Price Dynamics for Durable Goods

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Motivation

- Durables play a crucial role in business cycle fluctuations
  - ~60% of non-service consumption, all of investment
  - most volatile component of GDP

- Standard macro models assume marginal cost or constant markup pricing for durables
  - DSGE models with durables
  - Barsky, House and Kimball (2007)

- Endogenous price dynamics can affect the cyclical properties of durables

- Pass-through and markup dynamics with durable good pricing

- (Interesting time inconsistency problem)
Motivation
Gopinath, Itskhoki and Neiman (2011)

Figure: Change in US Import Values and Prices, 2008:07–2009:06
Main Findings

• Assumptions
  — Some degree of monopolistic power
  — Lack of commitment by firms
  — Discrete time periods between price setting

• Results
  1. Endogenous markup dynamics
     — markups decrease with the stock of durables
  2. ‘Countercyclical’ markups in response to cost shocks
     — incomplete pass-through
  3. ‘Procyclical’ markups in response to demand shocks
  4. Adjustment-cost-like effect on quantities
Literature

Durable Monopoly Pricing

• Coase conjecture
  — We focus on: $\Delta t \gg 0$, $\delta > 0$, dynamics

• Durable-good oligopoly pricing
  — We focus on: dynamics of markups, GE

• Macro models
  — Caplin and Leahy (2006), Parker (2001)
  — We focus on: general demand and market structures, GE
Demand

• Representative agent solves:

\[
\max_{\{C_t, D_t, X_t, \ldots\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U(C_t, D_t) \quad \text{s.t.} \quad P C_t C_t + P_t X_t \leq E_t \\
D_t = (1 - \delta)D_{t-1} + X_t
\]

Denote \( \Lambda_t \) the LM on expenditure constraint

— Partial durability, \( \delta \in (0, 1) \)
— Discrete time, \( \beta < 1 \)

• Optimal choice of \( D_t \) satisfies:

\[
u'(D_t; \xi_t) = P_t - \beta(1 - \delta) \mathbb{E}_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} P_{t+1} \right\}
\]

where \( u'(D_t, \xi_t) = U_D(C_t, D_t)/\Lambda_t \) and \( \xi_t \) is a stand-in for an arbitrary demand shock

• Approximation: \( \Lambda_t \approx \text{const} \) (implies constant interest rate)
Demand
Two special cases

- Constant-elasticity demand:
  \[ u'(D, \xi) = \xi \cdot D^{-1/\sigma} \]
  - in the limit \( \delta \to 1 \) results in constant markup pricing

- Linear demand:
  \[ u'(D, \xi) = a + \xi - bD \]
  - yields simple closed-form solutions
Market Structure

- Market structure:
  - Monopoly
  - Monopolistic competition
  - Homogenous-good Oligopoly
  - Next time: differentiated-good oligopoly

- Equilibrium concept:
  - Commitment (benchmark)
  - Discretion (Markov Perfect Equilibrium)
  - Not for now: reputational equilibria under oligopoly
Durable Good Monopoly
Commitment

- Optimal pricing with commitment

\[ V^C(D_{-1}) = \max_{\{P_t, X_t, D_t\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (P_t - W_t) X_t \]

subject to durable stock dynamics

\[ D_t = X_t + (1 - \delta) D_{t-1} \]

and durable-good demand

\[ u'(D_t, \xi_t) = P_t - \beta (1 - \delta) \mathbb{E}_t P_{t+1} \]

and initial condition \( D_{-1} = 0 \)
Commitment
(continued)

• First-order optimality:

\[ P_0 : \quad D_0 - (1 - \delta)D_{-1} = \lambda_0, \]
\[ P_t, t \geq 1 : \quad D_t - (1 - \delta)D_{t-1} = \lambda_t - (1 - \delta)\lambda_{t-1}, \]
\[ D_t, t \geq 0 : \quad (P_t - W_t) - \beta(1 - \delta)\mathbb{E}_t\{P_{t+1} - W_{t+1}\} = -\lambda_t u''(D_t, \xi_t), \]

where \( \lambda_t \) is LM on demand constraint

• Given initial condition \((D_{-1} = 0)\), we have \( \lambda_t \equiv D_t \) (commitment \( \sim \) leasing)

• Optimality condition:

\[ (P_t - W_t) - \beta(1 - \delta)\mathbb{E}_t\{P_{t+1} - W_{t+1}\} = -D_t u''(D_t, \xi_t) \equiv \frac{1}{\sigma_t} u'(D_t, \xi_t) \]
Commitment
(continued)

- Combining optimality condition with demand:

\[ u'(D_t, \xi_t) + D_t u''(D_t, \xi_t) = W_t - \beta(1 - \delta) \mathbb{E}_t W_{t+1} \]

\[ P_t - \beta(1 - \delta) \mathbb{E}_t P_{t+1} = u'(D_t, \xi_t) \]

- Contrast with marginal cost pricing:

\[ u'(D_t, \xi_t) = W_t - \beta(1 - \delta) \mathbb{E}_t W_{t+1} \]

**Proposition**

*Durable pricing with commitment features*

no endogenous dynamics:

- \( P_t \equiv \bar{P} \) when there are no shocks \((W_t \text{ and } \xi_t \text{ constant})\)
- \( D_{t-1} \) does not affect \( P_t \), controlling for \( W_t \) and \( \xi_t \)
- \( P_t \) inherits the exogenous persistence of \( W_t \) and \( \xi_t \)
Commitment

Two special cases

- **Constant-elasticity demand**
  - constant markup pricing
  \[ P_t = \frac{\sigma}{\sigma - 1} W_t \]

- **Linear demand**
  \[ P_t = \frac{1}{2} \left[ \frac{a}{1 - \beta(1 - \delta)} + \frac{\xi_t}{1 - \rho\xi\beta(1 - \delta)} + W_t \right] \]
  - response to cost shocks does not depend on \( \delta \)
  - level of markup increases with durability
Durable Good Monopoly

Discretion

• Time inconsistency problem:
  — demand depends on expected price tomorrow
  — firm wants to promise high price tomorrow
  — but tomorrow it fails to internalize the effect of price on previous-period demand
  — firm competes with itself across time and in the limit of continuous time firm loses all monopoly power (Coase)

• Solution concept:
  — consumers are infinitesimal, form rational expectations about future prices and purchase durables according to demand
  — the firm set today’s price to maximize value anticipating its inability to commit to future prices
  — accumulated stock of durables is the state variable
  — Markov Perfect Equilibrium

• Optimal price duration? Commitment versus flexibility
Discretion
(continued)

• Formally, the problem of the firm:

\[ V(D_{-1}, W, \xi) = \max_{(P, X, D)} \left\{ (P - W)X + \beta \mathbb{E}V(D, W', \xi') \right\} \]

s.t. \[ D = X + (1 - \delta)D_{-1}, \]
\[ u'(D, \xi) = P - \beta(1 - \delta)\mathbb{E}_t p(D, W', \xi') \]

• Equilibrium requirement:

\[ p(D_{-1}, W, \xi) = \arg \max_{(P, X, D)} \left\{ (P - W)X + \beta \mathbb{E}V(D, W', \xi') \right\} \]

is the equilibrium strategy of the firm given state variable
Discretion
(continued)

• Optimality condition:

\[(P_t - W_t) - \beta(1 - \delta) \mathbb{E}_t \{P_{t+1} - W_{t+1}\} \]

\[= (D_t - (1 - \delta)D_{t-1}) \frac{1}{-\varphi'(P_t, W_t, \xi_t)},\]

where demand slope is

\[\varphi'(P_t, W_t, \xi_t) = \frac{1}{u''(D_t, \xi_t) + \beta(1 - \delta) \mathbb{E}_t p'(D_t, W_{t+1}, \xi_{t+1})}\]

— Perturbation argument
— Lack of commitment (contrast with leasing)
— State variable dynamics:

\[D_t = \varphi(p(D_{t-1}, W_t, \xi_t), W_t, \xi_t) = f(D_{t-1}, W_t, \xi_t)\]
Discretion
General Results

Proposition

(a) Steady state:

\[ \bar{P} = \frac{\bar{\sigma}}{\bar{\sigma} - \delta \bar{\kappa}} \bar{W}, \]

where \( \bar{\sigma} \equiv \frac{-u'(\bar{D})}{Du''(\bar{D})} \), \( \bar{\kappa} \equiv 1 + \frac{\beta(1-\delta)p'(\bar{D})}{u''(\bar{D})} > 1 \), \( u'(\bar{D}) = [1 - \beta(1 - \delta)]\bar{P} \).

(b) Endogenous dynamics:

\( D_{t-1} \) is state variable for pricing at \( t \) and \( p'(\cdot, W, \xi) < 0 \).
Proposition

With linear demand and AR(1) demand and cost shocks, there exists a linear equilibrium:

\[
P_t = \bar{P} - \alpha (D_{t-1} - \bar{D}) + \gamma (W_t - \bar{W}) + \omega \xi_t,
\]
\[
D_t = \bar{D} + \phi (D_{t-1} - \bar{D}) - \psi (W_t - \bar{W}) + \chi \xi_t,
\]

with \(\alpha > 0\), \(\phi \in (0, 1 - \delta)\), \(\gamma \in (0.5, 1)\), \(\omega, \psi, \chi > 0\).

Corollary

(i) \(D_t\) increases over time, as prices and markups fall.

(ii) markups increase (\textit{procyclical}) with demand shocks and decrease (\textit{countercyclical}) with cost shocks.
Monopolistic competition

- $D$-good is a CES aggregator of varieties:

$$D_t = \left( \int_0^1 D_{it}^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}}$$

- Two alternative assumptions:

  (i) Durable aggregator: $D_t = X_t + (1 - \delta)D_{t-1}$.

  Constant markup pricing (Barsky et al., 2007)

  (ii) Durable varieties: $D_{it} = X_{it} + (1 - \delta)D_{i,t-1}$.

  Problem isomorphic to that of a monopolist with $\xi_t$ related to the equilibrium dynamics of $D_t$
Durable Good Oligopoly
Commitment (Cournot-Nash)

- Consider $N$ symmetric firms producing a homogenous durable good with constant marginal cost and no shocks

- Durable good dynamics

$$D_t = (1 - \delta)D_{t-1} + \sum_{i=1}^{N} x_{it}$$

- A given firm commits to a sequence $\{\tilde{x}_{it}\}$ given the symmetric strategy of the other $N - 1$ firms $\{x_t\}$. In equilibrium, $\tilde{x}_t = x_t$

- In equilibrium, $x_t = \frac{1}{N} (D_t - (1 - \delta)D_{t-1})$ and $\lambda_t = D_t/N \Rightarrow$

$$(P_t - W_t) - \beta(1 - \delta)\mathbb{E}_t\{P_{t+1} - W_{t+1}\} = -\frac{D_t}{N} u''(D_t, \xi_t)$$
Durable Good Oligopoly
Discretion (Cournot-MPE)

• Under discretion, both competition within firm over time and between firms at a given \( t \) reduces markups

• A firm chooses \( \tilde{x}(D_-) \) given the symmetric strategy \( x(D_-) \) of the other \( N-1 \) firms and equilibrium price next period \( p(D) \):

\[
\nu(D_-) = \max_{\tilde{x}, D, P} \{ (P - W)\tilde{x} + \beta \nu(D) \}
\]

s.t.
\[
D = (1 - \delta)D_- + (N - 1)x(D_-) + \tilde{x}
\]
\[
P = u'(D) + \beta(1 - \delta)E_p(D)
\]

• The solution to this problem in equilibrium yields:

\[
\tilde{x}(D_-) = x(D_-), \quad P = p(D_-), \quad D = f(D_-) = (1 - \delta)D_- + Nx(D_-)
\]
Durable Good Oligopoly
Discretion (Cournot-MPE)

- Optimality condition for a firm:
  \[(P - W) - \beta [(1 - \delta) + (N - 1)x'(D)] (P' - W')\]
  \[= \tilde{x}(D_-) (-u''(D) - \beta (1 - \delta)p'(D))\]

- Impose equilibrium:
  \[\tilde{x}(D_-) = x(D_-) = \frac{1}{N} (f(D_-) - (1 - \delta)D)\]

- Then equilibrium dynamics is characterized by
  \[u'(D_t) = P_t - \beta (1 - \delta)P_{t+1},\]
  \[(P_t - W) - \beta (P_{t+1} - W) \left( \frac{1 - \delta}{N} + \frac{N - 1}{N} f'(D_t) \right)\]
  \[= \frac{D_t - (1 - \delta)D_{t-1}}{N} \frac{1}{-\varphi'(P_t)},\]

where \(D_t = f(D_{t-1}), P_t = p(D_{t-1})\) and \(\varphi(\cdot) = f(p^{-1}(\cdot))\).
Proposition

With linear demand, there exists a linear oligopoly equilibrium:

\[ D_t = \bar{D} + \phi^{(N)}(D_{t-1} - \bar{D}) \quad \text{and} \quad P_t = \bar{P} - \alpha^{(N)}(D_{t-1} - \bar{D}). \]

\( \phi^{(N)} \) and \( \alpha^{(N)} \) decrease in \( N \).

— As number of firms increases, prices are closer to marginal cost and there is less endogenous dynamics
General Utility Functions

Approximation

- Steady state markup cannot be solved for without $p'(\bar{D})$.
- To compute the steady state markup exactly, we need to know all derivatives of the policy function $p(D)$ at $\bar{D}$.
- Similar problem arises in hyperbolic discounting
  - Krusell, Kuruscu, and Smith (2002)
  - Judd (2004)
  - Polynomial approximations
- In the case of durables, polynomial approximations should work perfectly.
- Each additional higher order term is suppressed by $\phi^n$. 
In the case of monopoly, the transition function $f(D)$ satisfies

$$
\frac{(1 - \beta (1 - \delta)) W - u'(f(D))}{f(D) - (1 - \delta) D} - u''(f(D))
$$

$$
= \beta (1 - \delta) \frac{(1 - \beta (1 - \delta)) W - u'(f(f(D))))}{f(f(D)) - (1 - \delta) f(D)} f'(f(D))
$$
• Express $f(D)$ as a power series.
• When Taylor expanded, the functional equation gives an infinite number of conditions for the derivatives of $f(D)$ at $\bar{D}$
  • The first one links $\bar{D}$ and $f'(\bar{D})$.
  • The second one links $\bar{D}$, $f'(\bar{D})$, and $f''(\bar{D})$.
  • The third one links $\bar{D}$ and the first three derivatives.
• Etc.
If we set $f^{(n)}(\bar{D})$ to zero and solve the system, we make only a small mistake proportional to $\phi^n$, where $\phi \equiv f'(\bar{D})$.

In practice, only a couple of terms will be needed.

When translated to the GE context, this means that it is possible to solve GE models with durables and discretion, for arbitrary utility functions.
Numerical Example

- $\beta = 0.9$
- $\delta = 0.2$
- Constant elasticity $\sigma = 2$
- Value function iteration on a grid, polynomial smoothing:

$$V(D-) = \max_D \left\{ (u'(D) + \beta (1 - \delta)p(D) - W)(D - (1 - \delta)D-) + \beta V(D) \right\}$$

Update $\tilde{V}(D-)$ and $D = \tilde{f}(D-)$, and calculate

$$\tilde{p}(D-) = u'(f(D-)) + \beta (1 - \delta)p(f(D-))$$

Polynomially smooth $f(\cdot)$ and $p(\cdot)$
Numerical Example
Dynamics with no shocks

Figure: Dynamic path of $D_t$
Numerical Example
Dynamics with no shocks

Figure: Dynamic path of $P_t$

$$p = \frac{\sigma}{\sigma - 1} w$$
Numerical Example

Unexpected permanent cost increase

Figure: Response of $P_t$
Numerical Example

Unexpected permanent cost increase

Figure: Response of markup, $P_t/W_t$
Numerical Example

Unexpected permanent cost increase

Figure: Response of $D_t$
Numerical Example

Unexpected permanent demand increase

Figure: Response of $P_t$ and markup $P_t/W_t$
Numerical Example

Unexpected permanent demand increase

Figure: Response of $D_t$
### Table: Statistical properties

<table>
<thead>
<tr>
<th>log(·)</th>
<th>σ (%)</th>
<th>ρ</th>
<th>corr(·, log $W_t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage, $W_t$</td>
<td>4.9</td>
<td>0.80</td>
<td>1.00</td>
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<tr>
<td>Price, $P_t$</td>
<td>5.1</td>
<td>0.90</td>
<td>0.88</td>
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<td>Markup, $P_t/W_t$</td>
<td>2.2</td>
<td>0.69</td>
<td>−0.19</td>
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<tr>
<td>Durable stock, $D_t$</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>— constant markup</td>
<td>15.5</td>
<td>0.79</td>
<td>−0.99</td>
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<tr>
<td>— discretion</td>
<td>12.2</td>
<td>0.95</td>
<td>−0.75</td>
</tr>
<tr>
<td>— ratio (disc/comm)</td>
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<tr>
<td>Durable purchases, $X_t$</td>
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<tr>
<td>— constant markup</td>
<td>70.7</td>
<td>−0.08</td>
<td>−0.31</td>
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<tr>
<td>— discretion</td>
<td>21.4</td>
<td>0.57</td>
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<tr>
<td>— ratio (disc/comm)</td>
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<td>0.16</td>
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Numerical Example

Stochastic cost shocks

Table: Pass-through

<table>
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<tr>
<th></th>
<th>$\log W_t$</th>
<th>$\log W_{t-1}$</th>
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<tr>
<td>$\log P_t$</td>
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<tr>
<td>$\log P_t$</td>
<td>0.65</td>
<td>0.34</td>
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<th>$\Delta \log W_t$</th>
<th>$\Delta \log W_{t-1}$</th>
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<td>$\Delta \log P_t$</td>
<td>0.61</td>
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<tr>
<td>$\Delta \log P_t$</td>
<td>0.63</td>
<td>0.15</td>
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## Numerical Example

Stochastic demand shocks

<table>
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<th>log(·)</th>
<th>$\sigma$ (%)</th>
<th>$\rho$</th>
<th>corr(·, log $\xi_t$)</th>
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<tbody>
<tr>
<td>Demand, $\xi_t$</td>
<td>4.8</td>
<td>0.77</td>
<td>1.00</td>
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<td>Price and markup, $P_t/W$</td>
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<td>$-0.18$</td>
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<tr>
<td>Durable stock, $D_t$</td>
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<tr>
<td>— constant markup</td>
<td>9.7</td>
<td>0.77</td>
<td>1.00</td>
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<tr>
<td>— discretion</td>
<td>7.2</td>
<td>0.94</td>
<td>0.66</td>
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<tr>
<td>— ratio (disc/comm)</td>
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<td>$-0.22$</td>
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<tr>
<td>Durable purchases, $X_t$</td>
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<td></td>
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<tr>
<td>— constant markup</td>
<td>36.1</td>
<td>$-0.03$</td>
<td>0.91</td>
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<tr>
<td>— discretion</td>
<td>13.6</td>
<td>0.56</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Table: Statistical properties
Conclusion

- Durable monopoly pricing results in endogenous dynamics
- Procyclical markups in response to demand shocks
- Countercyclical markups in response to cost shocks (incomplete pass-through)
- Oligopoly: endogenous dynamics dies out with $N$
- Next steps: general equilibrium, quantitative evaluation