Firms, Trade and Labor Market Dynamics*

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Abstract

Adjustment to trade liberalization is associated with substantial reallocation of labor across firms within sectors. This salient feature of the data is well captured by the new generation of trade models, which however assume frictionless and instantaneous adjustments in the labor market. A natural question is whether labor market frictions slow down this reallocation process, thereby dissipating the gains from trade along the transition path. In this paper, we develop a model with heterogeneous firms and Diamond-Mortensen-Pissarides type frictions in the labor market, in which we fully characterize the transitional dynamic response to a trade liberalization. The sunk cost of hiring workers makes low-productivity non-exporting firms reluctant to fire workers and exit in the short run, which in turn crowds out the new more-productive entrants. This depresses aggregate productivity and trade flows during the adjustment process. Yet, despite the lengthy dynamic adjustment, the consumer gains from trade in this economy are achieved instantaneously and do not depend on the extent of the labor market frictions. Lastly, the trade shock creates short-run winners and losers among the \textit{ex ante} homogenous workers, with \textit{bad} jobs concentrated in the non-exporting firms adversely affected by trade liberalization.

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1 Introduction

Adjustment to trade liberalization is associated with substantial reallocation of labor, both across sectors, but even more importantly across firms within sectors (see e.g. Levinson, 1999). The new generation of trade models, following Melitz (2003), captures well this salient feature of the data, predicting large post-trade-liberalization reallocation of workers from shrinking non-exporting firms facing increased export competition towards expanding exporting firms (see e.g. Eaton, Kortum, and Kramarz, 2011). Much of the analysis in this literature, however, is confined either to the steady state comparative statics, or to the case of the frictionless labor markets, in which labor transitions happen instantaneously and all workers benefit from trade independently of their employment history.

A natural question, then, is to what extent labor market frictions slow down this reallocation process across firms, depressing the productivity and reducing the gains from trade in the short run, during the transition period. Additionally, do labor market frictions result in unequal distributional consequences for labor market outcomes, in particular creating winners and losers from trade in an ex ante homogenous pool of workers? We address this questions by developing a tractable trade model with heterogeneous firms and Diamond-Mortensen-Pissarides (DMP) search frictions in the labor market (as described in e.g. Pissarides, 2000), in which we fully characterize the transitional dynamics in response to a trade liberalization.

The model features two symmetric countries with a non-traded and a traded sector, both subject to the same labor market frictions. Firms in the traded sector are large (multiworker), heterogeneous, monopolistic competitors facing both fixed and variable trade costs to access the foreign market, as in Melitz (2003). There is free entry of firms in both sectors, and upon entry the firms can post costly vacancies to attract workers. The unemployed workers are perfectly mobile across sectors and, upon choosing a sector, can search for a job and randomly meet vacant firms. Upon matching, a firm and its workers bargain about wages without commitment. In order to switch jobs, both within or across sectors, workers need to first separate into unemployment. In this economy, we study the dynamic response to an unanticipated bilateral reduction in the trade costs.

In the new steady state, the least productive of the previously active firms exit, the firms in the middle of the productivity distribution shrink their employment under pressure from foreign competition, while the most productive firms expand their export sales and employment. This leads to a productivity and welfare improvement economy-wide and for each and every worker. We show, however, that the transition dynamics to this new steady state can take a long time, with less productive incumbent firms reluctant to fire their workers and exit, and choosing instead to gradually shrink their employment subject to the natural attrition forces. Continued employment and sales by these firms crowds out the new more productive entrants.
This results in misallocation of labor across firms, which in turn reduces the productivity in the traded sector.\footnote{There is a second force affecting the sectoral productivity, namely the positive variety effect from the incumbents surviving in the short run (as emphasized, for example, in Alessandria, Choi, and Ruhl, 2013). We show that the misallocation effect dominates in the short run, resulting in reduced productivity, while the variety effect dominates towards the end of the transition, when the unproductive incumbent firms shrink below a small enough size.} Furthermore, as non-exporters are relatively more prevalent among the incumbents, international trade flows are depressed during the transition, and reach their new steady state level only gradually, as less productive incumbents are replaced by a more selected group of new entrants.

This labor market dynamics creates distributional consequences for workers, which are tied to the heterogeneous outcomes of their employers in response to the trade liberalization. Workers employed by expanding firms, as well as the unemployed workers, gain from trade. At the same time, workers employed by less productive incumbents, which need to shrink after trade liberalization, fare less well and may even loose from trade. Some of these workers have to experience a spell of unemployment if their firm exits or fires part of its labor force, however, the other such workers see a decline in their wages during the transition period.\footnote{The income losses for these workers are, of course, bounded above by their outside option of separating into unemployment and finding a new job.} Therefore, the interaction between firm heterogeneity and labor market frictions creates “good” and “bad” jobs during the transition in response to a trade liberalization. These heterogenous outcomes for observationally identical workers, tied to the outcomes of their employers after a trade liberalization episode are consistent with the recent empirical evidence (see Verhoogen, 2008; Amiti and Davis, 2011; Helpman, Itskhoki, Muendler, and Redding, 2012).

Despite these rich cross-sectional patters during the potentially lengthy transition, the consumer gains from trade are realized instantaneously and, furthermore, do not depend on the extent of labor market frictions. Both of these results are intriguing. We show that the \textit{proportional} long-run gains from trade are equal to the gains in the consumer surplus, which do not depend on the labor market frictions, although these frictions are important in determining the long-run levels of productivity and welfare (as well as comparative advantage, as in the static model of Helpman and Itskhoki, 2010).

The fact that the consumer gains from trade are realized instantaneously relies on two main assumptions that we adopt—the free entry condition for firms and the mobility of unemployed workers across the traded and non-trade sector. The mobility of workers ties down the labor market conditions in the traded sector to those in the rest of the economy, assuming the traded sector never exhausts the full economy-wide pool of unemployed. The free entry condition in the non-traded sector determines the labor market conditions, while the free entry condition in the traded sector ties down the product market competition to the
labor market conditions and trade costs to ensure expected zero profits for the new entrants. Under these circumstances, there exists a unique level of prices (and hence of the consumer surplus), which is consistent with the new lower level of trade costs throughout the whole transition period. This “free-entry” logic is similar to that in Atkeson and Burstein (2010) in the context of a model with technology adoption and a frictionless labor market.

The overall dynamic gains from trade in our model equal the difference between the consumer gains from trade and the household income loss due to the reduction in worker wages and firm profits during the transition period. We show quantitatively that the aggregate income loss component, although increasing in the extent of labor market frictions, is small relative to the consumer gains from trade. At the same time, since the decline in wage income is heterogeneous across workers, it can lead to considerable distributional consequence. We further show that the balance of income losses between firms and workers depends on the extent of labor market frictions. In particular, firms bear most of the losses in rigid labor markets due to the large sunk hiring costs, which they have incurred before the trade shock and which make them reluctant to destroy the matches and fire their employees. By consequence, beyond a certain level of labor market rigidity, further increase in labor market frictions shields workers from separation into unemployment and short-run income losses after a trade shock.

To make our model tractable, we adopt a number of strong assumptions, in particular by make the model linear along various dimensions, including firm entry and hiring costs. We view this as a useful environment to clearly isolate the qualitative forces shaping the dynamic heterogeneous adjustment to a trade shock in the cross-section of firms, focusing on the direct effects of the labor market frictions and separating it from other mechanisms that result in a period of transitional dynamics even in the absence of any labor market rigidities. In particular, labor market frictions in the model result in hiring costs and an associated S nature of the firm employment decisions, along with an inaction region, which is the source of the transitional dynamics in the labor market. In Section 6, we provide a detailed discussion of the assumptions and the direction in which the future quantitative work can relax them.

Related Literature The model in this paper builds on our earlier work, Helpman and Itskhoki (2010), in which we study the the long-run effects of labor market and trade reforms in countries with asymmetric labor market institutions and heterogeneous firms. The consequences of labor market frictions for transition dynamics in neoclassical trade models

3Felbermayr, Prat, and Schmerer (2011) and others also study the steady state effects of a trade liberalization in an economy with heterogeneous firms and search frictions in the labor market; Davis and Harrigan (2011) and Egger and Kreickemeier (2009) analyze the effects of other labor market frictions in a trade model with heterogeneous firms.
were studied by Davidson, Martin, and Matusz (1999), Kambourov (2009) and Coşar (2010). Labor market dynamics with heterogeneous firms were analyzed in Coşar, Guner, and Tybout (2011), Fajgelbaum (2013), Danziger (2013), Cacciatore (2013) and Felbermayr, Impullitti, and Prat (2014). Our paper is the first to fully characterize the transition dynamics in an economy with heterogeneous firms in response to an aggregate trade shock. In macro-labor literature, the dynamics of labor market with heterogeneous firms were studied in Acemoglu and Hawkins (2013), Elsby and Michaels (2013) and Schaal (2012).

2 Setup

Consider a world of two symmetric countries, each producing two goods—a non-traded homogenous good and a traded differentiated good. The differentiated good is produced by heterogeneous firms under monopolistic competition, and exporting is associated with both variable and fixed trade costs. The labor market in each sector is subject to a random search friction with wage bargaining upon matching. We setup the model in discrete time with short time intervals, and use the continuous-time approximation to simplify notation, while the appendix provides exact discrete-time expressions.

2.1 Households

Each country is populated by a unit continuum of identical infinitely-lived households with a per period utility function $u(q_0, Q)$ over the consumption of a homogenous good $q_0$ and a differentiated good $Q$, and with an annualized discount rate $r$. We suppress the dependence on time $t$ where it leads to no confusion. The differentiated good is a CES aggregator of individual varieties

$$Q = \left( \int_{\omega \in \Omega} q(\omega)^\beta d\omega \right)^{1/\beta}, \quad 0 < \beta < 1,$$

(1)

where $\Omega$ is the set of varieties $\omega$ available for consumption and $\varepsilon \equiv 1/(1 - \beta) > 1$ is the elasticity of substitution between the varieties.

We choose the non-traded homogenous good as numeraire setting $p_0 \equiv 1$.\textsuperscript{4} We make the following functional form assumption:

\textbf{Assumption 1} The utility function is quasi-linear:

$$u(q_0, Q) = q_0 + \frac{1}{\zeta} Q^\zeta \quad \text{ for } \quad (q_0, Q) \in \mathbb{R} \times \mathbb{R}_+,$$

(2)

\textsuperscript{4}Even if the homogenous good were tradable, in equilibrium with symmetric countries it is non-traded.
This is a strong assumption. However, it allows to focus our analysis on the dynamic effects stemming from labor market frictions, unconfounded by the effects of the curvature in the utility function on the timing of entry of firms, which operates independently of the labor market frictions. This assumption can be equivalently replaced by an assumption of a perfectly elastic supply of capital at interest rate \( r \), common in small-open-economy models. In either case, the assumption has a partial equilibrium flavor, which we think is a reasonable point of approximation, since only a small fraction of consumer expenditure is on tradables. Hence, a reduction in trade costs affects directly only a part of the economy, and the homogenous-good sector represents in our analysis the rest of the economy not affected directly by trade.

Under Assumption 1, the flow utility can be written as:

\[
\begin{align*}
\text{where the first term is income and the second term is consumer surplus from the differentiated good. Therefore, the lifetime utility equals the discounted present value of household income and consumer surpluses, and trade in general affects both components of the utility.}
\end{align*}
\]

Each household has (a measure) \( L \) of workers, who are allocated between the two sectors. Workers in each sector can be either employed or unemployed and searching for a job. Unemployed workers can frictionlessly reallocate between the two sectors, while employed workers need to first separate into unemployment before starting to search for a new job in either sector. Unemployed workers in both sectors face Diamond-Mortensen-Pissarides search frictions, as we describe in detail below. The workers receive an unemployment benefit \( b_u \) in units of the homogenous good while unemployed and searching for a job, which is financed by a lump sum tax on all households.

### 2.2 Non-traded sector

A match between a firm and a worker in this sector produces a constant flow \( \Delta \) of the homogenous good per period of length \( \Delta \). We assume that \( \Delta \) is small (e.g., \( \Delta = 1/12 \), corresponding to a period length of one month), rendering accurate the approximation around \( \Delta = 0 \).

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5 This parameter restriction implies that the differentiated varieties are better substitutes with each other than with the homogenous good.

6 With this utility function, the market clearing interest rate equals discount rate, \( r \), and hence expenditure in the indirect utility function can be assumed to equal income without loss of generality. Denote by \( P \) the price index of the differentiated good in units of the numeraire. A period’s expenditure equals \( q_0 + PQ = I \). The optimal choice of \( Q \) satisfies \( PQ = Q^\zeta \), and therefore the period’s flow utility is \( I - PQ + Q^\zeta / \zeta = I + \frac{1}{\zeta}Q^\zeta \).
The market for homogenous goods is competitive. A firm can enter this sector freely, and post costly vacancies to attract workers. We denote the expected cost to a firm of attracting a worker by \( b_0 \), equal to the cost of a vacancy divided by the vacancy-filling rate. In the appendix we show that assuming a Cobb-Douglas matching function between sectoral vacancies and unemployed searching for work in the sector, we have the following relationship:

\[
b_0 = a_0 x_0^\alpha,
\]  

(3)

where \( a_0 \) is a derived parameter increasing in the cost of a vacancy and decreasing in the productivity of the matching technology, and \( x_0 \) is the job finding rate—a measure of the sectoral labor market tightness.\(^7\) Upon matching, the firm and the worker Nash-bargain over the wage rate without commitment. A match is exogenously destroyed with a constant hazard rate \( s_0 \).

We make the following assumption:

**Assumption 2** The homogenous good sector is large enough relative to the differentiated good sector (i.e., \( L \) is large enough), that along the equilibrium path the stock of unemployed searching for a job in the homogenous-good sector is positive, \( U_{0,t} > 0 \), in every period \( t \).

With this assumption, in the appendix we prove the following proposition which characterizes the labor market equilibrium in the homogenous sector:

**Lemma 1** Under Assumption 2, the job finding rate \( x_0 \) and the hiring cost \( b_0 \) are positive, finite, constant over time, and solve

\[
\left[ 2(r + s_0) + x_0 \right] b_0 = 1 - b_u
\]

(4)

together with (3); and the economy-wide value to an unemployed worker is constant over time and given by:

\[ rJ_0^U = b_u + x_0 b_0. \]

(5)

Since unemployed are mobile across sectors, and given that some of the unemployed always search for work in the homogenous sector (Assumption 2), \( J_0^U \) in Lemma 1 characterizes the economy-wide value to unemployed from the partial equilibrium in the homogenous sector’s labor market. The driving force behind this result is the free entry condition for firms in the homogenous sector, which equalizes the value of a filled vacancy, \( J_0^F \), with the hiring cost, \( b_0 \). This, in turn, is only consistent with a single value of the labor market tightness,

\(^7\)Indeed, with a Cobb-Douglas matching function, the job finding rate is a power function of the vacancy-unemployment ratio—a conventional measure of the labor market tightness.
characterized by (3)–(4). Nash bargaining equalizes the surplus from the employment relationship between the firm and the worker. As a result, an unemployed worker receives $b_u$ and expects to find employment at rate $x_0$ with the surplus from employment given by $b_0$, as reflected in (5). In addition, we show that the equilibrium wage rate in the homogenous sector equals:

$$w_0 = b_u + (r + s_0 + x_0)b_0.$$  

(6)

Assumption 2 ensures that there always are unemployed workers searching for a homogenous-sector job, and therefore firms always enter and post vacancies in this sector. In other words, we require that the trade shock to the differentiated sector is never large enough to lead this sector to absorb all economy-wide unemployment, even for a single period. This arguably is a realistic assumption in view of the modest employment share of the tradable sector. By pinning down the economy-wide value to unemployed, Assumption 2 and Lemma 1 ensure a useful block-recursive structure of the model. In Section 4, we evaluate this assumption quantitatively, and relax it in Section 6.

2.3 Traded sector

A firm in the differentiated sector pays a sunk cost $f_e$ in terms of the numeraire (homogenous good) to enter the industry with a unique variety $\omega$ and draws its productivity $\theta$ from a known distribution $G(\theta)$. To simplify exposition, we adopt the assumption that the productivity distribution is Pareto, $G(\theta) = 1 - \theta^{-k}$, with the shape parameter $k > \varepsilon - 1$, but this is not required for our qualitative results. The firm then has access to a linear technology producing $y = \theta h$ units of its variety when it employs $h$ workers. The firm faces flow fixed cost of operation $f_d$, and it can additionally choose to export its variety by paying a flow fixed exporting cost $f_x$. All fixed costs are in terms of the numeraire. Exports are additionally subject to an iceberg variable trade cost $\tau \geq 1$, i.e. $\tau$ units of a good must be shipped out in order for one unit of the good to arrive to the foreign market.

We show in the appendix that, given the CES preference aggregator (1), the revenue function of a firm with productivity $\theta$ is given by:

$$R(h, \iota; \theta) = \Theta(\iota; \theta)^{1-\beta} h^\beta, \quad \Theta(\iota; \theta) \equiv \left[ 1 + \iota \tau^{-\frac{1}{1-\beta}} \left( \frac{Q^*}{Q} \right)^{-\frac{\beta}{1-\beta}} \right] Q^{-\frac{\beta}{1-\beta} \theta^\beta},$$  

(7)

where $\iota \in \{0, 1\}$ is an indicator of whether the firm exports, while $Q$ and $Q^*$ characterize the product market competition at home and in foreign. Note that the revenue of a non-exporting firm is simply $Q^{-\beta} y^\beta$, while an exporting firm (with $\iota = 1$) optimally splits its output $y = \theta h$ between the domestic and foreign markets, which results in its revenues shifted
outwards, as reflected by the square bracket in (7).

The firms have to pay a cost of $b$ units of the numeraire to hire one unit (measure one) of workers. Similarly to (3), the Cobb-Douglas matching function links this hiring cost to the job finding rate of workers in this sector, $x$:

$$ b = ax^\alpha. \tag{8} $$

Upon matching the firm bargains with its workers (without commitment) according to Stole and Zwiebel (1996). That is, the firm bargains bilaterally with each of its workers according to Nash, taking into account that the departure of the worker will cause a renegotiation of wages for all of its remaining workers. The bargaining is over the revenues of the firm once the employment decision and the per-period fixed production and exporting costs are sunk. In view of Lemma 1 and given the functional form of the revenue function in (7), we prove in the appendix the following result:

**Lemma 2** The outcome of the bargaining game between a firm and its $h$ workers is the wage schedule

$$ w(h, \nu; \theta) = \frac{\beta}{1 + \beta} \frac{R(h, \nu; \theta)}{h} + \frac{1}{2} rJ^U, \tag{9} $$

where the value to unemployed $J^U = J^U_0$ is characterized in Lemma 1.

Importantly, the wage schedule in Lemma 2 applies equally to both firms that expand and reduce their labor force, and independently of whether $h$ is the optimal level of employment of the firm. The bargained wage rate in (9) partly compensates the workers for the forgone flow value of unemployment ($rJ^U$) and in addition delivers them their share in surplus equal to a constant fraction of average revenues of the firm. Combining together (9) and (7), we can write the flow operating profit of the firm, gross of hiring costs, as:

$$ \varphi(h, \nu; \theta) = \frac{1}{1 + \beta} R(h, \nu; \theta) - \frac{1}{2} rJ^U h - f_d - \nu f_x. \tag{10} $$

A firm exogenously separates with a fraction of its workforce at an annualized rate $\sigma$ and it dies at an annualized rate $\delta$. We denote by $s = \sigma + \delta$ the overall exogenous rate for a worker employed in the differentiated sector to be separated into unemployment. A firm can fire some or all of its workers at no cost. Therefore, a firm needs to pay the hiring cost of

$$ C(h', h) = b \cdot \max \left\{ h' - (1 - \sigma \Delta) h, 0 \right\} \tag{11} $$

to change its employment from $h$ to $h'$ next period. Part of the hiring cost ($\sigma h \Delta$) is borne to
replace the exogenous labor force attrition, and the rest \((h' - h)\) is payed to increase the size of the labor force, while non-hiring firms (with \(h' \leq (1 - \sigma \Delta)h\)) incur no costs.

With this setup, we characterize the labor market equilibrium in the differentiated sector:

**Lemma 3** (a) The job finding rate \(x\) and the hiring cost \(b\) in the differentiated sector are constant, and satisfy \(xb = x_0b_0\); (b) The optimal employment of a hiring firm is given by

\[
h(\iota; \theta) = \Phi^{1/\beta} \Theta(\iota; \theta), \quad \text{where} \quad \Phi \equiv \left( \frac{2\beta}{1 + \beta} \frac{1}{b_u + [2(r + s) + x]b} \right)^{\frac{1}{1 - \alpha}}
\]  

(12)

and \(\Theta(\iota; \theta)\) is defined in (7).

The formal proof of this lemma is presented in the appendix, and here we describe the logic behind this result.\(^8\) A hiring firm equalizes the value from its marginal worker with the cost of hiring, \(b\). The splitting of the surplus in bargaining, in turn, ensures that the employment value to the workers equals the employment value to the firm. Therefore, the unemployed workers in the differentiated sector have the job finding rate of \(x\) and the gain in value of \(b\) upon employment, in comparison to, correspondingly, \(x_0\) and \(b_0\) in the homogenous sector (see Lemma 1). The indifference of unemployed workers between the two sectors then requires \(xb = x_0b_0\), which in view of (8) results in:

\[
x = x_0 \left( \frac{a}{a_0} \right)^{-\frac{1}{1 + \alpha}} \quad \text{and} \quad b = b_0 \left( \frac{a}{a_0} \right)^{\frac{1}{1 + \alpha}},
\]

(13)

where \(a/a_0\) is the inverse measure of the efficiency of the matching technology in the differentiated sector relative to the homogenous sector.

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\(^8\)The proof uses the recursive Bellman equation for a firm with productivity \(\theta\):

\[
J^F(h) = \max_{h'} \left\{ \varphi(h)\Delta - C(h', h) + \frac{1 - \delta \Delta}{1 + r \Delta} J^F(h') \right\}
\]

The inequalities in the first order condition reflect the \(sS\) nature of the labor force adjustment in this model. Provided a constant \(b\), we have \(J^F_h = J^F_{h,+} = \frac{1 + r \Delta}{1 - \delta \Delta} b\) for a hiring firm, which together with the Envelope Theorem characterizes optimal employment, \(\varphi'(h) = \frac{r + s}{1 - \delta \Delta} b\). The approximation with \(\Delta \approx 0\) yields \(J^F_h = b\) and \(\varphi'(h) = (r + s)b\).
The optimal employment rule (12) in the second part of Lemma 3 results from the equalization of the flow value from the marginal workers, \( \varphi'(h) \), characterized by (10), with the flow cost of hiring an extra worker, \((r + s)b\). The derived parameters \( \Phi \) is the summary statistic for the extent of labor market imperfections, and it decreases in the hiring cost \( b \). Indeed, more productive firms and exporters have larger optimal employment (due to higher \( \Theta(\iota; \theta) \)), while all firms are smaller in a more frictional labor market (due to lower \( \Phi \)).

Note that the employment of a firm is a jump variable. Due to the linearity of the hiring cost, the firm immediately jumps its employment up to the optimal level, and in a stationary environment the firm then maintains this employment level each period by hiring to exactly offset the attrition. At the same time, if the firm has more workers relative to the optimum, it does not fire them immediately, and waits until its labor force shrinks as a result of the exogenous attrition.

Substituting the optimal employment level (12) into the wage schedule (9), we obtain the equilibrium wage rate paid to all employed works by the hiring firms:

\[
w = b_u + (r + s + x)b,
\]

which parallels expressions (6) for the wage rate in the homogenous sector. Note that all hiring firms in this economy pay the same wages, independently of their productivity, by adjusting their employment on the extensive margin. The non-hiring firms and firing firms, in contrast, pay a lower wage rate, as their employment is above the optimal level.\(^9\)

Lastly, we introduce notation \( J^V(\theta) \) for the value function of a firm with productivity \( \theta \) and zero workforce in a given time period, which allows us to write the free entry condition as

\[
\int J^V(\theta)dG(\theta) \leq f_e,
\]

which holds with equality when there is entry of firms at that time period. Firms with \( V(\theta) \geq 0 \) hire workers and produce starting next period, while firms with \( V(\theta) < 0 \) exit immediately. The following result provides a sharp explicit expression for the value of the

\(^9\)Helpman and Itskhoki (2010) focus on the effects of cross-country differences in labor market frictions (\( \Phi \)) on the steady state comparative advantage and asymmetric gains from trade. Note that using (4) and Lemma 3(a), one can show that \( \Phi \) is decreasing in \((sb - s_0b_0)\). Therefore, countries with higher overall level of labor market frictions (high \( b = b_0 \)) have comparative advantage in sectors with greater labor market turnover (higher relative separation rate, \( s/s_0 \)), consistent with the evidence in Cuñat and Melitz (2011).

\(^{10}\)Indeed, from (9) and (7), the wage rate is a decreasing function of employment, and a firm chooses to reduce its labor force only if its current employment exceeds the desired level given by (12) and resulting in wage rate in (14). Furthermore, the wage rate paid by firing firms equals the flow value of unemployment, \( rJ^U = b_u + xb \), as employment in this case yields no surplus (i.e., \( J^F_h = J^E(h) - J^U = 0 \)). Therefore, wages paid by non-hiring firms fall within the range from \( b_u + xb \) paid by firing firms to \( b_u + (r + s + x)b \) paid by hiring firms.
Lemma 4 The value of a firm with productivity $\theta$ and zero employees, which hires workers in every future period, solves the following difference equation:

$$(r + \delta)J_{-1}^{V}(\theta) - \dot{J}^{V}(\theta) = \max_{\iota \in \{0,1\}} \left\{ \frac{1 - \beta}{1 + \beta} \Phi(\iota; \theta) - f_d - \iota f_x \right\},$$

(16)

where the $(-1)$-subscript denotes the previous period, $\dot{J}^{V}(\theta) \equiv (J^{V}(\theta) - J^{V}_{-1}(\theta))/\Delta$, and $\Theta(\iota; \theta)$ and $\Phi$ are defined in (7) and (12) respectively.

Lemma 4 applies for a general time path of aggregate state variables such as $b$ and $Q$, which affect $\Phi$ and $\Theta$ in (16). Furthermore, the reason the requirement of Lemma 4 that a firm hires in every future period is not overly restrictive, as it is satisfied, for example, for firms that simply maintain their employment level by hiring to offset attrition. Finally, equation (16) is derived under the assumption that the exporting fixed cost is not sunk, and a firm can choose whether to pay a fixed cost and export every period. We adopt this assumption to simplify exposition, but it is not necessary for our results. To save space, we provide the formal conditions for production and exporting under the special circumstances discussed below.

2.4 General equilibrium

To close the model, we need to characterize the aggregate employment and number of firms in the differentiated sector. We denote by $M$ the number (measure) of firms that have entered the differentiated sector, and did not die yet for an exogenous reason. Therefore, $M$ evolves according to:

$$M_+ = (1 - \delta \Delta)M + M^e,$$

where $M^e$ is the number of entrants in a given period. $M^e \geq 0$ holds with complementary slackness with the free entry condition (15). Note that the number of firms is a jump variable, an assumption that we relax in Section 6.

Further denote by $G(h, \theta)$ the joint cumulative distribution function of firm employment and productivity in a given time period among the $M$ currently active firms. The $M^e$ new entrants have zero employment until the following period. The aggregate employment in the differentiated sector is then:

$$H = M \int h dG(h, \theta),$$

(17)

The formal proof of this lemma is in the appendix, and it combines the Bellman equation for the value function of the firm with the optimal employment policy function of a hiring firm described in footnote 8.
The evolution of $\mathcal{G}(\cdot)$ is characterized by the firm’s employment policy functions described in footnote 8. Given the number of firms and their employment and exporting decisions we can use (1) to compute the consumption of the differentiated good, $Q$. Finally, this also allows us to recover the aggregate number of vacancies posted in the differentiated sector $V$, and the corresponding sectoral unemployment $U$ in order to maintain the constant labor market tightness $x$ given in (13). This determines the number of workers attached to the two sectors in a given period, $N = H + U$ and $N_0 = L - N$. We provide further details in the appendix.

3 Long-run Equilibrium

Consider a symmetric steady state in which all variables have the same values at home and in foreign, and in particular $Q = Q^*$. Due to the positive exogenous workforce attrition rate ($\sigma > 0$), all producing firms in steady state hire workers in every period to offset attrition, and therefore their employment is given by (12). Due to the positive firm death rate ($\delta > 0$), there is constant firm entry in steady state, and therefore the free entry condition holds with equality.

Now consider an entrant in a steady state environment. The conditions of Lemma 4 are satisfied for all firms that do not exit immediately, and therefore the steady-state value of an entrant with productivity $\theta$ and zero employment is given by

$$J^V(\theta) = \frac{1}{r + \delta} \max_{\iota \in \{0, 1\}} \left\{ \frac{1 - \beta}{1 + \beta} \Phi(\iota; \theta) - f_d - \iota f_x \right\}. \quad (18)$$

The solution to the maximization problem in (18) defined the exporting cutoff $\theta_x$ such that $\iota(\theta) \equiv 1_{\{\theta \geq \theta_x\}}$. We also define the production cutoffs, $\theta_d$, from $J^V(\theta_d) = 0$, such that all firms with $\theta \geq \theta_d$ hire workers and produce in the long run, while firms with $\theta < \theta_d$ exit the industry immediately upon entry. Using the definition of $\Theta(\iota; \theta)$ in (7), the two long-run cutoff conditions can be written as (see the appendix):

$$\frac{1 - \beta}{1 + \beta} \Phi Q^{\sigma - 1} \theta_d^{\beta - 1} = f_d, \quad (19)$$

$$\theta_x/\theta_d = \tau (f_x/f_d)^{1/(\varepsilon - 1)}, \quad (20)$$

and we choose the value of the fixed cost $f_x$ large enough that $\theta_x > \theta_d$. These cutoff conditions, together with the free entry condition (15), allow us to solve for the long-run values of
\((\theta_d, \theta_x, Q)\) as functions of the hiring cost \(b\) and trade costs \(\tau\).\(^{12}\)

Next, using (1) and (17), we can solve for the steady state employment in the differentiated sector (see the appendix):

\[
H = \Phi^{1-\beta} Q^\epsilon, \tag{21}
\]

and, additionally using the Pareto productivity assumption, the number of firms \(M\) in the differentiated sector:

\[
M = \frac{1}{k(r+\delta)\beta} \frac{\beta}{1+\beta} Q^\epsilon. \tag{22}
\]

Both aggregate sectoral employment and the number of firms are proportional to \(Q^\epsilon\), which equals total sales of the differentiated good.

Finally, note that \(sH\) is the total number of hires in the differentiated sector each period (\(\sigma H\) to replace attrition and \(\delta H\) by new entrants). In steady state the flow in and out of unemployment are equalized, and hence \(sH = xU\). Given the value of \(x\), this pins the total number of workers assigned to the two sectors in the economy: \(N = (1 + s/x)H\) and \(N_0 = L - N\). This is sufficient to recover the remaining equilibrium variables, in particular the output of the homogenous good and the amount of the homogenous good spent on fixed (entry, production, exports) and hiring costs. The production of the homogenous good is a linear function of the size of the economy, determined by \(L\), and we require that \(L\) is large enough to ensure positive consumption of the outside good. Since firms make zero profits on average, we measure the steady state welfare as the sum of the consumer surplus from the differentiated good, \((1 - \zeta)Q^\epsilon/\zeta\), and the employment income, \(I = w_0H_0 + wH\).\(^{13}\)

### 3.1 Long-run gains from trade

Given the equilibrium conditions described above, we can immediately prove the following comparative statics result across steady states:

\(^{12}\)Using (19)–(20) and (18), the free entry condition (15) can be simplified to:

\[
f_d \int_{\theta \geq \theta_d} [(\theta/\theta_d)^{\epsilon-1} - 1] \, dG(\theta) + f_x \int_{\theta \geq \theta_x} [(\theta/\theta_x)^{\epsilon-1} - 1] \, dG(\theta) = (r + \delta) f_e,
\]

which under the Pareto distribution further simplifies to

\[
\frac{1}{\epsilon - 1} [f_d \theta_d^{-\frac{\epsilon}{\epsilon - 1} - 1} + f_x \theta_x^{-\frac{\epsilon}{\epsilon - 1} - 1}] = (r + \delta) f_e.
\]

Note that all steady state equations closely parallel their analogs in the static model in Helpman and Itskovich (2010).

\(^{13}\)In case with symmetric labor market frictions in both sectors \((s = s_0\) and \(a = a_0)\), both the wage rate and the unemployment rate in both sectors are the same and equal to \(w = b_u + (r+s+x)b\) and \(u = s/(x+s)\) respectively. As a result, labor market income equals \(I = w \cdot xL/(x+s)\), where the second term is the economy-wide employment. Finally, since the expenditure on the differentiated good is equal to \(Q^\epsilon\), it is sufficient to require \(I > Q^\epsilon\), or equivalently \(L > (1 + s/x) \cdot Q^\epsilon/w\), to ensure positive consumption of the outside good in steady state.
Proposition 1 In a symmetric world economy, a bilateral reduction in trade costs leads to:
(a) an increase in $Q$, $H$ and $M$, with the proportional changes in these variables independent of the extent of labor market frictions; (b) assuming additionally symmetric labor markets across sectors ($s = s_0$ and $a = a_0$), the aggregate unemployment and labor income in terms of the homogenous good do not change with trade costs, and the steady state welfare gains from trade do not depend on the extent of the labor market frictions.

Proof: The export cutoff condition (20) and the free entry condition in footnote 12 determine the production and export cutoffs $\theta_d$ and $\theta_x$ as functions of the product market parameters and trade costs only, and independently from the labor market parameters. A reduction in $\tau$ results in an increase in $\theta_d$. Further, from the production cutoff condition (19) and equations (21) and (22), we immediately have:

$$
\left( \frac{Q'}{Q} \right)^{\zeta} = \frac{H'}{H} = \frac{M'}{M} = \left( \frac{\theta_d'}{\theta_d} \right)^{\frac{\beta_c}{\beta - \zeta}},
$$

where prime denotes the new steady state.

Next, from Lemmas 1 and 3, the labor market outcomes $x_0$, $x$, $b_0$ and $b$ do not depend on the trade costs, and therefore the reduction in trade costs leaves unchanged the labor market outcomes. With symmetric labor markets, we have $x_0 = x$ and $b_0 = b$ (see (13)), and the wage rates are equalized across all workers in both sectors, $w_0 = w = b_u + (r + s + x)b$ (see (6) and (14)), both before and after reduction in trade costs.

We adopt the following measure of the welfare gains from trade:

$$
GT' \equiv \frac{(I' - I) + \frac{1 - \zeta}{\zeta} \left[ (Q')^{\zeta} - Q^{\zeta} \right]}{\frac{1 - \zeta}{\zeta} Q^{\zeta}},
$$

i.e. the change in the consumer surplus plus the change in market income relative to the initial consumer surplus. Since $I' = I = wxL/(x + s)$, we have the steady state welfare gains from trade equal to the gains in the consumer surplus from the differentiated good, which as we showed does not depend on the extent of the labor market frictions.

14The only result here that requires the Pareto productivity assumption is the characterization of the change in the number of firms; without the Pareto assumption, $M'/M$ may be larger or smaller than $H'/H$. Under the Pareto productivity distribution we can additionally obtain a closed-form solution for the production cutoff:

$$
\theta_d = \left[ \frac{f_d}{f_c} \cdot \frac{1 + (f_d/f_x)^{k/(\varepsilon - 1) - 1} \tau^{-k}}{k/(\varepsilon - 1) - 1(r + \delta)} \right]^{-1/k}.
$$
Proposition 1 shows that the long-run welfare effects of a reduction in trade costs are not affected by the extent of the labor market rigidities and, in particular, do not depend on the hiring costs, $b$, and the level of the labor market tightness, $x$. The proportional change in the consumer surplus from the traded good, $Q^c$, depends only on the change in the trade costs and the product market parameters, but does not depend on whether the labor market is frictionless or not. This is not to say that labor market frictions are inconsequential in this economy, but instead they only affect the overall level of economic activity, equally before and after the reduction in trade costs. This can be seen from the production cutoff condition (19), where $\Phi$ summarizes the effect of the labor market frictions on the level of output given the production cutoff, which as we show in the proof of Proposition 1 depends only on the product market parameters. Furthermore, household income in terms of the non-traded good, $I$, does not change with the trade costs. This is because in both steady states firms make zero profits on average, while the labor income stays unchanged, as both the employment rate and the wage rate stay the same.\footnote{This is only the case with symmetric labor market frictions in both sectors, which is our benchmark. Outside this case, the sectoral unemployment rates are not the same, and the labor reallocation towards the traded sector resulting from trade liberalization changes the economy-wide employment rate (Helpman and Itskhoki, 2010). As a result, the labor market income is in general not constant and may go both up and down with the reduction in the trade costs.}

The measure of the welfare gains used in our analysis (see (24)) is equal to the proportional change in the consumer surplus from the traded good, if trade does not affect the market income, as is the case when we compare two steady states. More generally, the welfare gains from trade also depend on the trade-induced change in the market income in the economy, as we discuss in detail in our analysis of the transition dynamics. We scale the measure of the gains from trade by the consumer surplus from the traded-good before the reduction in trade costs. We choose this measure, as it parallels the measure of welfare gains in a one-sector model and makes it invariant to the size of the non-traded sector, $L$.

The sharp result of Proposition 1 on the irrelevance of the labor market frictions for the long-run welfare gains from trade is already intriguing, yet one may expect that most of the bite of the labor market frictions may occur during the transition dynamics. Indeed, labor market frictions may delay the increase in the traded-good output, $Q$, or result in transitional unemployment and a reduction in the market income during the transition. Furthermore, although the average employment per firm, $H/M$, stays constant across steady states, individual firms change their employment in different ways depending on their productivity. After the reduction in trade costs, the less productive non-exporting firms shrink, while the more productive exporting firms expand their employment (see the appendix). Labor market frictions slow down this reallocation, leading to misallocation of employment across firms, which
may depress the welfare gains from trade along the transition. We now turn to the formal analysis of these issues.

4 Dynamic Gains from Trade

We now study the dynamic transition of the economy from an initial steady state with a high variable trade cost, \( \tau \), to a new long-run equilibrium with a lower variable trade cost, \( \tau' < \tau \). The change in \( \tau \) happens at \( t = 0 \) and is one-time, permanent and unexpected. We denote with a prime the new steady state, without a prime and no subscript the initial steady state, and with a time subscript the dynamic evolution of the variables.

Before exploring the rich dynamic adjustment at the firm level with its impact on aggregate productivity and trade flows in the next section, we start our analysis here with a sharp analytical result characterizing the gains in the consumer surplus from trade—the unique source of the long-run gains from trade, as we have shown in Proposition 1:

**Proposition 2** Along the transition path, \( Q_t \geq Q' \) for all \( t \geq 0 \), which holds with equality in all periods when there is entry of firms. Therefore, if there is entry in every period, the gains from trade in the consumer surplus, \( \left( \frac{Q_t}{Q} \right)^\zeta \equiv \left( \frac{Q'}{Q} \right)^\zeta \) for all \( t \geq 0 \), are instantaneous, and, as follows from Proposition 1, independent from the labor market frictions.\(^{16}\)

**Proof:** First, from Lemma 1 and 3, we know that the labor market tightness, \( x \), and the hiring cost, \( b \), are constant in both sectors throughout the transition. This is achieved by the reallocation of unemployed from the homogenous to the differentiated sector in order to maintain the increased demand for labor in this sector. Assumption 2 ensures that there are enough unemployed economy-wide to satisfy this increased labor demand. This assumption imposes a joint upper bound on the relative size of the traded sector and the reduction in the trade costs, which we explore numerically in the following subsection.

Second, we prove that there are two possibilities depending on the parameters of the model and the size of the trade liberalization:

(i) Positive entry flow starting immediately at \( t = 0 \), \( M^e_0 > 0 \). In this case, \( M^e_t > 0 \) for all \( t \geq 0 \), \( Q_0 < Q' \) and \( Q_t = Q' \) for all \( t > 0 \). That it, output jumps up on impact (with a one period lag) to the new long-run level.

Since \( M^e_t > 0 \), the free entry condition (15) holds with equality for all \( t \geq 0 \). Since \( b_t = b \) and \( Q_t = Q' \) for all \( t > 0 \), every entrant that chooses to produce (i.e., with \( \theta \geq \theta'_d \)) does so in every period after entry and hence continuously hires workers to replace attrition.

\(^{16}\)Since entry into production happens with a one-period lag, which is required to hire workers, Proposition 2 formally applies for \( t \geq \Delta \), and by our convention we use an approximation \( t \geq 0 \), as we consider small \( \Delta \approx 0 \).
Therefore, the characterization of \( J^V(\theta) \) in Lemma 4 applies, with \( J^V(\theta) \) constant over time, and we can write the free entry condition as:

\[
\int_\theta \left[ \int_t^\infty e^{-(r+\delta)(t-t')} \max_{i} \left\{ \frac{1-\beta}{1+\beta} \Phi \Theta_i(t; \theta) - f_d - tf_x, 0 \right\} dt' \right] dG(\theta) = f_e, \tag{25}
\]

where we approximate a sum with small finite time increments \( \Delta \) with an integral. Taking the derivative with respect to time \( t \), we have:

\[
\int \max_{i} \left\{ \frac{1-\beta}{1+\beta} \Phi \Theta_i(t; \theta) - f_d - tf_x, 0 \right\} dG(\theta) = (r+\delta)f_e.
\]

Given the definition of \( \Theta_i \) in (7), the left-hand side of this equation is monotonically decreasing in \( Q_t \), and there exists a single value \( Q_t = Q' \) consistent with the above equality. This verifies our conjecture.

(ii) No entry initially at \( t = 0 \), \( M_0^e = 0 \). In this case, there exists \( T_e > 0 \) such that \( M_t^e = 0 \) and \( Q_t > Q_{t+\Delta} \geq Q' \) for \( t \in [0, T_e) \), and \( M_t^e > 0 \) and \( Q_t = Q' \) for \( t > T_e \). That is, output gradually declines (in finite time) to the new long-run level.

Since \( b_t = b \) for all \( t \geq 0 \) and \( Q_t \) is non-increasing, any entrant that chooses to produce at \( t > 0 \) also does so at all future dates. Similarly, every firm that chooses to export at \( t \geq 0 \) also does so at all future dates. Therefore, every entrant that chooses to hire workers, continuously hires them in all future periods, either to replace attrition or also to expand employment in cases when \( Q_t \) strictly declines. This again allows us to use the characterization of the value of an entrant from Lemma 4, however in this case the relationship in (25) holds initially as a strict inequality, and \( M_t^e = 0 \) to satisfy complementary slackness. Due to firm death (as well as labor force attrition for some shrinking incumbents), during this initial period output \( Q_t \) declines, until it reaches \( Q' \) immediately after some finite time \( T_e \), at which the free entry condition starts to hold with equality and firms start to enter, \( M_t^e > 0 \) for all \( t \geq T_e \).

Finally, there are no other possibilities for the behavior of entry and output during the transition dynamics. Indeed, if \( Q_0 < Q' \), there is no future date \( t > 0 \) at which \( Q_t > Q' \), i.e. overshoots the new long-run level. If such \( t > 0 \) existed, those firms that entered just before \( t \) would have made losses in expectation, violating the complementary slackness condition to the free entry condition (15). Furthermore, \( Q_t \) necessarily jumps up to \( Q' \) already at \( t = \Delta \), as otherwise entrants at \( t = 0 \) are making positive profits in expectation, violating the free entry condition (15). Lastly, if \( Q_0 > Q' \), then there cannot exist a future date \( t > 0 \) such that \( Q_t < Q' \). If such \( t \) existed, the firms could enter at \( t - \Delta \) and make positive profits, violating the free entry condition (15). ■
Proposition 2 is a powerful result since it shows that all gains from trade in the consumer surplus—the only source of the long-run gains from trade, as shown in Proposition 1—are realized immediately without delay and irrespectively of the extent of the labor market frictions, unless there is overshooting in the short run. The overshooting only happens in cases when there is no entry of firms in the short run—an empirically unlikely scenario, which we explore numerically in the following sections.

Of course, there is another sources of departure from the long-run gains from trade due to the after-shock transition—specifically, the dynamics of the household market income, which we also quantify numerically below. However, it is only the net present value of this income that matters for welfare, and hence a representative family experiences the full amount of gains from trade instantaneously with no delay.

The main force behind the result of Proposition 2 is the free entry condition of firms: both in the non-traded sector to ensure stable labor market tightness and hiring costs economy-wide, and more importantly in the traded sector to act as a buffer for product market competition. Indeed, the free entry condition pins down the single level of product market competition, \( Q_t = Q' \), consistent with the firm entry (given the hiring cost \( b \) and trade cost \( \tau' \)), and independently from the distribution of employment across incumbent firms. Indeed, the joint distribution of employment and productivity among the incumbents, \( G_0(h, \theta) \), which is a complex state variable in the model, is irrelevant for the result of the proposition, provided there is entry of new firms. The entry ceases only when there is no more slack left in the product market, i.e. \( Q_t = Q' \) for all \( t > 0 \).

The lack of the dynamic effects of the labor market frictions on the consumer surplus from trade does not imply, however, that the labor market frictions are inconsequential for the allocation of resources and productivity after the reduction in trade cost, as we show next.

5 Dynamic Adjustment to Trade

Proposition 2 requires no information on the micro-level allocation of employment across either incumbent firms or entrants, which masks the rich dynamic patterns of labor reallocation across firms within the traded sector, as well as across the two sectors. This firm-level reallocation, in turn, shapes the aggregate dynamics of employment, productivity and trade flows. In this section we provide a full characterization of this dynamic adjustment.

While the model admits a full analytical characterization, it is convenient to illustrate the forces in the model by means of specific numerical examples which are representative of various possible dynamic patterns that arise for different parameter values. The qualitative dynamic patterns are not sensitive to some of the parameters, and thus we choose to fix these
parameters at their conventional values.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Parameter</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td>$r$</td>
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<td></td>
</tr>
<tr>
<td>Exogenous separation rate</td>
<td>$s$</td>
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<td>$s_0 = s$</td>
</tr>
<tr>
<td>— Labor force attrition rate</td>
<td>$\sigma$</td>
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<tr>
<td>— Firm death rate</td>
<td>$\delta$</td>
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</tr>
<tr>
<td>Job finding rate</td>
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<td>$a_0 = a = 0.12$</td>
</tr>
<tr>
<td>Relative elasticity of matching</td>
<td>$\alpha$</td>
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<td></td>
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<tr>
<td>Unemployment benefit</td>
<td>$b_u$</td>
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</tr>
<tr>
<td>Pareto shape parameter</td>
<td>$k$</td>
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</tr>
<tr>
<td>CES within sector</td>
<td>$\varepsilon$</td>
<td>4</td>
<td>$\beta = 3/4$</td>
</tr>
<tr>
<td>Semi-elasticity across sectors</td>
<td></td>
<td>2</td>
<td>$\zeta = 1/2$</td>
</tr>
</tbody>
</table>

| Employment share in the traded sector | 14% | $L = 10$, $f_d = 0.05$ |
| Fraction of exitors                  | 25% | $(r + \delta)f_e/f_d = 2.7$ |
| Fraction of exporters                | 11% | $f_x/f_d = 1$ |
| Fraction of output exported           | 16% | $\tau = 1.75$ |

| Trade liberalization                 | $\tau' = 1.375$ |
| — Fraction of exporters              | 28% |
| — Fraction of output exported         | 28% |

Notes: All rates are annualized: for example, the separation rate is 20% per year and the job finding rate of 2 corresponds to an average unemployment duration of $1/x = 0.5$ years. $\alpha = 1$ is the ratio of the two elasticities of the matching function with respect to employment and vacancies respectively, which sum to one ensuring constant returns to scale in matching. The unemployment benefit $b_u = 0.4$ corresponds to a 45% replacement ratio. Semi-elasticity across sectors is equal to $1/(1 - \zeta) = 2$. The Pareto shape parameter for employment and sales in steady state is equal to $k/(\varepsilon - 1) = 1.33$. The middle panel shows the values of fixed costs $f_d$, $f_e$, and iceberg trade cost $\tau$ used to match the moments on exports and exit; “fraction of exitors” is the fraction of entrants that choose not to produce in the initial steady state.

Table 1 summarizes the benchmark values of parameters and corresponding empirical moments.\(^{17}\) We calibrate the productivity of the matching function—a key parameter controlling the extent of labor market frictions—to match the average unemployment duration of 6 month, corresponding to an annualized job finding rate of $x = 2$. This is more characteristic of the slower European rather than the more dynamic US labor market, and corresponds to an economy-wide unemployment rate of 9%. In addition, we show the sensitivity of the results to a wide range of variation in this parameter.\(^{18}\)

\(^{17}\)See the notes to Table 1 for additional details about the calibration.

\(^{18}\)More generally, the labor market frictions in this model are fully captured by a vector $(x, b, b_u, s)$, where
We make the traded sector relatively small, corresponding to the employment share of the manufacturing sector in the developed countries. We consider a 50% reduction in variable trade costs (i.e., \((\tau - \tau')/(\tau - 1) = 0.5\)), corresponding to a major increase both in the fraction of exporters in the traded sector and the share of output shipped abroad by the exporting firms. In particular, the initial steady state roughly corresponds to the openness measures for the United States, while the new steady state is in the ballpark for the more open large European countries. Additionally, we explore the sensitivity of the results to both smaller and larger trade shocks.

As we show below, qualitatively different adjustment patterns emerge depending on the extent of labor market frictions and the size of trade liberalization. When either trade liberalization is large enough or labor market is flexible enough, there is always a continuous entry of firms after trade liberalization, and therefore \(Q_t = Q'\) for all \(t > 0\). In particular, this is the case under our benchmark parameterization in Table 1. We, thus, proceed with the analysis under the assumption that entry is indeed continuous and \(Q_t \equiv Q'\), and consider the alternative case in Section 6.

### 5.1 Employment reallocation

It is useful to consider separately new entrants and incumbents, as their employment outcomes are not the same even if they have the same productivity. This is because the initial stock of employment at the moment of the trade shock potentially matters for both short-run and long-run outcomes, as the past hiring decisions are costly and sunk.

**Entrants** Consider first the firms that enter industry after trade liberalization. Their choices are shaped by the optimal employment policy (12), coupled with the production and exporting cutoffs which satisfy respectively (19) and (20) given \(\tau'\) and \(Q'\). We reproduce here this employment policy function for the new entrants, denoting it with a prime:

\[
h'(\theta) = \Phi^{1/\beta} \left[1 + \iota'(\theta)(\tau')^{1-\varepsilon} \right] (Q')^{-\frac{\alpha-\xi}{1-\beta}} \theta^{\frac{-\xi}{1-\beta}} \theta^{\frac{\beta}{1-\beta}} \quad \text{for} \quad \theta \geq \theta'_d,
\]

where \(\iota'(\theta) = \mathbb{I}\{\theta \geq \theta'_x\}\), and \(h'(\theta) = 0\) for \(\theta < \theta'_x\). The new entrants reach this level of employment instantaneously upon entry (or, more precisely, with a small time lag of \(\Delta\)). We define the initial steady state employment schedule \(h(\theta)\) by analogy with (26). It is easy to see that, like in a static Melitz model, the employment of low-productivity non-exporting

\(x\) and \(b\) are closely tied by (8). Reduction in the matching function efficiency (adjusted by the vacancy cost), i.e. an increase in \(a = a_0\), simultaneously reduce \(x\) and increases \(b\) (see Lemma 1 and equation (13)). An additional source of labor market rigidity is the unemployment benefit \(b_u\), while we treat the separation rate \(s\) as a technological parameter of the environment.
entrants shrinks after trade liberalization, due to the increased foreign competition, while the employment of high-productivity entrants that choose to export expands. The largest increase in employment is experienced by the new exporters with intermediate levels of productivity, \(\theta \in [\theta', \theta]\), as they did not export in the initial steady state.\(^{19}\)

**Incumbents** Next we consider incumbent firms with productivity \(\theta\) and employment stock \(h(\theta)\) at the time of the trade shock. Some of these firms expand their employment instantaneously after the shock, while others choose to reduce their size—instantaneously by firing workers or gradually over time through attrition, or both. The optimal choices depend both on the productivity of the firm (and the associated initial employment \(h(\theta)\)), the extent of labor market frictions (captured by \(x\)), and the size of the trade liberalization, as we now describe.

We show in the appendix that there exist cutoffs \(\bar{\theta}'_d \leq \theta'_d\) and \(\bar{\theta}'_x \leq \theta'_x\), with inequalities strict when away from the frictionless labor market limit, such that the incumbents with productivity \(\theta \geq \bar{\theta}'_d\) choose to stay in the industry in the long run and the incumbents with productivity \(\theta \geq \bar{\theta}'_x\) choose to export after trade liberalization. Given these choices, the long-run employment of the incumbents is still governed by (12), and we denote it with:

\[
\bar{h}'(\theta) = \Phi^{1/\beta} [1 + \tau'((\tau')^{1-\varepsilon}) (Q')^{-\frac{\beta-\xi}{1-\beta}} \theta^{\tau^b}] \quad \text{for} \quad \theta \geq \bar{\theta}'_d,
\]

and \(\bar{h}'(\theta) = 0\) for \(\theta < \bar{\theta}'_d\), where \(\tau'(\theta) \equiv \mathbb{I}\{\theta \geq \bar{\theta}'_x\}\). Note that \(\bar{h}'(\theta) > h'(\theta)\) for \(\theta \in (\bar{\theta}'_d, \theta'_d) \cup (\bar{\theta}'_x, \theta'_x)\), which is in general non-empty, and \(\bar{h}'(\theta) = h'(\theta)\) otherwise.

Intuitively, the incumbents have made a sunk investment in their employment stock before trade liberalization was anticipated, and it turns out to be an over-investment once trade costs have fallen unexpectedly. As a result, they face a choice between shrinking their employment versus maintaining it yet adjusting the extensive-margin decision to produce or export (altering the cutoff rules (19) and (20)). And, indeed, by continuity, firms with productivity just below the long-run cutoffs \(\theta'_d\) and \(\theta'_x\) prefer to stay in business and export respectively. The more severe are the labor market frictions, the larger are the sunk costs and hence the further away are the incumbent cutoffs \(\bar{\theta}'_d\) and \(\bar{\theta}'_x\) from the cutoffs for entrants \(\theta'_d\) and \(\theta'_x\).

\(^{19}\)Indeed, we have:

\[
\frac{h'(\theta)}{h(\theta)} = \frac{1 + \mathbb{1}\{\theta \geq \theta'_d\} (\tau')^{1-\varepsilon}}{1 + \mathbb{1}\{\theta \geq \theta_x\} \tau^{1-\varepsilon}} \left(\frac{Q'}{Q}\right)^{-\frac{\beta-\xi}{1-\beta}} \theta^{\tau^b} \quad \text{for} \quad \theta \geq \theta'_d,
\]

and this ratio equals zero for \(\theta \in [\theta_d, \theta'_d]\), as firms with productivity in this range choose to exit. Since \(Q' > Q\), the employment of non-exporting firms \((\theta'_d \leq \theta < \theta_x)\) shrinks. However, for exporters \((\theta > \theta_x)\) the effect of a reduction in trade costs, \(\tau' < \tau\), dominates, and their employment rises (see appendix). The employment of new exporters \((\theta'_x \leq \theta < \theta_x)\) then increases \textit{a fortiori}. Note that (20) and the free entry condition in
Now consider the incumbents with \( \theta \in [\theta_d, \tilde{\theta}_x] \). These firms do not stay in the long-run, but do not necessarily exit immediately or even fire any workers. There exist a cutoff \( \theta'_d \in [\theta_d, \tilde{\theta}_d] \), that firms with productivity \( \theta \in [\theta'_d, \tilde{\theta}_d] \) choose to produce in the short run until their employment declines to \( h'(\theta) \) at \( t = T(\theta) > 0 \), and then fire the remaining workers and exit.\(^{20}\) The exit employment level equalizes to zero the flow operating profit given in (10), i.e. solves \( \varphi(h'(\theta); \theta) = 0 \). The firms with productivity \( \theta < \theta'_d \) fire all workers and exit immediately upon trade liberalization.

Finally, we discuss the employment dynamics of the incumbent firms with \( \theta \geq \theta'_d \) between the instant of trade liberalization and convergence to the long run. At the instant of trade liberalization, some of these firms may choose to fire a subset of their workers to reduce their employment to \( h'(\theta) \), which equalizes the value of a marginal worker to zero, i.e. satisfies

\[ J^I(\theta) = \max \{ 0, J^I_d(\theta), J^I_x(\theta) \}, \]

where \( J^I_d(\theta) \) is the value of staying in the long run and exporting; \( J^I_x(\theta) \) is the value of staying in the long run and non-exporting; \( J^I(\theta) \) is the value of producing in the short run, but then eventually firing all remaining workers and exiting; and 0 corresponds to the outside option of firing all workers immediately and not producing at all after the trade liberalization, which exhaust all long-run strategies available to the firm. For our benchmark case, we plot these value function against the firm productivity \( \theta \) in Figure ?? in the appendix. The continuation value is the upper envelope, and the intersections of the relevant value functions determine the three cutoffs, \( \theta'_d, \theta'_d \) and \( \tilde{\theta}_x \), which we discussed in the text. Note that firms with \( \theta < \theta_d \) have no employment even before trade liberalization, \( h(\theta) = 0 \), and therefore always stay inactive.

Footnote 12 ensure that \( \theta'_x < \theta_x \).

\(^{20}\) Formally, we introduce in the appendix the continuation value function to an incumbent firm with productivity \( \theta \) and employment stock \( h(\theta) \) at the instant of trade liberalization:
$J_h^F(\tilde{h}'(\theta); \theta) = 0$. Therefore, immediately upon trade liberalization, the employment of the firms with $\theta \geq \bar{\theta}'_x$ jumps up to $h_0(\theta) = \bar{h}'(\theta)$, while the employment of the firms with $\theta \in [\bar{\theta}'_d, \bar{\theta}'_x]$ equals $h_0(\theta) = \min \{h(\theta), \tilde{h}'(\theta)\}$. This lower envelope characterizing the firing cutoff $\bar{\theta}'_d \in [\bar{\theta}'_d, \bar{\theta}'_x]$, such that the firms with $\theta \geq \bar{\theta}'_d$ do not fire workers on impact, while firms with $\theta \in [\bar{\theta}'_d, \bar{\theta}'_d]$ fire a subset of their workers on impact. Indeed, we prove in the appendix a single crossing property for $\tilde{h}'(\theta)$ and $h(\theta)$, and show that either $\bar{\theta}'_d \in [\bar{\theta}'_d, \bar{\theta}'_d]$ or $\bar{\theta}'_d = \bar{\theta}'_x$, i.e. either none of the staying firms fire workers on impact or all of them do, as their problems scale with productivity $\theta$. To summarize, when away from the frictionless labor market limit, we have the following ranking of the incumbent and entrant cutoffs:

$$\theta_d \leq \bar{\theta}'_d \leq \bar{\theta}'_d < \bar{\theta}'_x < \theta_x \quad \text{and} \quad \tilde{\theta}'_d \in [\bar{\theta}'_d, \bar{\theta}'_d] \cup \{\bar{\theta}'_x\},$$

as we illustrate in Figure 1.\(^{21}\)

![Figure 2: Employment as a function of productivity for incumbents and entrants](image)

**Note:** Benchmark parameterization with $x = 2$ (unemployment duration of 6 months and unemployment rate of 9%) in the left panel and $x = 24$ (unemployment duration of 1/2 months and unemployment rate of 1%) and a larger trade shock ($\tau' = 1.25$) in the right panel. With $x = 2$, no firm exits or fires workers on impact (i.e., $\bar{\theta}'_d = \tilde{\theta}'_d = \theta_d$), while with $x = 24$ some firms exit on impact, and some firms that stay in short run fire some workers on impact ($\theta_d < \bar{\theta}'_d < \bar{\theta}'_d < \theta_x$, as in Figure 1). The right panel zooms in on the range $[\theta_d, \theta_d']$.

After the initial period, the employment of all incumbent firms with $\theta < \bar{\theta}'_x$ declines due to

\(^{21}\)In the frictionless limit, the cutoffs collapse, and as in the static Melitz model we have: $\bar{\theta}'_d = \bar{\theta}'_d = \theta_d$ and $\tilde{\theta}'_d = \tilde{\theta}'_d = \theta_x$, so that all firms with $\theta < \bar{\theta}'_d$ fire all workers and exit immediately and all firms with $\theta \in [\bar{\theta}'_d, \theta_x]$ fire the workers to immediately reach their new long-run employment.
to natural attrition at rate $\sigma$, so that we have:

$$h_t(\theta) = \begin{cases} 
0, & \text{if } \theta < \theta_d', \\
 e^{-\sigma t} \cdot \min \{h(\theta), \tilde{h}'(\theta)\}, & \text{if } \theta \in [\theta_d', \tilde{\theta}_x') \text{ and } t \leq T(\theta), \\
\tilde{h}'(\theta), & \text{if } \theta \geq \tilde{\theta}_x',
\end{cases}$$

(28)

where $T(\theta)$ is the time at which either the exit employment $\tilde{h}'(\theta)$ is reached for firms with $\theta \in [\theta_d', \tilde{\theta}_d')$, or the new steady-state employment $\tilde{h}'(\theta)$ is reached for firms with $\theta \in [\tilde{\theta}_d', \tilde{\theta}_x')$.

Quantitatively, for our benchmark size of the trade liberalization of 50%, the new steady state employment for non-exporters is 11% below their initial employment level, and with the labor force attrition rate of $\sigma = 0.175$, it takes 7.5 months for the non-exiting firms to gradually reduce their employment to the new long-run level. The firms that eventually exit take much longer time—between 3.75 and 4.5 years—to gradually reduce their employment through attrition before they fire the remaining workers and exit the industry.

This completes our description of the employment dynamics of firms with different productivities after the trade shock, which we illustrate in Figure 2. We note the richness of employment dynamics already in this simple specification of the model without idiosyncratic productivity dynamics. The micro-level distribution of employment at each instant of the transition can be very different from the long-run distribution of the Melitz model, and even arbitrary far into the future after the shock the employment of some surviving incumbents differs from that in a Melitz model without labor market frictions. In other words, zooming into this economy, the allocation of labor across firms looks qualitatively different from that in the standard frictionless Melitz model. In light of this, the sharp result in Proposition 2—that gains in consumer surplus from trade are captured instantaneously and are independent from the extent of the labor market frictions—is particularly striking.

5.2 Entry, exit and firing

The discussion above allows us to construct the joint distribution of employment and productivity for incumbents, $G_0(h, \theta)$, given the value of the aggregate state variable ($b, \tau', Q'$) in period $t = 0$, i.e. under the assumption that entry is continuous and therefore $Q_0 = Q'$ (see Proposition 2). We now verify this assumption. In order to do so, we only need to check that,

$$T(\theta) = \frac{1}{\sigma} \log \min \{h(\theta), \tilde{h}'(\theta)\}$$

and

$$\bar{T} = \frac{1}{\sigma} \log \min \{h(\theta), \tilde{h}'(\theta)\},$$

where $T(\theta)$ is the time to exit and $\bar{T}$ is the time to the new steady-state employment for non-exporters, which does not depend on $\theta$ as the employment choices of all non-exiting firms scale with $\theta$.

22Formally, we can define
Reduction in trade costs, \( (\tau - \tau')/(\tau - 1) \)

Unemployment duration, \( 1/x \)

No Entry/

Overshooting

No Exit

or Firing

Exit/No

Firing

Exit and

Some Firing

Exit and

All Firing

Figure 3: Possible dynamic adjustment patterns

Note: Different qualitative patterns of adjustment to trade depending on the size of trade liberalization and the extent of labor market frictions (unemployment duration in years). Regions: (0) In the “No Entry/Overshooting” region, \( Q_t > Q' \) for some \( t > 0 \) and the characterization of Section 4 does not apply (see Section 6; (1) “No Exit or Firing” is the case with \( \theta'_d = \theta'_d = \theta_d \); (2) “Exit/No Firing”: \( \theta_d < \theta'_d < \theta'_d \); (3) “Exit and Some Firing” by exiting firms only: \( \theta_d < \theta'_d < \theta'_d \); (4) “Exit and All Firing”: \( \theta_d < \theta'_d < \theta'_d \) and \( \theta'_d = \theta'_d \). The region between (3) and (4) has \( \theta'_d = \theta'_d \), i.e. all exiting firms fire some workers on impact, but all staying firms do not fire any workers on impact. See Figure 1 for the definition of cutoffs. The two vertical dashed lines correspond to the main two sizes of trade liberalization we consider—the benchmark 50% (or \( \tau' = 1.375 \)) trade liberalization case and a larger trade liberalization of 67% (\( \tau' = 1.25 \)).

given \( G_0(h, \theta) \), the aggregate supply of the differentiated good by the incumbent firms does not exceed \( Q' \). In the alternative case, \( Q_0 > Q' \) and there is no entry at \( t = 0 \). We offer the formalities in the appendix. We now study how entry, exit and firing decisions of firms after a trade liberalization depend on the extent of the labor market frictions.

Figure 3 plots the alternative possible dynamic adjustment patterns depending on the size of the trade liberalization and the extent of labor market frictions.\(^{23}\) As the figure shows, when trade liberalization is small (less than a 35% reduction in trade costs), there is no entry of new firms on impact and consumer surplus overshoots in the short run, unless labor markets are extremely flexible (with unemployment duration of less than a month). When

\(^{23}\)The size of trade liberalization is measured as \( (\tau - \tau')/(\tau - 1) \), so that 0 corresponds no trade liberalization and 1 corresponds to a complete trade liberalization, i.e. no remaining iceberg trade costs (\( \tau' = 1 \)). The extent of labor market frictions is measured by expected unemployment duration, \( 1/x \), which we control by choice of the cost of vacancies relative to the matching function productivity, parameter \( a \).
trade liberalization exceeds 35%, there must be entry of new firms even if all existing firms chose not to exit or fire any workers.

Now consider a given size of the trade liberalization in excess of 35%. When labor markets are rigid, no firm chooses to exit or fire workers immediately. This is because the sunk costs of hiring the workers were substantial, and hence the surplus from the employment relationship is large and stays positive even despite the increased foreign competition in product market. As labor market frictions diminish, so does the surplus from the employment relationship, and some firms start to exit and fire workers on the impact of the trade liberalization. Specifically, first, some firms start to exit on impact ($\theta'_d > \theta_d$). Then, some of the eventually, but not immediately, exiting firms start to fire a part of their workers on impact ($\theta_d < \theta'_d < \bar{\theta}'_d$), until all of such firms fire some workers on impact ($\bar{\theta}'_d = \bar{\theta}'_x$). Finally, when labor markets are very flexible, even the staying firms choose to fire some, but not all, of their employees on impact of the trade liberalization ($\bar{\theta}'_d = \bar{\theta}'_x$).²⁴

Note that for both firing and exit to happen along the transition path, the model requires a rather flexible labor market: when the size of the trade liberalization is our benchmark 50%, exit starts to happen when unemployment duration is less than two months and firing starts to happen when it is around one month. As the size of the trade liberalization increases, both exit and firing become more likely, and happen even under more rigid labor markets. Indeed, a larger trade liberalization destroys more of the employment surplus for low-productivity incumbent firms, forcing them to cut employment or even exit immediately. However, with unemployment duration of slightly in excess of six months, the sunk cost of hiring and the surplus from employment are so large, that even a full trade liberalization does not trigger any exit or firing by firms.

### 5.3 Good jobs and bad jobs

In a model with heterogeneous firms, not all jobs fare equally well upon trade liberalization, even though in steady state all workers receive the same wages. Wages in terms of the numeraire non-traded good remain unchanged for all workers employed by the new entrants and by the exporting incumbents (with $\theta \geq \bar{\theta}'_x$) that expand after trade liberalization. Since the price index of the traded good declines, all these workers gain in real terms. So do the unemployed, who continue to receive the unemployment benefit in terms of the numeraire until they find a job in one of the entrants or expanding firms, while their prospects of finding

²⁴In the limit of the flexible labor market, as $x \to \infty$ and unemployment duration becomes zero, the model converges to the frictionless Melitz case, where all firms with productivity below $\theta'_d$ exit and all firms with $\theta \in [\theta'_d, \theta'_x)$ fire all of the excess workers immediately upon trade liberalization.
a job do not change \((x_0 \text{ and } x \text{ stay constant throughout the transition})\).\(^{25}\)

The bad jobs are concentrated among the incumbent non-exporters (with \(\theta < \bar{\theta}'_x\)) that need to shrink, and possibly exit altogether, following a trade liberalization. The (nominal) wages decline in these firms in the short run, as these firms end up with too high a labor force in the new more competitive environment (see the bargained wage schedule in (9), which declines in employment of the firm). As employment shrinks, the wages recover. In particular, in the firms that choose to stay in business in the new equilibrium, wages recover fully once the employment reaches the new long-run level.

![Figure 4: Change in the value of employed workers by firm type](image)

Note: The loss in the value of the employed workers is measured as \((\tilde{J}^E(\theta) - J^U - b)\), which we then normalize by the expected annual worker income in steady state (i.e., \(-0.02\) corresponds to a loss of 2% of annual income); \(\tilde{J}^E(\theta)\) is the continuation value of employment at a firm with productivity \(\theta\) at the instant of the trade shock, \(t = 0\), and \(b\) is the steady state worker surplus from employment. Parameterizations in the two panels correspond to those in Figure 2: (a) benchmark case in the left panel (with \(x = 2\) and \(\tau' = 1.375\)) and (b) a more flexible labor market \((x = 24)\) and a larger trade liberalization \((\tau' = 1.25)\) in the right panel. The two unmarked vertical dashed lines in the right panel correspond to \(\theta'_d\) and \(\bar{\theta}'_d\), with the following ranking of the cutoffs \(\theta_d < \theta'_d < \bar{\theta}'_d < \theta'_d\). In the left panel, no firm exits or fires workers on impact \((\theta'_d = \bar{\theta}'_d = \theta_d)\).

In Figure 4, we plot the loss in the present value of the employed workers (in terms of the numeraire good) depending on the productivity of their employer at the instant of the trade liberalization. The left panel considers the benchmark case, in which no firm exits or fires workers on impact, while some least productive firms exit eventually. The right panel considers the case with a more flexible labor market and a larger trade liberalization, which ensures both that the least productive firms exit on impact and that some other low-productivity firms

\(^{25}\)Similarly, those employed in the outside sector maintain their nominal income level, while seeing a reduction in the price index of the traded good.
fire a fraction of their employees on impact. In both cases, the value loss of the employed workers is weakly increasing in the productivity of their employer. In particular, there is no loss at all for workers employed by the exporters (with $\theta \geq \bar{\theta}'_d$). The loss in worker value is bounded above by the employment surplus, $J^E - J^U = b$, which is never binding in the left panel (with a more rigid labor market), but is binding for some least productive firms in the right panel (with a more flexible labor market).\footnote{Of course, the value of employment is much larger with more rigid labor markets: $b$ equals 27\% of annual income in the left panel with $x = 2$, while it is only 2.5\% in the right panel with $x = 24$.} Indeed, employed workers lose the full surplus when the firm shuts down and fires everyone (i.e., $\theta < \bar{\theta}'_d$), but also when it fires only some of the employees ($\theta \in [\theta'_d, \bar{\theta}'_d]$).\footnote{Clearly, all fired workers lose the full surplus, but so do their former co-workers, as all workers are symmetric and the marginal retained worker must fare as well as the marginal fired worker by the firm.} Workers maintain part of the employment surplus in firms that do not fire workers on impact ($\theta \geq \bar{\theta}'_d$). If the firm exits eventually ($\theta \in [\bar{\theta}'_d, \bar{\theta}_d]$), the value of employment is strictly increasing in the productivity of the firm (and thus in the length of time interval the firm stays in business), while the value of employment is the same in all firms that choose to stay in the long-run (with $\theta \in [\bar{\theta}'_d, \bar{\theta}_d]$), as their problem scales with productivity $\theta$.

The bad jobs in this model are directly linked to the ‘bad’ firms—the firms that end up with too many workers after the trade shock, and need to shrink and even possibly exit, whether on impact or eventually. These are the relatively low-productive non-exporting firms which suffer from the increased foreign competition.\footnote{The Eaton, Kortum, and Kramarz (2011) estimates suggest that over 90\% of French firms would shrink in response to a reduction in trade costs, emphasizing that the majority of the manufacturing workers may lose in nominal terms, and possibly resist trade liberalization. In our benchmark calibration, 76\% of firms, accounting for 23.5\% of manufacturing employment, cut their employment and wages after the trade shock.} The less productive is the firm, the larger is the loss in the value of its workers, while all workers in the most productive exporting firms necessarily gain in real terms. This prediction is consistent with a body of recent empirical work which links the welfare consequences of trade for workers to the performance of their employers (see Verhoogen, 2008; Amiti and Davis, 2011; Helpman, Itskhoki, Muendler, and Redding, 2012). The short-run predictions of the model are similar in spirit to the model with specific factors, if one reinterprets workers attached to particular firms as the specific factors whose well-being depends on the well-being of their employer (cf. Davidson, Martin, and Matusz, 1999). This contrasts with the lack of any such distributional consequences in the long run, where all workers fare equally well as they are no longer attached to particular employers.
5.4 Job destruction versus wage cuts

The discussion above already hints at the trade-off identified by the model: When the frictions in the labor market are considerable, trade shocks have the capacity to erode substantially the value for employed workers, yet the surplus from employment is large enough that no jobs are destroyed on impact and there is no excess separation into unemployment. On opposite, in the more flexible labor markets, a lot of jobs are destroyed on impact of the trade shock (by firms that either exit or fire workers), yet the employment surplus in this market is more modest and so is the reduction in the value for employed workers. This suggests, that for a given size of the trade shock, losses in the employment surplus and job destruction are likely to be negatively correlated as the labor market becomes more frictional.

Figure 5: Labor market rigidity, worker displacement, and labor income loss

Note: Benchmark parameter values as in Table 1, with the exception of the matching productivity parameter, which we vary to span various degrees of labor market rigidity, as captured by the unemployment duration, $1/x$, on the $x$-axis (same as in Figure 3). For a sharper illustration of the effects, we used a larger trade shock ($\tau' = 1.25$), yet all the qualitative patterns are the same with the benchmark trade shock ($\tau' = 1.375$). The green and blue lines are respectively the fractions of workers fired on impact of the trade shock (by exiting and shrinking firms) and all displaced workers (including those fired eventually when firms exit). The red line is the aggregate income loss of manufacturing workers as a share of the sector’s annual income, which we multiply by a factor of 10 to put on the same scale (so that 0.04 corresponds to 0.4% of annual manufacturing income). The dashed red line is the aggregate income loss by workers who are not fired immediately after the shock. The vertical dashed lines separate the adjustment regions in the parameter space (see Figure 3).

Figure 5 explore this relationship by plotting both the aggregate loss in the value of employment (i.e., the reduction in the present value of labor income relative to aggregate annual
income in the manufacturing sector) and the fraction of workers that lose their jobs after a
given trade shock, both on impact and eventually, as a function of the labor market rigidity.
In a frictionless labor market, workers experience no income loss, yet the fraction of the dis-
placed workers is the largest—equal to 7.1% of manufacturing employment, with 2.5% fired
by the staying firms and 4.6% by the exiting firms. Note that the total job destruction after
a given trade shock is bounded above by the frictionless case (i.e., 7.1% under current pa-
rameterization), while the fraction of adversely affected workers is considerably larger (23.5%
in this case), as it includes all workers employed by the staying non-exporting firms, even
those that keep their jobs indefinitely (until the exogenous separation shock), as they face
temporary wage cuts.

What changes with the labor market frictions is the composition of worker losses that
come the separation into unemployment versus the reduction in the present value of wages
(illustrated with the dashed line in the figure). As the labor market becomes frictional, the
share of displaced workers drops sharply: already with a 3-month unemployment duration, no
worker is fired on impact, and only 3.5% of workers are employed by the firms that eventually
exit the industry. In contrast, the labor income loss is increasing in the extent of labor market
rigidity, reaching its pick at 0.4% of annual manufacturing income when the unemployment
duration is between 2 and 3 months, and gradually plateauing after that at 0.3%.\textsuperscript{29} Hence,
the model generates either a significant job destruction or a significant labor income loss, but
not the two simultaneously. This prediction arises due to the flexibility of wages and absence
of inefficient job separations in the model.

5.5 Job creation and unemployment

Job destruction discussed above is offset by job creation simultaneously by entrants and
expanding incumbents-exporters. In fact, trade liberalization triggers a spike in job creation
in the manufacturing sector which expands its size across the two steady states. This is
accommodated by the reallocation of unemployed workers from the non-traded to the traded
sector. Most of the extra job creation happens in the first period, after which job creation
fluctuates little around its long-run level, which supports the continuous reallocation process
of workers from shrinking and dying firms to newly entering firms.\textsuperscript{30}

Consider now the dynamics of unemployment. Recall that under our benchmark param-
eterization with unemployment duration of 6 months, no firm fires workers on impact, and

\textsuperscript{29}As we discuss in more detail below, once unemployment durations reaches two months, further increases
in labor market rigidities (in this flexible-wage model) start to shield workers from income losses.
\textsuperscript{30}There is another (relatively small) spike in both job creation and job destruction when the incumbent
firms with low productivity start to fire their remaining workers and exit, clearing up way for new entrants.
hence the aggregate number of unemployed stays the same immediately after the trade shock. Unemployment in the traded sector spikes up on impact to maintain the constant labor market tightness in view of the extra job creation in the sector, yet it comes down close to its long-run level, equal to \( s/(x + s) \), already in the second period.\(^{31}\) There is also little fluctuation over time in the aggregate unemployment rate, as unemployed workers largely just move between the two sectors, without any substantial changes in the total number of unemployed.

Matters are slightly different when labor market is more flexible and some workers are fired at the instant of the trade shock. Indeed, as we showed in Figure 5, up to 7\% of workers can be fired on impact as labor market becomes frictionless. This results in an instantaneous increase in the stock of unemployed, and hence the aggregate unemployment rate. Recall, however, that with our benchmark size of the trade liberalization, immediate separation in unemployment only happens when the expected unemployment duration is less than one month, and hence the spike in the aggregate unemployment rate is very short lived.

### 5.6 Trade and productivity

The sunk hiring costs in a frictional labor market make incumbents reluctant to fire workers and exit the industry. This results in misallocation of labor across firms, but also leaves less room in the market for the new entrants. As a result, the selection forces after the trade shock are slower to operate. This has consequences both for aggregate productivity and aggregate trade flows along the transition path. We illustrate these aggregate dynamic effects in Figure 6 for our benchmark parameterization (Table 1).

Consider first the evolution of labor productivity in the traded sector, measured as the aggregate sector’s revenues per worker \( (Q^t_\zeta / H_t) \), which we plot in the left panel of Figure 6. From Proposition 1, this measure of productivity remains unchanged in the long run relative to the pre-trade liberalization equilibrium. Yet, in the short run labor productivity is depressed, until it overshoots its long-run level in the medium term. The productivity is low upon trade liberalization because of the misallocation of employment across firms, with too much labor employed by the relatively unproductive incumbents. As incumbent firms shrink due to the natural labor force attrition, the amount of misallocation goes down. Eventually, the consumer gains from extra variety—coming from the temporarily surviving incumbents that exit in the long run—more than offsets the losses from misallocation, causing the medium-run overshooting in productivity.

\(^{31}\) When the period in the model is one month, in our benchmark case the on-impact spike in the number of unemployed in the traded sector, in order to maintain constant labor market tightness, is sixfold. Therefore, given the benchmark size of the trade shock, Assumption 2 require that the traded sector accounts for at most 1/7 of the economy-wide employment before trade liberalization, which is easily satisfied for most developed countries. We provide further discussion of job creation and unemployment dynamics in the appendix.
Figure 6: Aggregate productivity and trade flow dynamics

Note: The left panel plots aggregate productivity in the traded sector, measured as the average sectoral revenue per worker \( (Q_t^e/H_t) \), over time \( t \) (in years) starting from the trade liberalization shock at \( t = 0 \). The right panel, similarly, plots the aggregate trade flow, measured as export revenues scaled by the traded sector’s expenditure \( Q_t^e \). Both panels correspond to the benchmark parameterization in Table 1; in particular, the job finding rate is \( x = 2 \) and the iceberg trade cost after the shock is \( \tau' = 1.375 \). In both panels, the vertical dashed lines correspond to the time it takes the incumbents to shrink to the new steady-state size (\( \bar{T} \approx 7.5 \) months) and for the least and most productive exiting firms to leave the industry (\( T(\theta_d) \approx 4 \) and \( T(\bar{\theta}'_d) \approx 4.5 \) years, respectively). The horizontal dashed line in the right panel indicates the amount of trade in the new steady state.

Note that the model features no long-run misallocation of labor, measured following Hsieh and Klenow (2009) as the dispersion of average revenue product of labor across firms.\(^{32}\) A trade shock, however, results in a spike in misallocation during the transition period, when non-exporting incumbent firms are too large and have lower marginal products of labor. The model is hence consistent with the empirical pattern that larger firms and exporters appear to be too small in relative terms. The testable implication of the model is that misallocation should be particularly pronounced among the incumbents relative to the new entrants after a major trade liberalization episode.

Consider now the aggregate trade flows, which we plot over time after the trade shock in the right panel of Figure 6. The trade flows jump on impact, but are depressed in the short run relative to their long-run level. This is because the surviving incumbents are on average less productive and less likely to export than the new entrants active in the industry. Over time, the employment share reallocates towards the new entrants, and the trade flows converge towards their long-run level.\(^{33}\) Therefore, our model with an adjustment costs in

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\(^{32}\)This follows from the optimal employment choice (12) substituted into the revenue function (7).

\(^{33}\)The full convergence does not happen in finite time, before all incumbents die, since the incumbent
the form of labor market frictions generates trade undershooting, i.e. a lower trade elasticity in the short-run relative to the long-run, with the amount of missing trade in the short run increasing with the labor market frictions.

In view of this discussion, it is further surprising that consumers can reap full benefits from trade liberalization right on impact and independently of the extent of the labor market frictions (Proposition 2), given that this frictions are consequential for the amount of trade during the potentially lengthy transition in the labor market. The extent to which the incumbents crowd out the new entrants in the short run is precisely offset, at each point in time, by the output and additional varieties produced by the incumbents. This is exactly what the free entry condition ensures, at the cost of a lower short-run productivity in the traded sector due to misallocation of the resources.

5.7 Income loss and gains from trade

Along with an increase in the consumer surplus from the traded good, the adjustment to the trade liberalization triggers an income loss (in terms of the numeraire good) both for firms and workers. Some workers are separated into unemployment on impact or in the aftermath of the trade shock and some other workers have to take a wage cut in the short run, as we analyzed in Section 5.3. We now discuss how the valuation of firms changes after the trade shock.

The firms make lower profits in the domestic market and larger profits in the foreign market if they choose to export. As a result, the non-exporting firms lose, in particular because of the sunk investment into hiring their labor force, which ends up being excessive after the trade liberalization. The continuation value for some of these firms may become negative and in this case they exit on impact of the trade shock. The most productive exporters necessarily gain in value, as they choose to increase their employment after the shock.

In the frictionless Melitz model with Pareto productivity, the gains and the losses in firm profits exactly offset each other, and the total value of the business sector remains unchanged. This is not the case when labor markets are frictional. The sunk employment costs result in the greater losses relative to gains in the firm value. These firm value losses are borne out by the household, who, as we assume, are the owners of the diversified portfolio of domestic firms. In regular times, this portfolio generates the normal return (equal to the discount rate \( r \)), but a trade liberalization results in a subnormal return on the domestic stock market, with the size of the capital loss increasing in the extend of the labor market rigidities. This constitutes

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production and exporting cutoffs are different from the new long-run cutoffs (see Figure 1). Taking into account the endogenous election forces, the incumbents are on average less productive and less likely to export relative to the surviving entrants.
another testable implication of the model.

Table 2: Decomposition of the dynamic gains from trade

<table>
<thead>
<tr>
<th>Trade shock, $\tau'$ (% change)</th>
<th>1.375 (50%)</th>
<th>1.25 (66.7%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job finding rate, $x$</td>
<td>2</td>
<td>24</td>
</tr>
<tr>
<td>(1) Annual gains in consumer surplus</td>
<td>5.60</td>
<td>5.60</td>
</tr>
<tr>
<td>(2) Capital-income loss</td>
<td>0.24</td>
<td>0.06</td>
</tr>
<tr>
<td>(3) Wage-income loss</td>
<td>0.09</td>
<td>0.12</td>
</tr>
<tr>
<td>(3') — due to unemployment</td>
<td>—</td>
<td>0.03</td>
</tr>
<tr>
<td>(4) Full gains from trade $= (1) - r[(2) + (3)]$</td>
<td>5.585</td>
<td>5.595</td>
</tr>
</tbody>
</table>

Note: All numbers are percentages relative to the annual consumer surplus from the traded good before the trade liberalization, $(1 - \zeta)/\zeta \cdot Q^C$. (1) is $(1 - \zeta)/\zeta \cdot [Q^C - Q^C]$ is the annualized gain the consumer consumer surplus; (2) and (3) are the net present value of the aggregate firm-profit and wage-income losses; (3a) is the part of the wage-income loss that comes from the separation into unemployment (job destruction) at the instant of the trade shock; (4) are the full dynamic gains from trade according to (29), which subtracts the flow values of (2) and (3) from (1). We consider two sizes of the trade shocks (50% and 66.7% reduction in the iceberg trade cost $\tau$) and two values of the labor market frictions ($x = 2$ and $x = 24$ corresponding to a 6 months and 0.5 months unemployment duration).

We can now return to the question of the dynamic welfare gains from trade, which combine the gains in consumer surplus characterized in Proposition 2 with the income losses of the households. The two sources of the household income loss are the capital-income loss from the ownership of the firms and the wage-income loss, which in particular includes income loss from the endogenous separation into unemployment upon the trade liberalization. We measure the full dynamic gains from trade according to:

$$GT \equiv -r(L^F + L^W) + \frac{1 - \zeta}{\zeta}[(Q')^C - Q^C] \frac{1 - \zeta}{\zeta}Q^C,$$

where $L^F$ and $L^W$ are the aggregate net present values of capital- and wage-income loss of the households, and we convert them into the annual flow terms using the discount rate $r$. The second term in the numerator is the annualized gain in the consumer surplus from the traded good. Table 2 and Figure 7 summarize the results.

Both Table 2 and Figure 7 show that the income losses are small relative to the gains in the consumer surplus. Given the annualized discount rate of $r = 0.05$, the income losses in the aftermath of the trade liberalization constitute only two hundreds of a percent (0.02%) of the lifetime gains in the consumer surplus. In the annualized terms, these losses are 20 times larger: that is, in the first year after the trade liberalization the (present value of) income losses associated with the labor market frictions is between 0.18% and 0.78% of the annual
Figure 7: Labor market rigidities and dynamics gains from trade

Note: The figure plots the full dynamic gains from trade (solid blue line) against the extent of labor market frictions (measured by the unemployment duration, $1/x$), and decomposes it into its components—gains in the consumer surplus (solid flat black line), wage-income loss (the area between the black line and the green dashed line), its component due to separation into unemployment (the area between the solid black line and the red dashed line), and the capital-income loss (the area between the dashed green line and the solid blue line). The size of the trade liberalization is 50% ($\tau' = 1.375$) and other parameters are as in Table 1. The vertical dashed lines indicate the regions of the parameter space with no firm exit and no firing by the non-exiting firms respectively.

Consumer surplus from the traded good, depending on the size of the trade liberalization and the extent of the labor market frictions. The losses from the separation into unemployment are either absent altogether or small relative to the overall wage-income loss in this economy. The ranking of capital- and wage-income loss depends on the extent of labor market frictions, as we discuss next.

Figure 7 illustrates that the full dynamic gains from trade are strictly below the gains in the consumer surplus when labor market is frictional, and monotonically decreasing in the extent of the labor market frictions. The decomposition of income losses into the capital and wage loss is, however, non-monotonic, with the wage-income loss dominating in the more flexible labor markets and the capital-income loss dominating in the more rigid labor markets. In fact, the wage-income loss starts to shrink in absolute terms as unemployment duration exceeds 1.5 months (in our calibration). The reason is that the labor market frictions, under flexible wage bargaining, shield workers from the separation into unemployment because of the firms’ sunk and costly ex ante investment in hiring. As a result, most losses in the aftermath of the
trade liberalization are borne out by the firms when the labor markets are rigid. Therefore, the model suggests that when a country can only choose between rigid and very rigid labor markets, the latter choice may result in lower losses for the labor force in response to aggregate shocks.\footnote{Of course, the ultimate residual claimants on firm income are households, and the aggregate households income losses increase in the extent of labor market frictions.}

\section{Discussion}

We construct a tractable model to study the dynamic adjustment to a trade shock in an economy with heterogeneous firms and labor market frictions, which impede the fast within-industry labor reallocation across firms. As a result of the labor market frictions, unproductive incumbent firms do not exit immediately after the trade shock, instead of giving way to the new more productive entrants. This slows down the Melitz selection forces triggered by the reduction in trade costs, and the industry productivity is depressed in the short run due to the misallocation of labor. Since labor is misallocated away from more productive exporting firms, the trade flows in the model are also depressed in the short run. Despite all of these departures from a frictionless allocation, the welfare gains from trade are realized instantaneously, and furthermore the proportional gains in the consumer surplus, which constitute the only source of the long-run gains from trade, are independent from the extent of the labor market frictions.

The tractability of our model relies on a number of strong assumptions, which however we view as empirically plausible in the context of the current framework, which aims to isolate the direct partial effect of labor market frictions. The general purpose of the assumptions is to rule out various sources of convexity in the model, which are not directly related to the adjustment (i.e., hiring) costs of labor reallocation across firms. Thus, we view our model as a benchmark limiting case to analyze the effects of the labor market frictions. We now discuss these assumptions and possible extensions of the model which relax them.

First, we assume quasi-linear utility in the non-traded good to rule out convexity in the utility function (Assumption 1). As we discuss, this is a mild assumption, which can be replaced by a constant interest rate assumption common in the small open economy literature. The purpose of this assumption is to allow all required entry (and hence fixed cost expenditure) to happen in one period without affecting the marginal utility of consumption. Yet, we emphasize that the curvature in the utility functions is not directly related to the labor market frictions, and its effects will be qualitatively the same even in a model with frictionless labor market.

Second, we allow for the unconstrained entry of firms at a constant fixed cost, which again
allows all of the required entry to happen in one period. Instead, one could imagine a model in which the pool of potential entrants is limited each period, or there are other types of decreasing returns in entry, so that the entry of firms is spread out in time. Once again, just like with curvature in the utility function, this force has nothing to do with the labor market frictions, and its effects should have the same qualitative effect even with a frictionless labor market. Both these assumptions can be easily relaxed numerically.

Third, we model the labor market friction as a linear adjustment costs at the firm level, so that when firms choose to hire workers, they immediately adjust their employment to the long-run optimal level, rather than gradually increasing their employment towards the target. This assumption arises from a constant-returns-to-scale matching function, both at the firm and at the industry level. While not being general, this constitutes a natural benchmark case, and the alternative cases can be explored numerically in the future work. The model can also easily accommodate the extension to the case with firing costs, as it would not affect the qualitative properties of the inaction region in the model.

Fourth, and most importantly, we assume that the traded sector is relatively small and that unemployed workers are homogeneous and perfectly mobile across sectors (Assumption 2). This assumption ensures that increased labor demand in the traded sector does not result in the complete reallocation of all economy-wide unemployment towards the traded sector, i.e. some unemployed continue to search for a job in the non-traded sector. As a result, the non-traded sector acts as a buffer for workers, and the labor market tightness and hiring costs can remain constant in both sectors during the transition period. Viewed through the prism of the adopted labor-market model, this assumption is trivially satisfied empirically, as there are always some job opening in each sector of the economy, independently of the sectoral shocks. Yet, the model focuses on the DMP-style labor market friction, which has a very particular structure: it takes a spell of unemployment to find a job, but the unemployed workers can frictionlessly move across sectors, and are equally likely to be matched with and hired by any firms in the traded sector. Thus the model fully ignores all mobility costs associated, for example, with sector- or firm-specific human capital of the workers. Numerically, it is easy to introduce into the model the costs of mobility across sectors, and in particular one may consider a special limiting case with no inter-sectoral mobility, in which dynamics are the same as in a one-sector trade model.

Fifth, we assume no idiosyncratic productivity evolution of the firms after entry. This greatly simplifies the analysis, as in the alternative case the state space is the full joint distribution $G_t(h, \theta)$, which makes the model intractable even numerically, without the use of

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**35 An additional required assumption is the linearity of production and matching, along with free entry, in the non-traded sector, which is the standard assumption in the DMP literature.**
a variant of Krusell-Smith approximation techniques (see Elsby and Michaels, 2013). While the idiosyncratic productivity shocks to the firms are clearly important empirically, there is no obvious reason to expect that they will necessarily amplify or moderate the role of the labor market frictions in shaping the response of the economy to an aggregate trade shock.

Finally, the last two assumptions which we have made are the symmetry of countries and the flexibility of wage setting via bargaining without commitment. The symmetry of counters ensures that trade is a positive shock for the sector at the aggregate, as the sector expends on net, albeit with heterogenous effects across firms and workers. The flexibility of the wage bargaining ensures that there are no inefficient separations into unemployment after the trade shock. A more complete quantitative model of the adjustment to trade in a frictional labor market may incorporate multiple sectors with sector-level comparative advantage (so that some of the sectors shrink on net after the trade liberalization), slow creation of new jobs due to decreasing return in firm entry and/or hiring, and rigid wages to generate a spike in inefficient job separations on impact of the trade shock. We leave this quantitative extension to future work.
A APPENDIX—PRELIMINARY

A.1 Matching function

Matching of unemployed and vacancies in each sector is governed by a Cobb-Douglas matching function, i.e. the (annualized) matching rate is given by:

\[ m = m(U, V) = \frac{1}{\tilde{a}} U^{\chi} V^{1-\chi}, \quad (A1) \]

where \( U \) is the stock of unemployed searching for a job and \( V \) is the stock of open vacancies, both in a given sector, and \( \tilde{a} \) is the inverse of the productivity of the matching function. Therefore, \( m \Delta \) is the flow of new vacancies during the period of length \( \Delta \). The job finding rate is given by

\[ x = \frac{m}{U} = \frac{1}{\tilde{a}} \left( \frac{V}{U} \right)^{1-\chi}, \]

while the vacancy-filling rate is

\[ g = \frac{m}{V} = \frac{1}{\tilde{a}} \left( \frac{V}{U} \right)^{-\chi} = (\tilde{a} x^{-\chi})^{\frac{1}{1-\chi}}. \]

We assume that the time period length \( \Delta \) is short enough so that the probabilities of finding a job (\( x \Delta \)) and filling a vacancy (\( g \Delta \)) are both well-defined (i.e., less than one). We denote by \( \gamma \Delta \) the flow cost of posting a vacancy during period \( \Delta \). Then the expected cost of filling a vacancy is

\[ b \equiv \frac{\gamma \Delta}{g \Delta} = \frac{\gamma}{g} = a x^\alpha, \quad (A2) \]

where \( a \equiv \gamma \tilde{a}^{1+\alpha} \) and \( \alpha \equiv \chi/(1 - \chi) \), and this equation corresponds to (3) and (8) in the text. In other words, \( b \) is a per-worker (expected) cost of a match: by paying \( b \) instantaneously, the firm fills (with certainty, by the law of large numbers) a measure one of vacancies by the end of the period \( \Delta \).\(^{36}\) We assume that the two sectors share the same \( \alpha \), but may differ in terms of \( \gamma \) and/or \( \tilde{a} \), and hence in terms of \( a \).

\(^{36}\)Consider a firm posting \( v \) vacancies for \( n \) periods, where \( t = n \Delta \) is the corresponding length of time (in years). Assuming \( t \) is small, so no discounting is needed, the total cost of this action is \( v(\gamma \Delta)n = vgt \). The expected yield of matches from this action is \( v(g \Delta)n = vgt \). Now if \( v \) is the measure of vacancies posted, then by law of large numbers \( vgt \) is the measure of workers met. Therefore, by paying \( (v\gamma t)/(vgt) = \gamma/g = b \), a firm can meet a measure one of workers during a period of arbitrary small time length \( t \); a corresponding flow payment per period is \((b/t) \cdot \Delta\), which equals \( b \) if all hiring is done in one period (i.e, if \( t = \Delta \)).
A.2 Labor market in the outside sector (Lemma 1)

We denote by \( J^U_0 \) and \( J^E_0 \) the values to unemployed and employed workers in the outside sector, which satisfy the following Bellman equations:\(^{37}\)

\[
J^U_0 = b_u \Delta + \frac{x_0 \Delta}{1 + r \Delta} J^E_{0,+} + \frac{1 - x_0 \Delta}{1 + r \Delta} \dot{J}^U_{0,+},
\]

\[
J^E_0 = w_0 \Delta + \frac{s_0 \Delta}{1 + r \Delta} J^U_{0,+} + \frac{1 - s_0 \Delta}{1 + r \Delta} \dot{J}^E_{0,+},
\]

where the + subscript denotes the next period’s variables.

We denote by \( J^V_0 \) and \( J^F_0 \) the values to a vacant and a filled job in the outside sector, which satisfy the following Bellman equations:

\[
J^V_0 = -\gamma \Delta + \frac{1 - \delta_0 \Delta}{1 + r \Delta} \left[ g_0 \Delta \cdot J^F_{0,+} + (1 - g_0 \Delta) \dot{J}^V_{0,+} \right],
\]

\[
J^F_0 = (1 - w_0) \Delta + \frac{1 - \delta_0 \Delta}{1 + r \Delta} \left[ \sigma_0 \Delta \cdot J^V_{0,+} + (1 - \sigma_0 \Delta) \dot{J}^F_{0,+} \right],
\]

where \( \delta_0 \) is the death rate of firms and \( \sigma_0 \) is the exogenous separation rate with workers, so that from the point of view of workers the overall exogenous separation rate is \( s_0 \) defined by \( (1 - s_0 \Delta) = (1 - \delta_0 \Delta)(1 - \sigma_0 \Delta) \). All our results hold in the special case of \( \delta_0 = 0 \) and \( s_0 = \sigma_0 \), however, we introduce \( \delta_0 = \delta \) for symmetry with the differentiated sector to simplify some discrete-time expressions, and the differences between the two cases disappear altogether as \( \Delta \to 0 \).

Given free entry and unbounded pool of potential entrants in the homogenous sector, we must have \( J^V_0 \leq 0 \) in all periods, which holds with equality in all periods when firms post positive vacancies, \( V_0 > 0 \). Note that \( U_{0,t} > 0 \) and \( V_{0,t} = 0 \) is inconsistent with equilibrium, since in this case the vacancy is filled instantaneously (and costlessly in the limit as \( \Delta \to 0 \)). Therefore, Assumption 2 implies \( V_{0,t} > 0 \) in all periods along the equilibrium path, and, therefore,

\[
J^V_0 \equiv 0
\]

(A7) then implies that the present value of a filled job in the homogenous sector next period equals the current period hiring cost:

\[
\frac{1 - \delta \Delta}{1 + r \Delta} J^F_{0,+} = b_0,
\]

along the whole equilibrium path.

Upon matching, the firm and the worker determine wages according to Nash bargaining with equal weights and without commitment. This means that in each period when the match is not exogenously destroyed, we have:

\[
J^E_0 - J^U_0 = J^F_0 - J^V_0.
\]

(A9)

\(^{37}\)When \( \Delta \approx 0 \), the following approximation is accurate:

\[
r \dot{J}^U_0 = b_u + x_0 (J^E_0 - J^U_0) + \dot{J}^U_0,
\]

where \( \dot{J}^U_0 \equiv (J^U_{0,+} - J^U_0)/\Delta \). This equation is a generalized version of (5) in the text. By analogy, similar approximations can be used for other value functions.
We also combine (A3)–(A4) and (A5)–(A6) to obtain:

\[
J^E_0 - J^U'_0 = (w_0 - b_u)\Delta + \frac{1 - (s_0 + x_0)\Delta}{1 + r\Delta} (J^E_{0,+} - J^U'_{0,+}),
\]

\[
J^F_0 - J^V'_0 = (1 - w_0)\Delta + \frac{1 - s_0\Delta}{1 + r\Delta} (J^F_{0,+} - J^V'_{0,+}) + \left(\frac{1 - \delta\Delta}{1 + r\Delta} J^V_{0,+} - J^V'_0\right).
\]

We can sum these two equations to eliminate the wage rate, \(w_0\), and, using (A7)–(A9), obtain a dynamic equation for \(b_0\):

\[
\frac{1 + r\Delta}{1 - \delta\Delta} 2b_{0,-1} = (1 - b_u)\Delta + \frac{2(1 - s_0\Delta - x_0\Delta) b_0}{1 - \delta\Delta},
\]

or equivalently:

\[
\frac{2(r + s_0) + x_0\Delta}{1 - \delta\Delta} b_0 = 1 - b_u + 2 + r\Delta b_0 - b_{0,-1},
\]

(A10)

where the \((-1)\)-subscript indicates the previous period.\(^{38}\) Given (A2), the unique stationary solution of this difference equation has \(b_0\) constant, and therefore:

\[
\frac{2(r + s_0) + x_0\Delta}{1 - \delta\Delta} b_0 = 1 - b_u,
\]

(A11)

which corresponds to (4) in the text, after taking the \(\Delta \approx 0\) approximation. Along the explosive solutions of (A10), \(b_0\) converges to zero or becomes unbounded in finite time, both of which are inconsistent with the optimizing behavior (of either workers or firms).

With \((x_0, b_0)\) pinned down by (A2) and (A11), the rest of the equilibrium in the homogenous sector is characterized from (A3)–(A9). In particular, we have:

\[
w_0 = b_u + \frac{(r + s_0 + x_0) b_0}{1 - \delta\Delta},
\]

\[
\pi_0 = 1 - w_0 = \frac{(r + s_0) b_0}{1 - \delta\Delta},
\]

\[
rJ^U_0 = b_u + \frac{x_0 b_0}{1 - \delta\Delta} + \frac{1}{1 + r\Delta} J^U_{0,+} - J^U'_0.
\]

The last equation also has a unique stationary solution

\[
\frac{rJ^U_0}{1 + r\Delta} = b_u + \frac{x_0 b_0}{1 - \delta\Delta},
\]

(A12)

with non-stationary solutions violating the no-bubble condition. This equation corresponds to (5) in the text, where we use the approximation that \(\Delta \approx 0\). These derivations provide a proof of Lemma 1.

\(^{38}\)In the limit with \(\Delta \to 0\), this becomes an ordinary differential equation in \(b_0\) (provided the relationship \(b_0 = a_0 x_0^2\)):

\[
[2(r + s_0) + x_0] b_0 = 1 - b_u + 2b_0,
\]

which has a unique non-explosive path with a constant \(b_0\).
A.3 Differentiated sector

Product market A firm splits its output between domestic and foreign markets:

\[ y = q + \nu \tau q^*, \]

where \( \tau \) is the iceberg trade costs and \( \nu \in \{0, 1\} \) is the indicator of whether the firm exports. Given the CES aggregator (1) and the utility function (2), the demand for the demand for a good in a given market satisfies

\[ q = Q \left( \frac{p}{P} \right)^{-\frac{1}{1-\beta}} \quad \text{and} \quad P = Q^{-(1-\zeta)}, \]

which results in a revenue

\[ pq = Q^{-(\beta-\zeta)} q^\beta. \]

Therefore, the firm’s optimal revenue from serving the two markets given output \( y \) is given by:

\[ R(y, \nu) = \max_{(q, q^*)}: \quad q + \nu \tau q^* = y \left\{ Q^{-(\beta-\zeta)} q^\beta + \nu (Q^*)^{-(\beta-\zeta)} (q^*)^\beta \right\} = \left[ 1 + \nu \tau^{-\frac{\beta}{1-\beta}} \left( \frac{Q^*}{Q} \right)^{-\frac{\beta-\zeta}{1-\beta}} \right] Q^{-(\beta-\zeta)} y^\beta, \]

which corresponds to (7) after we substitute \( y = \theta h \) in. The quantities supplied to each market are:

\[ q = \frac{1}{1 + \nu \tau^{-\frac{\beta}{1-\beta}} \left( \frac{Q^*}{Q} \right)^{-\frac{\beta-\zeta}{1-\beta}}} y \quad \text{and} \quad q^* = \frac{\nu \tau^{-\frac{1}{1-\beta}} \left( \frac{Q^*}{Q} \right)^{-\frac{\beta-\zeta}{1-\beta}} y}{1 + \nu \tau^{-\frac{\beta}{1-\beta}} \left( \frac{Q^*}{Q} \right)^{-\frac{\beta-\zeta}{1-\beta}}}. \]

Firm’s problem The Bellman equation for the problem of the firm is given by:

\[ J^F(h) = \max_{h'} \left\{ \varphi(h) \Delta - b [h' - (1 - \sigma \Delta) h]^{+} + \frac{1 - \delta \Delta}{1 + r \Delta} J^F_{h'} (h') \right\}, \]

where \( J^F(h) \) and \( J^F_{h'} (h') \) are the values of the firm in this and next periods with employment \( h \) and \( h' \) respectively; the flow per-period operating revenue gross of hiring cost is denoted by \( \varphi(h) \Delta \); we have substituted in the hiring cost \( C(h', h) \) from (11); and \( [\cdot]^{+} \equiv \max\{\cdot, 0\} \). A firm dies at rate \( \delta \), loses labor at an exogenous rate \( \sigma \), and discounts at rate \( r \). We assume \((1 - s \Delta) \equiv (1 - \delta \Delta)(1 - \sigma \Delta)\), which for small \( \Delta \) is approximately equivalent to \( s = \delta + \sigma \), as we state in the text.

The first order condition for the choice of \( h' \) is given by:

\[ \frac{1 - \delta \Delta}{1 + r \Delta} J^F_{h',+} (h') = \begin{cases} b, & \text{when } h' > (1 - \sigma \Delta) h, \\ \in [0, b], & \text{when } h' = (1 - \sigma \Delta) h, \\ 0, & \text{when } h' < (1 - \sigma \Delta) h, \end{cases} \]

where the \( h \)-subscript denotes a partial derivative with respect to employment. The Envelope theo-
rem is given by:

\[ J_F^h(h) = \varphi'(h) \Delta + \begin{cases} 
(1 - \sigma \Delta)b, & \text{when } h' > (1 - \sigma \Delta)h, \\
1 - s \Delta J_{h,+}(h'), & \text{when } h' = (1 - \sigma \Delta)h, \\
0, & \text{when } h' < (1 - \sigma \Delta)h,
\end{cases} \]

where the intermediate case corresponds to the inaction region in which \( h' = (1 - \sigma \Delta)h \) and the hiring cost \( C(h', h) = 0 \); in this region, the continuation value is \( \frac{1 - s \Delta}{1 + r \Delta} J_{h,+}(h') \), and its derivative with respect to \( h \) is \( \frac{1 - s \Delta}{1 + r \Delta} J_F^h(h) \). Combining the Envelope theorem with the first order condition, we obtain

\[ J_F^h(h) = \varphi'(h) \Delta + \frac{1 - s \Delta}{1 + r \Delta} J_{h,+}(h'), \]

(A16)

which holds for all possible firm’s hiring decisions (hiring, firing, inaction).

**Wage schedule (Lemma 2)** The firm bargains with all its workers each period according to the Stole and Zwiebel (1996) bargaining protocol, that is it Nash bargains bilaterally with each worker with equal bargaining weights internalizing the effect of potential worker’s departure on the wage rebargaining with all other workers. The Bellman equation characterizing the value to the worker of employment at a given firm with labor force \( h \) is given by:

\[ J^E(h) - J^U = w(h) \Delta + \frac{1 - s \Delta}{1 + r \Delta} (J^E_{h,+}(h') - J^U_{h,+}) + \left( \frac{1}{1 + r \Delta} J^U_{h,+} - J^U \right), \]

Stole and Zweibel surplus division implies:

\[ J^E(h) - J^U = J^F_h(h) \]

(A17)

for all \( h \) and in each period of time (that is, including next period for workers that stay with the firm, independently of firm’s hiring decision \( h' \)). Combining the above two expressions with (A16), we obtain a static differential equation in \( h \) for \( w(h) \):

\[ \varphi'(h) = w(h) - \Delta^U, \quad \text{where} \quad \Delta^U \equiv \frac{1}{\Delta} \left( J^U - \frac{1}{1 + r \Delta} J^U_{h,+} \right). \]

Note that the values of continued employment within the firm have dropped out from both sides, and therefore the wage determination is effectively static given the dynamics of the value of unemployment. The flow revenue function is defined as:

\[ \varphi(h) \equiv R(h) - w(h)h - f_d - \varphi f_x, \]

where we suppressed \((\varphi; \theta)\) notation inside the revenue function defined in (7), as well as inside the wage schedule. Taking the derivative and substituting in the differential equation, we have:

\[ R'(h) - w'(h)h - w(h) = w(h) - \Delta^U. \]
Rearranging, we have \(2w(h) + w'(h)h = R'(h) + \Delta^U\), which we can integrate analytically by multiplying both sides by \(h\):

\[
w(h)h^2 = \int_0^h \left( R'(\tilde{h}) + \Delta^U \right) \tilde{h} d\tilde{h} = \frac{\beta}{1 + \beta} R(h)h + \frac{1}{2} \Delta^U h^2,
\]

where we have used the power function structure of, \(R(h) = \Theta^1 - ^{}h^\beta\), which implies \(R'(h) = \beta \Theta^1 - ^{}h^\beta - 1\). Note that we have set the constant of integration to zero to ensure that the wage bill with zero workers is zero, \(w(h)h|_{h=0} = 0\). Dividing by \(h^2\), we obtain:

\[
w(h) = \frac{\beta}{1 + \beta} \frac{R(h)}{h} + \frac{1}{2} \Delta^U.
\]  
(A18)

This corresponds to (14) in the text after we impose \(J^U_+ = J^U\), so that \(\Delta^U = rJ^U/(1 + r\Delta)\), and use the approximation with \(\Delta \approx 0\). This derivation provides a proof of Lemma 2. Substituting (A18) into the definition of \(\varphi(h)\), we have:

\[
\varphi(h) = \frac{1}{1 + \beta} R(h) - \frac{1}{2} \Delta^U h - f_d - \nu_f,
\]  
(A19)

which corresponds to (10) in the text, again using approximation \(\Delta \approx 0\) for \(\Delta^U\).

**Value to unemployed (Lemma 3(a))** is characterized by a Bellman equation similar to (A3):

\[
J^U - \frac{1}{1 + r\Delta} J^U_+ = b_u \Delta + \frac{x \Delta}{1 + r \Delta} E(J^E(h') - J^U_+),
\]  
(A20)

where the expectation \(E\) is taken across all potential employers. However, for all hiring firms (i.e., all potential employers) we have from Stole and Zwiebel bargaining (A17) together with firm optimization (A15):

\[
J^E(h') - J^U_+ = J^E_{h,+}(h') = \frac{1 + r \Delta}{1 - \delta \Delta} b.
\]

Substituting this into the Bellman equation (A20), we have:

\[
J^U - \frac{1}{1 + r \Delta} J^U_+ = \left( b_u + \frac{xb}{1 - \delta \Delta} \right) \Delta.
\]  
(A21)

Since there always are some of the unemployed in the homogenous sector (Assumption 2), the indifference condition implies that \(J^U = J^U_0\), which is constant over time (Lemma 1). Therefore, we have, using (A12):

\[
J^U - \frac{1}{1 + r \Delta} J^U_+ = J^U_0 - \frac{1}{1 + r \Delta} J^U_{0,+} = \frac{rJ^U_0 \Delta}{1 + r \Delta} = \left( b_u + \frac{x_0 b_0}{1 - \delta \Delta} \right) \Delta.
\]

Combining the two expressions above results in \(xb = x_0 b_0\). In view of Lemma 1 and relationship (8), this implies that \((x, b)\) must be constant over time, just like \((x_0, b_0)\). This derivation provides a proof for the first part of Lemma 3. Furthermore, combining \(xb = x_0 b_0\) with (8) results in (13).
Optimal hiring (Lemma 3(b))  Consider a firm hiring in a current period, as well as in the next period. Then the first order condition (A15) together with the Envelope theorem (A16) can be written as:

$$\varphi'(h') \Delta = \frac{1 + r \Delta}{1 - \delta \Delta} b - (1 - \sigma \Delta) b_+, \quad (A22)$$

and $h' > (1 - \sigma \Delta) h$. From the first part of Lemma 3, we know that $b$ is constant over time, and therefore we can rewrite:

$$\varphi'(h') = \frac{r + s}{1 - \delta \Delta} b,$$

where we have used $(1 - \sigma \Delta)(1 - \delta \Delta) = (1 - s \Delta)$, and similarly for $h$. Using the function form of $\varphi(h)$ defined in (A19), we have the optimal employment given by:

$$\frac{\beta}{1 + \beta} \Theta_{1-\beta} h_{\beta-1} = \frac{1}{2} \Delta U + \frac{r + s}{1 - \delta \Delta} b = \frac{1}{2} \left[ b_u + \frac{2(r + s) + x}{1 - \delta \Delta} b \right],$$

where we used the definition of $\Delta U$ and (A21) according to which:

$$\Delta U = \frac{r \Delta U}{1 + r \Delta} = b_u + \frac{xb}{1 - \delta \Delta}.$$  

Solving for optimal employment results in:

$$h = \left( \frac{2 \beta}{1 + \beta} \left[ b_u + \frac{2(r + s) + x}{1 - \delta \Delta} b \right]^{-1} \right)^{\frac{1}{1-\beta}} \Theta, \quad (A23)$$

which corresponds to (12), under the approximation $\Delta \approx 0$. This derivation completes the proof of Lemma 3. Finally, substituting (A23) into the wage schedule (A18) and using $R(h) = \Theta_{1-\beta} h_{\beta}$ from (7), we obtain the wage rate paid by the hiring firms:

$$w = b_u + \frac{(r + s + x) b}{1 - \delta \Delta}, \quad (A24)$$

which corresponds to (14) in the text under the approximation $\Delta \approx 0$.

Value of a hiring firm (Lemma 4)  Now consider a firm which enters at $t$ and hires in every period after entry, $h' > (1 - \sigma \Delta) h$. Specializing the Bellman equation (A14) for this case, we have:

$$J^F(h) = \varphi(h) \Delta - b [h' - (1 - \sigma \Delta) h] + \frac{1 - \delta \Delta}{1 + r \Delta} J^F(h'),$$

Note that hiring in the consecutive periods of time is a typical outcome for firms, unless there is a sharp movement in aggregate variables. This is because the optimal employment evolves continuously in the aggregate variables, and a hiring firm in one period will likely need to replace, at least, partly the attrition next period, unless its optimal employment declines sharply with some aggregate variable.
where \( h \) and \( h' \) satisfy the optimality condition (A15) for the hiring case, that is
\[
\frac{1 + r\Delta}{1 - \delta\Delta} h - b = \varphi'(h)\Delta + (1 - \sigma\Delta) b,
\]
where the \((-1)\)-subscript corresponds to the previous period. Multiplying this expression by \( h \) and subtracting from the Bellman equation above, we have:
\[
J_F(h) - \frac{1 + r\Delta}{1 - \delta\Delta} b h = [\varphi(h) - \varphi'(h)h] \Delta + \frac{1 - \delta\Delta}{1 + r\Delta} J_F^+'(h') - bh',
\]
which defines a difference equation for \( J_F(h) - \frac{1 + r\Delta}{1 - \delta\Delta} b h \). Using the functional form for \( \varphi(h) \) defined in (A19) and substituting optimal employment from (A23), we have:
\[
\varphi(h) - \varphi'(h)h = \frac{1 - \beta}{1 + \beta} \Phi \Theta - f_d - \iota f_x,
\]
where \( \Phi \equiv \left( \frac{2\beta}{1 + \beta} \left[ b_u + \frac{2(r + s) + x}{1 - \delta\Delta} b \right]^{-1} \right)^{-\frac{1}{1 + \beta}} \) and corresponds to the definition of \( \Phi \) (12) in the text under the approximation \( \Delta \approx 0 \). Finally, we rewrite (A25) as:
\[
J_{V-1} = \frac{1 - \delta\Delta}{1 + r\Delta} \left[ \frac{1 - \beta}{1 + \beta} \Phi \Theta - f_d - \iota f_x \right] \Delta + \frac{1 - \delta\Delta}{1 + r\Delta} J_V,
\]
where the value of a vacant firm equals
\[
J_V = \frac{1 - \delta\Delta}{1 + r\Delta} J_F^+(h') - bh',
\]
since a firm with zero employees can pay a cost of \( bh' \) in the current period to obtain the value \( J_F^+(h') \) next period, conditional on surviving with probability \((1 - \delta\Delta)\). After an algebraic manipulation, and using approximation \( \Delta \approx 0 \), (A26) correspond to (16) in the text.\footnote{As \( \Delta \to 0 \), the following approximation is accurate (for optimal employment \( h \)):
\[
J_F(h) = bh + J_V, \quad (r + \delta) J_V - J_V = \frac{1 - \beta}{1 + \beta} \Phi \Theta - f_d - \iota f_x.
\]}

A.4 Steady state

In steady state both \( \Phi \) and \( \Theta \) are constant and therefore \( \theta_d \) and \( \theta_x \) are constant. Firms have natural attrition of labor force and hire in every period to offset it. Additionally, firms die at an exogenous rate \( \delta \) and are replaced by new entrants, i.e. there is positive entry every period. As a result, Lemma 4 applies and the value of an entrant with productivity \( \theta \) and zero employment is given by
\[
J_V(\theta) = \max \left\{ 0, \frac{1 - \delta\Delta}{r + \delta} \max_{\iota \in \{0,1\}} \left[ \frac{1 - \beta}{1 + \beta} \Phi \Theta(\iota; \theta) - f_d - \iota f_x \right] \right\}.
\]
The two max operators in (A27) define the entry/exit and the export cutoffs:

\[
\max_{\iota \in \{0,1\}} \left[ \frac{1 - \beta}{1 + \beta} \Phi(\iota; \theta_d) - \iota f_x \right] = f_d,
\]

\[
\frac{1 - \beta}{1 + \beta} \Phi[\Theta(1; \theta_x) - \Theta(0; \theta_x)] = f_x.
\]

Additionally, as in Melitz (2003), we assume that \( \tau \left( \frac{f_x}{f_d} \right)^{(1-\beta)/\beta} > 1 \), which as we show below (see (A31)) ensures \( \theta_x > \theta_d \). Under these circumstances, and using the definition of \( \Theta \) in (7), we can rewrite the two cutoff conditions above as:

\[
\frac{1 - \beta}{1 + \beta} \Phi \left[ Q^{-\frac{\beta - \zeta}{1 - \beta}} \theta_d^{\frac{\beta}{1 - \beta}} \right] = f_d, \quad \text{(A28)}
\]

\[
\frac{1 - \beta}{1 + \beta} \Phi \left[ Q^{-\frac{\beta - \zeta}{1 - \beta}} \theta_x^{\frac{\beta}{1 - \beta}} \right] = f_x. \quad \text{(A29)}
\]

We can use (A28)–(A29) to rewrite the value function of an entrant in (A27) as:

\[
J^V(\theta) = \frac{1 - \delta}{r + \delta} \left[ f_d \left[ \left( \frac{\theta}{\theta_d} \right)^{\frac{\beta}{1 - \beta}} - 1 \right]^+ + f_x \left[ \left( \frac{\theta}{\theta_x} \right)^{\frac{\beta}{1 - \beta}} - 1 \right]^+ \right],
\]

where \([\cdot]^+ \equiv \max\{\cdot, 0\}\). Given this expression and the positive entry flow of firms, the free entry condition is

\[
\int J^V(\theta) dG(\theta) = f_e,
\]

which under the Pareto distributional assumption can be simplified to:

\[
\frac{1}{1 - \beta} \left[ f_d \theta_d^{-\beta} + f_x \theta_x^{-\beta} \right] = \frac{r + \delta}{1 - \delta} f_e. \quad \text{(A30)}
\]

Under the symmetry of the two countries (implying \( Q = Q^* \)), the free entry condition (A30) together with the two cutoff conditions (A28)–(A29) determine \( (\theta_d, \theta_x, Q) \) in steady state, just like in the static model. Taking the ratio of the two cutoff conditions we have:

\[
\frac{\theta_x}{\theta_d} = \tau \left( \frac{f_x}{f_d} \right)^{\frac{1 - \beta}{\beta}}. \quad \text{(A31)}
\]

Note that (A30)–(A31) allow us to solve for \( (\theta_d, \theta_x) \) and then recover \( Q \) from (A28).

Given \( Q \) and the cutoffs, we can characterize individual decisions of every firm, as well as aggregate mass of firms \( M \) operating at every instant and aggregate employment \( H \) in the differentiated sector. In particular, we can rewrite (17) as:

\[
H = M \int_{\theta_d} \theta dG(\theta) = M \Phi^{1/\beta} Q^{-\frac{\beta - \zeta}{1 - \beta}} \int_{\theta_d} \left[ 1 + \mathbf{1}_{\{\theta \geq \theta_x\}} \tau^{-\frac{\beta}{1 - \beta}} \right] \theta^{-\frac{\beta}{1 - \beta}} dG(\theta)
\]

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and (1) as:

\[ Q^\beta = M \left( \int_{\theta_d}^{\infty} q_d(\theta)^\beta dG(\theta) + \int_{\theta_x}^{\infty} q_x(\theta)^\beta dG(\theta) \right) \]
\[ = M \Phi Q^{-\beta} \frac{1}{\hat{\tau}^{\frac{2}{1-\beta}}} \int_{\theta_d}^{\infty} \left[ 1 + 1_{\{\theta \geq \theta_x\}} \frac{\theta}{\hat{\tau}^{\frac{\beta}{1-\beta}}} \right] \frac{\theta^{2-\beta}}{\hat{\tau}^{1-\beta}} dG(\theta), \]

where we have used (A23) for employment \( h(\theta), \) \( y(\theta) = \theta h(\theta), \) and the split of \( y(\theta) \) into \( q_d(\theta) \) and \( q_x(\theta) \) from (A13). Combining the expressions for \( Q \) and \( H \) we have:

\[ H = \Phi \frac{1-\beta}{\hat{\tau}} Q^\beta, \]

corresponding to (21) in the text. Finally, \( M \) can be recovered from either expression. Additionally, under the Pareto distribution \( \theta, \) we can simplify the integral in the expression for \( H: \)

\[ H = M \Phi^{1/\beta} Q^{-\frac{\beta-1}{\beta-\beta}} \theta_d^{\frac{\beta}{1-\beta}} \int_{\theta_d}^{\infty} \left[ 1 + 1_{\{\theta \geq \theta_x\}} \frac{\theta}{\hat{\tau}^{\frac{\beta}{1-\beta}}} \right] \left( \theta/\theta_d \right)^{\frac{\beta}{1-\beta}} dG(\theta) \]
\[ = M \Phi^{1/\beta} \frac{1 + \beta}{1-\beta} f_d \left[ \int_{\theta_d}^{\infty} (\theta/\theta_d)^{\frac{\beta}{1-\beta}} dG(\theta) + f_x \int_{\theta_x}^{\infty} (\theta/\theta_x)^{\frac{\beta}{1-\beta}} dG(\theta) \right] \]
\[ = M \Phi^{1/\beta} \frac{1 + \beta}{1-\beta} \frac{k}{k-1} \left[ f_d \theta_d^{-k} + f_x \theta_x^{-k} \right] = M \Phi^{1/\beta} \frac{1 + \beta}{\beta} \frac{r + \delta}{1 - \delta \Delta} f_e, \]

where the second line uses the cutoff condition (A28) and substitutes in (A31) to split the integral into two parts, the third line integrates using the properties of Pareto and the last equality utilizes the free entry condition (A30). Combining the two above expressions, we obtain (22) in the text.

**A.4.1 Comparative statics across steady states**

Consider a reduction in \( \tau. \) We now characterize changes in both aggregate and firm-specific variables across the two steady states. To obtain analytical results for large changes in \( \tau, \) we adopt the Pareto distribution. Then we can plug in the cutoff ratio (A31) into the free entry (A30) to solve for \( \theta_d \) explicitly as a function of \( \tau: \)

\[ \theta_d = \left[ \frac{(1-\beta) k - 1}(r + \delta)/(1 - \delta \Delta) \cdot \frac{f_e}{f_d} \right]^{1/k}, \quad (A32) \]

so that \( \theta_d \) is an increasing function of \( \tau \) and each change in \( \theta_d \) can be linked to a corresponding change in \( \tau. \) Therefore, we can express changes in other variables as function of the change in \( \theta_d: \)

\[ \frac{Q'}{Q} = \left( \frac{\theta_d'}{\theta_d} \right)^{\frac{\beta}{\beta-\beta}} \quad \text{and} \quad \frac{H'}{H} = \frac{M'}{M} = \left( \frac{\theta_d'}{\theta_d} \right)^{\frac{\beta \zeta}{\beta-\beta}}, \]

which follow from (A28), (21) and (22).\(^{41}\) Finally, note from (A32) that neither level nor change in \( \theta_d \) depend on the level of search frictions (reflected in equilibrium values of \( x \) and \( b \)), and therefore

\(^{41}\)The number of active firms, equal to \( M \theta_d^{-k} \), increases when \( \tau \) decreases if and only if \( \beta \zeta > k(\beta - \zeta). \)
the effect of a change in \( \tau \) on the change in the aggregate variables such as \( Q \), \( H \) and \( M \) also does not depend on the frictions in the labor market.

Finally, consider the firm-specific employment outcomes:

\[
    h(\theta) = \Phi^{1/\beta} \left[ 1 + 1_{(\theta \geq \theta_x)} \tau \right] Q^{-\beta} \theta^{\beta} \\
    = \frac{1 + \beta}{1 - \beta} \Phi^{1/\beta} \left[ f_d(\theta/\theta_d)^{\beta} + f_x(\theta/\theta_x)^{\beta} \right], \quad \text{for } \theta \geq \theta_d,
\]

where the second line substitutes in the cutoff condition (A28) and the uses the cutoff ratio (A31) to split the two terms. For \( \theta < \theta_d \), \( h(\theta) = 0 \). Comparing the two steady state with \( \tau \) and \( \tau' < \tau \), we have four types of firms: (i) firms with \( \theta \in [\theta_d, \theta_d'] \) exit; (ii) firms with \( \theta \in [\theta_d', \theta_x'] \) reduce employment and stay in the industry as non-exporters; (iii) firms with \( \theta \in [\theta_x', \theta_x] \) increase employment and start to export; and (iv) firms with \( \theta \geq \theta_x \) continue to export, reduce domestic sales, increase export sales and increase overall employment:

\[
    \frac{h'(\theta)}{h(\theta)} = \left( \frac{\theta_d}{\theta_d'} \right)^{\beta} \cdot \left[ \begin{array}{c} 0 \quad = \quad 0, \quad \text{for } \theta \in [\theta_d, \theta_d'], \\
1 \quad < \; 1, \quad \text{for } \theta \in [\theta_d', \theta_x'], \\
\frac{1 + \tau' - \beta/(1-\beta)}{1 + \tau - \beta/(1-\beta)} \quad > \; 1, \quad \text{for } \theta \in [\theta_x', \theta_x], \\
\frac{1 + \tau' - \beta/(1-\beta)}{1 + \tau - \beta/(1-\beta)} \quad > \; 1, \quad \text{for } \theta \geq \theta_x. \end{array} \right]
\]

Note that \( \theta_x \) decreases when \( \theta_d \) increases to satisfy the free entry condition (A30), and hence \( \theta_x' < \theta_x \). The last inequality is ensured by the parameter restriction \( k > \beta/(1-\beta) \) and the free entry condition (A30), which requires that \( f_d \theta_d^{-k} + f_x \theta_x^{-k} \) is constant, and hence \( f_d \theta_d^{-\beta/(1-\beta)} + f_x \theta_x^{-\beta/(1-\beta)} \) must increase with \( \theta_d \), and hence so does \( h(\theta) \) for every \( \theta \geq \theta_x \). Finally, it is easy to see that the employment increase for the firms with \( \theta \in [\theta_x', \theta_x] \) is strictly larger than for the firms with \( \theta \geq \theta_x \).

TO BE COMPLETED

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References


