How Expensive is Commitment?

Oleg Itskhoki

itskhoki@fas.harvard.edu
www.people.fas.harvard.edu/~itskhoki

Preliminary and Incomplete

Macro Lunch
Harvard University

November 15, 2006
Introduction:
Contemporary Macro Models

- Benchmark Neoclassical Model: Representative Agent Stochastic Growth Model (RBC model)
  - Fails empirically on multiple dimensions, especially in the international context (BKK [1992])
Introduction:
Contemporary Macro Models

- Benchmark Neoclassical Model: Representative Agent Stochastic Growth Model (RBC model)
  - Fails empirically on multiple dimensions, especially in the international context (BKK [1992])

- To fix it we add frictions, or augment the benchmark model with constraints:
  - Incomplete markets
  - Transportation costs
  - Sticky prices
  - Limited Commitment
  - Informational frictions
  - etc.
Motivation: Limited Commitment

- Limited Commitment (also Time Inconsistency)
  - very popular friction
  - introduced by Kydland and Prescott [1977]
  - APS [1990] methodology of solving dynamic models with LC
Motivation:

Limited Commitment

- Limited Commitment (also Time Inconsistency)
  - very popular friction
  - introduced by Kydland and Prescott [1977]
  - APS [1990] methodology of solving dynamic models with LC

- Extensive Recent Macro Literature:
  - Endogenously Incomplete Markets:
    - reduces capital flows and the extent of equilibrium risk sharing
    - leads to positively correlated investment
    - Kehoe and Levine [1993], Kocherlakota [1996], Kehoe and Perri [2002]
  - Capital Outflows in Bad States: Atkeson [1991]
  - Fiscal Amplification: Aguiar, Amador and Gopinath [2006]
  - Non-zero Capital Taxes in the Long-Run: Benhabib and Rustichini [1997], Phelan and Stacchetti [2001]
This Paper... 

- However often when LC models are simulated they do not provide interesting dynamics in the long-run: commitment problem is fully resolved in the long-run.
- Computational papers often need to assume impatience: see Sleet [2006]
• However often when LC models are simulated they do not provide interesting dynamics in the long-run: commitment problem is fully resolved in the long-run

• Computational papers often need to assume impatience: see Sleet [2006]

• I provide a theoretical result about these models: under what conditions LC friction goes away or remains in the long-run:
  – one-sided limited commitment
  – ability to accumulate (risk-free) assets
  – patience: $\beta(1 + r) \geq 1$

• The focus of the paper at the moment are SOE models
Main Idea and Results

- Asset accumulation serves the role of collateral for SOE
- This technology is feasible and does not require to waist resources
Main Idea and Results

- Asset accumulation serves the role of collateral for SOE
- This technology is feasible and does not require to waist resources

- It is optimal when patient: $\beta(1 + r) \geq 1$
- Moreover, it is nearly costless
- Leads to bounded or unbounded asset accumulation
Main Idea and Results

- Asset accumulation serves the role of collateral for SOE
- This technology is feasible and does not require to waist resources
  - It is optimal when patient: $\beta(1 + r) \geq 1$
  - Moreover, it is nearly costless
  - Leads to bounded or unbounded asset accumulation
- Not true when the economy is impatient: commitment problem is never resolved then
- Similar on the transition path for a growing economy
  \[ \beta \frac{u_{c,t+j}}{u_{c,t}} < \beta \]
Intuition for the Result

- When $\beta(1 + r) = 1$ an economy with commitment is indifferent on the margin whether to accumulate assets or not.

- If the limited commitment constraint is more tight for low levels of assets, the countries would want to accumulate forever (unless the constraint would no longer ever be binding).

- It is also the intuition for why this accumulation is nearly costless.

- Why the economy would not do so when it is impatient even marginally?
  - infeasibility of the first best allocation
  - first order optimality
• Technically, the result is similar to that of Chamberlain and Wilson [1988→2000] and Aiyagari [1994] but is more general (a more general form of the constraint)
  — Martingale Convergence Theorem for the sequence of the scaled value functions (which in this case form a sub-martingale)
Related Paper

- Technically, the result is similar to that of Chamberlain and Wilson [1988–2000] and Aiyagari [1994] but is more general (a more general form of the constraint)
  - Martingale Convergence Theorem for the sequence of the scaled value functions (which in this case form a sub-martingale)

- Conceptually, the result is similar to back-loading argument in Acemoglu, Golosov and Tsyvinski [2006]:
  - The cost of back-loading is second-order while it provides first order gains from relaxing the IC constraints in all periods prior to the deferred payment
  - all are in deterministic set-ups
Implications I

What does this result tell us about the models?

- Many LC models may have interesting predictions but potentially only for off-the-equilibrium dynamics
- LC friction by itself may not be enough
- What is a reasonable additional ingredient?
- When is one-sided LC a reasonable assumption?
Implications II

What does this result tell us about the world?

- Reinforces the Bulow-Rogoff [1989] result: we should expect small countries not only not to borrow but also to save extensively abroad
Implications II

What does this result tell us about the world?

- Reinforces the Bulow-Rogoff [1989] result: we should expect small countries not only not to borrow but also to save extensively abroad

- Why this does not happen?
  - Accumulation technology is imperfect?
  - LC friction is not important?
  - Still on transition path? Development Traps?
  - Impatience?
Outline

1. Introduction

2. Theoretical Results
   - Warm-up
   - General Set-Up
   - Assumptions
   - Results

3. Applications
   - Endogenously Incomplete Markets
   - Optimal Capital Taxation
   - Atkeson Model
Warm-up: CW Result

Consider an endowment SOE characterized by the following Bellman Equation:

$$V(a, z) = \max_{c,a'} \left\{ u(c) + \beta \mathbb{E}\{ V(a', z') | z \} \right\}$$

subject to the standard resource constraint:

$$c + a' = (1 + r)a + y(z)$$

and the natural debt limit constraint:

$$a' \geq a$$

→ exogenously incomplete markets
Warm-up: CW Result

• Optimality condition:

\[ V_a(a, z) \geq \beta (1 + r) \mathbb{E} \{ V_a(a', z') \mid z \} \]

with equality when \( a' > \underline{a} \)
Warm-up: CW Result

- Optimality condition:

\[ V_a(a, z) \geq \beta(1 + r) \mathbb{E}\left\{ V(a', z') \right\} \]

with equality when \( a' > a \)

- Therefore, \( \left\{ [\beta(1 + r)]^t V(a_t, z_t) \right\}_{t=0}^\infty \) is a sub-martingale and by MCT converges (a.s.) since \( V_a(\cdot) \geq 0 \). For \( \beta(1 + r) \geq 1 \) this implies convergence of \( \left\{ V(a_t, z_t) \right\}_{t=0}^\infty \)
Warm-up: CW Result

- Optimality condition:

\[ V_a(a, z) \geq \beta(1 + r) \mathbb{E}\{ V_a(a', z') | z \} \]

with equality when \( a' > a \)

- Therefore, \( \{ [\beta(1 + r)]^t V_a(a_t, z_t) \}_{t=0}^\infty \) is a sub-martingale and by MCT converges (a.s.) since \( V_a(\cdot) \geq 0 \). For \( \beta(1 + r) \geq 1 \) this implies convergence of \( \{ V_a(a_t, z_t) \}_{t=0}^\infty \)

- Under some regularity conditions, \( u_c(c_t) \propto V_a(a_t, z_t) \), which implies convergence of \( \{ u_c(c_t) \}_{t=0}^\infty \)
Warm-up: CW Result

- **Optimality condition:**
  \[ V_a(a, z) \geq \beta(1 + r)\mathbb{E}\{V_a(a', z')|z\} \]
  with equality when \( a' > a \)

- Therefore, \( \{[\beta(1 + r)]^t V_a(a_t, z_t)\}_{t=0}^{\infty} \) is a sub-martingale and by MCT converges (a.s.) since \( V_a(\cdot) \geq 0 \). For \( \beta(1 + r) \geq 1 \) this implies convergence of \( \{V_a(a_t, z_t)\}_{t=0}^{\infty} \)

- Under some regularity conditions, \( u_c(c_t) \propto V_a(a_t, z_t) \), which implies convergence of \( \{u_c(c_t)\}_{t=0}^{\infty} \)

- Either \( c_t \to \bar{c} < \infty \) or \( c_t \to \infty \)
  - The first option is feasible only if \( y_t \to \bar{y} \), since \( c_t + a_{t+1} = (1 + r)a_t + y_t \)
  - If \( y_t \) is mean stationary but does not converge, \( a_t \to \infty \)
  - A problem for SOE models (Schmitt-Grohé and Uribe [2003])
General Set-Up

Bellman Equation:

$$V(a, \eta, z) = \max_{(\xi, a', \eta') \in \Omega} \left\{ u(\xi) + \beta \mathbb{E}\{V(a', \eta', z')|z}\right\}$$

subject to the technological constraint:

$$a' - (1 + r)a - F(\xi, \eta, \eta'; z) \leq 0 \quad \text{(TC)}$$

and the incentive compatibility constraint

$$\forall z' \in \mathbb{Z} \quad V(a', \eta', z') \geq U(a', \eta', z') \quad \text{(IC)}$$

- $\xi$ is the control variable
- $\eta$ is the endogenous state variable other than risk-free assets $a$
- $z$ is the exogenous state variable
- $U(\cdot)$ is the value after deviation
- $\tilde{\gamma}(z') \equiv \frac{\pi(z')}{1+r} \cdot \gamma(z')$ is LM on (IC) and $\lambda$ is LM on (TC)
Example

• Utility:

\[ u(\xi) \equiv u(c, \ell) = \frac{1}{1-\sigma} c^{1-\sigma} - \frac{1}{1+\phi} \ell^{1+\phi} \]

• Technology:

\[ F(\xi, \eta, \eta'; z) \equiv F(c, \ell, k, k'; z) = z \cdot f(k, \ell) + (1 - \delta)k - k' - c \]

• Value after Deviation:

\[ U(k, z) = \max_{c, \ell, k' \geq 0} \left\{ u(c, \ell) + \beta \mathbb{E}\{U(k', z') \mid z}\right\} \]

subject to \( \tilde{F}(c, \ell, k, k'; z) \geq 0 \)
Assumptions

1. **Utility:** is increasing in $\xi_1$ and concave in $\xi$

2. **Technology:** $F(\cdot)$ is
   - (i) concave in $(\xi, \eta, \eta')$
   - (ii) increasing in $\eta$ and $z$, decreasing in $\eta'$
   - (iii) $u_{\xi_j}(\cdot) \cdot F_{\xi_j}(\cdot) \leq 0$

3. **Deviation Value:** increasing and concave in $(a, \eta, z)$

4. **IC Constraint:** $V(a, \eta, z) - U(a, \eta, z) \geq 0$ is
   - (i) convex in $a$
   - (ii) binding for low enough $a$
   - (iii) slack for high enough $a$
   - (iv) monotone in $a$: $V_a(\cdot) > U_a(\cdot)$

5. **Stationarity:** $\{z_t\}$ is mean stationary and has a non-degenerate conditional distributions
Unconstrained Allocation

• Envelop Theorem for asset accumulation:

\[-(1 + r) u_{\xi_j}(\xi) / F_{\xi_j}(\xi, \eta, \eta'; z) = V^*_a(a, \eta, z)\]

\[(1 + r) u_c(c, \ell) = V^*_a(a, k, z)\]

• Optimality condition for asset accumulation:

\[V^*_a(a, \eta, z) = \beta(1 + r) \mathbb{E}\{V^*_a(a', \eta', z')|z\}\]

\[u_c(c, \ell) = \beta(1 + r) \mathbb{E}\{u_c(c', \ell')|z\}\]

• Optimality Condition for \(\eta\):

\[\beta \mathbb{E}_t \{\lambda_{t+1} F_{\eta, t+1}\} + \lambda_t F_{\eta', t} = 0\]

\[\beta \mathbb{E}_t \{u_{c, t+1} \cdot [z_{t+1} f_k, t+1 + (1 - \delta)]\} = u_{c, t}\]
Preliminary Results

Lemma 1: Sub-Martingale Property

— Optimal asset accumulation:

\[ V_a(a, \eta, z) = \beta(1 + r) \mathbb{E}\{ V_a(a', \eta', z') | z \} + \]
\[ + \mathbb{E}\left\{ \gamma(z') \cdot \left[ V_a(a', \eta', z') - U_a(a', \eta', z') \right] | z \right\} \geq 0 \]

\[ > 0 \]

\[ \rightarrow V_a(a, \eta, z) \geq \beta(1 + r) \mathbb{E}\{ V_a(a', \eta', z') | z \} \]
Preliminary Results

Lemma 1: Sub-Martingale Property

— Optimal asset accumulation:

\[ V_a(a, \eta, z) = \beta(1 + r) \mathbb{E}\{ V_a(a', \eta', z') | z \} + \]
\[ + \mathbb{E} \left\{ \gamma(z') \cdot \left[ V_a(a', \eta', z') - U_a(a', \eta', z') \right] | z \right\} \geq 0 \]
\[ > 0 \]

\[ \rightarrow V_a(a, \eta, z) \geq \beta(1 + r) \mathbb{E}\{ V_a(a', \eta', z') | z \} \]

Result 1: Martingale Convergence Theorem

\[ \left\{ [\beta(1 + r)]^t V_a(a_t, \eta_t, z_t) \right\}_{t=0}^\infty \]

converges to a non-negative constant along every equilibrium path.
The Main Theorem

\[ V_{a,t} = \beta(1 + r)^{T} \mathbb{E}_t V_{a,t+T} + \sum_{j=1}^{T} \beta(1 + r)^{j-1} \mathbb{E}_t \{ \gamma_{t+j} \cdot [V_{a,t+j} - U_{a,t+j}] \} \]
The Main Theorem

\[ V_{a,t} = [\beta(1 + r)]^\tau \mathbb{E}_t V_{a,t+\tau} + \sum_{j=1}^\tau [\beta(1 + r)]^{j-1} \mathbb{E}_t \{ \gamma_{t+j} \cdot [V_{a,t+j} - U_{a,t+j}] \} \]

Theorem

(a) \( \beta(1 + r) \geq 1 \): the commitment problem is fully resolved and the unconstrained allocation is achieved in the long-run
- (IC) is not binding, or \( \gamma_t \to 0 \), as \( t \to \infty \)
- \( [V(a_t, \eta_t, z_t) - V^*(a_t, \eta_t, z_t)] \to 0 \), as \( t \to \infty \)
- \( \eta_t \to \eta_t^* \), as \( t \to \infty \)

(b) \( \beta(1 + r) < 1 \): the commitment problem is never fully resolved and the first best cannot be achieved
- (IC) binds, or \( \gamma_t + \tau > 0 \), with positive probability
- \( V_t < V_t^* \)
- \( \eta_t + \tau \neq \eta_t^* + \tau \) when \( \gamma_t + \tau > 0 \), as long as \( V_{\eta_t, t+1} \neq U_{\eta_t, t+1} \)
The Main Theorem

\[ V_{a,t} = [\beta(1 + r)]^T E_t V_{a,t+\tau} + \sum_{j=1}^{\tau} [\beta(1 + r)]^{j-1} E_t \{ \gamma_{t+j} \cdot [V_{a,t+j} - U_{a,t+j}] \} \]

Theorem

(a) \( \beta(1 + r) \geq 1 \): the commitment problem is fully resolved and the unconstrained allocation is achieved in the long-run
   - (IC) is not binding, or \( \gamma_t \to 0 \), as \( t \to \infty \)
   - \( [V(a_t, \eta_t, z_t) - V^*(a_t, \eta_t, z_t)] \to 0 \), as \( t \to \infty \)
   - \( \eta_t \to \eta^*_t \), as \( t \to \infty \)

(b) \( \beta(1 + r) < 1 \): the commitment problem is never fully resolved and the first best cannot be achieved
   - (IC) binds, or \( \gamma_{t+\tau} > 0 \), with positive probability
   - \( V_t < V^*_t \)
   - \( \eta_{t+\tau} \neq \eta^*_{t+\tau} \) when \( \gamma_{t+\tau} > 0 \), as long as \( V_{\eta,t+1} \neq U_{\eta,t+1} \).
Proposition

Consider the case $\beta (1 + r) = 1$. Let $F_{\xi_1} \equiv \text{const.}$ Then:

(i) $\xi_1$ either is eventually a constant or converges to infinity (a.s.)

(ii) In the later case, $a$ necessarily converges to infinity as well

(iii) In the former case, $a$ remains at the lower bound of the region for which (IC) is not binding and perfect smoothing is feasible
Additional Results

Proposition
Consider the case $\beta(1 + r) = 1$. Let $F_{\xi_1} \equiv \text{const.}$ Then:

(i) $\xi_1$ either is eventually a constant or converges to infinity (a.s.)
(ii) In the later case, $\eta$ necessarily converges to infinity as well
(iii) In the former case, $\eta$ remains at the lower bound of the region for which (IC) is not binding and perfect smoothing is feasible

Proposition
Consider the case $\beta(1 + r) \in (1 - \varepsilon, 1)$. For small enough $\varepsilon$, the first best allocation for $\eta$ is achieved in some states of the world but never in all states of the world.

Remark: This has yet to be proven!
Welfare Costs

Proposition

Consider the case $\beta(1 + r) \geq 1$.

(a) If $u_{\xi_1} \equiv \text{const}$ (i.e., risk-neutrality), the welfare cost is exactly zero. That is, limited commitment does not reduce welfare.
Welfare Costs

Proposition

Consider the case $\beta(1 + r) \geq 1$.

(a) If $u_{\xi_1} \equiv \text{const}$ (i.e., risk-neutrality), the welfare cost is exactly zero. That is, limited commitment does not reduce welfare.

(b) If $u_{\xi_1} \xi_1 < 0$ (i.e., risk-aversion), then the welfare costs are positive but second order. That is, it is feasible not to distort the expected present value of the stream of $\xi_1$, while its allocation across time is distorted.
Welfare Costs

Proposition
Consider the case $\beta(1 + r) \geq 1$.

(a) If $u_{\xi_1} \equiv \text{const}$ (i.e., risk-neutrality), the welfare cost is exactly zero. That is, limited commitment does not reduce welfare.

(b) If $u_{\xi_1}\xi_1 < 0$ (i.e., risk-aversion), then the welfare costs are positive but second order. That is, it is feasible not to distort the expected present value of the stream of $\xi_1$, while its allocation across time is distorted.

Proposition
For the case $\beta(1 + r) < 1$, the welfare cost increases as $\beta$ falls for a given $r$. The welfare cost is continuous at $\beta(1 + r) = 1$. That is, there is a discontinuous change in allocation at $\beta(1 + r) = 1$ but the change in welfare cost is continuous.
• Generalize the set-up to a closed economy: make \( r \) endogenous

• As long as \( \beta(1 + r_t) \) converges to a number greater than 1, my results still hold

• The general equilibrium effect of incentive constraints is likely to be increased savings and reduced interest rate (like in Aiyagari [1994])
Applications

Application I: Optimal Capital Taxation without Commitment

Application II: Endogenously Incomplete Markets Model, or Risk-Sharing without Commitment

Application II: Endogenously Incomplete Markets

- Models of Endogenously Incomplete Markets:
  - Kocherlakota [1996]
  - Kehoe and Perri [2002]

  are models with two-sided lack of commitment

- Models of one-sided lack of commitment are commonly used for SOE’s
  - Is this a reasonable assumption?
  - Can this setting arise naturally?

- Lack of commitment reduces the extend of risk-sharing and capital flows
Application II:
Endogenously Incomplete Markets

Utility in Autarky: \[ U(z) = u(y(z)) + \beta \mathbb{E}\{U(z')|z}\] 

Constrained Optimal Value: 
\[ V(a, z) = \max_{a', c} \left\{ u(c) + \beta \mathbb{E}\{V(a'(z'), z')|z\} \right\} \]
subject to 
\[ \forall z' \in \mathbb{Z} \quad a'(z') = (1 + r)(a - c) + y(z) + d(z), \]
\[ \mathbb{E}\{d(z')|z\} = 0, \]
\[ \forall z' \in \mathbb{Z} \quad V(a'(z'), z') \geq U(z') \]
Application II: Endogenously Incomplete Markets

Proposition

(a) First Best is characterized by perfect consumption smoothing
(b) For low $a$, (IC) cannot be satisfied
(c) For high $a$, (IC) is slack and consumption is smooth
(d) For intermediate $a$, (IC) limits the extent of risk sharing
(e) For $\beta (1 + r) \geq 1$ the economy accumulates assets till it reaches perfect consumption smoothing
(f) For $\beta (1 + r) < 1$, the economy remains in the region of $a$ for which (IC) has positive conditional probability of being binding and, thus, perfect consumption smoothing is not achieved
Application II:
Endogenously Incomplete Markets

Figure: Value Functions: $\beta(1 + r) = 1$ (left) and $\beta(1 + r) < 1$ (right)
Application II:
Endogenously Incomplete Markets

Figure: Equilibrium Dynamics: $\beta(1 + r) = 1$
Application II: Endogenously Incomplete Markets

Figure: Equilibrium Dynamics: $\beta(1 + r) < 1$
Application I:
Optimal Capital Taxation

- Ramsey Taxation with Commitment
  - Zero Capital Tax in the Long-Run (Chamley-Judd Result)

- Zero taxes in the Long-Run with the absence of commitment
  - Benhabib and Rustichini [1997]
  - Phelan and Stacchetti [2001]

- Zero Taxation Result would be restored if one allows for risk-free asset accumulation
  - Domínguez [2006]
  - Reis [2006]

- Optimal Capital Taxation in a SOE fits my set-up
  - e.g., Aguiar, Amador and Gopinath [2006]
Application I: Optimal Capital Taxation

- Ramsey Taxation with Commitment
  - Zero Capital Tax in the Long-Run (Chamley-Judd Result)

- Non-zero taxes in the Long-Run with the absence of commitment
  - Benhabib and Rustichini [1997]
  - Phelan and Stacchetti [2001]

- Zero Taxation Result would be restored if one allows for risk-free asset accumulation
  - Domínguez [2006]
  - Reis [2006]

- Optimal Capital Taxation in a SOE fits my set-up
  - e.g., Aguiar, Amador and Gopinath [2006]
Application I: Optimal Capital Taxation

- Ramsey Taxation with Commitment
  - Zero Capital Tax in the Long-Run (Chamley-Judd Result)

- Non-zero taxes in the Long-Run with the absence of commitment
  - Benhabib and Rustichini [1997]
  - Phelan and Stacchetti [2001]

- Zero Taxation Result would be restored if one allows for risk-free asset accumulation
  - Domínguez [2006]
  - Reis [2006]
Application I: Optimal Capital Taxation

- Ramsey Taxation with Commitment
  - Zero Capital Tax in the Long-Run (Chamley-Judd Result)

- Non-zero taxes in the Long-Run with the absence of commitment
  - Benhabib and Rustichini [1997]
  - Phelan and Stacchetti [2001]

- Zero Taxation Result would be restored if one allows for risk-free asset accumulation
  - Domínguez [2006]
  - Reis [2006]

- Optimal Capital Taxation in a SOE fits my set-up
  - e.g., Aguiar, Amador and Gopinath [2006]
Application III:
Atkeson’s [1991] Model

International Lending with Moral Hazard and Risk of Repudiation:

- Stochastic Production Economy with Moral Hazard:
  \[ Y_t \in \{ Y_1, \ldots, Y_N \} \equiv Y \]
  \[ g(Y; I) \equiv \Pr\{ Y_{t+1} = Y \in Y | I_t = I \} \]

- Competitive international state contingent lending:
  \[ b_t = \delta \sum_{Y_{t+1} \in Y} d_{t+1}(Y_{t+1})g(Y_{t+1}; I_t) \]

- State variable: \( Q_t \) – wealth after repayment on the contract
  \[ c_t + I_t - b_t \leq Q_t \equiv Y_t - d_t(Y_t) \]

- Incentive Compatible Repayment: \( V(Q_{t+1}) \geq U(Y_{t+1}) \)
Application III:
Atkeson’s [1991] Model

Autarky:
\[ b_{t+j} \equiv d_{t+j+1}(\cdot) \equiv 0, \quad \forall j \geq 0 \]

\[ \Rightarrow \quad U(Q) = \max_{I \in [0, Q]} \left\{ (1 - \delta)u(Q - I) + \delta \sum_{Y \in \mathbf{Y}} U(Y)g(Y; I) \right\} \]

Complete Markets benchmark:
\[ W(Q) = \max_{I \in [0, Q+b], b, d(\cdot)} \left\{ (1 - \delta)u(Q - I + b) + \right. \]
\[ \left. + \delta \sum_{Y \in \mathbf{Y}} W[Y - d(Y)]g(Y; I) \right\} \]

subject to
\[ b = \delta \sum_{Y \in \mathbf{Y}} d(Y)g(Y; I) \]
Application III:
Atkeson’s [1991] Model

Atkeson Equilibrium:

\[ V(Q) = \max_{l,b,d(\cdot)} \left\{ (1 - \delta)u[Q + b - l] + \delta \sum_{Y \in Y} V[Y - d(Y)]g[Y; l(d(\cdot))] \right\}, \]
subject to zero profit condition:

\[ b = \delta \sum_{Y \in Y} d(Y)g(Y; l) \]

incentive compatibility of investment:

\[ l = \arg \max_{l} \left\{ (1 - \delta)u(Q + b - l) + \delta \sum_{Y \in Y} V[Y - d(Y)]g(Y; l) \right\} \]
and incentive compatibility of repayment:

\[ V[Y - d(Y)] \geq U(Y) \quad \forall Y \in Y \]
Capital Outflows

Proposition

*If investment choice is internal (not a corner solution) then for low enough Q there would be capital outflows*

\[ b(Q') < d(Y'|Q) \quad \text{for} \quad Q' = Y' - d(Y'|Q) \]

*for the lowest output realization when the no repudiation constraint becomes binding.*
Capital Outflows

Proposition

If investment choice is internal (not a corner solution) then for low enough $Q$ there would be capital outflows

$$b(Q') < d(Y' | Q) \quad \text{for} \quad Q' = Y' - d(Y' | Q)$$

for the lowest output realization when the no repudiation constraint becomes binding.

- **Remark**: However, investment choice is likely to be at the corner for $Q$ very low (especially if $\min\{Y\} \approx 0$) and hence the proposition looses a lot in terms of generality.
- Tsyrennikov [2006]
Application III:
Atkeson’s [1991] Model

Figure: Equilibrium Dynamics for $\beta(1 + r) = 1$