Price Dynamics for Durable Goods

Michal Fabinger
Harvard

Gita Gopinath
Harvard

Oleg Itskhoki
Princeton

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Motivation

- Durables play a crucial role in business cycle fluctuations
  - $\sim$60% of non-service consumption, all of investment
  - most volatile component of GDP
- Standard macro models assume marginal cost or constant markup pricing for durables
  - DSGE models with durables
  - Barsky, House and Kimball (2007)
- Endogenous price dynamics can affect the cyclical properties of durables
- Pass-through and markup dynamics with durable good pricing
- (Interesting time inconsistency problem)

Motivation

Gopinath, Itskhoki and Neiman (2011)

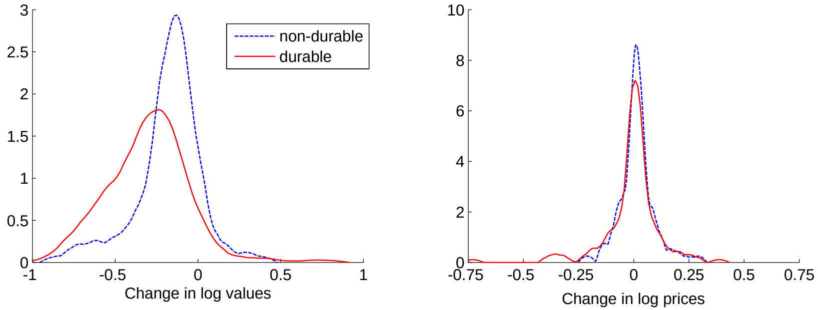

Figure: Change in US Import Values and Prices, 2008:07–2009:06

Main Findings

- Assumptions
  - — Some degree of monopolistic power
  - — Lack of commitment by firms
  - — Discrete time periods between price setting

- Results

  1. Endogenous markup dynamics
     - — markups decrease with the stock of durables
  2. 'Countercyclical' markups in response to cost shocks
     - — incomplete pass-through
  3. 'Procyclical' markups in response to demand shocks
  4. Adjustment-cost-like effect on quantities

Literature

Durable Monopoly Pricing

- Coase conjecture
  - Coase (1972), Stokey (1981), Bulow (1982), Gul et al. (1986), Bond and Samuelson (1984)
  - We focus on: $\Delta t \gg 0$, $\delta > 0$, dynamics
- Durable-good oligopoly pricing
  - Gul (1987), Esteban (2003), Esteban and Shum (2007)
  - We focus on: dynamics of markups, GE
- Macro models
  - Caplin and Leahy (2006), Parker (2001)
  - We focus on: general demand and market structures, GE

Demand

- Representative agent solves:

$$\max_{\{C_t, D_t, X_t, \dots\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U(C_t, D_t) \quad \text{s.t.} \quad \begin{array}{l} P_{Ct} C_t + P_t X_t \leq E_t \\ D_t = (1-\delta) D_{t-1} + X_t \end{array}$$

  Denote Λ_t the LM on expenditure constraint

  - Partial durability, $\delta \in (0, 1)$
  - Discrete time, $\beta < 1$

- Optimal choice of D_t satisfies:

$$u'(D_t; \xi_t) = P_t - \beta(1-\delta)\mathbb{E}_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} P_{t+1} \right\},$$

  where $u'(D_t, \xi_t) = U_D(C_t, D_t)/\Lambda_t$ and ξ_t is a stand-in for an arbitrary demand shock

- Approximation: $\Lambda_t \approx \mathit{const}$ (implies constant interest rate)

Demand

Two special cases

- Constant-elasticity demand:

$$u'(D,\xi) = \xi \cdot D^{-1/\sigma}$$

  — in the limit $\delta \to 1$ results in constant markup pricing

- Linear demand:

$$u'(D,\xi) = a + \xi - bD$$

  — yields simple closed-form solutions

Market Structure

- Market structure:
  - Monopoly
  - Monopolistic competition
  - Homogenous-good Oligopoly
  - Next time: differentiated-good oligopoly
- Equilibrium concept:
  - Commitment (benchmark)
  - Discretion (Markov Perfect Equilibrium)
  - Not for now: reputational equilibria under oligopoly

Durable Good Monopoly
Commitment

- Optimal pricing with commitment

$$V^C(D_{-1}) = \max_{\{P_t, X_t, D_t\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (P_t - W_t) X_t$$

subject to durable stock dynamics

$$D_t = X_t + (1-\delta) D_{t-1}$$

and durable-good demand

$$u'(D_t, \xi_t) = P_t - \beta(1-\delta)\mathbb{E}_t P_{t+1}$$

and initial condition $D_{-1} = 0$

Commitment
(continued)

- First-order optimality:

$$\begin{aligned} P_0: \quad & D_0 - (1-\delta)D_{-1} = \lambda_0, \\ P_t, t \geq 1: \quad & D_t - (1-\delta)D_{t-1} = \lambda_t - (1-\delta)\lambda_{t-1}, \\ D_t, t \geq 0: \quad & (P_t - W_t) - \beta(1-\delta)\mathbb{E}_t\{P_{t+1} - W_{t+1}\} \\ & \qquad = -\lambda_t u''(D_t, \xi_t), \end{aligned}$$

where λ_t is LM on demand constraint

- Given initial condition ($D_{-1} = 0$), we have $\lambda_t \equiv D_t$ (commitment $\sim$ leasing)

- Optimality condition:

$$(P_t - W_t) - \beta(1-\delta)\mathbb{E}_t\{P_{t+1} - W_{t+1}\} = \underbrace{-D_t u''(D_t, \xi_t)}_{\equiv \frac{1}{\sigma_t} u'(D_t, \xi_t)}$$

Commitment

(continued)

- Combining optimality condition with demand:

$$u'(D_t,\xi_t) + D_t u''(D_t,\xi_t) = W_t - \beta(1-\delta)\mathbb{E}_t W_{t+1}$$
$$P_t - \beta(1-\delta)\mathbb{E}_t P_{t+1} = u'(D_t,\xi_t)$$

- Contrast with marginal cost pricing:

$$u'(D_t,\xi_t) = W_t - \beta(1-\delta)\mathbb{E}_t W_{t+1}$$

Proposition

Durable pricing with commitment features
no endogenous dynamics:

— $P_t \equiv \bar{P}$ *when there are no shocks* (W_t *and* ξ_t *constant*)

— D_{t-1} *does not affect* P_t, *controlling for* W_t *and* ξ_t

— P_t *inherits the exogenous persistence of* W_t *and* ξ_t details

- Constant-elasticity demand

  $\longrightarrow$ constant markup pricing

$$P_t = \frac{\sigma}{\sigma - 1} W_t$$

- Linear demand

$$P_t = \frac{1}{2}\left[\frac{a}{1-\beta(1-\delta)} + \frac{\xi_t}{1-\rho_\xi \beta(1-\delta)} + W_t\right]$$

  — response to cost shocks does not depend on δ

  — level of markup increases with durability

Durable Good Monopoly

Discretion

- Time inconsistency problem:
  - demand depends on expected price tomorrow
  - firm wants to promise high price tomorrow
  - but tomorrow it fails to internalize the effect of price on previous-period demand
  - firm competes with itself across time and in the limit of continuous time firm loses all monopoly power (Coase)

- Solution concept:
  - consumers are infinitesimal, form rational expectations about future prices and purchase durables according to demand
  - the firm set today's price to maximize value anticipating its inability to commit to future prices
  - accumulated stock of durables is the state variable
  - Markov Perfect Equilibrium

- Optimal price duration? Commitment versus flexibility

Discretion
(continued)

- Formally, the problem of the firm:

$$
\begin{aligned}
V(D_{-1}, W, \xi) &= \max_{(P,X,D)} \left\{ (P - W)X + \beta \mathbb{E} V(D, W', \xi') \right\} \\
\text{s.t.} \qquad D &= X + (1-\delta) D_{-1}, \\
u'(D, \xi) &= P - \beta(1-\delta)\mathbb{E}_t p(D, W', \xi')
\end{aligned}
$$

- Equilibrium requirement:

$$
p(D_{-1}, W, \xi) = \arg \max_{(P,X,D)} \left\{ (P - W)X + \beta \mathbb{E} V(D, W', \xi') \right\}
$$

is the equilibrium strategy of the firm given state variable

Discretion
(continued)

- Optimality condition:

$$(P_t - W_t) - \beta(1-\delta)\mathbb{E}_t\{P_{t+1} - W_{t+1}\}$$
$$= \big(D_t - (1-\delta)D_{t-1}\big)\frac{1}{-\varphi'(P_t, W_t, \xi_t)},$$

  where demand slope is

$$\varphi'(P_t, W_t, \xi_t) = \frac{1}{u''(D_t, \xi_t) + \beta(1-\delta)\mathbb{E}_t p'(D_t, W_{t+1}, \xi_{t+1})}$$

  — Perturbation argument
  — Lack of commitment (contrast with leasing)
  — State variable dynamics:

$$D_t = \varphi\big(p(D_{t-1}, W_t, \xi_t), W_t, \xi_t\big) = f(D_{t-1}, W_t, \xi_t)$$

Proposition

(a) *Steady state:*

$$\bar{P} = \frac{\bar{\sigma}}{\bar{\sigma} - \delta\bar{\kappa}} W,$$

where $\bar{\sigma} \equiv \frac{-u'(\bar{D})}{\bar{D}u''(\bar{D})}$, $\bar{\kappa} \equiv 1 + \frac{\beta(1-\delta)p'(\bar{D})}{u''(\bar{D})} > 1$, $u'(\bar{D}) = [1-\beta(1-\delta)]\bar{P}$.

(b) *Endogenous dynamics:*

D_{t-1} *is state variable for pricing at* t *and* $p'(\cdot, W, \xi) < 0$.

Proposition

With linear demand and AR(1) demand and cost shocks, there exists a linear equilibrium:

$$P_t = \bar{P} - \alpha(D_{t-1} - \bar{D}) + \gamma(W_t - \bar{W}) + \omega\xi_t,$$
$$D_t = \bar{D} + \phi(D_{t-1} - \bar{D}) - \psi(W_t - \bar{W}) + \chi\xi_t,$$

with $\alpha > 0$, $\phi \in (0, 1 - \delta)$, $\gamma \in (0.5, 1)$, $\omega, \psi, \chi > 0$. details

Corollary

(i) D_t *increases over time, as prices and markups fall.*

(ii) *markups increase (procyclical) with demand shocks and decrease (countercyclical) with cost shocks.*

Monopolistic competition

- D-good is a CES aggregator of varieties:

$$D_t = \left(\int_0^1 D_{it}^{\frac{\sigma-1}{\sigma}} \mathrm{d}i \right)^{\frac{\sigma}{\sigma-1}}$$

- Two alternative assumptions:

  (i) Durable aggregator: $D_t = X_t + (1-\delta)D_{t-1}$.

  Constant markup pricing (Barsky et al., 2007)

  (ii) Durable varieties: $D_{it} = X_{it} + (1-\delta)D_{i,t-1}$.

  Problem isomorphic to that of a monopolist with ξ_t related to the equilibrium dynamics of D_t

Durable Good Oligopoly

Commitment (Cournot-Nash)

- Consider N symmetric firms producing a homogenous durable good with constant marginal cost and no shocks

- Durable good dynamics

$$D_t = (1-\delta)D_{t-1} + \sum_{i=1}^{N} x_{it}$$

- A given firm commits to a sequence $\{\tilde{x}_{it}\}$ given the symmetric strategy of the other $N-1$ firms $\{x_t\}$. In equilibrium, $\tilde{x}_t = x_t$

- In equilibrium, $x_t = \frac{1}{N}\big(D_t - (1-\delta)D_{t-1}\big)$ and $\lambda_t = D_t/N \Rightarrow$

$$(P_t - W_t) - \beta(1-\delta)\mathbb{E}_t\{P_{t+1} - W_{t+1}\} = -\frac{D_t}{N}u''(D_t, \xi_t)$$

- Under discretion, both competition within firm over time and between firms at a given t reduces markups

- A firm chooses $\tilde{x}(D_-)$ given the symmetric strategy $x(D_-)$ of the other $N-1$ firms and equilibrium price next period $p(D)$:

$$v(D_-) = \max_{\tilde{x},D,P} \{(P-W)\tilde{x} + \beta v(D)\}$$

$$\text{s.t.} \qquad D = (1-\delta)D_- + (N-1)x(D_-) + \tilde{x}$$

$$P = u'(D) + \beta(1-\delta)\mathbb{E}p(D)$$

- The solution to this problem in equilibrium yields:

$$\tilde{x}(D_-) = x(D_-), \qquad P = p(D_-),$$

$$D = f(D_-) = (1-\delta)D_- + Nx(D_-)$$

- Optimality condition for a firm:

$$(P - W) - \beta\big[(1-\delta) + (N-1)x'(D)\big](P' - W') \\ = \tilde{x}(D_-)\big(-u''(D) - \beta(1-\delta)p'(D)\big)$$

- Impose equilibrium:

$$\tilde{x}(D_-) = x(D_-) = \frac{1}{N}\big(f(D_-) - (1-\delta)D\big)$$

- Then equilibrium dynamics is characterized by

$$u'(D_t) = P_t - \beta(1-\delta)P_{t+1},$$

$$(P_t - W) - \beta(P_{t+1} - W)\left(\frac{1-\delta}{N} + \frac{N-1}{N}f'(D_t)\right) \\ = \frac{D_t - (1-\delta)D_{t-1}}{N}\frac{1}{-\varphi'(P_t)},$$

where $D_t = f(D_{t-1})$, $P_t = p(D_{t-1})$ and $\varphi(\cdot) = f\big(p^{-1}(\cdot)\big)$.

Proposition

With linear demand, there exists a linear oligopoly equilibrium:

$$D_t = \bar{D} + \phi^{(N)}(D_{t-1} - \bar{D}) \quad \textit{and} \quad P_t = \bar{P} - \alpha^{(N)}(D_{t-1} - \bar{D}).$$

$\phi^{(N)}$ *and* $\alpha^{(N)}$ *decrease in* N. details

— As number of firms increases, prices are closer to marginal cost and there is less endogenous dynamics

- Steady state markup cannot be solved for without $p'(\bar{D})$.
- To compute the steady state markup exactly, we need to know all derivatives of the policy function $p(D)$ at $\bar{D}$.
- Similar problem arises in hyperbolic discounting
    - Krusell, Kuruscu, and Smith (2002)
    - Judd (2004)
    - Polynomial approximations
- In the case of durables, polynomial approximations should work perfectly.
- Each additional higher order term is suppressed by ϕ^n.

- In the case of monopoly, the transition function $f(D)$ satisfies

$$\frac{(1-\beta(1-\delta))W-u'(f(D))}{f(D)-(1-\delta)D}-u''(f(D))$$

$$=\beta(1-\delta)\frac{(1-\beta(1-\delta))W-u'(f(f(D)))}{f(f(D))-(1-\delta)f(D)}f'(f(D))$$

- Express $f(D)$ as a power series.
- When Taylor expanded, the functional equation gives an infinite number of conditions for the derivatives of $f(D)$ at $\bar{D}$
- The first one links $\bar{D}$ and $f'(\bar{D})$.
- The second one links $\bar{D}$, $f'(\bar{D})$, and $f''(\bar{D})$.
- The third one links $\bar{D}$ and the first three derivatives.
- Etc.

- If we set $f^{(n)}(\bar{D})$ to zero and solve the system, we make only a small mistake proportional to ϕ^n, where $\phi \equiv f'(\bar{D})$
- In practice, only a couple of terms will be needed.
- When translated to the GE context, this means that it is possible to solve GE models with durables and discretion, for arbitrary utility functions.

Numerical Example

- $\beta = 0.9$
- $\delta = 0.2$
- Constant elasticity $\sigma = 2$
- Value function iteration on a grid, polynomial smoothing:

$$V(D_-) = \max_D \Big\{ \big(u'(D) + \beta(1-\delta)p(D) - W\big)\big(D - (1-\delta)D_-\big) + \beta V(D) \Big\}$$

Update $\tilde{V}(D_-)$ and $D = \tilde{f}(D_-)$, and calculate

$$\tilde{p}(D_-) = u'\big(f(D_-)\big) + \beta(1-\delta)p\big(f(D_-)\big)$$

Polynomially smooth $f(\cdot)$ and $p(\cdot)$

Numerical Example

Dynamics with no shocks

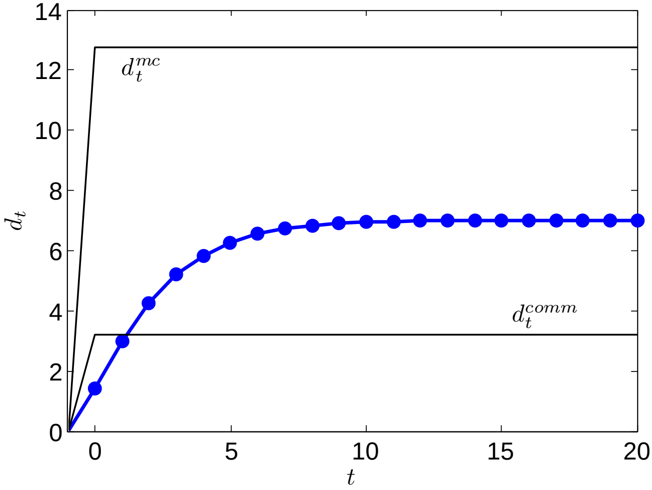

Figure: Dynamic path of D_t

Numerical Example

Dynamics with no shocks

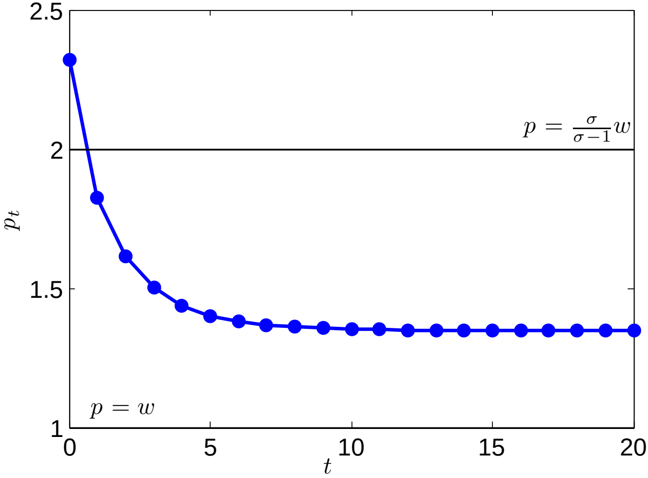

Figure: Dynamic path of P_t

Numerical Example

Unexpected permanent cost increase

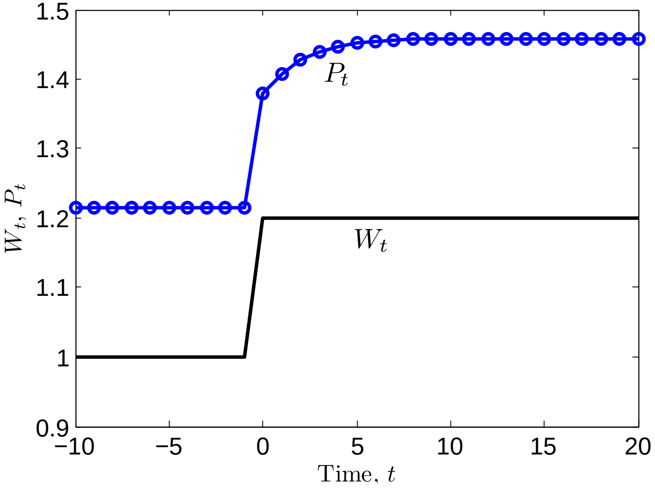

Figure: Response of P_t

Numerical Example

Unexpected permanent cost increase

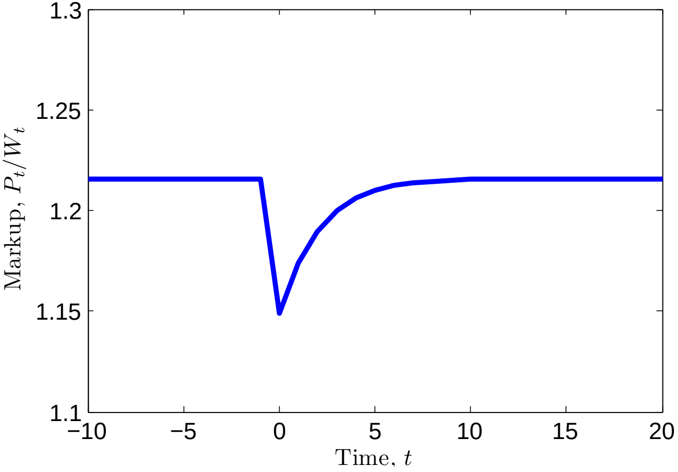

Figure: Response of markup, P_t/W_t

Numerical Example

Unexpected permanent cost increase

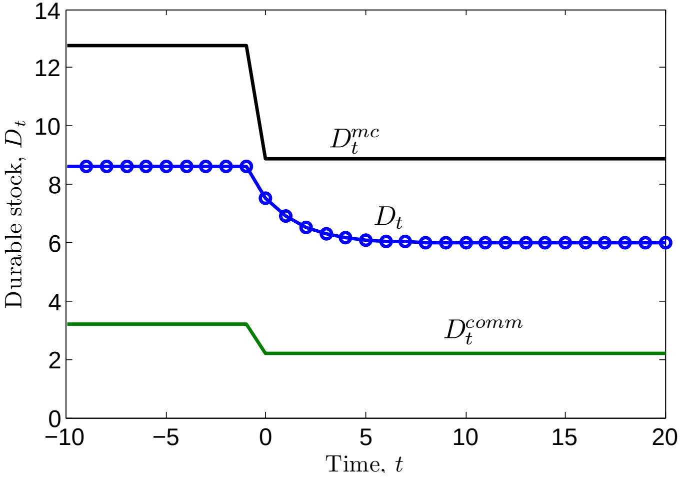

Figure: Response of D_t

Numerical Example

Unexpected permanent demand increase

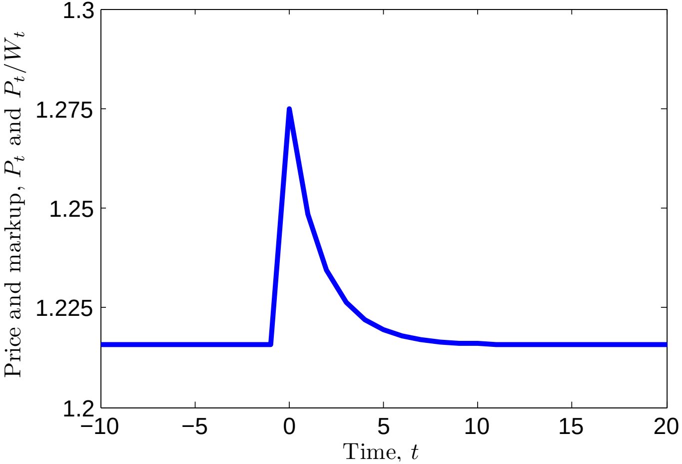

Figure: Response of P_t and markup P_t/W_t

Numerical Example

Unexpected permanent demand increase

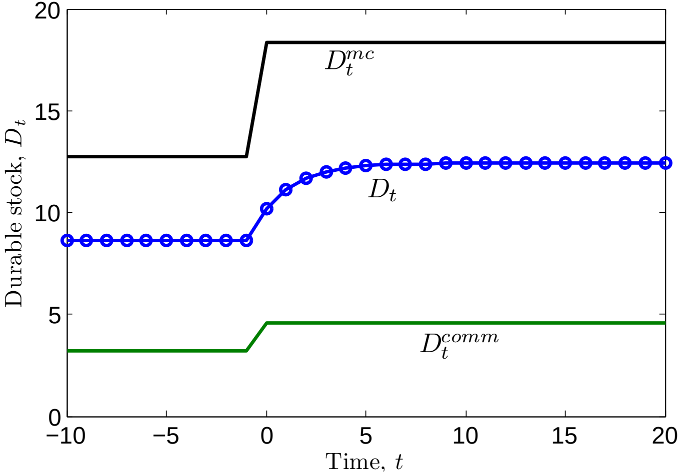

Figure: Response of D_t

Table: Statistical properties

$\log(\cdot)$	σ (%)	ρ	$\mathrm{corr}(\cdot, \log W_t)$
Wage, W_t	4.9	0.80	1.00
Price, P_t	5.1	0.90	0.88
Markup, P_t/W_t	2.2	0.69	-0.19
Durable stock, D_t			
— constant markup	15.5	0.79	-0.99
— discretion	12.2	0.95	-0.75
— ratio (disc/comm)			0.29
Durable purchases, X_t			
— constant markup	70.7	-0.08	-0.31
— discretion	21.4	0.57	-0.91
— ratio (disc/comm)			0.16

Numerical Example

Stochastic cost shocks

Table: Pass-through

	$\log W_t$	$\log W_{t-1}$
$\log P_t$	0.91	
$\log P_t$	0.65	0.34
	$\Delta \log W_t$	$\Delta \log W_{t-1}$
$\Delta \log P_t$	0.61	
$\Delta \log P_t$	0.63	0.15

Numerical Example

Stochastic demand shocks

Table: Statistical properties

$\log(\cdot)$	σ (%)	ρ	$\mathrm{corr}(\cdot, \log \xi_t)$
Demand, ξ_t	4.8	0.77	1.00
Price and markup, P_t/W	1.9	0.79	-0.18
Durable stock, D_t			
— constant markup	9.7	0.77	1.00
— discretion	7.2	0.94	0.66
— ratio (disc/comm)			-0.22
Durable purchases, X_t			
— constant markup	36.1	-0.03	0.91
— discretion	13.6	0.56	0.55

Conclusion

- Durable monopoly pricing results in endogenous dynamics
- Procyclical markups in response to demand shocks
- Countercyclical markups in response to cost shocks (incomplete pass-through)
- Oligopoly: endogenous dynamics dies out with N
- Next steps: general equilibrium, quantitative evaluation