the sector-level regressions, with column (4) corresponding to columns (1)–(2) of Table 1 in the empirical section.

Table 7: Pass-through heterogeneity across firms

<table>
<thead>
<tr>
<th>Dep. var.: $\Delta \log P_{it}$</th>
<th>Without size interaction</th>
<th>With size interaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \log MC_{it}$</td>
<td>0.569</td>
<td>0.644</td>
</tr>
<tr>
<td>$\Delta \log MC_{it} \times L_{it}$</td>
<td>—</td>
<td>−0.279</td>
</tr>
<tr>
<td>$\Delta \log P_{i,t}$</td>
<td>0.421</td>
<td>0.350</td>
</tr>
<tr>
<td>$\Delta \log P_{i,t} \times L_{it}$</td>
<td>—</td>
<td>0.287</td>
</tr>
</tbody>
</table>

Table 7 reports the results from firm-level regressions of change in log firm prices ($\Delta \log P_{it}$) on the
change in log firm marginal cost (\(\Delta \log MC_{it}\)) and the change in the firm’s log competitor price index (\(\Delta \log P_{-i,t}\), calculated according to 32). The second column adds two additional interaction terms of the right-hand side variables with the ‘large’ dummy \((L_{it})\) for the firm belonging to the top quartile (20%) in terms of the market share to capture the heterogeneity in the markup elasticity across firms. The two columns of the table correspond to column (3) of Tables 3 and 4 respectively. We find very similar results in the first column, with coefficients on own marginal cost and competitors prices equal to 0.57 and 0.42 in the simulated dataset versus 0.70 and 0.36 in the Belgian dataset. In the second column, we see that the respective coefficients change to 0.65 and 0.35 for the smallest 75% of firms in terms of market share, while they equal 0.36 \((= 0.64 - 0.28)\) and 0.64 \((= 0.35 + 0.29)\) for the largest 25% of firms. This corresponds to somewhat smaller variations than in our Belgian dataset, where the smallest firms have about 90–95% pass-through of idiosyncratic marginal cost shocks, while for the largest firms it is lower by 35–40%.

5.4 Counterfactuals

6 A Model with Large Firms
A Data Appendix

Data Sources We draw on the three main data sources for the period 1995 to 2008. One, the production data at the firm-product level (PRODCOM) is from the National Bank of Belgium, collected by Statistic Belgium (part of the Federal Government Department of Economics). These data report production values and quantities at the firm-product level in the manufacturing sector, where the product is defined at the PC 8-digit. It is a survey of all firms with a minimum of 10 employees, covering at least 90% of production in each NACE 4-digit (that is, the first 4 digits of the PC 8-digit code), which includes around 1,700 manufacturing product codes in any one year. We only keep PC codes that are classified as manufactured goods - these are products for which the first 4-digits of the PC8 codes are in the range of 1500 to 3699. We drop all PC8 codes in petroleum (NACE 2-digit code 23) and industrial services. Firms are required to report total values and quantities but are not required to report the breakdown between domestic and exports. Therefore, to get a measure of domestic values and quantities we merge on the export data from customs and subtract total export values and quantities from total production values and quantities sold.

Two, the international data are from the National Bank of Belgium, with the intra-EU trade data collected by the Intrastat Inquiry and the extra-EU transactions data by Customs. These data are reported at the firm level by destination and source country for each product classified at the 8-digit combined nomenclature (CN) in values and quantities, with around 10,000 distinct products. The first 6-digits of the CN codes correspond to the World Harmonized System (HS). All transactions that involve a change of "ownership with compensation" (codes 1 and 11) are in our sample. These data include all extra-EU transactions of firms with trade greater than 1,000 euros or whose weights are more than 1,000 kilograms - these thresholds were reduced in 2006; and intra-EU trade with a higher threshold of 250,000 euros, with both these thresholds raised somewhat in 2006.

Three, data on firm characteristics are from the Belgian Business Registry, covering all incorporated firms. These data are used to construct measures of total costs and total factor productivity. They are available on an annual frequency at the firm level. Each firm reports their main economic activity within a 5-digit NACE industry. However, there is no product level data within firms available from this source.

Merging the trade and production data The production and trade data are easily merged using a unique firm identifier. But the merging of the firm’s products in the production and customs data is a bit more complicated.

First, we had to aggregate the monthly PRODCOM data to the annual frequency. To avoid large jumps in annual values due to nonreporting for some months by some firms, we only keep a firm’s observation in period \( t \) if there was positive production reported for at least one product in each month. In some cases the firm reported positive values but the quantities were missing. For these cases, in order to construct domestic unit values we impute the quantity sold from the average value to quantity ratio in the months where both values and quantities were reported - this only affected a small proportion of the observations, 3% of the observations, accounting for 1% of the production value. With this adjustment,
we aggregated the data to the annual level.

Second, there is the task of converting the highly disaggregated trade data that is at the CN 8-digit level with the more aggregated PC 8-digit PC codes. To match these two datasets, we use the concordance provided by Eurostat - these mappings may be one-to-one, many-to-one, one-to-many, or many-to-many. We use the files developed by Ilke, et al. to identify these mappings. While CN-to-PC conversion is straightforward for one-to-one and many-to-one mappings, conversion for one-to-many and many-to-many mappings required grouping of some PC codes. There were 77 such groupings, which account for approximately 4% of the observations and production value.

Third, in order to construct the domestic unit values, where we net out exports from total production values and quantities, we need to ensure that the quantities in the two data sets are comparable. So we drop observations where the units that match in the two data sets are less than 95 percent of the total export value and the firm’s export share is greater than 5% within a firm-PC-year observation. The rationale for doing this is that if the export share (exports as a ratio of production) is really small then the domestic unit value won’t be affected very much if we don’t subtract all of the firm’s exports.

Fourth, some PC codes change over time. Here, we only make an adjustment if the code is a one-to-one change between two years. We do not take into account changes in PC codes that involve splitting into multiple codes or multiple PC codes combining into one code. Effectively, these changes in the PC codes are treated as though they are new products.
B Model Appendix

B.1 Derivation of demand

Denote by $\lambda$ and $\mu$ the Lagrange multipliers on demand aggregator (10) and the expenditure constraint (13) respectively. The first order conditions for $C$ and $C_j$ are respectively:

$$1 = \lambda \sum_{j \in \Omega} A_j \Upsilon' \left( \frac{NC_j}{\xi_j C} \right) \frac{C_j}{\xi_j C^2},$$

$$\mu P_j = \lambda A_j \Upsilon' \left( \frac{NC_j}{\xi_j C} \right) \frac{1}{\xi_j C}.$$

Denote by $P \equiv 1/\mu$, which is the ideal price index such that $PC = E$ under the optimal consumption allocation, and by

$$D \equiv \frac{C}{\lambda} = \sum_{j \in \Omega} \frac{A_j C_j}{\xi_j C} \Upsilon' \left( \frac{NC_j}{\xi_j C} \right).$$

With this notation, we can rewrite the optimality conditions to obtain the product demand function:

$$C_j = \frac{\xi_j C}{N} \cdot \psi \left( \frac{P_j / \gamma_j}{P / D} \right), \quad \gamma_j \equiv A_j / \xi_j, \quad \psi(\cdot) \equiv \Upsilon'^{-1}(\cdot).$$

Given $P = E/C$, $P$ and $D$ are determined from the two constraints on the problem (10) and (13), which can be rewritten as:

$$\frac{1}{N} \sum_{j \in \Omega} A_j \Upsilon \left( \psi \left( \frac{P_j / \gamma_j}{P / D} \right) \right) = 1,$$

$$\frac{1}{N} \sum_{j \in \Omega} \frac{\xi_j P_j}{P} \psi \left( \frac{P_j / \gamma_j}{P / D} \right) = 1,$$

which we reproduce in the main text as (16) and (17). This fully characterizes the solution to the consumer’s problem and hence the demand schedule. Note that equation (17) is simply the statement that the sum of market shares in the industry equals 1, since the market share of a product is given by:

$$s_j = \frac{P_j C_j}{PC} = \frac{\xi_j P_j}{NP} \cdot \psi \left( \frac{P_j / \gamma_j}{P / D} \right) = \frac{\xi_j P_j \psi \left( \frac{P_j / \gamma_j}{P / D} \right)}{\sum_{i \in \Omega} \xi_i P_i \psi \left( \frac{P_i / \gamma_i}{P / D} \right)},$$

where we substituted demand (15) for $C_j$ and expressed $P$ out using (17). In the CES case, we have $\psi(x) = \left( \frac{x}{\sigma-1} \right)^{-\sigma}$, and the expression for market share simplifies to:

$$s_j = \frac{\left( A_j \xi_j^{1-\sigma} \right) P_j^{1-\sigma}}{\sum_{i \in \Omega} \left( A_i \xi_i^{1-\sigma} \right) P_i^{1-\sigma}} = \frac{A_j^{\sigma} \xi_j^{1-\sigma}}{N} \left( \frac{P_j}{P} \right)^{1-\sigma},$$

where $P$ is defined in (??).
Finally, we defined the elasticity and the super-elasticity of demand:

\[
\tilde{\sigma}_j = \tilde{\sigma}(x_j) \equiv -\frac{d \log \psi'(x_j)}{d \log x} = -\frac{x_j \psi'(x_j)}{\psi(x_j)},
\]

\[
\tilde{\varepsilon}_j = \tilde{\varepsilon}(x_j) \equiv -\frac{x_j \psi'(x_j)}{\psi(x_j)}.
\]

### B.2 Large firms

Denote by \( Z \equiv D/P \) and take a full log differential of (16)–(17) with respect to \((P_i, P, Z)\) for some \( i \in \Omega \) and holding \( P_j \) for all \( j \neq i \) constant:

\[
\frac{d \log Z}{d \log P_i} = \frac{\xi_i P_i}{NP} \left[ \psi \left( \frac{Z P_i}{\gamma_i} \right) + \frac{Z P_i}{\gamma_i} \psi' \left( \frac{Z P_i}{\gamma_i} \right) \right] + \frac{d \log Z}{d \log P_i} \sum_{j \in \Omega} \frac{\xi_j P_j Z P_j'}{NP} \frac{Z P_j}{\gamma_j} \psi' \left( \frac{Z P_j}{\gamma_j} \right),
\]

where in manipulating the differential of (16) we used the fact that \( \Upsilon'(\psi(x)) \equiv x \) by definition of \( \psi(\cdot) \) as the inverse function of \( \Upsilon'(\cdot) \). Using the definition of the market share \( s_j \) and the elasticity of demand \( \tilde{\sigma}_j \), we can rewrite:

\[
\frac{d \log Z}{d \log P_i} = -\frac{D \xi_i P_i}{NP} \frac{Z P_i}{\gamma_i} \tilde{\sigma}_i = \frac{s_i \tilde{\sigma}_i}{\sum_{j \in \Omega} s_j \tilde{\sigma}_j},
\]

where we have:

\[
\frac{d \log C_j}{d \log P_j} = -\eta s_j - \tilde{\sigma}_j \left[ 1 - \frac{s_j \tilde{\sigma}_j}{\sum_{i \in \Omega} s_i \tilde{\sigma}_i} \right],
\]

and therefore price-setting satisfies:

\[
P_j = M_j MC_j, \quad M_j = \frac{\tilde{\sigma}_j \left[ 1 - \frac{s_j \tilde{\sigma}_j}{\sum_{i \in \Omega} s_i \tilde{\sigma}_i} \right] + \eta s_j}{\tilde{\sigma}_j \left[ 1 - \frac{s_j \tilde{\sigma}_j}{\sum_{i \in \Omega} s_i \tilde{\sigma}_i} \right] + \eta s_j - 1}.
\]
As $s_j \to 0$, we have $M_j = \tilde{\sigma}_i / (\tilde{\sigma}_i - 1)$. When $\varepsilon \to 0$ and hence $\tilde{\sigma}_j \equiv \sigma$ for all $j$, we have:

$$M_j = \frac{\sigma (1 - s_j) + \eta s_j}{\sigma (1 - s_j) + \eta s_j - 1}.$$ 

We need to derive:

$$\Gamma_j \equiv -\frac{d \log M_j}{d \log P_j},$$
$$\Gamma_P \equiv \frac{d \log M_j}{d \log P},$$
$$\Gamma_D \equiv \frac{d \log M_j}{d \log D}.$$ 

### B.3 Klenow and Willis demand

Figure 1 plots these cross-sectional relationships (for $\sigma = 4$ and various values of $\varepsilon$), from which we can draw a number of useful lessons. Figure 1a shows that for $\varepsilon > 0$ there is a finite choke-off price above which firms cannot sell positive quantities; this choke-off price corresponds to the level at which markups equals 1 in Figure 1c and, consequently, the price is equal to marginal cost (intersects $45^\circ$-line) in Figure 1f. Figure 1b illustrates that for low enough prices the elasticity of demand is less than unity, $\sigma_i < 1$, which is inconsistent with firm optimization; therefore, optimizing firms always choose a price at least to ensure demand with unit-elasticity, $\sigma(x) = 1$—this can be seen in Figure 1c as the markup goes to infinity, in Figure 1e as the pass-through goes to zero, and in Figure 1f as the price asymptotes (on the left) and becomes insensitive to the marginal cost. Finally, Figure 1e shows that the maximal pass-through rates (for the smallest firms) are low when $\varepsilon$ is large (below 60% for $\varepsilon = 3$ and below 45% for $\varepsilon = 6$); when $\varepsilon$ is small ($=1$), the pass-through varies moderately between 60% and 80%—this means we need an intermediate level of $\varepsilon \in [1.5, 2.5]$ to match the data.
Figure 1: Klenow-Willis specification of Kimball demand
B.4 Special case of $\bar{\varepsilon} = \bar{\sigma}$

With $\bar{\sigma} = \bar{\varepsilon} > 1$, we can take the integral defining $\Upsilon(y)$ analytically, as $\Gamma(1, y) = \int_0^\infty e^{-t}dt = e^{-y}$.

Therefore, in this case, we have:

$$y_i = \psi(x_i) = 1 - \sigma \log \left( \frac{\sigma}{\sigma - 1} x_i \right), \quad x_i = \frac{P_i/\gamma_i}{P/D},$$

$$\Upsilon(y_i) = 1 + (\sigma - 1) \left[ 1 - \exp \left\{ \frac{(1 - y_i)}{\sigma} \right\} \right]$$

and thus

$$\Upsilon(\psi(x_i)) = \sigma (1 - x_i).$$

Substituting this into (16)–(17), we have (in the monopolistic competition limit):

$$\frac{\sigma}{|\Omega|} \int_{i \in \Omega} A_i \left( 1 - \frac{P_i/\gamma_i}{P/D} \right) di = 1,$$

$$\frac{1}{|\Omega|} \int_{i \in \Omega} \frac{\xi_i P_i}{P} \left[ 1 - \sigma \log \left( \frac{\sigma}{\sigma - 1} \frac{P_i/\gamma_i}{P/D} \right) \right] di = 1.$$

The first of these defines the ratio $P/D$:

$$\frac{P}{D} = \frac{\sigma \cdot \mathbb{E}\{\xi_i P_i\}}{\sigma \cdot \mathbb{E}\{A_i\} - 1},$$

where $\mathbb{E}\{\cdot\}$ denotes a population average of a variable. Using the expression $P/D$, we can express out the price index $P$ from the second condition as:

$$P = \mathbb{E}\{\xi_i P_i\} \cdot \left[ 1 - \sigma \mathbb{E}\left\{ \frac{\xi_i P_i}{\mathbb{E}\{\xi_i P_i\}} \cdot \log \left( \frac{\xi_i P_i}{\mathbb{E}\{\xi_i P_i\}} \cdot \frac{1}{\sigma \cdot \mathbb{E}\{A_i\} - 1} \right) \right\} \right].$$

It is natural to impose the following normalization: $\mathbb{E}\{A_i\} = \frac{1}{|\Omega|} \int_{i \in \Omega} A_i di = 1$. In that case, the expression simplify to:

$$\frac{P}{D} = \frac{\sigma}{\sigma - 1} \mathbb{E}\{\xi_i P_i\},$$

$$P = \mathbb{E}\{\xi_i P_i\} \cdot \left[ 1 - \sigma \mathbb{T}\{\xi_i P_i\} + \frac{\mathbb{E}\{\xi_i P_i \log A_i\}}{\mathbb{E}\{\xi_i P_i\}} \right],$$

where $\mathbb{T}\{\xi_i P_i\}$ is the Theil inequality index for $\{\xi_i P_i\}$ defined as

$$\mathbb{T}\{\xi_i P_i\} = \frac{1}{|\Omega|} \int_{i \in \Omega} \frac{\xi_i P_i}{\mathbb{E}\{\xi_i P_i\}} \cdot \log \left( \frac{\xi_i P_i}{\mathbb{E}\{\xi_i P_i\}} \right) di.$$
References


https://sites.google.com/site/stevenpennings/.