Exchange Rate Disconnect in General Equilibrium*

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August 5, 2019

Abstract

We propose a dynamic general equilibrium model of exchange rate determination, which simultaneously accounts for all major exchange rate puzzles. This includes the Meese-Rogoff disconnect puzzle, the PPP puzzle, the terms-of-trade puzzle, the Backus-Smith puzzle, and the UIP puzzle. We build on a standard international real business cycle model augmented with a financial sector with noise traders and risk-averse intermediaries, which results in equilibrium UIP deviations due to limits to arbitrage. We show that financial UIP shocks result in persistent near-martingale processes for exchange rates and ensure empirically relevant comovement properties between exchange rates and macro variables, including excess exchange-rate volatility relative to aggregate consumption and output. In contrast, conventional productivity and monetary shocks, while successful in explaining the international business cycle comovement, result in counterfactual exchange rate dynamics with insufficient volatility. When combined together, the two sets of shocks reproduce both the exchange rate disconnect behavior and the empirical business cycle properties. The transmission mechanism relies on a conventional Taylor rule, home bias in consumption, and muted pass-through of exchange rates into prices and quantities due to pricing to market and weak substitutability between home and foreign goods. Nominal rigidities improve the quantitative performance of the model, yet are not necessary for the exchange rate disconnect, as monetary shocks are not the key drivers of the exchange rate.

*We thank Ariel Burstein, Charles Engel, Robert Kollmann, Cédric Tille and Eric van Wincoop for insightful discussions, Mark Aguiar, Andy Atkeson, Adrien Auclert, V.V. Chari, John Cochrane, Giancarlo Corsetti, Javier Cravino, Ian Dew-Becker, Michael Dooley, Wenxin Du, Konstantin Egorov, Sebastian Fanelli, Emmanuel Farhi, Jordi Galí, Gita Gopinath, Pierre-Olivier Gourinchas, Galina Hale, Lars Hansen, Tarek Hassan, Hanno Lustig, Maury Obstfeld, Fabrizio Perri, Hélène Rey, Kenneth Rogoff, Esteban Rossi-Hansberg, Jesse Schreger, Vania Stavrakeva, Jón Steinsson, John Taylor, Chris Tonetti, Jaume Ventura, Adrien Verdelhan, Mark Watson and seminar/conference participants for helpful comments, and Nastya Burya and Gordon Ji for outstanding research assistance.

1 Introduction

Equilibrium exchange rate dynamics is a foundational topic in international macroeconomics (Dornbusch 1976, Obstfeld and Rogoff 1995). At the same time, *exchange rate disconnect* remains among the most challenging and persistent puzzles in macroeconomics (Obstfeld and Rogoff 2001). The term disconnect narrowly refers to the lack of correlation between exchange rates and other macro variables, but the broader puzzle is more pervasive and nests a number of additional empirical patterns, which stand at odds with conventional international macro models. We define the broader exchange rate disconnect to include:

1. Meese and Rogoff (1983) puzzle: the *nominal* exchange rate follows a random-walk-like process, which is *not* robustly correlated, even contemporaneously, with macroeconomic fundamentals (see also Engel and West 2005). Furthermore, the exchange rate is an order of magnitude more volatile than macroeconomic aggregates such as consumption, output and inflation.
2. Purchasing Power Parity (PPP) puzzle (Rogoff 1996): the *real* exchange rate tracks very closely the nominal exchange rate at most frequencies and, in particular, exhibits a similarly large persistence and volatility as the nominal exchange rate. Mean reversion, if any, takes a very long time, with half-life estimates in the range of 3-to-5 years, much in excess of conventional durations of price stickiness (see also Chari, Kehoe, and McGrattan 2002, henceforth CKM).
3. *Terms of trade* are positively correlated with the real exchange rate, yet exhibit a markedly lower volatility, in contrast with the predictions of standard models (Atkeson and Burstein 2008).
4. Backus and Smith (1993) puzzle: the international risk-sharing condition that relative consumption across countries should be strongly positively correlated with the real exchange rates (implying high relative consumption in periods of low relative prices) is sharply violated in the data, with a mildly negative correlation and a markedly lower volatility of relative consumption (see Kollmann 1995 and also Corsetti, Dedola, and Leduc 2008).
5. Forward premium puzzle (Fama 1984), or the violation of the *uncovered interest rate parity* (UIP): cross-currency interest rate differentials are not balanced out by expected depreciations, and instead predict exchange rate appreciations (albeit with a very low R^2), resulting in positive expected returns on *currency carry trades* (see also Engel 1996).

We summarize the above puzzles as a set of moments characterizing comovement between exchange rates and macro variables in *developed* countries under a *floating* regime, and use them as quantitative goals in our analysis (see Table 2).

Existing general equilibrium international macro models either feature these puzzles, or attempt to address one puzzle at a time, often at the expense of aggravating the other puzzles, resulting in a lack of a unifying framework that exhibits satisfactory exchange rate properties. This is a major challenge for the academic and policy discussion, since exchange rates are the core prices in any international model, and failing to match their basic properties jeopardizes the conclusions one can draw from the analysis. In particular, would the conclusions in the vast literatures on currency unions, international policy spillovers and international transmission of shocks survive in a model with realistic exchange rate properties? Furthermore, what are the implications of such a model for the numerous micro-level empirical studies that treat exchange rate shocks as a source of exogenous variation?

The goal of this paper is to offer a unifying theory of exchange rates that can simultaneously account for all stylized facts introduced above. Our theory of the exchange rate relies on a standard transmission mechanism and emphasizes the role of shocks in the financial market. Conventional productivity and monetary shocks, while successful in explaining the business cycle comovement, result in counterfactual exchange rate dynamics, reflected in the puzzles above. In contrast, we show that shocks to the uncovered interest parity (UIP) condition simultaneously resolve all exchange rate puzzles, delivering the empirically-relevant comovement properties between exchange rates and macro variables. Furthermore, the UIP shock generates a large gap in the volatility of the exchange rates and macro variables, while the other shocks lead to comparable volatilities. As a result, a combination of conventional shocks with the UIP shock delivers simultaneously exchange rate disconnect behavior and standard international business cycle comovement of the macro variables (as in e.g. Backus, Kehoe, and Kydland 1992, henceforth BKK). In particular, the full model captures two additional prominent international business cycle puzzles — the weak correlation of consumption and the positive correlation of investment across countries (see Kehoe and Perri 2002).

The two building blocks in our analysis consist of a model of the financial sector, which results in equilibrium UIP violation, and a general equilibrium transmission mechanism with conventional ingredients. In particular, in the financial sector, we adopt a segmented market model of limits to arbitrage with noise traders, following De Long, Shleifer, Summers, and Waldmann (1990), Jeanne and Rose (2002; henceforth JR) and Gabaix and Maggiori (2015; henceforth GM), in which households trade local bonds only, and their net foreign asset positions are intermediated by financial arbitrageurs. The arbitrageurs are risk averse, and demand a risk premium in order to intermediate the positions of both households and noise traders, which results in equilibrium UIP deviations.[1] At the same time, covered interest parity (CIP) holds in the model, as departures from it would result in risk-free arbitrage opportunities.

The transmission mechanism, in turn, features: (i) a conventional *monetary policy* rule that stabilizes domestic inflation, (ii) significant *home bias* in consumption, consistent with the empirical trade shares in GDP, and (iii) a combination of *pricing to market* and *weak substitutability* between home and foreign goods. Combined together, these ingredients mute the pass-through of exchange rate fluctuations into aggregate prices and quantities, resulting in the *volatility disconnect* between exchange rates and macro variables. We discipline the parameters of the transmission mechanism with the recent empirical estimates, without targeting any specific exchange rate moments. Interestingly, *nominal rigidities* are not essential to generate the qualitative disconnect behavior, yet they improve the quantitative fit of the model by further muting the exchange rate transmission.

[1] *Exogenous* UIP shocks are commonly used in the international macro literature (see e.g. Devereux and Engel 2002, Kollmann 2005, Farhi and Werning 2012), and can be viewed to emerge from exogenous asset demand, as in the literature following Kouri (1976, 1983). Alternative models of *endogenous* UIP deviations include models with incomplete information, expectational errors and heterogeneous beliefs (Evans and Lyons 2002, Gourinchas and Tornell 2004, Bacchetta and van Wincoop 2006, Burnside, Han, Hirshleifer, and Wang 2011), financial frictions (Adrian, Etula, and Shin 2015, Camanho, Hau, and Rey 2018), habits, long-run risk and rare disasters (Verdelhan 2010, Colacito and Croce 2013, Farhi and Gabaix 2016), and alternative formulations of segmented markets (Alvarez, Atkeson, and Kehoe 2009). We show that the disconnect mechanism requires that UIP deviations are not associated with *large contemporaneous* shocks to productivity or money supply.

We complement the quantitative analysis with analytical results in the context of a simplified version of the model, which allows us to dissect the disconnect mechanism. In particular, we consider a special case of the model without capital and nominal rigidities, which admits a tractable analytical solution, yet maintains the main disconnect properties of a richer quantitative environment. The analytical framework delivers three main qualitative insights.

First, we characterize equilibrium exchange rate dynamics, which emerges as an interplay between equilibrium forces in the financial and in the real (product and factor) markets. In particular, we show that a demand shock for the foreign-currency bonds results in a UIP violation and a slow but persistent expected appreciation in order to ensure equilibrium in the financial market. In turn, this needs to be balanced out by an *unexpected* depreciation on impact in order to satisfy the intertemporal budget constraint. The more persistent is the shock, the larger is the initial depreciation, and thus the closer is the behavior of the nominal exchange rate to a random walk. Other persistent shocks, including productivity, result in a similar near-random-walk behavior.[2] However, unlike productivity and other macro shocks, financial UIP shocks generate *excess volatility* in exchange rates relative to macro aggregates, which is an essential feature of exchange rate disconnect (see also Devereux and Engel 2002). We further show that, as the economy becomes less open to international trade, UIP shocks result in more volatile exchange rate fluctuations with vanishingly small effects on the rest of the economy.[3]

Second, we address the equilibrium properties of the real exchange rate, and in particular the PPP puzzle, which is often viewed as the prime evidence in support of long-lasting real effects of nominal rigidities (as surveyed in Rogoff 1996). However, in view of the moderate empirical durations of nominal prices, calibrated sticky price models are incapable of generating persistent PPP deviations observed in the data (see CKM). The baseline assumption in this analysis is that monetary shocks are the main drivers of the nominal exchange rate, and that nominal rigidity is the key part of the transmission mechanism into the real exchange rate. We suggest an entirely different perspective, which deemphasizes nominal rigidities, and instead shifts focus to the nature of the shock process. We argue that the behavior of the real exchange rate — both in the time series (see e.g. Blanco and Cravino 2018) and in the cross-section (see e.g. Kehoe and Midrigan 2008) — is neither evidence in favor or against sticky prices, but instead suggests that monetary shocks *cannot* be the key source of exchange rate fluctuations. We show that financial shocks drive both nominal and real exchange rates in concert, resulting in volatile and persistent behavior of both variables, thus reproducing the PPP puzzle. The only two essential ingredients of the transmission mechanism for this result are the monetary policy rule, which stabilizes domestic inflation, and home bias in consumption, which limits the response of aggregate prices to the exchange rate.[4]

Third, we address the Backus-Smith puzzle, namely the comovement between consumption and the

[2]This provides a general equilibrium analogy to the famous Engel and West (2005) result, which derives exclusively from the equilibrium in the financial market. While our result appears similar, the economic mechanism is nonetheless distinct.

[3]Indeed, consider the extreme case of a demand shock for foreign bonds in an economy which cannot trade goods internationally. The full equilibrium adjustment in this case is achieved exclusively by means of exchange rate movements.

[4]Consumer price stabilization can account for the PPP puzzle even in response to productivity shocks (*cf.* Eichenbaum, Johannsen, and Rebelo 2018); however, this results in counterfactual predictions for alternative measures of the real exchange rate, in particular those based on relative wages, as well as in the other exchange rate puzzles.

real exchange rate. Our approach crucially shifts focus from risk sharing (in the financial market) to expenditure switching (in the goods market) as the key force shaping this comovement. We show that expenditure switching robustly implies a negative correlation between relative consumption and the real exchange rate, as is the case in the data. An exchange rate depreciation increases global demand for domestic goods, which in light of the home bias requires a reduction in domestic consumption.[5] Indeed, this force is present in all models with expenditure switching and goods market clearing, yet it is usually dominated by the direct effect of shocks on consumption.[6] With a financial shock as the key source of exchange rate volatility, there is no direct effect, and expenditure switching is the only force affecting consumption, resulting in the empirically relevant direction of comovement. Put differently, in order to reproduce the empirical Backus-Smith comovement, the real exchange rate depreciations must be mostly triggered by relative demand shocks for foreign-currency assets rather than supply shocks of domestically-produced goods. Finally, the transmission mechanism with home bias and muted pass-through into prices and quantities ensures that the response of aggregate consumption is very mild, much smaller than that of the exchange rate.

In addition, we show that the model with UIP shocks reproduces the comovement properties of the exchange rate with interest rates, and in particular the forward premium puzzle. Indeed, a demand shock for foreign-currency bond is compensated in equilibrium with a lower return (a UIP deviation) due to both an increase in the relative home interest rate and an expected appreciation. This leads to a negative *Fama coefficient* in the regression of exchange rate changes on interest rate differentials, albeit with a vanishingly small R^2 as UIP shocks become more persistent and the exchange rate becomes closer to a random walk. The disconnect transmission mechanism further ensures that interest rates, just like consumption, are an order of magnitude less volatile than the exchange rate. While the fact that UIP shocks can match the forward premium moments is, perhaps, least surprising, these moments offer a direct way to discipline the properties of this shock process. The contribution of the paper is then to show that the same shock reproduces a rich array of additional exchange rate comovement patterns with other macro variables, via the general equilibrium mechanism of the model.[7]

The rest of the paper is organized as follows. In Section 2, we describe the full modeling framework, and in particular the model of the financial sector, which is our only departure from a conventional

[5] Perhaps more intuitively, the same general equilibrium mechanism can be restated as follows: financial shocks that lead home households to delay their consumption, which is biased towards domestically-produced goods, require an exchange rate depreciation to shift global expenditure towards these goods in order to clear the goods market.

[6] For example, productivity shocks (or also expansionary monetary shocks) increase the supply of domestic goods, reducing their prices (and hence depreciating the exchange rate) and increasing consumption, which induces a counterfactual correlation pattern. Alternative mechanisms in the literature (see e.g. Corsetti, Dedola, and Leduc 2008, Benigno and Thoenissen 2008, Kocherlakota and Pistaferri 2007, Colacito and Croce 2013, Karabarbounis 2014) either mute the direct effect of productivity shocks on consumption or reverse the sign of the exchange rate response, as we discuss further below.

[7] Our framework is related to the international DSGE models in Devereux and Engel (2002) and Kollmann (2001, 2005), with which it shares several key elements, including the UIP shock, as well as to the open-economy wedge accounting (see Guillén 2013, Eaton, Kortum, and Neiman 2015). What sets our analysis apart is the emphasis on simultaneously resolving a broad range of exchange rate disconnect puzzles using a concise and tractable framework. In particular, we show that a *small-scale* model with just two shocks — to productivity and UIP — can robustly explain the comovement of all international macro variables. Our work is also related to the VAR literature which studies impulse responses of international macro variables to structural shocks (Eichenbaum and Evans 1995, Scholl and Uhlig 2008, Stavrakeva and Tang 2015, Schmitt-Grohé and Uribe 2018). In view of large uncertainty around *conditional* moments in the data, e.g. the existence of delayed overshooting (see Kim and Roubini 2000), we adopt the *unconditional* moments as our primary targets (see Nakamura and Steinsson 2018).

international business cycle environment. Section 3 then explores the disconnect mechanism with a sequence of analytical results in the context of a simplified model environment. Section 4 presents the quantitative results, emphasizing the fit of the full model of both exchange rate moments and conventional international business cycle moments. This section also explores the contribution of various shocks, both conventional and financial, as well as the role of nominal rigidities. Section 5 concludes and the appendix provides detailed derivations and proofs.

2 Modeling Framework

We build on a standard international real business cycle (IRBC) model with domestically-held capital, home bias in consumption, and productivity shocks. The monetary policy is conducted according to a conventional Taylor rule targeting inflation, and the *baseline* specification features no nominal rigidities — all prices and wages are flexible. The only departure from a conventional IRBC model is a segmented financial market, which features noise traders and risk-averse arbitrageurs, who intermediate the bond holdings of the households by participating in international carry trades.

There are two symmetric countries — home (Europe) and foreign (US, denoted with a $*$) — each with its own nominal unit of account in which the local prices are quoted: for example, the home wage rate is W_t euros and the foreign is W_t^* dollars. The nominal exchange rate $\mathcal{E}_t$ is the price of dollars in terms of euros, hence an increase in $\mathcal{E}_t$ signifies a nominal devaluation of the euro (the home currency). In our description, we focus on the home country, and relegate technical derivations to Appendix A.2.

2.1 Model setup

Households A representative home household maximizes the discounted expected utility over consumption and labor:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+1/\nu} L_t^{1+1/\nu} \right), \tag{1}$$

where σ is the relative risk aversion and ν is the Frisch elasticity of labor supply, and the results are robust to alternative utility specifications (e.g., GHH preferences). The flow budget constraint is given by:

$$P_t C_t + P_t Z_t + \frac{B_{t+1}}{R_t} \leq W_t L_t + R_t^K K_t + B_t + \Pi_t, \tag{2}$$

where P_t is the consumer price index and W_t is the nominal wage rate, B_t is the quantity of the local bond paying out one unit of the home currency next period, R_t is the gross nominal interest rate (i.e., $1/R_t$ is the price of the bond), R_t^K is the nominal rental rate of capital, and Π_t are dividends. Finally, Z_t is the gross investment into the domestic capital stock K_t, which accumulates according to a standard rule with depreciation δ and quadratic capital adjustment costs with parameter κ.[8]

[8]Specifically, $K_{t+1} = (1-\delta)K_t + \left[Z_t - \frac{\kappa}{2}\frac{(\Delta K_{t+1})^2}{K_t}\right]$, where the term in brackets is investment net of adjustment costs.

Household optimization results in the standard labor supply condition and Euler equation for bonds:

$$C_t^{\sigma} L_t^{1/\nu} = W_t/P_t, \tag{3}$$

$$1 = \beta R_t \, \mathbb{E}_t \left\{ (C_{t+1}/C_t)^{-\sigma} P_t/P_{t+1} \right\}, \tag{4}$$

as well as a standard Euler equation for capital accumulation provided in Appendix A.2. We assume that households trade only local-currency bonds, and hold the entire home capital stock, as well as own home firms (*cf.* Heathcote and Perri 2013). The foreign households are symmetric, with their behavior characterized by parallel optimality conditions. In particular, they trade only foreign-curency bonds B^*_{t+1}, hold the entire foreign capital stock K^*_t and own foreign firms which pay Π^*_t as dividends.

Expenditure and demand The domestic households allocate their within-period consumption expenditure P_tC_t between home and foreign varieties of the goods:

$$P_tC_t = P_{Ht}C_{Ht} + P_{Ft}C_{Ft} = \int_0^1 \Big[P_{Ht}(i)C_{Ht}(i) + P_{Ft}(i)C_{Ft}(i) \Big] \mathrm{d}i \tag{5}$$

to minimize expenditure on aggregate consumption, defined implicitly by a Kimball (1995) aggregator:

$$\int_0^1 \left[(1-\gamma) g\bigg(\frac{C_{Ht}(i)}{(1-\gamma)C_t} \bigg) + \gamma g\bigg(\frac{C_{Ft}(i)}{\gamma C_t} \bigg) \right] \mathrm{d}i = 1, \tag{6}$$

where $\gamma \in [0, 1/2)$ is the *trade openness* parameter, which can be due to a combination of home bias in preferences, trade costs and non-tradable goods (see Obstfeld and Rogoff 2001).[9] We adopt the more general Kimball aggregator, rather than the conventional CES demand structure, to allow for pricing to market, as we discuss below.[10]

The aggregator function $g(\cdot)$ in (6) has the following properties: $g'(\cdot) > 0$, $g''(\cdot) < 0$ and $-g''(1) \in (0,1)$, and two normalizations: $g(1) = g'(1) = 1$. The solution to the optimal expenditure allocation results in the following homothetic demand schedules:

$$C_{Ht}(i) = (1-\gamma) h\left(\frac{P_{Ht}(i)}{\mathcal{P}_t} \right) C_t \qquad \text{and} \qquad C_{Ft}(j) = \gamma h\left(\frac{P_{Ft}(j)}{\mathcal{P}_t} \right) C_t, \tag{7}$$

where $h(\cdot) = g'^{-1}(\cdot) > 0$ and satisfies $h(1) = 1$ and $h'(\cdot) < 0$. The function $h(\cdot)$ controls the curvature of the demand schedule, and we denote its point elasticity with $\theta \equiv -\frac{\partial \log h(x)}{\partial \log x}\big|_{x=1} = -h'(1) > 1$. Note that the demand for the foreign good collapses to zero as $\gamma \to 0$, and more generally γ characterizes the expenditure share on foreign goods around a symmetric equilibrium, $\frac{P_{Ft}C_{Ft}}{P_tC_t}\big|_{P_{Ft}=P_{Ht}} = \gamma$.

The consumer price level P_t and the auxiliary variable $\mathcal{P}_t$ in (7) are two alternative measures of average prices in the home market, which are defined implicitly by (5) and (6) after substituting in

[9] The aggregate implications of the model do not depend on whether home bias emerges on the extensive margin due to non-tradables or on the intensive margin due to trade costs or preferences; hence, we do not explicitly introduce non-tradables.

[10] The CES demand is nested as a special case of the Kimball aggregator (6) with $g(z) = 1 + \frac{\theta}{\theta-1}\big(z^{1-1/\theta} - 1\big)$, resulting in the demand schedule $h(x) = x^{-\theta}$ and price index $P_t = \mathcal{P}_t = \big(\int_0^1 \big[(1-\gamma)P_{Ht}(i)^{1-\theta} + \gamma P_{Ft}(i)^{1-\theta}\big] \mathrm{d}i \big)^{1/(1-\theta)}$.

the demand schedules (7) (see the expressions in Appendix A.2). In a symmetric equilibrium with $P_{Ht}(i) = P_{Ft}(j) = \mathcal{P}_t$ for all i, j, we have $P_t = \mathcal{P}_t$. More generally, P_t and $\mathcal{P}_t$ are first-order equivalent, and differ at most by a second order term in the dispersion of relative prices. We summarize the properties of the optimal expenditure allocation in:

Lemma 1 *Around a symmetric equilibrium with $\bar{P}_H(i) = \bar{P}_F(j)$ for all $i, j \in [0,1]$, the elasticity of demand equals θ, the expenditure share on foreign goods equals γ, and the log-linear approximation to the price index is a weighted average of individual prices:*

$$p_t \equiv \mathrm{d}\log P_t = \mathrm{d}\log \mathcal{P}_t \qquad \textit{and} \qquad p_t = \int_0^1 \left[(1-\gamma)p_{Ht}(i) + \gamma p_{Ft}(i)\right] \mathrm{d}i, \tag{8}$$

where $p_t \equiv \mathrm{d}\log P_t$ denotes the log deviation of the price index from its steady-state value.

Trade openness γ and the elasticity of substitution between home and foreign goods θ play an important role for the quantitative properties of the transmission mechanism.

The expenditure allocation of the foreign households is characterized by a symmetric demand system. In particular, the demand for home and foreign goods by foreign households is given by:

$$C^*_{Ht}(i) = \gamma h\left(\frac{P^*_{Ht}(i)}{\mathcal{P}^*_t}\right) C^*_t \qquad \text{and} \qquad C^*_{Ft}(j) = (1-\gamma) h\left(\frac{P^*_{Ft}(j)}{\mathcal{P}^*_t}\right) C^*_t, \tag{9}$$

where $P^*_{Ht}(i)$ and $P^*_{Ft}(j)$ are the foreign-currency prices of the home and foreign goods in the foreign market, and $\mathcal{P}^*_t$ is an auxiliary variable characterizing the average level of prices in the foreign market, which is approximated in a symmetric way to (8). The foreign consumer price level P^*_t similarly characterizes the minimum expenditure necessary to purchase one unit of the foreign good C^*_t in the foreign market, and the *real exchange rate* is the relative consumer price level in the two countries:

$$\mathcal{Q}_t \equiv \frac{P^*_t \mathcal{E}_t}{P_t}. \tag{10}$$

An increase in $\mathcal{Q}_t$ corresponds to a real depreciation, that is a decrease in the relative price of the home consumption basket.

Production Home output is produced by a given pool of identical firms (hence we omit indicator i) according to a Cobb-Douglas technology in labor L_t, capital K_t and intermediate inputs X_t:

$$Y_t = \left(e^{a_t} K_t^{\vartheta} L_t^{1-\vartheta}\right)^{1-\phi} X_t^{\phi}, \tag{11}$$

where ϑ is the elasticity of the value added with respect to capital and ϕ is the elasticity of output with respect to intermediates, which determines the equilibrium expenditure share on intermediate goods.[11]

[11] The presence of intermediates is *not* essential for the qualitative results, however, is needed to properly capture the degree of trade openness in our calibration. For greater analytical tractability, we focus on a constant-returns-to-scale Cobb-Douglas production, but the results are robust to decreasing returns to scale and non-unitary elasticity of substitution between inputs.

The aggregate *value-added productivity* follows an AR(1) process in logs:

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a, \qquad \varepsilon_t^a \sim iid(0,1), \tag{12}$$

where $\rho_a \in [0,1]$ is the persistence parameter and $\sigma_a \geq 0$ is the volatility of the innovation.

Intermediates are the same bundle of home and foreign varieties as the final consumption bundle (6), and hence their price index is also given by P_t. Therefore, the marginal cost of production is:

$$MC_t = \tfrac{1}{\varpi}\big[e^{-a_t}(R_t^K)^{\vartheta} W_t^{1-\vartheta}\big]^{1-\phi} P_t^{\phi}, \tag{13}$$

where $\varpi \equiv \phi^{\phi}\big[(1-\phi)\vartheta^{\vartheta}(1-\vartheta)^{1-\vartheta}\big]^{1-\phi}$. The firms optimally allocate expenditure between capital, labor and intermediates. In particular, the optimal labor demand condition can be written as:

$$W_t L_t = (1-\vartheta)(1-\phi) MC_t\, Y_t, \tag{14}$$

and analogous demand conditions hold for capital and intermediates. The expenditure on intermediates $P_t X_t$ is further split between the domestic and foreign varieties, $X_{Ht}(i)$ and $X_{Ft}(j)$, in parallel with the household consumption expenditure in (7). The same is true for the allocation of the household expenditure on the investment good, $P_t Z_t$.

Profits and price setting Even though the production function is symmetric for every firm i, each firm faces a downward sloping demand for its variety. The firm maximizes profits from serving the home and foreign markets:

$$\Pi_t(i) = \big(P_{Ht}(i) - MC_t\big)\, Y_{Ht}(i) + \big(P_{Ht}^*(i)\mathcal{E}_t - MC_t\big)\, Y_{Ht}^*(i) \tag{15}$$

by setting the home market price $P_{Ht}(i)$ and the foreign market price $P_{Ht}^*(i)$, by convention expressed in home and foreign currency respectively. The supply to the home-market is split between home demand for consumption, intermediate and investment goods, $Y_{Ht}(i) = C_{Ht}(i) + X_{Ht}(i) + Z_{Ht}(i)$, which all satisfy the demand schedules analogous to (7), and similarly in the foreign market.

The aggregate profits of the domestic firms, $\Pi_t = \int_0^1 \Pi_t(i)\mathrm{d}i$, are distributed to the domestic households. We assume no entry or exit of firms. Therefore, our model captures the short and the medium run, namely the horizons from *one month* up to *five years*, where empirically extensive margins play a negligible role in the aggregate (see e.g. Bernard, Jensen, Redding, and Schott 2009).

The firms set prices $P_{Ht}(i)$ and $P_{Ht}^*(i)$ flexibly by maximizing profits in (15) state-by-state. This results in the markup pricing rules, with a common price across all domestic firms $i \in [0,1]$ in a given destination market and expressed in the destination currency:

$$P_{Ht}(i) = P_{Ht} = \mu\left(\frac{P_{Ht}}{\mathcal{P}_t}\right)\cdot MC_t \qquad \text{and} \qquad P_{Ht}^*(i) = P_{Ht}^* = \mu\left(\frac{P_{Ht}^*}{\mathcal{P}_t^*}\right)\cdot \frac{MC_t}{\mathcal{E}_t},$$

where $\mu(x) \equiv \frac{\tilde{\theta}(x)}{\tilde{\theta}(x)-1}$ is the markup function and $\tilde{\theta}(x) = -\frac{\partial \log h(x)}{\partial \log x}$ is the demand elasticity schedule.

Log-linearizing the markup function $\mu(\cdot)$ around a symmetric equilibrium with $P_{Ht} = \mathcal{P}_t = P_t$, we prove in Appendix A.2:

Lemma 2 *The log-linear approximation around a symmetric equilibrium to the optimal price-setting rules for all domestic firms $i \in [0,1]$ is given by:*

$$p_{Ht}(i) \equiv p_{Ht} = (1-\alpha)mc_t + \alpha p_t, \tag{16}$$

$$p^*_{Ht}(i) \equiv p^*_{Ht} = (1-\alpha)(mc_t - e_t) + \alpha p^*_t, \tag{17}$$

where $\alpha \equiv \frac{\Gamma}{1+\Gamma} \in [0,1)$ is the strategic complementarity *elasticity and $\Gamma \equiv -\frac{\partial \log \mu(x)}{\partial x}\big|_{x=1} \geq 0$ is the point elasticity of the markup function.*

Price setting according to (16)–(17) results in the deviation from the *law of one price* (LOP), that is $p^*_{Ht} + e_t - p_{Ht} = \alpha q_t \neq 0$ whenever $q_t \neq 0$ and $\alpha > 0$. The optimal price of the firm depends on both its marginal cost and the average price of its local competitors with the weights shaped by the elasticity of the markup function Γ, which results in *strategic complementarities* in price setting, $\alpha > 0$. As a result, firms optimally choose different markups for the two markets, and adjust them differentially in response to shocks. These patterns of *pricing to market* are both pronounced in the data and emerge in a variety of models of price setting (see Amiti, Itskhoki, and Konings 2019). The complementary quantity $(1-\alpha)$ is the cost pass-through elasticity, which is in general incomplete. Note that under CES demand with monopolistic competition, the optimal markup is constant and therefore $\Gamma = 0$, resulting counterfactually in no strategic complementarities $\alpha = 0$, complete pass-through and no pricing to market. An important property of a general Kimball demand is that the elasticity Γ, and consequently the parameter α, can be taken as independent parameters from the elasticity of demand θ.[12]

Good and factor market clearing The labor market clearing requires that L_t equals simultaneosly the labor supply of the households in (3) and the labor demand of the firms in (14), and equivalently for L^*_t in foreign. Similarly, equilibrium in the capital market requires that K_t (and K^*_t) equals simultaneously the capital supply of the households and the capital demand of the local firms.

The goods market clearing requires that the total production by the home firms is split between supply to the home and foreign markets respectively, $Y_t = Y_{Ht} + Y^*_{Ht}$, and satisfies the local demand in each market for the final, intermediate and capital goods:

$$Y_{Ht} = C_{Ht} + X_{Ht} + Z_{Ht} = (1-\gamma)h\left(\frac{P_{Ht}}{\mathcal{P}_t}\right)\left[C_t + X_t + Z_t\right], \tag{18}$$

$$Y^*_{Ht} = C^*_{Ht} + X^*_{Ht} + Z^*_{Ht} = \gamma h\left(\frac{P^*_{Ht}}{\mathcal{P}^*_t}\right)\left[C^*_t + X^*_t + Z^*_t\right], \tag{19}$$

where we substituted in the demand schedules (7) and (9) and used the fact that intermediate and capital goods are defined by the same Kimball aggregator (6) as the consumption good. Symmetric market clearing conditions hold for the foreign-produced output supplied to the two markets, $Y^*_t = Y_{Ft} + Y^*_{Ft}$.

[12] Indeed, α depends on the curvature of demand, while θ characterizes its slope. We provide an illustration in Appendix A.2.

Lastly, we combine the household budget constraint (2) with profits (15), aggregated across all home firms, as well as the market clearing conditions above to obtain the home country budget constraint:

$$\frac{B_{t+1}}{R_t} - B_t = NX_t \qquad \text{with} \qquad NX_t = \mathcal{E}_t P^*_{Ht} Y^*_{Ht} - P_{Ft} Y_{Ft}, \tag{20}$$

where NX_t denotes net exports expressed in units of the home currency (see derivation in Appendix A.2). *Terms of trade*, or the relative price at which home exchanges its exports for imports, are given by:

$$\mathcal{S}_t \equiv \frac{P_{Ft}}{P^*_{Ht}\mathcal{E}_t}. \tag{21}$$

The foreign country faces a symmetric budget constraint to (20), which is redundant by a version of *Walras law* that applies in this economy.

Monetary policy rule The government is present in the economy only by means of the monetary policy rule, as the fiscal authority is fully passive. This is without loss of generality as, in view of Ricardian equivalence, the net foreign asset position of the country B_{t+1} should be regarded as the consolidated position of the public and the private sectors. The monetary policy is implemented by means of a conventional Taylor rule:

$$i_t = \rho_m i_{t-1} + (1-\rho_m)\phi_\pi \pi_t + \sigma_m \varepsilon^m_t, \tag{22}$$

where $i_t \equiv \log R_t$ is the log nominal interest rate, $\pi_t = \Delta \log P_t$ is the inflation rate, and $\varepsilon^m_t \sim iid(0,1)$ is the monetary policy shock with volatility parameter $\sigma_m \geq 0$. The parameter ρ_m characterizes the persistence of the monetary policy rule and $\phi_\pi > 1$ is the Taylor rule parameter which guarantees that the Taylor principle is satisfied.

Nominal rigidities Our baseline specification features no nominal rigidities, yet in the quantitative Section 4, we consider an extension of the model with Calvo sticky prices and wages, which we introduce in a conventional way (see e.g. Clarida, Galí, and Gertler 2002, Galí 2008, and Appendix A.4). We denote with λ_p and λ_w the Calvo probabilities of price and wage non-adjustment respectively.

2.2 Financial market

Our model of the financial sector builds on JR and GM, and features three types of agents: households, noise traders and professional intermediaries.[13] The home and foreign households trade their local currency bonds only, and hence cannot directly trade assets with each other. In particular, the home households demand at time t a quantity B_{t+1} of the home-currency bonds, as appears in their budget constraint (2). Similarly, foreign households demand a quantity B^*_{t+1} of the foreign-currency bonds.

[13]We follow JR in modeling the financial intermediaries, who take limited asset positions due to exposure to the exchange rate risk, rather than due to financial constraints as in GM. In contrast, we follow GM in modeling the segmented participation of the households. Lastly, the exogenous liquidity needs of the noise trader are akin to the exogenous 'portfolio flows' in GM, but can equally emerge from biased expectations about the exchange rate, $\mathbb{E}^n_t \mathcal{E}_{t+1} \neq \mathbb{E}_t \mathcal{E}_{t+1}$, as in JR.

Both B_{t+1} and B^*_{t+1} can take positive or negative values, depending on whether the households save or borrow respectively.

In addition to the household *fundamental* demand for currency (bonds), the financial market features a source of *liquidity* currency demand from (a measure) n symmetric *noise* traders, which evolves independently of the expected return of currency and the other macroeconomic fundamentals. In particular, noise traders follow a zero-capital strategy by taking a long position in the foreign currency and shorting equal value in the home currency, or vice versa if they have an excess demand for the home currency. The overall position of the noise traders is

$$\frac{N^*_{t+1}}{R^*_t} = n\left(e^{\psi_t} - 1\right) \tag{23}$$

dollars invested in the foreign-currency bonds and respectively $\frac{N_{t+1}}{R_t} = -\mathcal{E}_t \frac{N^*_{t+1}}{R^*_t}$ euros in the home-currency bonds. We refer to the noise trader shock ψ_t as the *financial shock*, and assume it follows an exogenous AR(1) process:

$$\psi_t = \rho_\psi \psi_{t-1} + \sigma_\psi \varepsilon^\psi_t, \qquad \varepsilon^\psi_t \sim iid(0,1), \tag{24}$$

where $\rho_\psi \in [0,1]$ is the persistence of the financial shock and $\sigma_\psi \geq 0$ is its volatility.

The trades of the households and the noise traders are intermediated by (a measure) m symmetric risk-averse arbitrageurs, or market makers. These intermediaries adopt a zero-capital *carry trade* strategy by taking a long position in the foreign-currency bonds and a short position of equal value in the home-currency bonds, or vice versa. The return on this carry trade is therefore:

$$\tilde{R}^*_{t+1} = R^*_t - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \tag{25}$$

per one dollar invested in the foreign-currency bonds and $\mathcal{E}_t$ euros sold of the home-currency bonds at time t. We denote the size of individual positions by d^*_{t+1}, which may take positive or negative values, and assume that intermediaries maximize the CARA utility of the real return on their investment in units of the foreign consumption good:

$$\max_{d^*_{t+1}} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \frac{\tilde{R}^*_{t+1}}{P^*_{t+1}} \frac{d^*_{t+1}}{R^*_t} \right) \right\}, \tag{26}$$

where $\omega \geq 0$ is the risk aversion parameter.[14] In aggregate, all m intermediaries invest $\frac{D^*_{t+1}}{R^*_t} = m \frac{d^*_{t+1}}{R^*_t}$ dollars in foreign-currency bonds, and take an offsetting position of $\frac{D_{t+1}}{R_t} = -\mathcal{E}_t \frac{D^*_{t+1}}{R^*_t}$ euros in home-currency bonds, resulting in a zero-capital portfolio at time t.

Both currency bonds are in zero net supply, and therefore financial market clearing requires that

[14] CARA utility provides tractability, as it results in portfolio choice that does not depend on the level of wealth of the intermediaries (see Appendix A.2), thus avoiding the need to carry it as an additional state variable; the tradeoff of working with CARA-utility, however, is that intermediaries need to be short-lived, maximizing the one-period return on their investment.

the positions of the households, noise traders and intermediaries balance out:

$$B_{t+1} + N_{t+1} + D_{t+1} = 0 \qquad \text{and} \qquad B^*_{t+1} + N^*_{t+1} + D^*_{t+1} = 0. \tag{27}$$

As both noise traders and intermediaries hold zero-capital positions, financial market clearing (27) implies a balanced position for the home and foreign households combined, $\frac{B_{t+1}}{R_t} + \mathcal{E}_t \frac{B^*_{t+1}}{R^*_t} = 0$. In other words, the financial market merely intermediates the intertemporal borrowing between home and foreign households, without them directly trading any assets. Lastly, the trades of intermediaries and noise traders result in income (or losses), which we assume are returned to the foreign households at the end of each trading period, as a lump-sum payment together with the dividends of the foreign firms.[15]

In equilibrium, the intermediaries absorb the demand for home and foreign currency of both households and noise traders. If intermediaries were risk neutral, $\omega = 0$, they would do so without a risk premium, resulting in the *uncovered interest parity* (UIP), or equivalently a zero expected real return on the carry trade, $\mathbb{E}_t\{\tilde{R}^*_{t+1}/P^*_{t+1}\} = 0$. However, risk-averse intermediaries are not willing to take a risky carry trade without an appropriate compensation, resulting in equilibrium risk premia and deviations from the UIP. We characterize the equilibrium in the financial market in:

Lemma 3 *The equilibrium condition in the financial market, log-linearized around a symmetric steady state with $\bar{B} = \bar{B}^* = 0$, $\bar{R} = \bar{R}^* = 1/\beta$, $\bar{\mathcal{Q}} = 1$ and a finite nonzero $\omega\sigma_e^2/m$, is given by:*

$$i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} = \chi_1 \psi_t - \chi_2 b_{t+1}, \tag{28}$$

where $i_t - i^*_t \equiv \log(R_t/R^*_t)$, $b_{t+1} \equiv B_{t+1}/\bar{Y}$, and the coefficients $\chi_1 \equiv \frac{n}{\beta}\frac{\omega\sigma_e^2}{m}$ and $\chi_2 \equiv \bar{Y}\frac{\omega\sigma_e^2}{m}$, with $\sigma_e^2 \equiv \mathrm{var}_t(\Delta e_{t+1})$ denoting the conditional volatility of the nominal exchange rate.

The equilibrium condition (28) is the *modified UIP* in our model with imperfect financial intermediation, where the right-hand side corresponds to the departures from the UIP. Condition (28) arises from the combination of the financial market clearing (27) with the solution to the portfolio choice problem of the intermediaries (26), as we formally show in Appendix A.2. Intuitively, the optimal portfolio of the intermediaries D^*_{t+1} is proportional to the expected log return on the carry trade, $i_t - i^*_t - \mathbb{E}_t\Delta e_{t+1}$, scaled by the risk absorption capacity of the intermediary sector, $\omega\sigma_e^2/m$, i.e. the product of their effective risk aversion (ω/m, the price of risk) and the volatility of the carry trade return (σ_e^2, the exchange rate risk). As $\omega\sigma_e^2/m \to 0$, the risk absorption capacity of the intermediaries increases, and the UIP deviations disappear in the limit as $\chi_1, \chi_2 \to 0$. With $\omega\sigma_e^2/m > 0$, the UIP deviations remain first order and hence affect the first-order equilibrium dynamics. In this case, any shock that affects net exports — and hence net foreign assets b_{t+1} (see (31)), which need to be intermediated by the financial sector — results in endogenous UIP violations. Note that both $\psi_t > 0$ and $b_{t+1} < 0$ correspond to the excess demand for the foreign-currency bond — by noise traders and households, respectively — and hence

[15] This generates an additional income of $\tilde{R}^*_{t+1}(D^*_{t+1} + N^*_{t+1})/R^*_t$ dollars for the foreign households. As a result of this transfer, the foreign country budget constraint becomes the same as the home country budget constraint (20), despite foreign households facing a generally different rate of return $R^*_t \neq R_t$. See Appendix A.2 for details, where we also show that this transfer is second order, and hence does not affect the first-order dynamics of the equilibrium system.

result in a negative expected return on the foreign currency bond.[16]

Covered interest parity We consider briefly the equilibrium pricing of the currency forwards by the financial sector, and the resulting *covered interest parity* (CIP). Consider a period t forward price $\mathcal{F}_t$ of one unit of foreign currency in units of home currency at $t+1$. The financial intermediaries price it at $\mathcal{F}_t = \mathcal{E}_t R_t / R_t^*$, as any other price would result in riskless arbitrage, and in the absence of financial constraints the intermediaries would take unbounded positions buying or selling such forward contracts.[17] Therefore, CIP holds in equilibrium, even though UIP is generally violated. In this model, the imperfection in the financial market emerges due to market segmentation and the limited risk absorption capacity of the risk-averse intermediaries. Profiting from the UIP deviations requires taking a carry trade risk, which commands an equilibrium risk premium, while the departures from CIP would generate riskless arbitrage opportunities. This is in contrast with the models with financial constraints, where the departures from both UIP and CIP are not due to risk, but to the binding constraints on the balance sheet of the financial intermediaries. We opt in favor of this modeling approach as empirically the size of the CIP deviations is at least an order of magnitude smaller than the expected departures from the UIP.[18]

3 The Disconnect Mechanism

In this section, we consider a special case of the general model environment, which admits a tractable analytical solution, yet does not compromise the qualitative ability of the model to explain exchange rate disconnect. This allows us to fully dissect the disconnect mechanism, which remains at play in a richer quantitative environment in Section 4. We consider the *flexible-price* version of the model with the following additional simplifications:

R1. $\vartheta = 0$ in the production function (11), and hence the model features no capital or investment, saving us one dynamic state variable in each country.

R2. $\phi_\pi \to \infty$ in the Taylor rule (22), implying a monetary policy rule that completely stabilizes prices and ensures zero inflation, and hence the log deviations of the price level $p_t = p_t^* \equiv 0$ for all t.[19]

[16] In Itskhoki and Mukhin (2017), we discuss various alternative microfoundations for the UIP shock ψ_t in (28), ranging from complete-market models of risk-premia (e.g., Verdelhan 2010, Colacito and Croce 2013, Farhi and Gabaix 2016) to models of heterogeneous beliefs and expectational errors (e.g., Evans and Lyons 2002, Gourinchas and Tornell 2004, Bacchetta and van Wincoop 2006). All models resulting in a variant of (28) with $\chi_1 > 0$ (and $\chi_2 \geq 0$) produce similar equilibrium exchange rate dynamics, and can only be discriminated with additional financial moments (e.g., covered interest pairty, comovement of exchange rates with additional asset classes). Consistent with the recent work of Lustig and Verdelhan (2018), we emphasize the importance of financial frictions in explaining exchange rates, and in particular focus on the role of segmented markets.

[17] Indeed, one unit of home currency at t together with a forward contract yields $R_t/\mathcal{F}_t$ units of foreign currency at $t+1$, or alternatively it can yield $R_t^*/\mathcal{E}_t$ units of foreign currency. If $R_t/\mathcal{F}_t > R_t^*/\mathcal{E}_t$, the intermediaries would be taking unbounded long positions in forwards and home-currency bonds and corresponding unbounded short positions in foreign-currency bonds, and vice versa. We formalize this argument by generalizing the portfolio problem (26) in Appendix A.2.

[18] Du, Tepper, and Verdelhan (2018) document that the average annualized CIP deviations were a negligible 2 basis points prior to 2007, and since then increased tenfold to 20 basis points, yet this is still an order of magnitude smaller than the expected UIP deviations, which are around 200 basis points, or 2%.

[19] This offers a good point of approximation to both the model with a conventional Taylor rule with finite $\phi_\pi > 1$, as well as to the data for OECD countries with floating exchange rate regimes, where inflation is an order of magnitude less volatile than exchange rates. We explore the effects of a switch to a peg, the Mussa (1986) puzzle, in Itskhoki and Mukhin (2019).

R3. $\chi_2 = 0$ and we normalize $\chi_1 = 1$ in (28).[20] This case simplifies the dynamic roots of the equilibrium system without changing its *quantitative* properties (the general solution with endogenous $\chi_1, \chi_2 > 0$ is provided in Appendix A.3).

We solve the model analytically by log-linearization, and all the expressions that follow are in terms of log deviations from a symmetric steady state (see Appendix A.3). For concreteness, we focus on two shocks — the relative productivity shocks, $\tilde{a}_t \equiv a_t - a_t^*$, and the financial shock ψ_t, both following independent AR(1) processes with equal persistence ρ. Our qualitative results obtained for a general set of shocks — including monetary, government spending, price markup and labor wedge shocks — as long as the set of shocks features the financial shock ψ_t (see Itskhoki and Mukhin 2017).

While this section emphasizes the qualitative properties of the disconnect mechanisms, our goal is nonetheless *quantitative* in that we want to establish the ability of our baseline model to be consistent with a rich set of moments describing the comovement between exchange rates and macro variables. In order to evaluate the quantitative implications of the model, we fix the main parameters at their conventional values, *without* targeting any of the exchange rate moments. The key parameters of the transmission mechanism are the home bias in expenditure (low γ), strategic complementarities in price setting ($\alpha > 0$) and weak substitutability between home and foreign goods ($\theta > 1$, but small). The other parameters of the model, including relative risk aversion σ and labor supply elasticity ν, are less consequential for the quantitative results. The qualitative results of the model hold in general, independently of the specific parameter values.

Calibration We set $\gamma = 0.07$ to be consistent with the 0.28 trade (imports plus exports) to GDP ratio of the United States, provided the intermediate input share $\phi = 0.5$.[21] We further use the estimate of Amiti, Itskhoki, and Konings (2019) of the elasticity of strategic complementarities $\alpha = 0.4$, which is also in line with much of the markup and pass-through literature and corresponds to the own cost shock pass-through elasticity of $1 - \alpha = 0.6$ (see survey in Gopinath and Itskhoki 2011). Finally, we follow the recent estimates of Feenstra, Luck, Obstfeld, and Russ (2018) and set the elasticity of substitution $\theta = 1.5$, which is also the number used in the original calibrations of Backus, Kehoe, and Kydland (1994) and CKM.[22] We further set the relative risk aversion $\sigma = 2$, the Frish elasticity of labor supply $\nu = 1$ and the *quarterly* discount factor $\beta = 0.99$. For the persistence parameter common across shocks, we assume $\rho = 0.97$, a value that is consistent with both high empirical persistence of relative GDP and the cross-country interest rate differentials, as we discuss further in Section 4.

[20] Formally, from Lemma 3, the limit with $\chi_1 > 0$ and $\chi_2 \to 0$ emerges when $\bar{Y}/m \to 0$ as n/m stays bounded away from zero, that is when the size of the financial sector (number of both noise traders n and arbitrageurs m) increases relative to the size of the real economy. The further normalization of $\chi_1 = 1$ is simply a rescaling of the volatility units of the shock ψ_t.

[21] This value of the trade-to-GDP ratio is also characteristic of the other large developed economies (Japan and the Euro Zone). In Appendix A.2, we derive the relationship between the value of the trade-to-GDP ratio and the value of γ (steady state imports-to-expenditure ratio), which we set to be four times smaller. Intuitively, imports in a symmetric steady state are half of total trade (imports plus exports), and GDP (final consumption) is about one half of the total expenditure with the other half allocated to intermediate inputs ($\phi = 0.5$). This value of ϕ is consistent with both aggregate input-output matrices and firm level data on intermediate expenditure shares in total sales. The decomposition of gross exports for the U.S., the E.U. and Japan in Koopman, Wang, and Wei (2014) suggests this proportion holds for trade flows as well.

[22] The *macro* elasticity of substitution between the aggregates of home and foreign goods is indeed the relevant elasticity for our analysis, while the estimates of the *micro* elasticity at more disaggregated levels are typically larger, around 3 or 4. The quantitative performance of our model does not deteriorate significantly for elasticities of substitution as high as 3.

Equilibrium relationships We briefly discuss three equilibrium relationships, which are essential for understanding the dynamics of the model. First, the assumption on monetary policy (R2) ensures stable price levels, $p_t = p_t^* \equiv 0$, and therefore equality between the real and the nominal exchange rates:

$$q_t \equiv p_t^* + e_t - p_t = e_t. \tag{29}$$

In the absence of nominal frictions, the monetary policy rule simply selects a path of the nominal exchange rate, which tracks the real exchange rate, without affecting the equilibrium path of the latter.

The second equation combines the modified UIP (28) with the equilibrium interest rate differential to derive a condition for the expected nominal depreciation, $\mathbb{E}_t\Delta e_{t+1}$. The household Euler equations (4) result in $i_t - i_t^* = \sigma\mathbb{E}_t\{\Delta c_{t+1} - \Delta c_{t+1}^*\}$, in light of zero inflation implied by (R2). Furthermore, equilibrium in the product market results in a relationship between consumption, productivity and the real exchange rate, $c_t - c_t^* = \kappa_a\tilde{a}_t - \gamma\kappa_q q_t$, where $\kappa_q, \kappa_a > 0$ are derived parameters of the model that do not depend on β and ρ (see Section 3.3). Putting these three equations together yields:

$$\mathbb{E}_t\Delta e_{t+1} = -\frac{1}{1+\gamma\sigma\kappa_q}\chi_1\psi_t - \frac{\sigma\kappa_a(1-\rho)}{1+\gamma\sigma\kappa_q}\tilde{a}_t, \tag{30}$$

where we made use of (R2) and (R3). Equation (30) characterizes equilibrium in the financial market. In particular, the financial shock ψ_t that increases demand for foreign-currency bonds, in part decreases the relative foreign interest rate, $i_t^* - i_t$, and in part causes an expected appreciation of the home currency, both of which make holding foreign-currency bonds less attractive, returning the financial market to equilibrium.[23]

Lastly, the log-linearized flow budget constraint of the country (20) is given by:[24]

$$\beta b_{t+1} - b_t = nx_t = \gamma\big[\lambda_q q_t - \lambda_a\tilde{a}_t\big], \tag{31}$$

where the left-hand side is the evolution of the net foreign assets (NFA), while the right-hand side is the equilibrium expression for net exports of the country, which *decreases* with domestic demand ($c_t - c_t^*$, and hence $\tilde{a}_t$) and *increases* with expenditure switching towards the home goods under a weaker exchange rate (higher q_t). The derived parameters λ_a and λ_q are positive and do not depend on β and ρ, with $\lambda_q > 0$ ensured by the *Marshall-Lerner condition*, which holds in this model.

3.1 Equilibrium exchange rate dynamics

The equilibrium exchange rate dynamics are shaped by the interplay between financial and macroeconomic forces. The equilibrium in the financial market requires that the modified UIP condition (28) holds, which implies (30) and imposes discipline on the future expected appreciations and depreciations of the nominal exchange rate. Indeed, the equilibrium in the financial market is not affected by the *level*

[23] In turn, a mean-reverting positive productivity shock causes intertemporal substitution accommodated by a reduction in the interest rate, which in turn requires an expected appreciation ($\mathbb{E}_t\Delta e_{t+1} < 0$) to satisfy the modified UIP condition (28).

[24] Net exports, like net foreign assets, are zero in a symmetric steady state, and we denote $nx_t \equiv NX_t/\bar{Y}$, in parallel with b_t.

of the exchange rate, only by its *expected change*. This can be formally seen by solving (30) forward to express the exchange rate e_t as a function of the expected future shocks and its long-run expectation $\mathbb{E}_t e_\infty$, which remains indeterminate from the equilibrium conditions in the financial market.

In contrast to the financial market, the product market equilibrium depends on the level of the exchange rate. In particular, a depreciated (real) exchange rate causes expenditure switching towards home-produced goods and an increase in net exports, as can be seen from (31). Then, given the restriction (30) on the path of the exchange rate imposed by the financial market, the equilibrium value of the exchange rate, $e_t = q_t$, is uniquely determined by the intertemporal budget constraint of the country, which obtains from (31) under the No-Ponzi-Game Condition (NPGC) on net foreign assets.

Combining (30) and (31) in this way results in an equilibrium cointegration relationship between the exchange rate, the net foreign assets of the country b_t, and the exogenous shocks:[25]

$$e_t = -\frac{1-\beta}{\gamma\lambda_q} b_t + \frac{1}{1+\gamma\sigma\kappa_q}\frac{\beta}{1-\beta\rho}\chi_1\psi_t + \left[\frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}\frac{\beta(1-\rho)}{1-\beta\rho} + \frac{\lambda_a}{\lambda_q}\frac{1-\beta}{1-\beta\rho}\right]\tilde{a}_t. \tag{32}$$

Intuitively, the exchange rate is stronger (lower e_t) the greater are the net foreign assets b_t, and it *depreciates* with the financial shock ψ_t, which creates additional demand for foreign currency, as well as with the relative productivity shock $\tilde{a}_t$, which results in additional supply of home goods. This pins down the *unique* equilibrium path of the exchange rate, as a deviation from this value of e_t shifts the whole expected path of the exchange rate, and hence all expected trade surpluses on the right-hand side of (31), violating the intertemporal budget constraint.

Finally, we can solve for the equilibrium dynamic process for b_{t+1} and e_t by combining (32) with the flow budget constraint (31). In particular, the nominal exchange rate follows the following process:

$$\Delta e_t = \frac{1}{1+\gamma\sigma\kappa_q}\frac{\beta\chi_1}{1-\beta\rho}\left[\psi_t - \frac{1}{\beta}\psi_{t-1}\right] + \frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}\frac{\beta(1-\rho)}{1-\beta\rho}\left[\tilde{a}_t - \frac{1}{\beta}\tilde{a}_{t-1}\right] + \frac{\lambda_a}{\lambda_q}\frac{1-\beta}{1-\beta\rho}\tilde{\varepsilon}^a_t, \tag{33}$$

where $\tilde{\varepsilon}^a_t \equiv \sigma_a(\varepsilon^a_t - \varepsilon^{a^*}_t)$ is the innovation of the relative productivity shock $\tilde{a}_t$. The exchange rate process described by (33) is an ARIMA(1,1,1) — or equivalently, an ARMA(1,1) for Δe_t.[26]

Proposition 1 *In the baseline model, the equilibrium exchange rate e_t follows an ARIMA(1,1,1) described in* (33)*, which characterizes the equilibrium exchange rate path, and non-fundamental solutions do not exist. The autoregressive root of this process is equal to ρ, and the process becomes indistinguishable from a random walk as $\beta\rho \to 1$. Greater $\beta\rho$ and smaller trade openness γ result in a more volatile response of the exchange rate to a financial shock ψ_t, which dominates the volatility of Δe_t as $\beta\rho$ increases.*

The dynamics of the equilibrium exchange rate (33) are shaped by parameters β and ρ, while the other parameters of the model affect the proportional volatility scalers of the shocks. Interest-

[25] Solving (30) and (31) forward, using the fact that shocks are AR(1), and imposing NPGC $\lim_{T\to\infty}\beta^T b_{t+T+1}=0$, results in:

$$e_t = \mathbb{E}_t e_\infty + \frac{1}{1+\gamma\sigma\kappa_q}\left[\frac{\chi_1\psi_t}{1-\rho} + \sigma\kappa_a\tilde{a}_t\right] \qquad \text{and} \qquad b_t + \gamma\lambda_q\sum_{j=0}^\infty \beta^j\mathbb{E}_t e_{t+j} - \frac{\gamma\lambda_a}{1-\beta\rho}\tilde{a}_t = 0,$$

which together yield (32), as well as the solution for NFA: $\Delta b_{t+1} = \frac{\gamma\lambda_q}{1+\gamma\sigma\kappa_q}\frac{1}{1-\beta\rho}\chi_1\psi_t + \left[\frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q} - \frac{\lambda_a}{\lambda_q}\right]\frac{\gamma\lambda_q(1-\rho)}{1-\beta\rho}\tilde{a}_t$.

[26] Appendix A.3 offers an alternative derivation, using the Blanchard and Kahn (1980) technique, for the more general case with both $\chi_1, \chi_2 \geq 0$, which are endogenously determined in equilibrium, confirming the assumptions of Lemma 3.

ingly, the exchange rate volatility is higher in more closed economies, and it is maximized in the autarky limit $\gamma \to 0$ (see Figure 1a). This is in line with the data, where the more open economies have indeed less volatile exchange rates, even after controlling for country size and other characteristics (see e.g. Hau 2002). Intuitively, a more open economy cannot sustain the same amount of exchange rate volatility without it causing a more volatile response of the macro variables to the shocks, as we discuss further below.

Figure 1b provides the impulse response of the exchange rate to the financial shock ψ_t. An increase in demand for foreign-currency bonds, $\psi_t > 0$, results in an instantaneous depreciation of the home currency, while also predicting an expected appreciation according to (30), akin to the celebrated *overshooting* dynamics in Dornbusch (1976). This exchange rate path ensures both equilibrium in the financial market (via expected appreciation) and a balanced country budget (via instantaneous depreciation). The impulse response to the relative productivity shock $\tilde{a}_t$ is essentially identical (see Appendix Figure A1): an increase in productivity and resulting supply of home goods lead to an instantaneous depreciation needed to equilibrate the intertemporal budget constraint in view of the increased domestic production and consumption, with the currency gradually appreciating thereafter as the productivity shock wears out. For both types of shocks, a large instantaneous depreciation is followed by small but persistent expected future appreciations.

Furthermore, as the shocks become more persistent, the impulse responses of e_t become closer to a step function of a random walk for both types of shocks. More generally, for β and ρ close to 1, but $\beta\rho < 1$, the exchange rate exhibits the following *near-random-walk* properties:

(i) The autocorrelation of exchange rate changes, $\mathrm{corr}(\Delta e_t, \Delta e_{t-1})$, is arbitrarily close to zero, and in our baseline calibration it has a median estimate of -0.02 and not statistically different from zero in 30-year-long samples.

(ii) The contribution of the predictable component $\mathbb{E}_t \Delta e_{t+1}$ to the variance of Δe_{t+1} is negligible, or equivalently, the unexpected component, $\mathrm{var}_t(\Delta e_{t+1}) = \mathrm{var}(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})$, dominates the variance $\mathrm{var}(\Delta e_{t+1})$, as we illustrate in Figure 1c at different horizons.

(iii) The volatility of the exchange rate becomes unboundedly large relative to the volatility of the financial shock ψ_t, $\frac{\mathrm{var}(\Delta e_t)}{\mathrm{var}(\psi_t)} \to \infty$, and the contribution of ψ_t (relative to $\tilde{a}_t$) dominates the variance of the exchange rate.

The financial shock is not unique in delivering a near-random-walk behavior for the exchange rate. In fact, any persistent fundamental shock achieves this.[27] The differential implications of the financial and productivity shocks will become clear below, when we consider the comovement between the exchange rate and macro variables. The only material difference between the financial and productivity shocks from the point of Proposition 1 is that, as both shocks become more persistent, the effect of the financial shock on exchange rate increases without bound, while the effect of the productivity shock remains bounded even in the limit. As a result, *small but persistent* shocks to currency demand in the

[27] This is reminiscent of the result in Engel and West (2005), which however derives exclusively from the equilibrium condition in the financial market. In contrast, our solution relies on the full general equilibrium, and in particular endogenizes the real exchange rate dynamics; as a result, our Proposition 1 is not nested by their Theorem (see earlier draft, Itskhoki and Mukhin 2017, for details).

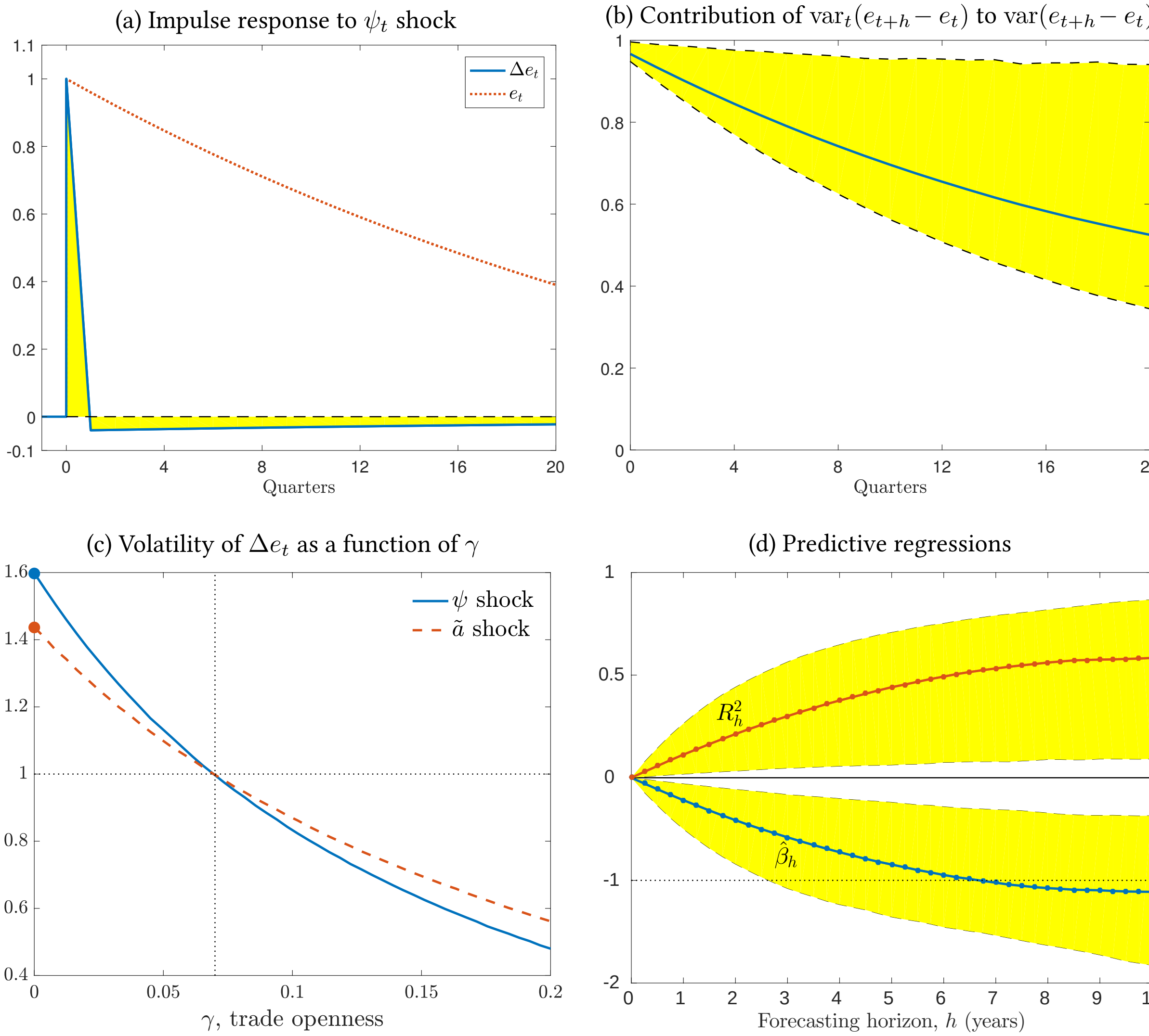

Figure 1: Properties of the equilibrium exchange rate process

Note: Calibrated baseline model. Panels (a), (b) and (d) are conditional on the ψ_t shock (see Appendix Figure A1 for $\tilde{a}_t$ shock). Panel (a): impulse response of both Δe_t and e_t to a ψ_t-shock innovation; (b): $\frac{\text{var}_t(e_{t+h}-e_t)}{\text{var}(e_{t+h}-e_t)} = \frac{\text{var}(e_{t+h}-\mathbb{E}_t e_{t+h})}{\text{var}(e_{t+h}-e_t)}$ at various horizons $h \geq 1$; (c): $\text{std}(\Delta e_t)$ as a function of γ, conditional on ψ_t and $\tilde{a}_t$, normalizing to one the calibrated value at $\gamma = 0.07$; (d): $\hat{\beta}_h$ and R_h^2 from the predictive regression $\mathbb{E}\{e_{t+h} - e_t | q_t\} = \alpha_h + \beta_h q_t$, at different horizons $h \geq 1$. Lines in (b) and (d) plot medians across simulations with 120 quarters each; shaded areas provide the 5%–95% range across simulations.

financial market have the ability to generate a volatile near-random-walk equilibrium response of the exchange rate, while small productivity shocks have proportionally small effects on the exchange rate.[28]

Equation (30) suggests departures from a random walk behavior and implies predictability of the nominal exchange rate. Indeed, there exists empirical evidence on the departure of the exchange rate process from a pure random walk (see e.g. Bacchetta and van Wincoop 2006, Engel 2016, Lustig, Stathopoulos, and Verdelhan 2016). In a recent paper, Eichenbaum, Johannsen, and Rebelo (2018) em-

[28]This is because random-walk productivity shocks have no direct effect on the intertemporal Euler equation (see (41)). In contrast, rare-disaster, long-run-risk or news productivity shocks are more similar in their properties to the financial shock ψ_t.

phasize the predictability of nominal exchange rate changes $(e_{t+h} - e_t)$ with the contemporaneous value of the real exchange rate q_t, which becomes stronger with the horizon h. Figure 1d plots these projection coefficients $\hat{\beta}$ and corresponding R^2 from the simulated paths of exchange rates in our baseline model, reproducing closely the empirical findings of Eichenbaum, Johannsen, and Rebelo (2018): (i) the projection coefficients $\hat{\beta}_h$ are about zero for small h and become increasingly more negative as h increases, crossing -1 after about 6 years; and (ii) R^2_h also starts around zero and increases towards 0.6 for large h. This pattern holds similarly for financial and productivity shocks, and does not rely on stationarity of the nominal or real exchange rate. Therefore, our model reproduces simultaneously the near-random-walk behavior and the subtle departures from a pure random walk in the nominal exchange rate observed in the data.

3.2 The PPP puzzle

We next explore the equilibrium dynamics of the real exchange rate (RER) and the associated purchasing power parity (PPP) puzzle, which we broadly interpret as the *close comovement* between the nominal and the real exchange rates, and a volatile near-random-walk behavior of both variables. In the data, all notions of the real exchange rate — consumer-price, producer-price and wage-based — comove nearly perfectly with the nominal exchange rate at short and medium frequencies and have long half-lives, as we report in Table 1 and illustrate in Appendix Figure A2. The close comovement involves both nearly perfect correlations at various horizons and nearly equal volatilities, as well as nearly equal persistence for all exchange rate series.

As we have seen, a monetary policy that stabilizes the domestic consumer prices results immediately in a close comovement — in fact, perfect comovement in the limit — between nominal and consumer-price real exchange rates, e_t and q_t (see (29)). Furthermore, Proposition 1 implies that both nominal and real exchange rates follow very persistent near-random-walk processes in response to both productivity and financial shocks. In particular, if one were to fit an AR(1) process for the real exchange rate, as is conventionally done in the PPP puzzle literature (see Rogoff 1996), one would be challenged to find evidence of mean reversion and would infer very long half-lives for the real exchange rate process (see Table 1 and Appendix A.3).[29]

To derive the equilibrium relationships between various notions of real exchange rates, we first combine the results in Lemmas 1 and 2 with the expression for the marginal cost (13), to solve for the equilibrium real wage:

$$w_t - p_t = a_t - \frac{1}{1-\phi}\frac{\gamma}{1-2\gamma}q_t. \tag{34}$$

The real wage increases with the productivity of the economy a_t and with its relative purchasing power captured by the real exchange rate appreciation (decline in q_t) in proportion with the openness of the economy γ. This allows us to characterize the wage-based and producer-price real exchange rates

[29]Under our baseline parametrization, the model reproduces the 3-to-5 year half-lives of the real exchange rate in finite (30-year-long) samples. While real exchange rate follows an integrated ARIMA(1,1,1) process when $\chi_2 = 0$, this process becomes a mean-reverting stationary ARMA(2,1) for any $\chi_2 > 0$, as we show in Appendix A.3. Despite this qualitative difference in terms of long-run stationarity, the two processes are indistinguishable in finite samples when χ_2 is small. See Müller and Watson (2018, 2019) for the recent evidence on the (lack of) long-run stationarity of the real exchange rate.

Table 1: Properties of the exchange rates

	Data	Data	Baseline Model	
			$\tilde{a}_t$ shock	ψ_t shock
	$\text{corr}(\bullet, \Delta e_t)$	$\text{std}(\bullet)/\text{std}(\Delta e_t)$		
Δq_t	0.99	0.99	1.00	1.00
Δq_t^P	0.97	0.97	1.10	1.10
Δq_t^W	0.99	1.02	0.03	1.32
Δs_t	0.20	0.25	0.26	0.26
	$\rho(q_t)$:	0.94	0.93	0.93

Note: Empirical exchange rate moments are quarterly, for the U.S. against the PPP-weighted average of France, Germany, and the U.K.; the moments for the terms of trade are from Obstfeld and Rogoff (2001) and Gopinath et al. (2018) (see Data Appendix A.5). $\rho(q_t)$ is the autocorrelation of q_t. Model moments are medians across simulations, each with 120 quarters.

as follows:

$$q_t^W \equiv w_t^* + e_t - w_t = -\tilde{a}_t + \left[1 + \frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\right] q_t, \tag{35}$$

$$q_t^P \equiv p_{Ft}^* + e_t - p_{Ht} = \left[1 + (1-\alpha)\frac{2\gamma}{1-2\gamma}\right] q_t. \tag{36}$$

From (36), which holds independently from the source of the shocks, we immediately see that the model reproduces a close comovement between consumer- and producer-price RERs. Furthermore, as openness γ decreases and strategic complementarities α become stronger, q_t and q_t^P are not just tightly correlated, but also have approximately the same volatility. Intuitively, a monetary policy that stabilizes domestic consumer prices also stabilizes domestic producer prices, as the difference between the two is not very large in economies with substantial home bias and strategic complementarities in price setting.

This logic does *not* hold for the wage-based real exchange rate q_t^W in (35). In particular, productivity shocks a_t and a_t^* drive a wedge between price and wage inflation. In other words, a monetary policy that stabilizes prices in response to productivity shocks results in volatile wages, as real wages (34) must reflect productivity. This tradeoff is not present, however, if the financial shocks are the key driving force behind the exchange rates. Indeed, financial shocks generate a volatile and persistent nominal exchange rate, which — under a monetary policy that stabilizes price levels — translates into equally volatile and persistent real exchange rates, independently of whether they are measured using consumer or producer prices, or wages. We illustrate these results in Table 1 and summarize them in:

Proposition 2 *In the baseline model, the equilibrium relationships between the nominal and real exchange rates are given by* (29), (35) *and* (36)*:* (i) *Home bias (small γ) and monetary policy that stabilizes consumer prices ensure perfect comovement between e_t, q_t and q_t^P, independently of the source of the shocks.* (ii) *A financial shock ψ_t, in addition, results in a perfect comovement between q_t and q_t^W, and, when $\beta\rho \to 1$, produces an arbitrarily volatile and persistent behavior of all real exchange rate series, with arbitrarily large half-lives in large finite samples.*

In other words, the combination of (a) conventional monetary policy, (b) significant home bias and (c) financial shocks allows the model to reproduce the empirical behavior of all the exchange rate series. The absence of the direct effect of the financial shock on the product and labor markets, translates price stability into both nominal and real wage stability, while international relative prices and wages comove closely with the nominal exchange rate, exhibiting a high degree of persistence. Greater openness of the economies leads to larger feedback effects of the exchange rate into domestic relative prices, as the foreign value added plays a bigger role in the domestic consumption basket (see e.g. (34)). Importantly, these properties of the model obtain under flexible prices and wages, and hence do not rely on nominal rigidities. While wage and price stickiness is arguably a salient feature of the real world, Proposition 2 shows that the empirical behavior of exchange rates is well captured to a first order by a flexible-price model, provided that financial shocks account for a considerable portion of exchange rate volatility.

In view of this result, a natural question is why the PPP puzzle posed such a challenge to the literature? From the definition of the real exchange rate, the close comovement between q_t and e_t implies that price levels p_t and p_t^* must, in turn, move little with the nominal exchange rate e_t. The PPP puzzle literature has largely focused on one conceptual possibility, namely that price levels move little due to nominal rigidities, assuming monetary shocks are the main drivers of the nominal exchange rate. The issue with this approach is that monetary shocks necessarily imply cointegration between relative nominal variables — $(w_t - w_t^*)$, $(p_t - p_t^*)$ and e_t — which results in mean reversion in the real exchange rate q_t. The speed of this mean reversion is directly controlled by the duration of nominal price stickiness, which is empirically insufficient to generate long half lives, characteristic of the real exchange rate (see CKM).

We focus on the other conceptual possibility, namely that prices are largely disconnected from exchange rates, or in other words the low exchange rate pass-through into CPI inflation, even in the long run, due to substantial home bias (small γ). Importantly, this mechanism requires that the main drivers of the exchange rate are not productivity or monetary shocks, which introduce a wedge between nominal and real exchange rates independently of the extent of the home bias. In contrast, the financial shock ψ_t drives no wedge between nominal and real exchange rates, even in the long run. Home bias is thus the only crucial part of the transmission mechanism here, leaving nominal rigidities, real rigidities α, and the extent of expenditure switching θ largely irrelevant.[30]

Terms of trade We now briefly comment on the properties of the terms of trade (ToT), another international relative price of great importance. As emphasized by Atkeson and Burstein (2008), conventional models imply a counterfactually volatile ToT relative to RER, while in the data ToT are substantially more stable — about four times less volatile than RER (see Table 1 and Appendix Figure A2).

[30] As a result, our model is consistent with the recent cross-sectional and time-series evidence, which poses a challenge for the conventional monetary model: Kehoe and Midrigan (2008) show a missing correlation between price durations and the volatility and persistence of the sectoral real exchange rates (see also Imbs, Mumtaz, Ravn, and Rey 2005, Carvalho and Nechio 2011), while Blanco and Cravino (2018) show that *reset-price RER* is as volatile and persistent as the conventional RER. Neither of this is evidence against price stickiness *per se*, but rather it is evidence against monetary shocks as the key driver of the nominal exchange rate. Furthermore, our model is also in line with the empirical findings in Engel (1999) and Betts and Kehoe (2008), as it is the *tradable* component that drives the volatility of the overall RER (see Appendix A.3.3, where we also discuss the Balassa-Samuelson effect).

Combining the definition of ToT (21) with the results in Lemmas 1 and 2, we obtain the equilibrium relationship between ToT and RER:[31]

$$s_t = \frac{1 - 2\alpha(1-\gamma)}{1-2\gamma} q_t. \tag{37}$$

From (37) and (36), we can see how conventional models without strategic complementarities ($\alpha = 0$), and hence without LOP deviations, imply that the ToT equal the producer-price RER, and both are more volatile than the consumer-price RER, $s_t = q_t^P = q_t/(1-2\gamma)$, as consumption bundles are more similar across countries than production bundles. This is, however, empirically counterfactual, and as explained by Atkeson and Burstein (2008) is not necessarily the case in models with *pricing to market* (PTM), arising from strategic complementarities in price setting ($\alpha > 0$):

Proposition 3 *The empirical patterns* $\mathrm{std}(\Delta s_t) \ll \mathrm{std}(\Delta q_t) \approx \mathrm{std}(\Delta q_t^P)$ *and* $\mathrm{corr}(\Delta s_t, \Delta q_t) > 0$ *obtain when strategic complementarities in price setting are significant, but not too strong:* $\frac{\gamma}{1-\gamma} \ll \alpha < \frac{1}{2(1-\gamma)}$.

When $\alpha > \frac{\gamma}{1-\gamma}$, the model reproduces the empirically relevant case, in which RER is considerably more volatile than ToT. Pricing to market smoothes the response of ToT to changes in producer prices q_t^P, and hence makes ToT less volatile, as export prices mimic the local competition.[32] Under our baseline parameterization with $\alpha = 0.4$, the terms of trade are about a quarter as volatile as the real exchange rate, yet the two are still positively correlated (see Table 1).

3.3 The Backus-Smith puzzle

We now study the relationship between aggregate consumption and the real exchange rate, emphasizing the role of *expenditure switching* in the product market, as opposed to the *international risk sharing* in the financial market (the Backus-Smith condition).[33] Specifically, we consider in turn the labor and product market clearing conditions. First, combining labor supply (3) and labor demand (14), and using the expression for the equilibrium real wage (34), the equilibrium in the labor market attains when:

$$y_t + \sigma\nu c_t = (1+\nu)a_t - \frac{\nu+\phi}{1-\phi}\frac{\gamma}{1-2\gamma} q_t. \tag{38}$$

A real depreciation (an increase in q_t) reduces the real wage (recall (34)) and hence labor supply, resulting in lower output y_t. In turn, higher productivity increases output both directly and indirectly (due to increased labor supply), while higher consumption reduces labor supply due to the income effect.

[31] This equation results from the two useful intermediate relationships: (i) $q_t = (1-\gamma)q_t^P - \gamma s_t$ reflects that the relative consumer prices q_t differ from the relative producer prices q_t^P by the relative price of imports s_t; and (ii) $s_t = q_t^P - 2\alpha q_t$ states that ToT reflect the relative producer prices adjusted for the law of one price deviations of exports (recall Lemma 2).

[32] Very strong PTM, just like local currency pricing (LCP), can turn the positive correlation between ToT and RER negative, which is empirically counterfactual, as emphasized by Obstfeld and Rogoff (2000). Also note from (37) that flexible-price models cannot generically reproduce the low empirical correlation between ToT and RER, as we discuss in Section 4.

[33] In complete market models, efficient international risk-sharing requires a positive correlation between relative consumption and RER, $\sigma(c_t - c_t^*) = q_t$. Instead, our model features incomplete markets and a shock to *risk-sharing* ψ_t, which implies $\mathbb{E}_t\{\sigma(\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1}\} = \psi_t$, and does *not* impose a particular correlation between consumption and RER.

Second, the product market clearing condition, which derives from (18)–(19), results in:

$$y_t = (1-\phi)\big[(1-\gamma)c_t + \gamma c_t^*\big] + \phi\big[(1-\gamma)y_t + \gamma y_t^*\big] + \gamma\left[2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} - \phi\right]q_t. \tag{39}$$

Note that in the closed economy ($\gamma = 0$), we simply have $y_t = c_t$. In an open economy, home production y_t is split between final consumption and intermediate use at home and abroad, as reflected by the first two terms on the right-hand side of (39). The remaining term combines the positive effect of a real depreciation due to expenditure switching from foreign to home goods and an associated negative effect due to substitution away from the more expensive intermediate inputs towards local labor. The magnitude of the expenditure switching effect is proportional to $\theta(1-\alpha)$, a product of the exchange rate pass-through into prices $(1-\alpha)$ and the elasticity of substitution θ.

Combining (38)–(39) with their foreign counterparts, we obtain the equilibrium relationship between consumption, productivity and the real exchange rate:

$$c_t - c_t^* = \kappa_a \tilde{a}_t - \gamma\kappa_q q_t, \tag{40}$$

where $\kappa_a, \kappa_q > 0$ and κ_q increases in the expenditure switching effect, $\theta(1-\alpha)$. This relationship, in combination with Proposition 1, which characterizes the equilibrium exchange rate, leads to the following result:

Proposition 4 (i) *A financial shock ψ_t results simultaneously in a real depreciation and a reduction in the relative home consumption, with the relative volatility of the consumption response declining to zero as $\gamma \to 0$.* (ii) *In contrast, a productivity shock $\tilde{a}_t$ increases consumption and depreciates the real exchange rate, with both effects having the same order of magnitude independently of the value of trade openness γ.*

It follows from part (i) of Proposition 4 (and directly from (40)) that financial shocks result in a negative correlation between relative consumption and the real exchange rate, both in levels and in growth rates. That is, our model predicts that consumption is low when prices are low, in relative terms across countries. This violates the pattern of the efficient international risk sharing, predicted by the Backus-Smith condition, yet it is consistent with the robust empirical patterns. Furthermore, the proposition shows that as the economy becomes less open to trade, the response of consumption to the financial shock becomes muted, even as the real exchange rate becomes more volatile (recall Proposition 1 and Figure 1a) — an essential property for the model of exchange rate disconnect. For our baseline parameterization, the consumption response to a financial shock is about 6 times less volatile than that of the real exchange rate, in line with the data (see e.g. CKM).

What is most striking about this simple resolution of the celebrated Backus-Smith puzzle is that it derives from conventional labor and product market clearing conditions, which are ubiquitous in international general equilibrium models. Indeed, the negative relationship between consumption and the real exchange rate is a *robust* feature of the expenditure switching mechanism. Real exchange rate depreciation switches expenditure towards home goods, and in order to clear the markets home output needs to rise and home consumption needs to fall, in view of the home bias.[34]

Nonetheless, this property of our model stands in stark contrast with predictions of both productivity-driven IRBC models and monetary-shock-driven New Keynesian open economy (NKOE) models, even when those models feature incomplete asset markets, as the second part of Proposition 4 illustrates. In these models, real depreciation is associated with increased supply of domestic output and thus of domestic consumption — either due to higher productivity or an expansionary monetary shock — while the expenditure switching effect is only a byproduct (e.g., see the two terms in (40)).[35] As a result, real depreciation in these models is associated with an empirically counterfactual domestic consumption boom, independently of the completeness of asset markets. The mechanism in our financial model is different, as real depreciations, caused by increased demand for foreign assets rather than increased supply of home goods, have *only* indirect expenditure switching effects on the real economy, resulting in lower consumption.

3.4 The Forward Premium puzzle

We finally turn to the equilibrium properties of the interest rates. Substituting (30) into the modified UIP condition (28), we obtain the solution for the equilibrium interest rate differential:

$$i_t - i_t^* = \frac{\gamma\sigma\kappa_q}{1+\gamma\sigma\kappa_q}\chi_1\psi_t - \frac{\sigma\kappa_a(1-\rho)}{1+\gamma\sigma\kappa_q}\tilde{a}_t. \tag{41}$$

A mean-reverting productivity shock reduces the interest rate as a result of intertemporal substitution, while a demand shock for foreign-currency bonds raises the interest rate differential to equilibrate the asset market, with the magnitude of this response declining towards zero as $\gamma \to 0$. Equation (41) further implies that the interest rate differential follows an AR(1) process with persistence ρ.

Using Proposition 1, we can characterize the joint properties of the interest rates and the nominal exchange rate. In particular, we are interested in the properties of the *Fama regression*, that is the projection of the exchange rate change Δe_{t+1} on the interest rate differential $(i_t - i_t^*)$. We prove:

Proposition 5 (i) *Conditional on a productivity shock $\tilde{a}_t$, the Fama coefficient $\beta_F = 1$. Conditional on a financial shock ψ_t, the Fama coefficient is negative, $\beta_F = -\frac{1}{\gamma\sigma\kappa_q} < 0$.* (ii) *As $\beta\rho \to 1$:*

- $\beta_F \to -\frac{1}{\gamma\sigma\kappa_q} < 0$ *and the R^2 in the* Fama regression *becomes arbitrarily small;*
- *the volatility of $(i_t - i_t^*)$ relative to Δe_{t+1} becomes arbitrarily small;*

[34] While we solved for equilibrium consumption given the exchange rate, one can also do the reverse. Equilibrium relationship (40) can then be interpreted as a *Keynes transfer effect*: a financial shock makes home households postpone their consumption resulting in a lower relative demand for home goods, which requires an exchange rate depreciation to clear the goods market (see e.g. Pavlova and Rigobon 2008, Caballero, Farhi, and Gourinchas 2008).

[35] Note that the expenditure switching effect is proportional to trade openness γ, and hence is weak for small γ, explaining why it *cannot* typically overturn the direct effect of productivity (and monetary) shocks on consumption. However, news shocks about future productivity or long-run risk shocks as in Colacito and Croce (2013), just like financial shocks, can trigger large exchange rate movements, while having a negligible direct effect on consumption. Alternatively, the Backus-Smith puzzle can be resolved if the real exchange rate appreciates with a positive productivity shock, either due to Balassa-Samuelson forces (e.g., Benigno and Thoenissen 2008), persistent productivity growth rates and/or low elasticity of substitution between home and foreign goods ($\theta < 1$), as in Corsetti, Dedola, and Leduc (2008). These alternative mechanisms are, however, at odds with other exchange rate puzzles, including Meese-Rogoff and PPP puzzles discussed above.

- *the persistence of* Δe_{t+1} *relative to* $(i_t - i_t^*)$ *becomes arbitrarily small;*
- *the Sharpe ratio of the* Carry trade*becomes arbitrarily small.*

Conditional on financial shocks, positive interest rate differentials predict expected exchange rate appreciations — a pattern of *UIP deviations* known as the *Forward Premium puzzle* (Fama 1984). The productivity shocks are unable to reproduce this empirical pattern. However, provided that financial shocks play an important role in the dynamics of the exchange rate, the model reproduces a negative unconditional Fama coefficient. At the same time, the predictive ability of the interest rate differentials for future devaluations is very weak in the data (see e.g. Valchev 2016), and our model captures this with a vanishingly small R^2 in the Fama regression, as shocks become more persistent and the exchange rate becomes closer to a pure random walk. The model also captures the pronounced differences in the statistical properties of $i_t - i_t^*$ and Δe_{t+1}, with the former following a smooth and persistent process and the latter being close to a volatile white noise. Finally, the UIP shock in our model does not result in counterfactually large returns on the Carry trade, which under our baseline parameterization has a Sharpe ratio of around 0.2, in line with the empirical patterns.[36]

4 Quantitative Analysis

In this section, we return to the full model of Section 2 and study its quantitative properties. The goal is threefold. First, we show that our baseline exchange rate disconnect results in Section 3 are robust to the introduction of capital and nominal rigidities in prices and wages, under a conventional Taylor-rule monetary regime. Second, we show that a multi-shock version of the model matches not only the relative volatilities, but also the correlations between exchange rates and other macro variables. Finally, we show that matching the exchange rate moments comes at no cost in terms of the model's ability to match the standard international business cycle moments. In particular, while financial shocks account for much of the exchange rate volatility, standard productivity and monetary shocks remain the key drivers of consumption, investment and output. As a result, the relative volatilities and correlations of macro aggregates in our model are similar to those achieved in the earlier international business cycle literature following BKK.

Data The first column of Table 2 shows the empirical moments that are the focus of our quantitative analysis. For comparability, we follow CKM and estimate the moments for the U.S. relative to the PPP-weighted sum of France, Germany, Italy and the U.K., using quarterly data from 1973–1994. The empirical moments are similar for the longer period that we extend to 2017 (see Appendix A.5). In our analysis, we use the first differences of the variables, and the results are similar for the HP-filtered series. Additionally, for the moments that involve interest rates, we rely on the estimates in Hassan and Mano

[36] A carry trade is a zero-capital investment strategy, which shorts the low interest rate currency and longs the high interest rate currency. For concreteness, following Lustig and Verdelhan (2011), we focus on a strategy with an intensity (size of the positions taken, x_t) proportional to the expected return, i.e. $x_t = i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$ (see Appendix A.3). The unconditional Sharpe ratio of the carry trade in the data is about 0.5, but at least half of it comes from the cross-sectional country fixed effects not modeled in our framework, which instead focuses on the time-series properties. Our empirical target for the Sharpe ratio of 0.2 corresponds to the "forward premium trade" in Hassan and Mano (2014).

(2014) and Valchev (2016). Finally, due to the high variability of the Backus-Smith correlation across countries and periods, we use the average estimates from Corsetti, Dedola, and Leduc (2008), which is representative of the conventional value for this correlation.

Calibration For the main parameters of the model, we keep the same values as in Section 3, as summarized in Appendix Table A1. For the additional parameters, we use the conventional capital share in value added $\vartheta = 0.3$ and a quarterly capital depreciation rate $\delta = 0.02$. For each specification of the model, we recalibrate the capital adjustment costs parameter κ to match the relative volatility of investment, $\frac{\text{std}(\Delta z_t)}{\text{std}(\Delta gdp_t)} = 2.5$.

We use a conventional Taylor rule (22) with inflation sensitivity parameter $\phi_\pi = 2.15$ following the estimates in Clarida, Galí, and Gertler (2000). We set the interest rate smoothness parameter $\rho_m = 0.95$ to match the empirical persistence of interest rates.[37] In the modified UIP equation (28), we use a small positive number for $\chi_2 = 0.001$, which ensures stationarity of the model (see e.g. Schmitt-Grohé and Uribe 2003), and is also consistent with the high persistence of the U.S. current account with autocorrelation $\rho\big(\Delta b_t\big) = 0.95$ in the data.

In the sticky-price version of the model, we assume that prices adjust on average once a year, and thus set $\lambda_p = 0.75$, while wages adjust on average every six quarters, $\lambda_w = 0.85$, following the conventional calibrations in the literature (see Galí 2008). Thus, the range of the models that we consider includes both the flexible price benchmark and specifications with considerable extent of price and wage stickiness. For our baseline sticky price specification, we follow CKM and adopt local-currency pricing (LCP), and we explore the alternative price setting regimes (PCP and DCP) as robustness.

The model features three exogenous shocks — two country-specific productivity shocks (a_t, a_t^*) and a financial shock ψ_t — for which we need to calibrate the covariance matrix. We assume that ψ_t is orthogonal with (a_t, a_t^*), while a_t and a_t^* are correlated and have a common variance. We choose the relative volatility of the productivity shock, σ_a/σ_ψ, to match the Backus-Smith correlation $\text{corr}\,(\Delta q_t, \Delta c_t - \Delta c_t^*) = -0.4$, while the cross-country correlation of productivity shocks is calibrated to match $\text{corr}\,(\Delta gdp_t, \Delta gdp_t^*) = 0.35$.[38] In addition, we consider a version of the sticky-price model with monetary (Taylor rule) shocks $(\varepsilon_t^m, \varepsilon_t^{m*})$ instead of the productivity shocks, and we discipline their relative volatility σ_m/σ_ψ and cross-country correlation in the same way. Note that the moments of interest in Table 2 are for the most part not targeted directly by our calibration, which adopts conventional values for most model parameters following the broader macro literature (see Appendix Table A1).

Single-shock models We start by evaluating four different specifications of the model, each with a single type of shocks — a financial shock ψ_t under both flexible and sticky prices, productivity shocks in a standard flexible-price IRBC model, and monetary shocks in an NKOE model with sticky prices and

[37] In the data, the interest rates are very persistent, with $\rho(i_t) = 0.97$, and the interest rate differentials are somewhat less persistent, $\rho(i_t - i_t^*) = 0.9$. Our choice of $\rho_m = 0.95$, given the persistence of shocks $\rho = 0.97$, allows the model to be approximately in line with both of these moments across various model specifications that we consider in Table 2.

[38] We normalize the effective volatility of the financial shock $\chi_1\sigma_\psi = 1$, as our results focus on the relative volatilities of the variables. Scaling σ_a and $\chi_1\sigma_\psi$ proportionally does not effect the results reported in Table 2, and allows us to match the volatility of the nominal exchange rate in any specification of the model.

wages. This partial analysis helps us dissect which shocks allow us to match which moments in the full quantitative model. We report the results in columns 2–5 of Table 2, with the exchange-rate-related moments summarized in panels A–C and the international business cycle moments in panel D.

We start with the exchange rate moments, which confirm the various analytical results of Section 3. First, all four single-shock specifications of the model match the near random-walk behavior of the nominal exchange rate. Consistent with Proposition 1, it is the persistence of the shock rather than its type that ensures that the exchange rate is a near-martingale. At the same time, it is *only* the financial shock ψ_t that has the ability to replicate the empirical disconnect in the volatilities of the exchange rate and the macro variables. In the data, exchange rates are about 5 times more volatile than GDP and 6 times more volatile than consumption. Both versions of the model with the ψ_t shock consistently reproduce this gap in the volatility. In fact, they predict that macro variables are an order of magnitude less volatile than exchange rates. As we explain in Section 3, home bias in the goods market allows to sustain large movements in exchange rates without passing-through the volatility into the macro variables.[39] In contrast, the effect of productivity and monetary shocks on macro aggregates is of the same order of magnitude as on the exchange rates, inconsistent with the disconnect in volatilities.

Next, we consider the properties of the real exchange rate. First, note that the Taylor rule that targets inflation ensures a close comovement of the CPI-based real exchange rate with the nominal exchange rate, *independently* from the type of the shock or price stickiness. However, in line with Proposition 2, only the models with the financial shock are consistent with a broader set of the PPP moments. In particular, consistent with the PPP puzzle literature, the monetary model (NKOE) *cannot* match the persistence of the real exchange rate — predicting its half-life to be under one year, considerably below the empirical estimates of about three years. While this is not an issue for the IRBC model, this model produces a wedge between the CPI-based and the wage-based real exchange rates, which moves with the productivity shocks. Specifically, the IRBC model predicts a negative correlation between the nominal and the wage-based real exchange rates, in contrast with the data (recall Appendix Figure A2). The models with the ψ_t shock, on the other hand, have no difficulty in simultaneously matching the persistence of the real exchange rate and nearly perfect comovement between both measures of the real exchange rate and the nominal exchange rate. In contrast to the conventional wisdom, price stickiness is *not* crucial to resolve the PPP puzzle, but it *does* help to increase the relative volatility of the real exchange rate towards one.

Consistent with Propositions 4 and 5, only the financial shock models are consistent with a negative Backus-Smith correlation and a negative Fama regression coefficient, again independently of the presence of nominal rigidities. In contrast, despite incomplete asset markets and an endogenous risk-premium, which evolves with the net foreign asset position b_{t+1} (recall (28)), the correlation between relative consumption and the real exchange rate and the Fama regression coefficient are both close to one for productivity and monetary shocks alike. These properties again favor the models with financial shocks. Importantly, such models also have a good fit for the other financial moments, such as the Carry trade Sharpe ratio and the low volatility and high persistence of the interest rates. Furthermore,

[39] Note that the pass-through of exchange rate fluctuations (induced by the financial shock) is higher into consumption under flexible prices and into output under sticky prices.

Table 2: Quantitative Models

Moments	Data	Single-type shocks				Multi-shock models		
		Financial shock		IRBC	NKOE	IRBC	IRBC$^+$	NKOE
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A. Exchange rate disconnect:								
$\rho(\Delta e)$	≈ 0	-0.02 (0.09)	-0.03 (0.09)	0.01 (0.09)	-0.06 (0.09)	-0.02 (0.09)	-0.03 (0.09)	-0.03 (0.09)
$\sigma(\Delta e)/\sigma(\Delta gdp)$	5.2	17.7	9.8	0.58	1.31	2.9	3.4	3.8
$\sigma(\Delta e)/\sigma(\Delta c)$	6.3	7.5	17.5	0.96	2.18	3.9	5.8	6.6
B. Real exchange rate and the PPP:								
$\rho(q)$	0.94	0.93 (0.04)	0.91 (0.05)	0.94 (0.04)	0.82 (0.06)	0.93 (0.04)	0.91 (0.05)	0.90 (0.05)
$\sigma(\Delta q)/\sigma(\Delta e)$	0.99	0.74	0.98	1.40	0.95	0.83	0.98	0.97
$\text{corr}(\Delta q, \Delta e)$	0.99	1.00	1.00	1.00	0.99	0.99	1.00	1.00
$\sigma(\Delta q^W)/\sigma(\Delta e)$	1.01	1.04	1.00	0.56	0.95	1.07	1.00	1.00
$\text{corr}(\Delta q^W, \Delta e)$	0.99	1.00	1.00	-0.99	0.99	0.96	1.00	1.00
C. Backus-Smith and Forward premium:								
$\text{corr}(\Delta q, \Delta c - \Delta c^*)$	-0.40	-1.00	-0.95 (0.01)	1.00	1.00	-0.40 (0.08)	-0.40 (0.08)	-0.40 (0.08)
Fama β	< 0	-2.0 (1.4)	-3.4 (2.7)	1.6 (0.8)	1.4 (0.8)	-1.7 (1.5)	-2.2 (2.1)	-1.9 (1.3)
Fama R^2	0.02	0.03 (0.02)	0.03 (0.02)	0.08 (0.03)	0.03 (0.02)	0.02 (0.02)	0.02 (0.02)	0.02 (0.02)
Carry trade SR	0.20	0.23 (0.04)	0.22 (0.04)	0.01 (0.10)	-0.03 (0.09)	0.23 (0.04)	0.22 (0.04)	0.22 (0.04)
$\sigma(i - i^*)/\sigma(\Delta e)$	0.06	0.08	0.05	0.17	0.12	0.08	0.06	0.06
$\rho(i - i^*)$	0.90	0.93	0.98	0.95	0.82	0.94	0.98	0.94
$\rho(i)$	0.97	0.93	0.98	0.95	0.83	0.94	0.98	0.89
D. International business cycle moments:								
$\sigma(\Delta c)/\sigma(\Delta gdp)$	0.82	2.37	0.56	0.60	0.60	0.74	0.59	0.57
$\text{corr}(\Delta c, \Delta gdp)$	0.64	-1.00	-0.82	1.00	1.00	0.85	0.80	0.75
$\text{corr}(\Delta z, \Delta gdp)$	0.81	-1.00	-0.20	1.00	1.00	0.81	0.86	0.86
$\text{corr}(\Delta gdp, \Delta gdp^*)$	0.35	-1.00	-1.00	0.35	0.35	0.35	0.35	0.35
$\text{corr}(\Delta c, \Delta c^*)$	0.30	-1.00	-1.00	0.40	0.38	0.14	0.39	0.40
$\text{corr}(\Delta z, \Delta z^*)$	0.27	-1.00	-1.00	0.45	0.39	0.10	0.54	0.55
$\frac{\sigma_a}{\chi_1 \sigma_\psi}$ or $\frac{\sigma_m}{\chi_1 \sigma_\psi}$		–	–	–	–	3.3	2.5	0.38
ρ_{a,a^*} or ρ_{m,m^*}		–	–	0.28	0.34	0.30	0.37	0.56
Nominal rigidities		–	✓	–	✓	–	✓	✓

Note: Panels A–D report the simulation results, with entries being median values of moments across $10,000$ simulations of 120 quarters, and brackets reporting (when relevant) the standard deviations across simulations. The bottom panel describes the model specifications. Columns 2, 4, 6 feature flexible prices and wages; columns 3, 5, 7, 8 feature both sticky wages and LCP sticky prices. Shocks: financial ψ_t in columns 2–3, 6–8; productivity (a_t, a_t^*) in columns 4, 6–7; monetary $(\varepsilon_t^m, \varepsilon_t^{m*})$ in columns 5, 8. In columns 4–8, correlation of shocks matches $\text{corr}(\Delta gdp, \Delta gdp^*) = 0.35$; in columns 6–8, in addition, the relative volatility of shocks matches $\text{corr}(\Delta q, \Delta c - \Delta c^*) = -0.4$ (see bottom panel). See data description in the text.

consistent with the near random-walk behavior of the exchange rate, the R^2 in the Fama regression is close to zero.[40]

While the financial shock model is highly successful in matching exchange rate moments, it is clearly dominated by the productivity and monetary shock models in terms of the standard international business cycle moments, as we report in panel D of Table 2. In particular, the financial shock induces counterfactual negative correlations between GDP and its domestic components (consumption and investment), as well as negative correlations between macro variables across countries. Furthermore, it does not allow to match the relative volatility of consumption and GDP. In contrast, and consistent with the previous literature, this is not an issue for either IRBC or NKOE models with productivity and monetary shocks, which reproduce the empirical positive comovement of macro aggregates within and across countries.

Multi-shock models We finally turn to the full quantitative model. In column 6 of Table 2, we report the results for the IRBC model with productivity and financial shocks and no nominal rigidities. Column 7 adds sticky wages and LCP sticky prices to the same specification, and we label it 'IRBC$^+$'. Finally, column 8 replaces productivity shocks with monetary shocks, keeping nominal rigidities as in column 7. The bottom line is that all three specifications are successful at *simultaneously* matching the exchange rate moments in panels A–C and the international business cycle moments in panel D.

Indeed, the multi-shock model inherits the ability of the financial-shock model to match the exchange rate moments and the capacity of the standard IRBC and NKOE models in matching the international business cycle moments. In particular, the multi-shock models generate volatile and persistent nominal and real exchange rates, which all comove nearly perfectly together, a negative Backus-Smith correlation and a negative Fama coefficient, while still allowing the main macro aggregates (GDP, consumption and investment) to be positively correlated with each other and across countries. Therefore, the multi-shock model faces no trade-off in matching the exchange rate and business cycle moments simultaneously, despite the failure of all single-shock models in one or the other task. Furthermore, note that the presence of the segmented financial market allows our multi-shock model to resolve two prominent international business cycle puzzles — the weak correlation of consumption and the positive correlation of investment across countries (see Kehoe and Perri 2002), without directly targeting these moments in the calibration. The only moment that we target directly — by choosing the relative variance of the productivity (monetary) and financial shocks — is the negative Backus-Smith correlation between the relative consumption growth and the real exchange rate.

To provide an intuitive explanation to this, perhaps surprising, result, Table 3 describes the variance decomposition of the contribution of the shocks to various macro variables. In particular, the table reports the contribution of the financial shock ψ_t to the unconditional variation of nominal and real exchange rates, consumption and GDP, while the remaining shares are accounted for by the other shocks in each of the specifications. Across specifications, financial shocks account for almost all of

[40]In the earlier draft, Itskhoki and Mukhin (2017), we also show how a multi-shock version of this model matches the additional moments on the intertemporal comovement of interest rates and exchange rates documented by Bacchetta and van Wincoop (2010), Engel (2016) and Valchev (2016).

Table 3: Contribution of ψ_t to macroeconomic volatility

		IRBC (6)	IRBC$^+$ (7)	NKOE (8)
Nominal exchange rate	$\text{var}(\Delta e)$	96%	98%	94%
Real exchange rate	$\text{var}(\Delta q)$	87%	97%	94%
Consumption	$\text{var}(\Delta c)$	20%	10%	12%
GDP	$\text{var}(\Delta gdp)$	1%	11%	14%

Note: Variance decompositions correspond to the quantitative models in columns (6)–(8) of Table 2. The entries are the % contributions of the financial shock ψ_t to the unconditional variances of the macro variables, with the remaining shares accounted for by the other shocks in each model specification.

the nominal exchange rate volatility and about 90% of the real exchange rate volatility. At the same time, these shocks account for only about 10-to-20% of the volatility in consumption and output. Home bias in the goods market, coupled with stable inflation and low effective substitutability of home and foreign products (recall the role of γ and $\theta(1-\alpha)$ in Propositions 2 and 4), limit the pass-through of the large exchange rate volatility into the macro variables. As a result, the macro variables are both stable in comparison with the exchange rates and are primarily driven by the productivity and monetary shocks, which exert strong direct effect on these variables, yet contribute little to the exchange rate volatility (recall Proposition 1).

Recall that in our calibration we target exchange rate moments from developed OECD countries under floating exchange rate regimes. In non-OECD countries, the volatility contribution of monetary, productivity and commodity-price shocks is arguably more pronounced (see e.g. Aguiar and Gopinath 2007), as well as pegs and partial pegs are more ubiquitous (see e.g. Reinhart and Rogoff 2004).[41] In the model, this makes the exchange rate puzzles less pronounced, as it reduces the relative role of the financial shock in shaping the dynamics of the exchange rate. This is in line with the empirical evidence that the Meese-Rogoff disconnect, PPP, the Backus-Smith and UIP conditions fair less badly in developing countries (see e.g. Rogoff 1996, Bansal and Dahlquist 2000) and under pegged regimes (for a detailed analysis of the peg, see Itskhoki and Mukhin 2019).

Returning to the main quantitative results in columns 6–8 in Table 2, what we find particularly surprising is that the quantitative success of the model is not sensitive, to a first approximation, to the presence or absence of nominal rigidities and to the natural of shocks, provided that the financial shock ψ_t is included in the mix. This emphasizes the robustness of the disconnect mechanism laid out in Section 3, as well as the reason why the earlier literature was challenged to explain the behavior of the exchange rates (e.g., the PPP and the Backus-Smith puzzles). Specifically, it is not the failures of the flexible-price or sticky-price transmission mechanisms, but rather the focus on productivity and monetary shocks as the key drivers of the exchange rates. Instead, we argue that these shocks are fundamentally inconsistent with the exchange rate *disconnect* behavior, which calls for the financial

[41] On the role of commodity prices in shaping the exchange rates see e.g. Chen and Rogoff (2003), Casas, Díez, Gopinath, and Gourinchas (2016) and Ayres, Hevia, and Nicolini (2019).

shocks as the key ingredient in a model of exchange rates.[42] Having said this, we do note that the sticky price versions of the model — IRBC$^+$ and NKOE — do improve the fit of the model on the margin, by further dampening the pass-through of exchange rate volatility into prices and quantities.

Terms of trade and net exports In conclusion, we briefly comment on the international transmission mechanism across various specifications of the model. While all versions of the model are comparable in their fit of the exchange rate and business cycle moments in Table 2, their ability to match the behavior of the terms of trade and net exports varies across specifications, without a clear winner. In particular, we compare the flexible-price IRBC model with six sticky-price specifications, which differ in terms of shocks (productivity vs monetary, in addition to the financial shock ψ_t) and the type of export price stickiness. We consider three conventional price setting regimes — in producer (PCP), consumer (LCP) and dominant (DCP) currencies respectively, as we formally describe in Appendix A.4. All sticky-price specifications additionally feature sticky wages, and in each case we recalibrate the second moments of the shocks to match the correlation moments described earlier.

Table 4: Terms of trade and net exports

Moments	Data	IRBC	IRBC$^+$			NKOE		
			PCP	LCP	DCP	PCP	LCP	DCP
$\sigma(\Delta q)/\sigma(\Delta e)$	0.99	0.83	0.85	0.98	0.91	0.83	0.97	0.90
$\sigma(\Delta s)/\sigma(\Delta e)$	0.25	0.22	0.86	0.81	0.08	0.85	0.81	0.06
$\text{corr}(\Delta s, \Delta e)$	$\approx$0.20	0.98	0.98	-0.94	0.59	0.97	-0.94	0.63
$\sigma(\Delta nx)/\sigma(\Delta q)$	0.10	0.26	0.27	0.17	0.22	0.25	0.16	0.20
$\text{corr}(\Delta nx, \Delta q)$	≈ 0	0.97	0.99	0.99	0.99	0.97	0.97	0.97

Note: Additional moments and model specifications; IRBC and the two LCP columns correspond to columns 6–8 in Table 2, respectively; PCP, LCP and DCP stand for producer, local and dominant currency price stickiness, respectively; IRBC$^+$ features productivity shocks and NKOE features monetary (Taylor rule) shocks, in addition to financial shock ψ_t in both cases.

We report the results in Table 4, which identifies the two PCP versions of the model as clear losers, as they lag in matching all three types of moments — the real exchange rate volatility, the behavior of the terms of trade and of the net exports. The flexible price IRBC model comes in second-to-last, with a good fit of the terms of trade volatility due to the pricing-to-market mechanism. The LCP and DCP specifications compete for the first place, with LCP being more successful in matching the volatility of the real exchange rate and net exports, while DCP having a clear lead in its ability to match the behavior of the terms of trade (as emphasized recently by Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Møller 2018).[43] The DCP mechanism captures both the stability of the terms of trade, as well as their imperfect correlation with the real exchange rate — properties that both PCP and LCP lack.

[42] In Itskhoki and Mukhin (2017), we provide a formal argument why the conventional shocks — such as productivity, monetary, markup and government spending shocks — cannot generate an exchange rate disconnect behavior, and why a financial shock ψ_t to international risk sharing is necessary; additionally, we discuss there alternative interpretations of this shock, as it can both arise fully in the financial market or be induced by the fundamental macro shocks (see footnote 16).

[43] Note that all specifications imply a counterfactually high correlation between the real exchange rate and net exports, which is nearly zero in the data, suggesting the need either for additional home-bias or trade-cost shocks, or for slow adjustment in the trade quantities (the J-*curve*; also see e.g. Fitzgerald, Yedid-Levi, and Haller 2019).

This, however, leads the DCP specification to yield volatile relative prices of imported to domestically-produced goods, and hence an insufficiently volatile real exchange rate and excessively volatile net exports. Both of this issues are taken care of under LCP, which produces stable relative prices of imported and domestically-produced goods. In order to match all these moments simultaneously, it is likely necessary to generalize the model with either mixed-currency pricing at the border (see Amiti, Itskhoki, and Konings 2018) or combine DCP export price stickiness with a model of local distribution margin and LCP retail price stickiness for imported goods (see Auer, Burstein, and Lein 2018).

5 Conclusion

We propose a parsimonious general equilibrium model of exchange rate determination, which offers a unifying resolution to the main exchange rate puzzles in international macroeconomics. In particular, we show that introducing a financial shock into an otherwise standard IRBC or NKOE model allows it to match a rich set of moments describing the comovement between exchange rates and macro variables, without compromising the model's ability to explain the main international business cycle properties. We exploit the analytical tractability of the model to dissect the underlying exchange rate disconnect mechanism, which we show is robust and relies only on home bias in consumption, a conventional Taylor monetary policy rule and muted exchange rate pass-through due to pricing to market and weak substitutability between home and foreign goods. While improving somewhat the quantitative fit, nominal rigidities are not essential for the qualitative ability of the model to explain exchange rates.

With this general equilibrium model, one can reconsider the conclusions in the broad international macro literature plagued by exchange rate puzzles. In particular, our analysis shows that these puzzles do not necessarily invalidate the standard international transmission mechanism for monetary and productivity shocks, including international spillovers from monetary policy (see e.g. Corsetti, Dedola, and Leduc 2010, Egorov and Mukhin 2019). We emphasize instead that these conventional shocks *cannot* be the main drivers of the *unconditional* behavior of exchange rates. In contrast, our findings likely challenge the conventional normative analysis in open economies, and in particular the studies of the optimal exchange rate regime and capital controls. Pegging the exchange rate may simultaneously reduce monetary policy flexibility (Friedman 1953), yet improve international risk-sharing by offsetting the noise-trader risk (Jeanne and Rose 2002, Devereux and Engel 2003). Furthermore, a microfoundation for financial shocks is essential, as they may endogenously interact with the monetary policy (see e.g. Alvarez, Atkeson, and Kehoe 2007, Itskhoki and Mukhin 2019).

In addition, our framework can be used as a theoretical foundation for the vast empirical literature, which relies on exchange rate variation for identification (see e.g. Burstein and Gopinath 2012). Similarly, it can serve as a point of departure for the equilibrium analysis of the *international price system* (Gopinath 2016, Mukhin 2017) and the *global financial cycle* (Rey 2013). The model also offers a simple general equilibrium framework for nesting the financial sector in an open economy environment. This may prove particularly useful for future explorations into the nature of financial shocks, disciplined with additional comovement properties between exchange rates and financial variables (e.g., see recent work by Jiang, Krishnamurthy, and Lustig 2018, Engel and Wu 2019).

A Appendix

A.1 Additional Figures and Tables referenced in the text

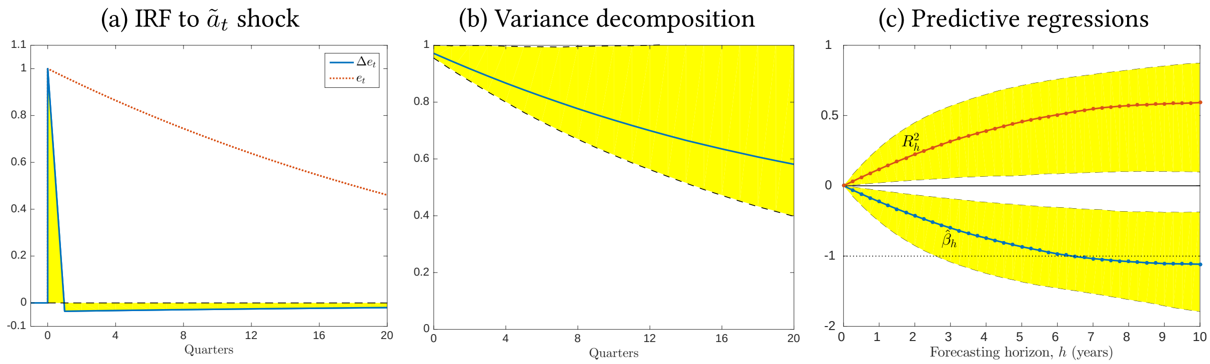

Figure A1: TFP shock

Note: Same as Figure 1 (panels a, b and d), but for productivity shock $\tilde{a}_t$ instead of financial shock ψ_t.

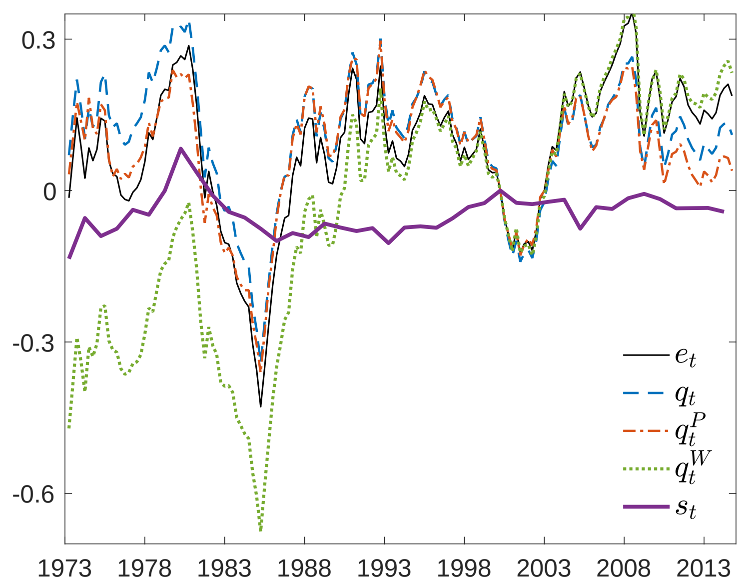

Figure A2: NER, RERs and ToT

Note: The figure plots quarterly NER e_t, CPI-based RER q_t, PPI-based RER q_t^P, wage-based RER q_t^W for the U.S. against the PPP-weighted sum of France, Germany, and the U.K., as well as the annual terms of trade s_t for the U.S. against the rest of the world. All series are in logs and normalized to zero in 2000:Q1. See Table 1 and Data Appendix A.5. Note how the low-frequency movement in q_t^W differ from those of q_t pre-1990, reflecting fast relative real wage growth in the US in this period, consistent with the role of relative productivity $\tilde{a}_t$ in (35).

Table A1: Model parameters

Parameter		Value	Comment
Conventional parameters			
Discount factor, quarterly	β	0.99	
Relative risk aversion	σ	2	
Macro Frisch elasticity	ν	1	
Intermediate share	ϕ	0.5	
Capital share in value added	ϑ	0.3	
Depreciation rate, quarterly	δ	0.02	
Transmission mechanism			
Trade openness	γ	0.07	Trade-to-GDP ratio $= 0.30$
Elasticity of substitution	θ	1.5	Feenstra et al. (2018)
Strategic complementarity	α	0.4	Amiti et al. (2019)
Monetary parameters			
Taylor rule coefficient	ϕ_π	2.15	CGG
Interest rate smoothing	ρ_m	0.95	$\rho(i_t), \rho(i_t - i_t^*) \approx 0.95$
Calvo probability for prices	λ_p	0.75	average duration of 4 quarters
Calvo probability for wages	λ_w	0.85	average duration of 6 quarters
Calibrated parameters			
Persistence of shocks	ρ	0.97	$\rho(gdp_t), \rho(i_t), \rho(\psi_t)$
Coefficient on NFA in UIP (28)	χ_2	0.001	$\rho(\Delta b_t) \approx 0.95$
Std. of the UIP shock	$\chi_1\sigma_\psi$	1	normalization
Std. productivity (monetary) shocks	σ_a (σ_m)	$\longrightarrow$	$\text{corr}(\Delta c_t - \Delta c_t^*, \Delta q_t) = -0.4$
Corr. productivity (monetary) shocks	ϱ_{a,a^*} (ϱ_{m,m^*})	$\longrightarrow$	$\text{corr}(\Delta gdp_t, \Delta gdp_t^*) = 0.35$
Capital adjustment cost parameter	κ	$\longrightarrow$	$\text{std}(\Delta z_t)/\text{std}(\Delta gdp_t) = 2.5$

Note: See the discussion of calibration in Sections 3 and 4. The values of κ, σ_a, σ_m, ϱ_{a,a^*} and ϱ_{m,m^*} vary across specifications to keep the targeted moments unchanged. Note that $\text{corr}(\Delta gdp_t, \Delta gdp_t^*)$ can only be matched in the specifications which feature either productivity or monetary shocks, and $\text{corr}(\Delta c_t - \Delta c_t^*, \Delta q_t)$ can only be matched in the multi-shock specifications. The empirical value of the persistence of the UIP shock ψ_t, $\rho = 0.97$, is disciplined by the high persistence of the interest rate (differential), as well as the recent direct estimates of the exogenous demand shocks to currencies in Koijen and Yogo (2019). The model requires $\chi_2 \gtrsim 0$ in (28) in order to be consistent with the high empirical persistence of the current account (Δb_t), which declines in χ_2.

Table A2: Empirical Moments

Moments	CKM	IM	Moments	CKM	IM
A. Exchange rate disconnect:			D. International business cycle moments:		
$\rho(\Delta e)$	0.3	0.3	$\sigma(\Delta c)/\sigma(\Delta gdp)$	0.82	0.81
$\sigma(\Delta e)/\sigma(\Delta gdp)$	5.2	6.5	$\text{corr}(\Delta c, \Delta gdp)$	0.64	0.63
$\sigma(\Delta e)/\sigma(\Delta c)$	6.3	8.0	$\text{corr}(\Delta z, \Delta gdp)$	0.81	0.75
B. Real exchange rate and the PPP:			$\text{corr}(\Delta gdp, \Delta gdp^*)$	0.35	0.42
$\rho(q)$	0.96	0.94	$\text{corr}(\Delta c, \Delta c^*)$	0.30	0.40
$\sigma(\Delta q)/\sigma(\Delta e)$	0.99	0.97	$\text{corr}(\Delta z, \Delta z^*)$	0.27	0.32
$\text{corr}(\Delta q, \Delta e)$	0.99	0.99	E. Terms of trade and net exports moments:		
C. Backus-Smith correlation:			$\sigma(\Delta nx)/\sigma(\Delta q)$	0.01	0.09
$\text{corr}(\Delta q, \Delta c - \Delta c^*)$	−0.20	−0.17	$\text{corr}(\Delta nx, \Delta q)$	−0.01	0.35

Note: CKM and IM correspond respectively to estimates obtained for the periods 1973–1994 (as in CKM) and our estimates for 1981–2017. See Data Appendix A.5.

A.2 Derivations for Section 2

Household optimization Substituting the capital accumulation equation from footnote 8 into the budget constraint (2), the Lagrangian for household utility maximization (1) is:

$$\max_{\{C_t,L_t,B_{t+1},K_{t+1}\}} \sum_{t,s^t} \beta^t \pi(s^t) \Bigg\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+1/\nu}}{1+1/\nu} + \lambda_t \left[W_t L_t - P_t C_t + B_t - \frac{B_{t+1}}{R_t} + P_t K_t \left(\frac{R_t^K}{P_t} - \delta - \frac{\Delta K_{t+1}}{K_t} - \frac{\kappa}{2}\left(\frac{\Delta K_{t+1}}{K_t}\right)^2 \right) \right] \Bigg\},$$

where $\pi(s^t)$ is probability of state $s^t = (s_0, s_1, \ldots, s_t)$ at time t and $\beta^t \pi(s^t)\lambda_t$ is the Lagrange multiplier on the flow budget constraint in state s^t at time t (note that we suppress the dependence of variables on s^t for brevity). The optimality conditions are:

$$\begin{aligned}
C_t^{-\sigma} &= \lambda_t P_t, \\
L_t^{1/\nu} &= \lambda_t W_t, \\
\lambda_t &= \beta R_t \mathbb{E}_t \lambda_{t+1}, \\
1 + \kappa \frac{\Delta K_{t+1}}{K_t} &= \beta \mathbb{E}_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left[\frac{R_{t+1}^K}{P_{t+1}} + (1-\delta) + \kappa \frac{\Delta K_{t+2}}{K_{t+1}} + \frac{\kappa}{2}\left(\frac{\Delta K_{t+2}}{K_{t+1}}\right)^2 \right].
\end{aligned}$$

Solving out $\lambda_t = C_t^{-\sigma}/P_t$, we arrive at the optimality conditions (3)–(4), as well as the Euler equation for capital:

$$\eta_t = \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left[\frac{R_{t+1}^K}{P_{t+1}} - \delta + \eta_{t+1} + \frac{(\eta_{t+1}-1)^2}{2\kappa} \right] \right\}, \tag{A1}$$

where $\eta_t \equiv 1 + \kappa \frac{\Delta K_{t+1}}{K_t}$ is the (q-theory) market price of one unit of capital in units of the home consumption good, with the last term in (A1) arising from the quadratic adjustment costs.

Expenditure minimization. Price level. Proof of Lemma 1. The household expenditure minimization problem for a given consumption level C_t is given by

$$\min_{\{C_{Ht}(i),C_{Ft}(i)\}} P_t C_t = \int_0^1 \Big[P_{Ht}(i) C_{Ht}(i) + P_{Ft}(i) C_{Ft}(i) \Big] \mathrm{d}i$$

subject to (6). The optimality conditions are:

$$P_{Jt}(i) = \mathcal{R}_t g'\left(\frac{C_{Jt}(i)}{\gamma_J C_t}\right) \frac{1}{C_t}, \quad J \in \{H, F\},$$

where $\gamma_H \equiv 1-\gamma$ and $\gamma_F \equiv \gamma$, and $\mathcal{R}_t$ is the Lagrange multiplier on the consumption aggregator constraint. Denoting $\mathcal{P}_t = \mathcal{R}_t / C_t$ and $h(\cdot) \equiv g'^{-1}(\cdot)$, we obtain the demand schedules (7):

$$C_{Jt}(i) = \gamma_J\, h\left(\frac{P_{Jt}(i)}{\mathcal{P}_t}\right) C_t, \tag{A2}$$

where $\mathcal{P}_t$ and P_t are defined implicitly by the following system:

$$1 = \int_0^1 \left[(1-\gamma) g\Big(h\Big(\frac{P_{Ht}(i)}{\mathcal{P}_t}\Big)\Big) + \gamma g\Big(h\Big(\frac{P_{Ft}(i)}{\mathcal{P}_t}\Big)\Big) \right] \mathrm{d}i, \tag{A3}$$

$$P_t = \int_0^1 \left[(1-\gamma) P_{Ht}(i) h\left(\frac{P_{Ht}(i)}{\mathcal{P}_t}\right) + \gamma P_{Ft}(i) h\left(\frac{P_{Ft}(i)}{\mathcal{P}_t}\right) \right] \mathrm{d}i. \tag{A4}$$

Equation (A3) arises from the definition of the Kimball consumption aggregator (6), and uniquely determines $\mathcal{P}_t$, as $h'(\cdot) < 0$. Given $\mathcal{P}_t$, equation (A4) determines the value of P_t, which ensures that the sum of all market shares is one, given that $P_t C_t$ is total expenditure. Indeed,

$$S_{Jt}(i) = \frac{C_{Jt}(i)P_{Jt}(i)}{P_t C_t} = \gamma_J \frac{P_{Jt}(i)}{P_t} h\left(\frac{P_{Jt}(i)}{\mathcal{P}_t}\right)$$

is the market share of domestic (for $J = H$) or foreign (for $J = F$) variety i in the home market.

When all $P_{Ht}(i) = P_{Ft}(j) = \mathcal{P}_t$ for all i, j and for some $\mathcal{P}_t$, then $P_t = \mathcal{P}_t$, given our normalization that $g(1) = g'(1) = 1$, which implies $h(1) = 1$. More generally, P_t and $\mathcal{P}_t$ offer two alternative generalized averages of prices of the varieties in the domestic market, and the difference between P_t and $\mathcal{P}_t$ is second order in the dispersion of prices. Indeed, taking the first order approximation to (A3)–(A4) around a symmetric equilibrium described above, we have:

$$p_t \equiv \mathrm{d}\log P_t = \int_0^1 \left[(1-\gamma)p_{Ht}(i) + \gamma p_{Ft}(i)\right] \mathrm{d}i \quad \text{and} \quad \mathrm{d}\log \mathcal{P}_t = \int_0^1 \left[(1-\gamma)p_{Ht}(i) + \gamma p_{Ft}(i)\right] \mathrm{d}i = p_t,$$

where $\mathrm{d}\log P_t$ and $\mathrm{d}\log \mathcal{P}_t$ denote the log-deviations from some symmetric steady state. In the derivations, we used the facts that $g'(1) = h(1) = 1$ and $h'(1) = -\theta$. This confirms that P_t and $\mathcal{P}_t$ differ at most by a second-order term in the dispersion of the vector $\big(\{p_{Ht}(i) - p_t\}_i, \{p_{Ft}(j) - p_t\}_j\big)$, which is an identical zero in a symmetric steady state. Lastly, note that the expenditure share on foreign goods in a symmetric equilibrium is given by $\int_0^1 S_{Ft}(i)\mathrm{d}i = \gamma$, which completes the proof of Lemma 1. ■

Price setting. Proof of Lemma 2. Monopolistically competitive firms set prices flexibly to maximize profits (15) subject to the demand schedule (7). The price setting problem of a representative home firm i partitions into price setting in the home and foreign markets separately:

$$P_{Ht}(i) = \arg\max_{P_{Ht}(i)} \left\{(P_{Ht}(i) - MC_t)(1-\gamma)h\left(\frac{P_{Ht}(i)}{\mathcal{P}_t}\right)C_t\right\} = \mu\left(\frac{P_{Ht}(i)}{\mathcal{P}_t}\right) \cdot MC_t, \tag{A5}$$

$$P^*_{Ht}(i) = \arg\max_{P^*_{Ht}(i)} \left\{(P^*_{Ft}(i)\mathcal{E}_t - MC_t)\gamma h\left(\frac{P^*_{Ht}(i)}{\mathcal{P}^*_t}\right)C^*_t\right\} = \mu\left(\frac{P^*_{Ht}(i)}{\mathcal{P}^*_t}\right) \cdot \frac{MC_t}{\mathcal{E}_t}, \tag{A6}$$

where $\mu(x) \equiv \frac{\tilde{\theta}(x)}{\tilde{\theta}(x)-1}$ is the optimal markup function and $\tilde{\theta}(x) = -\frac{\partial \log h(x)}{\partial \log x}$ is the elasticity of the demand schedule, and $\mathcal{P}^*_t$ is the auxiliary average price in the foreign market. Note that all home firms charge the same price in each of the markets, P_{Ht} and P^*_{Ht} repsectively, yet the prices may differ across markets, $P_{Ht} \neq P^*_{Ht}\mathcal{E}_t$, violating the law of one price. This happens *iff* $\mathcal{P}_t \neq \mathcal{P}^*_t$.

Note that P_{Ht} and P^*_{Ht} also correspond to the price indexes of the home good aggregator in the home and foreign markets respectively, defined in parallel with the overall price index P_t in (A4). Given the same prices, we also have the same quantities across domestic firms, in particular $P_{Ht}C_{Ht} = P_{Ht}(i)C_{Ht}(i)$ for all i, where C_{Ht} is the aggregate consumption index of all home goods in the home market defined analogously to the aggregate consumption C_t in (6). The same property applies in the foreign market, and for foreign goods in both markets. Finally, we have the aggregate expenditure in the home and foreign markets given by $P_tC_t = P_{Ht}C_{Ht} + P_{Ft}C_{Ft}$ and $P^*_tC^*_t = P^*_{Ht}C^*_{Ht} + P^*_{Ft}C^*_{Ft}$ respectively.

Next, consider the full log-differential of the optimal price setting equations around a symmetric equilibrium in both markets:

$$p_{Ht} = -\Gamma(p_{Ht} - p_t) + mc_t,$$
$$p^*_{Ht} = -\Gamma(p^*_{Ht} - p^*_t) + (mc_t - e_t),$$

where small letters denote log-deviations from the symmetric equilibrium and we used the fact that $\mathrm{d}\log \mathcal{P}_t = p_t$ (and same in foreign) and that

$$\Gamma \equiv -\frac{\partial \log \mu(x)}{\partial \log x}\Big|_{x=1} = \frac{\epsilon}{\theta - 1},$$

where we used the properties $\mu(x) = \frac{\tilde{\theta}(x)}{\tilde{\theta}(x)-1}$ and $\theta = \tilde{\theta}(1)$, and we defined the super elasticity of demand $\epsilon = \frac{\partial \log \tilde{\theta}(x)}{\partial \log x}\big|_{x=1}$. From the definition of $\tilde{\theta}(x)$, it follows that

$$\epsilon = \left[1 - \frac{h'(x)x}{h(x)} + \frac{h''(x)x}{h'(x)}\right]\Big|_{x=1} = 1 + \theta + \frac{h''(x)x}{h'(x)}\Big|_{x=1} = 1 + \theta - h''(1)/\theta,$$

and therefore, given the slope of demand θ, ϵ characterizes the curvature (i.e. the second derivative, $h''(1)$) of the demand schedule. We assume that the demand schedule $h(\cdot)$ is log-concave, that is $\epsilon \geq 0$, and therefore $\Gamma \geq 0$, that is the markup decreases with the relative price of the firm, and hence increases with its market share. Note that the $\theta > 1$ requirement corresponds to the second-order condition for the optimal price.

Solving the equations above for p_{Ht} and p^*_{Ht}, we arrive at (16)–(17). Note that the coefficient $\alpha = \frac{\Gamma}{1+\Gamma} = \frac{\epsilon}{\epsilon+\theta-1} \in [0, 1)$, as $\theta > 1$ and $\epsilon \geq 0$. In particular, $\alpha = 0$ *iff* $\epsilon = 0$, which also implies $\tilde{\theta}(x) \equiv \theta = const$, that is a constant elasticity (CES) demand. This completes the proof of Lemma 2. ■

Example 1: CES Consider the case of CES demand, which obtains when $g(z) = 1 + \frac{\theta}{\theta-1}\left(z^{\frac{\theta-1}{\theta}} - 1\right)$, which is normalized to satisfy $g(1) = g'(1) = 1$. In this case, $g'(z) = z^{-1/\theta}$ and $h(x) = x^{-\theta}$, so that $g(h(x)) = -\frac{1}{\theta-1} + \frac{\theta}{\theta-1}x^{1-\theta}$. As a result, equations (A3)–(A4) can be solved to yield:

$$P_t = \mathcal{P}_t = \left[\int_0^1 \Big[(1-\gamma)P_{Ht}(i)^{1-\theta} + \gamma P_{Ft}(i)^{1-\theta}\Big]\mathrm{d}i\right]^{1/(1-\theta)}.$$

Note that in the CES model $\tilde{\theta}(x) = -\frac{\partial \log h(x)}{\partial \log x} \equiv \theta = const$, implying $\epsilon = 0$, and therefore $\mu(x) \equiv \frac{\theta}{\theta-1} = const$ and $\Gamma = \alpha = 0$.

Example 2: Klenow and Willis (2016) Consider the demand structure implicitly defined by the demand schedule $h(x) = [1 - \epsilon \log(x)]^{\theta/\epsilon}$ for some elasticity parameter $\theta > 1$ and super-elasticity parameter $\epsilon > 0$. This demand structure has been originally developed by Klenow and Willis (2016) and was later used in Gopinath and Itskhoki (2010) in the context of exchange rate transmission. Note that it is indeed the case that $h(1) = 1$, $h'(1) = -\theta$, $\tilde{\theta}(x) = -\frac{\partial \log h(x)}{\partial \log x} = \frac{\theta}{1-\epsilon \log x}$ and $\frac{\partial \log \tilde{\theta}(x)}{\partial \log x}\big|_{x=1} = \frac{\epsilon}{1-\epsilon \log x}\big|_{x=1} = \epsilon$. Therefore, parameter θ controls the local slope of the demand schedule, while parameter ϵ controls its local curvature, and the two can be chosen independently. As $\varepsilon \to 0$, the demand schedule converges to CES demand, $h(x) \to x^{-\theta}$. The preference aggregator $g(\cdot)$ corresponding to the Klenow-Willis demand schedule $h(\cdot)$ is well-defined, but is not an analytical function. Therefore, there is no analytical characterization of $\mathcal{P}_t$ and P_t in the general case, yet the general result that P_t and $\mathcal{P}_t$ are both first-order equivalent to the sales-weighted average industry price (Lemma 1) still holds.[44] With this demand structure, we have $\mu(x) = \frac{\tilde{\theta}(x)}{\tilde{\theta}(x)-1} = \frac{\theta}{\theta-1+\epsilon \log x}$, which results in $\Gamma = -\frac{\partial \log \mu(x)}{\partial \log x}\big|_{x=1} = \frac{\epsilon}{\theta-1} > 0$. Therefore, indeed, Γ and α are shaped by the primitive parameters of demand ϵ and θ, and any value of $\alpha = \frac{\epsilon}{\epsilon+\theta-1} > 0$ can be obtained independently of the value of θ by setting $\epsilon = \frac{\alpha}{1-\alpha}(\theta - 1) > 0$. In the CES limit, as $\epsilon \to 0$, we have $\alpha \to 0$.

Country budget constraint. Walras law. Aggregating firm profits (15) across all domestic firms:

$$\begin{aligned}\Pi_t &= (P_{Ht} - MC_t)\,Y_{Ht} + (P^*_{Ht}\mathcal{E}_t - MC_t)\,Y^*_{Ht} \\ &= P_{Ht}Y_{Ht} + \mathcal{E}_t P^*_{Ht}Y^*_{Ht} - MC_tY_t \\ &= P_{Ht}Y_{Ht} + \mathcal{E}_t P^*_{Ht}Y^*_{Ht} - W_tL_t - R^K_tK_t - P_tX_t,\end{aligned}$$

where we used in the second line the fact that total output $Y_t = Y_{Ht} + Y^*_{Ht}$ and in the third line the expressions for the aggregate production inputs demand, namely (14) for labor and analogous conditions for capital and

[44] A special case with a tractable analytical solution obtains when $\theta = \epsilon$. In this case, $\mathcal{P}_t$ is a simple weighted average of prices $\bar{P}_t$, while P_t equals to $\bar{P}_t$ adjusted downwards by ϵ times the measure of price dispersion (namely, the Theil index), as consumers benefit from a greater price dispersion holding the average price constant.

intermediates:

$$R_t^K K_t = (1-\phi)\vartheta MC_t Y_t \qquad \text{and} \qquad P_t X_t = \phi MC_t Y_t. \tag{A7}$$

We next substitute Π_t into the household budget constraint (2), resulting after rearranging in:

$$\frac{B_{t+1}}{R_t} - B_t = P_{Ht}Y_{Ht} + \mathcal{E}_t P_{Ht}^* Y_{Ht}^* - P_t(C_t + X_t + Z_t).$$

Finally, we use the fact that total domestic expenditure can be split into the expenditure on the home and the foreign goods, $P_t(C_t + X_t + Z_t) = P_{Ht}Y_{Ht} + P_{Ft}Y_{Ft}$, as ensured by expenditure minimization (5) and market clearing (18) and the foreign counterpart to (19), which implies $Y_{Ft} = C_{Ft} + X_{Ft} + Z_{Ft}$. As a result, we can rewrite:

$$\frac{B_{t+1}}{R_t} - B_t = \mathcal{E}_t P_{Ht}^* Y_{Ht}^* - P_{Ft}Y_{Ft} \equiv NX_t, \tag{A8}$$

which yields (20) in the text.

A parallel expression to (A8) for foreign yields:

$$\frac{B_{t+1}^*}{R_t^*} - B_t^* = NX_t^* + \tilde{R}_t^* \frac{D_t^* + N_t^*}{R_{t-1}^*},$$

where $NX_t^* = -\frac{NX_t}{\mathcal{E}_t} = \frac{P_{Ft}}{\mathcal{E}_t}Y_{Ft} - P_{Ht}^* Y_{Ht}^*$ and the last term is the period t realized income (or loss) from the aggregate carry trade position of the financial sector, i.e. the intermediaries and the noise traders combined, transferred lump-sum to the foreign households (see footnote 15). Indeed, note that $\frac{D_t^*+N_t^*}{R_{t-1}^*}$ is the dollar exposure of the financial sector from $t-1$ to t and $\tilde{R}_t^* = R_{t-1}^* - R_{t-1}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}$ is the realized return at t per one dollar invested in a carry trade at $t-1$.

Using the market clearing condition in the financial sector (27), we have $D_t^* + N_t^* = -B_t^*$, and therefore we can rewrite the foreign country budget constraint as

$$NX_t^* = \frac{B_{t+1}^*}{R_t^*} - B_t^* + \frac{B_t^*}{R_{t-1}^*}\tilde{R}_t^* = \frac{B_{t+1}^*}{R_t^*} - B_t^* \frac{R_{t-1}}{R_{t-1}^*}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t},$$

where the second equality substitutes in the definitions of the carry trade return $\tilde{R}_t^*$ from (25).

Lastly, since the financial sector holds a zero-capital position, this implies a zero-capital position for the home and foreign households combined, that is $\frac{B_{t+1}}{R_t} + \mathcal{E}_t\frac{B_{t+1}^*}{R_t^*} = 0$ at all t. Applying this market clearing condition at $t-1$ and t to the foreign budget constraint yields after rearranging:

$$\frac{B_{t+1}}{R_t} - B_t = -\mathcal{E}_t NX_t^* = NX_t,$$

which is exactly equivalent to the home country budget constraint (20). Note that this represents a version of *Walras Law* in our economy with the financial sector, making the foreign budget constraint a redundant equation in the equilibrium system.

A.2.1 Financial market

Proof of Lemma 3 The proof of the lemma follows two steps. First, it characterizes the solution to the portfolio problem (26) of the intermediaries. Second, it combines this solution with the financial market clearing (27) to derive the equilibrium condition (28).

(a) Portfolio choice: *The solution to the portfolio choice problem* (26) *when the time periods are short is given by:*

$$\frac{d_{t+1}^*}{P_t^*} = -\frac{i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega\sigma_e^2}, \tag{A9}$$

where $i_t - i_t^* \equiv \log(R_t/R_t^*)$, $\sigma_e^2 \equiv \mathrm{var}_t(\Delta e_{t+1})$ *and* $\sigma_{e\pi^*} = \mathrm{cov}_t(\Delta e_{t+1}, \Delta p_{t+1}^*)$.

Proof: The proof follows Campbell and Viceira (2002, Chapter 3 and Appendix 2.1.1). Consider the objective in the intermediary problem (26) and rewrite it as:

$$\max_{d^*_{t+1}} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \big(1 - e^{x^*_{t+1}}\big) e^{-\pi^*_{t+1}} \frac{d^*_{t+1}}{P^*_t} \right) \right\}, \tag{A10}$$

where we used the definition of $\tilde{R}^*_{t+1}$ in (25) and the following algebraic manipulation:

$$\frac{\tilde{R}^*_{t+1}}{P^*_{t+1}} \frac{d^*_{t+1}}{R^*_t} = \frac{\tilde{R}^*_{t+1}/R^*_t}{P^*_{t+1}/P^*_t} \frac{d^*_{t+1}}{P^*_t} = \frac{1 - \frac{R_{t+1}}{R^*_t} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}}{e^{\pi^*_{t+1}}} \frac{d^*_{t+1}}{P^*_t} = \left(1 - e^{x^*_{t+1}}\right) e^{-\pi^*_{t+1}} \frac{d^*_{t+1}}{P^*_t}$$

and defined the log Carry trade return and foreign inflation rate as

$$x^*_{t+1} \equiv i_t - i^*_t - \Delta e_{t+1} = \log(R_t/R^*_t) - \Delta \log \mathcal{E}_{t+1} \qquad \text{and} \qquad \pi^*_{t+1} \equiv \Delta \log P^*_{t+1}.$$

When time periods are short, (x^*_{t+1}, π^*_{t+1}) correspond to the increments of a vector normal diffusion process $(\mathrm{d}\mathcal{X}^*_t, \mathrm{d}\mathcal{P}^*_t)$ with time-varying drift $\boldsymbol{\mu}_t$ and time-invariant conditional variance matrix $\boldsymbol{\sigma}$:

$$\begin{pmatrix} \mathrm{d}\mathcal{X}^*_t \\ \mathrm{d}\mathcal{P}^*_t \end{pmatrix} = \boldsymbol{\mu}_t \mathrm{d}t + \boldsymbol{\sigma} \mathrm{d}B_t, \tag{A11}$$

where B_t is a standard two-dimensional Brownian motion. Indeed, as we show below, in equilibrium x^*_{t+1} and π^*_{t+1} follow stationary linear stochastic processes (ARMAs) with correlated innovations, and therefore

$$(x^*_{t+1}, \pi^*_{t+1}) \,\big|\, \mathcal{I}_t \sim \mathcal{N}(\boldsymbol{\mu}_t, \boldsymbol{\sigma}^2),$$

where $\mathcal{I}_t$ is the information set at time t, and the drift and variance matrix are given by:

$$\boldsymbol{\mu}_t = \mathbb{E}_t \begin{pmatrix} x^*_{t+1} \\ \pi^*_{t+1} \end{pmatrix} = \begin{pmatrix} i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} \\ \mathbb{E}_t \pi^*_{t+1} \end{pmatrix} \quad \text{and} \quad \boldsymbol{\sigma}^2 = \mathrm{var}_t \begin{pmatrix} x^*_{t+1} \\ \pi^*_{t+1} \end{pmatrix} = \begin{pmatrix} \sigma_e^2 & -\sigma_{e\pi^*} \\ -\sigma_{e\pi^*} & \sigma^2_{\pi^*} \end{pmatrix},$$

where $\sigma_e^2 \equiv \mathrm{var}_t(\Delta e_{t+1})$, $\sigma^2_{\pi^*} \equiv \mathrm{var}_t(\Delta p^*_{t+1})$ and $\sigma_{e\pi^*} \equiv \mathrm{cov}_t(\Delta e_{t+1}, \Delta p^*_{t+1})$ are time-invariant (annualized) conditional second moments. Following Campbell and Viceira (2002), we treat (x^*_{t+1}, π^*_{t+1}) as discrete-interval differences of the continuous process, $(\mathcal{X}^*_{t+1} - \mathcal{X}^*_t, \mathcal{P}^*_{t+1} - \mathcal{P}^*_t)$.

With short time periods, the solution to (A10) is equivalent to

$$\max_{d^*} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \big(1 - e^{\mathrm{d}\mathcal{X}^*_t}\big) e^{-\mathrm{d}\mathcal{P}^*_t} \frac{d^*}{P^*_t} \right) \right\}, \tag{A12}$$

where $(\mathrm{d}\mathcal{X}^*_t, \mathrm{d}\mathcal{P}^*_t)$ follow (A11). Using Ito's Lemma, we rewrite the objective as:

$$\begin{aligned}
&\mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \Big(-\mathrm{d}\mathcal{X}^*_t - \tfrac{1}{2}(\mathrm{d}\mathcal{X}^*_t)^2 \Big) \Big(1 - \mathrm{d}\mathcal{P}^*_t + \tfrac{1}{2}(\mathrm{d}\mathcal{P}^*_t)^2 \Big) \frac{d^*}{P^*_t} \right) \right\} \\
&= \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \Big(-\mathrm{d}\mathcal{X}^*_t - \tfrac{1}{2}(\mathrm{d}\mathcal{X}^*_t)^2 + \mathrm{d}\mathcal{X}^*_t \mathrm{d}\mathcal{P}^*_t \Big) \frac{d^*}{P^*_t} \right) \right\} \\
&= -\frac{1}{\omega} \exp\left(\left[\omega \big(\mu_{1,t} + \tfrac{1}{2}\sigma_e^2 + \sigma_{e\pi^*} \big) \frac{d^*}{P^*_t} + \frac{\omega^2 \sigma_e^2}{2} \left(\frac{d^*}{P^*_t} \right)^2 \right] \mathrm{d}t \right),
\end{aligned}$$

where the last line uses the facts that $(\mathrm{d}\mathcal{X}^*_t)^2 = \sigma_e^2 \mathrm{d}t$ and $\mathrm{d}\mathcal{X}^*_t \mathrm{d}\mathcal{P}^*_t = -\sigma_{e\pi^*} \mathrm{d}t$, as well as the property of the expectation of an exponent of a normally distributed random variable; $\mu_{1,t}$ denotes the first component of the drift vector $\boldsymbol{\mu}_t$. Therefore, maximization in (A12) is equivalent to:

$$\max_{d^*} \left\{ -\omega \big(\mu_{1,t} + \tfrac{1}{2}\sigma_e^2 + \sigma_{e\pi^*} \big) \frac{d^*}{P^*_t} - \tfrac{1}{2}\omega^2 \sigma_e^2 \left(\frac{d^*}{P^*_t} \right)^2 \right\} \quad \text{w/solution} \quad \frac{d^*}{P^*_t} = -\frac{\mu_{1,t} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega \sigma_e^2}.$$

This is the portfolio choice equation (A9), which obtains under CARA utility in the limit of short time periods, but note is also equivalent to the exact solution under mean-variance preferences. The extra terms in the numerator correspond to Jensen's Inequality corrections to the expected real log return on the carry trade. ■

(b) Equilibrium condition: To derive the modified UIP condition (28), we combine the portfolio choice solution (A9) with the market clearing condition (27) and the noise-trader currency demand (23) to obtain:

$$B^*_{t+1} + R^*_t n\big(e^{\psi_t} - 1\big) - mP^*_t \frac{i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega\sigma_e^2} = 0. \tag{A13}$$

The market clearing conditions in (27) together with the fact that both intermediaries and noise traders take zero capital positions, that is $\frac{D_{t+1}+N_{t+1}}{R_t} = -\mathcal{E}_t \frac{D^*_{t+1}+N^*_{t+1}}{R^*_t}$, results in the equilibrium balance between home and foreign household asset positions, $\frac{B_{t+1}}{R_t} = -\mathcal{E}_t \frac{B^*_{t+1}}{R^*_t}$. Therefore, we can rewrite (A13) as:

$$\frac{i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega\sigma_e^2/m} = \frac{R^*_t}{P^*_t} n\big(e^{\psi_t} - 1\big) - \frac{R^*_t}{R_t}\frac{Y_t}{\mathcal{Q}_t}\frac{B_{t+1}}{P_t Y_t},$$

where we normalized net foreign assets by nominal output $P_t Y_t$ and used the definition of the real exchange rate $\mathcal{Q}_t$ in (10). We next log-linearize this equilibrium condition around a symmetric equilibrium with $\bar{R} = \bar{R}^* = 1/\beta$, $\bar{B} = \bar{B}^* = 0$, $\mathcal{Q} = 1$, and $\bar{P} = \bar{P}^* = 1$ and some $\bar{Y}$. As shocks become small, the (co)variances σ_e^2 and $\sigma_{e\pi^*}$ become second order and drop out from the log-linearization. We adopt the asymptotics in which as σ_e^2 shrinks ω/m increases proportionally leaving the risk premium term $\omega\sigma_e^2/m$ constant, finite and nonzero in the limit.[45] As a result, the log-linearized equilibrium condition is:

$$\frac{1}{\omega\sigma_e^2/m}\Big(i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1}\Big) = \frac{n}{\beta}\psi_t - \bar{Y} b_{t+1}, \tag{A14}$$

where $b_{t+1} = \frac{1}{\bar{P}\bar{Y}} B_{t+1} = -\frac{1}{\bar{P}\bar{Y}} B^*_{t+1}$. This corresponds to the modified UIP condition (28) in Lemma 3, which completes the proof of the lemma. ■

Income and losses in the financial market Consider the income and losses of the non-household participants in the financial market — the intermediaries and the noise traders:

$$\frac{D^*_{t+1} + N^*_{t+1}}{R^*_t}\tilde{R}^*_{t+1} = \big(md^*_{t+1} + R^*_t n(e^{\psi_t} - 1)\big)\big(1 - e^{x_{t+1}}\big),$$

where we used the definition of $\tilde{R}^*_{t+1}$ in (25) and the log Carry trade return $x_{t+1} \equiv i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} = \log(R_t/R^*_t) - \Delta \log \mathcal{E}_{t+1}$. Using the same steps as in the proof of Lemma 3, we can approximate this income as:

$$\left(-m\frac{\mathbb{E}_t x_{t+1}}{\omega\sigma_e^2} + \frac{n}{\beta}\psi_t\right)\big(-x_{t+1}\big) = m\left(\frac{\mathbb{E}_t x_{t+1}}{\omega\sigma_e^2} - \frac{n}{\beta m}\psi_t\right)x_{t+1} = -\bar{Y} b_{t+1} x_{t+1},$$

where the last equality uses (A14). Therefore, while the UIP deviations (realized x_{t+1} and expected $\mathbb{E}_t x_{t+1}$) are first order, the income and losses in the financial markets are only second order, as $B_{t+1} = \bar{P}\bar{Y} b_{t+1}$ is first order around $\bar{B} = 0$. Intuitively, the income and losses in the financial market are equal to the realized UIP deviation

[45] Note that σ_e^2/m is the quantity of risk per intermediary and ω is their aversion to risk; alternatively, ω/m can be viewed as the effective risk aversion of the whole sector of intermediaries who jointly hold all exchange rate risk. Our approach follows Hansen and Sargent (2011) and Hansen and Miao (2018), who consider the continuous-time limit in the models with ambiguity aversion. The economic rationale of this asymptotics is not that second moments are zero and effective risk aversion ω/m is infinite, but rather that risk premia terms, which are proportional to $\omega\sigma_e^2/m$, are finite and nonzero. Indeed, the first-order dynamics of the equilibrium system results in well-defined second moments of the variables, including σ_e^2, as in Devereux and Sutherland (2011) and Tille and van Wincoop (2010); an important difference of our solution concept is that it allows for a non-zero first-order component of the return differential, namely a non-zero expected Carry trade return. We characterize the equilibrium σ_e^2 in Appendix A.3.

times the gross portfolio position — while both are first order, their product is second order, and hence negligible from the point of view of the country budget constraint.

Covered interest parity Consider the extension of the portfolio choice problem (26) of the intermediaries with the additional option to invest in the CIP deviations:

$$\max_{d^*_{t+1}, d^{F*}_{t+1}} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \left[\frac{\tilde{R}^*_{t+1}}{P^*_{t+1}} \frac{d^*_{t+1}}{R^*_t} + \frac{R^{F*}_t}{P^*_{t+1}} \frac{d^{F*}_{t+1}}{R^*_t} + \frac{R^*_t}{P^*_{t+1}} \mathcal{W}^*_t \right] \right) \right\},$$

where the return on one dollar invested in the CIP deviation (long foreign-currency bond, short foreign-currency bond, plus a forward) is:

$$R^{F*}_t = R^*_t - \frac{\mathcal{E}_t}{\mathcal{F}_t} R_t,$$

since 1 dollar at t buys R^*_t units of foreign-currency bonds and $\mathcal{E}_t R_t$ units of home-currency bonds, and hence $\frac{d^{F*}_{t+1}}{R^*_t} \gtrless 0$ is the period-t dollar size of this position. Note that we also allowed for nonzero dollar wealth $\mathcal{W}^*_t$ of the intermediaries, which is by default invested into the 'riskless' foreign-currency bond. Both CIP investment and wealth investment are subject to the foreign inflation risk only, but no risk of nominal return, unlike the carry trade d^*_{t+1}. Note that the CIP investment, just like the carry trade, requires no capital at time t. Lastly, note that intermediaries may be pricing the forward without trading it, or trading it with the noise traders; as long as the households have access to the home-currency bond only, and not the forward, this does not change the macro equilibrium outcomes of the model.

The first order optimality condition of the intermediaries with respect to the CIP investment is:

$$R^{F*}_t \cdot \mathbb{E}_t \left\{ \frac{1}{P^*_{t+1} R^*_t} \exp\left(-\omega \left[\frac{\tilde{R}^*_{t+1}}{P^*_{t+1}} \frac{d^*_{t+1}}{R^*_t} + \frac{R^{F*}_t}{P^*_{t+1}} \frac{d^{F*}_{t+1}}{R^*_t} + \frac{R^*_t}{P^*_{t+1}} \mathcal{W}^*_t \right] \right) \right\} = 0.$$

However, since the expectation term is strictly positive for any $d^{F*}_{t+1} \in (-\infty, \infty)$, this condition can be satisfied only if $R^{F*}_t = 0$. If $R^{F*}_t > 0$, the intermediaries will take an unbounded position in the CIP trade, $d^{F*}_{t+1} = \infty$, and vice versa.

A.2.2 Equilibrium system

We summarize here the equilibrium system of the general flexible-price model in Section 2 by breaking it into blocks. The version of the model with sticky prices and wages is described in Appendix A.4.

1. **Labor market**: Labor supply (3) and its exact foreign counterpart. Labor demand in (14), used together with the definition of the marginal cost (13), and its exact foreign counterparts. Labor market clearing ensures that L_t (L^*_t respectively) satisfies simultaneously labor demand and labor supply at the equilibrium wage rate W_t (W^*_t respectively).
2. **Capital market**: Euler equation for capital (A1) determines supply of capital and the firm capital demand is given by the first-order condition:

   $$R^K_t K_t = (1-\phi)\vartheta MC_t Y_t, \tag{A15}$$

   where marginal cost MC_t is defined in (13). The equilibrium rental rate of capital R^K_t ensures that K_t satisfies simultaneously the demand and supply of capital. Identical equations characterize equilibrium in the foreign capital market.

   The home gross investment Z_t obtains from the capital dynamics equation described in footnote 8, which we rewrite here as:

   $$Z_t = \Big[K_{t+1} - (1-\delta)K_t\Big] + \frac{\kappa}{2} \frac{(\Delta K_{t+1})^2}{K_t}, \tag{A16}$$

   where the first term is net investment and the second term is adjustment cost. The foreign investment Z^*_t satisfies a symmetric equation.

3. **Goods prices**: Price setting is characterized by (A5) and (A6) for home firms in the two markets, and symmetric equations characterize price setting by foreign firms. The price indexes P_t and $\mathcal{P}_t$ are defined implicitly by (A4)–(A3) respectively (see Lemma 1). As a result, equilibrium prices of all varieties supplied from a given country to a given market are the same: $P_{Jt}(i) = P_{Jt}$ and $P^*_{Jt}(i) = P^*_{Jt}$ for all $i \in [0,1]$ and $J \in \{H, F\}$ (see Lemma 2).
4. **Goods market**: As a result of price setting, the quantities supplied by all firms from a given country to a given market are also the same: $Y_{Jt}(i) = Y_{Jt}$ and $Y^*_{Jt}(i) = Y^*_{Jt}$ for all $i \in [0,1]$ and $J \in \{H, F\}$. The total demand for home and foreign goods satisfies:

   $$Y_t = Y_{Ht} + Y^*_{Ht} \qquad \text{and} \qquad Y^*_t = Y_{Ft} + Y^*_{Ft}, \tag{A17}$$

   where the sources of demand for home goods are given in (18) and (19), and the counterpart sources of demand for foreign goods are given by:

   $$Y_{Ft} = C_{Ft} + X_{Ft} + Z_{Ft} = \gamma h \left(\frac{P_{Ft}}{\mathcal{P}_t} \right) \left[C_t + X_t + Z_t \right], \tag{A18}$$

   $$Y^*_{Ft} = C^*_{Ft} + X^*_{Ft} + Z^*_{Ft} = \gamma h \left(\frac{P^*_{Ft}}{\mathcal{P}^*_t} \right) \left[C^*_t + X^*_t + Z^*_t \right], \tag{A19}$$

   where X_t is the intermediate good demand by the home firms:

   $$P_t X_t = \phi M C_t Y_t, \tag{A20}$$

   with MC_t defined in (13), and a symmetric equation characterizes the intermediate good demand by the foreign firms X^*_t.

   The total supply (production) of the home goods Y_t satisfies the production function (11), with log productivity a_t that follows an exogenous shock process (12).[46] A symmetric equation and foreign productivity process a^*_t characterize foreign production Y^*_t.
5. **Asset market**: The only traded assets are home- and foreign-currency bonds, which are in zero net supply according to market clearing (27). The demand for home-currency bonds by home households B_{t+1} satisfies the Euler equation (4) given the nominal interest rate R_t. Similarly, the demand for foreign-currency bonds by foreign households B^*_{t+1} satisfies a symmetric Euler equation given foreign nominal interest rate R^*_t. The demand for bonds by noise traders and arbitrageurs are characterized by (23) and Lemma 3 respectively. The noise trader shock follows an exogenous process (24). No other assets are traded.

   Nominal interest rates are set by the monetary authorities according to the Taylor rule (22) — where $i_t \equiv \log R_t$, $\pi_t \equiv \Delta \log P_t$ and an exogenous random shock ε^m_t — and its foreign counterpart.
6. **Country budget constraint**: The home-country flow budget constraint (20) derives from the combination of the household budget constraint and firm profits. The flow budget constraint (20), together with the household Euler equation (4) and its foreign counterpart, establishes a condition on the path of consumption and nominal exchange rate, $\{C_t, C^*_t, \mathcal{E}_t\}$, with the real exchange $\mathcal{Q}_t$ and terms of trade $\mathcal{S}_t$ defined by (10) and (21) respectively. The foreign flow-budget constraint is redundant by Walras Law. See Appendix A.2.

A.2.3 Symmetric steady state

In a symmetric steady state, exogenous shocks $a_t = a^*_t = \varepsilon^m_t = \varepsilon^{m*}_t = \psi_t \equiv 0$, and state variables $\bar{B} = \bar{B}^* = \bar{NX} = 0$. This is the unique steady state in a model with $\chi_2 > 0$ in (28), which also ensures stationarity of the model around this steady state. We also for concreteness normalize $\bar{P} = \bar{P}^* = 1$. Then, from Euler equations (4) and (A1) and their foreign counterparts, we have:

$$\bar{R} = \bar{R}^* = \bar{R}^K + 1 - \delta = \bar{R}^{K*} + 1 - \delta = \frac{1}{\beta}.$$

[46] Note that the input demand equations (14), (A15) and (A20) together with the definition of the marginal cost (13) imply the production function in (11).

By symmetry, the exchange rates and terms of trade satisfy

$$\bar{\mathcal{E}} = \bar{\mathcal{Q}} = \bar{\mathcal{S}} = 1,$$

and all individual prices are equal 1 (the price level). Denote the steady state markup with $\bar{\mu} \geq 1$, so that the steady state marginal costs $\bar{MC} = \bar{MC}^* = 1/\bar{\mu}$, which allows to solve for $\bar{W} = \bar{W}^*$ given $\bar{R} = 1/\beta$ and $\bar{P} = 1$ from (13) as a function of model paramters.

Next, product and factor market clearing in a symmetric steady state requires:

$$\bar{Y} = \bar{C} + \bar{X} + \delta\bar{K},$$
$$\bar{Y} = \left(\bar{K}^{\vartheta}\bar{L}^{1-\vartheta}\right)^{1-\phi}\bar{X}^{\phi},$$

$$\bar{X} = \tfrac{\phi}{\bar{\mu}}\bar{Y},$$
$$\left(\frac{1-\beta}{\beta} + \delta\right)\bar{K} = \tfrac{(1-\phi)\vartheta}{\bar{\mu}}\bar{Y},$$
$$\bar{C}^{\sigma}\bar{L}^{1/\nu} = \tfrac{(1-\phi)(1-\vartheta)}{\bar{\mu}}\frac{\bar{Y}}{\bar{L}},$$

since $\bar{Z} = \delta\bar{K}$. These equations allow to solve for $(\bar{Y}, \bar{C}, \bar{L}, \bar{K}, \bar{X})$ and their symmetric foreign counterparts as a function of the model parameters.

Lastly, we define the following useful ratios in a symmetric steady state:

$$\zeta \equiv \frac{\text{GDP}}{\text{Output}} = \frac{P(C+Z)}{PY} = \frac{\bar{C} + \delta\bar{K}}{\bar{Y}} = \frac{\bar{Y} - \bar{X}}{\bar{Y}} = 1 - \frac{\phi}{\bar{\mu}}, \tag{A21}$$

$$\gamma \equiv \frac{\text{Import}}{\text{Expenditure}} = \frac{P_F Y_F}{P_H Y_H + P_F Y_F} = \frac{\bar{Y}_F}{\bar{Y}} = \gamma, \tag{A22}$$

$$\frac{\text{Import+Export}}{\text{GDP}} = \frac{\mathcal{E}P_H^* Y_H^* + P_F Y_F}{P(C+Z)} = \frac{2\bar{Y}_F}{\bar{Y} - \bar{X}} = \frac{2\gamma}{\zeta}. \tag{A23}$$

The steady state markup is $\bar{\mu} = \frac{\theta}{\theta-1}(1-\varsigma)$, where ς is the subsidy that offsets the markup distortion, conventional in the normative macro literature. To avoid the need to calibrate an extra parameter, we assume $\varsigma = 1/\theta$, so that $\bar{\mu} = 1$ and $\zeta = 1 - \phi$, or in words the share of intermediates in output equals the elasticity of the production function with respect to intermediate inputs. The qualitative and quantitative results below are not sensitive to the departures from $\zeta = 1 - \phi$.

A.3 Derivations for Section 3

A.3.1 Log-linearized system

We describe here the log-linearized system used for analytical characterization in Section 3, without capital (R1) and in the limiting case of monetary policy fully stabilizing the price levels (R2), but without imposing (R3) on the modified UIP condition (28), treating both of its coefficients as endogenous to show the validity of restriction (R3). We log-linearize the equilibrium system around the symmetric steady state, with both the system and the steady state described in Appendix A.2.2. We take advantage of the block-recursive structure of the equilibrium system, and characterize the solution in blocks.

Exchange rates and prices The price block contains the definitions of the price index from Lemma 1 and its foreign counterpart:

$$p_t = (1-\gamma)p_{Ht} + \gamma p_{Ft}, \tag{A24}$$
$$p_t^* = \gamma p_{Ht}^* + (1-\gamma)p_{Ft}^*, \tag{A25}$$

as well as the price setting equations (16)–(17) in Lemma 2 and their foreign counterparts.

Substituting in the log of the marginal cost (13) and its foreign counterpart yields:

$$p_{Ht} = (1-\alpha)(1-\phi)(w_t - p_t - a_t) + p_t, \tag{A26}$$
$$p^*_{Ht} = (1-\alpha)[(1-\phi)(w_t - p_t - a_t) + p_t - e_t] + \alpha p^*_t, \tag{A27}$$
$$p^*_{Ft} = (1-\alpha)(1-\phi)(w^*_t - p^*_t - a^*_t) + p^*_t, \tag{A28}$$
$$p_{Ft} = (1-\alpha)[(1-\phi)(w^*_t - p^*_t - a^*_t) + p^*_t + e_t] + \alpha p_t. \tag{A29}$$

Note that small letters denote log-deviations from steady state, and therefore the marginal-cost-related constants drop out from equations (A26)–(A29). In addition, we use the logs of the real exchange rate (RER) and the terms of trade (ToT) defined in (10) and (21):

$$q_t = p^*_t + e_t - p_t, \tag{A30}$$
$$s_t = p_{Ft} - p^*_{Ht} - e_t, \tag{A31}$$

as well as the wage-based and PPI-based real exchange rates:

$$q^W_t = w^*_t + e_t - w_t, \tag{A32}$$
$$q^P_t = p^*_{Ft} + e_t - p_{Ht}. \tag{A33}$$

We solve (A24)–(A33) for equilibrium prices and exchange rates. In particular, we have:

$$s_t = q^P_t - 2\alpha q_t,$$
$$q_t = (1-\gamma)q^P_t - \gamma s_t,$$

where αq_t equals the equilibrium LOP deviation for both home- and foreign-produced goods:

$$\alpha q_t = p^*_{Ht} + e_t - p_{Ht} = p^*_{Ft} + e_t - p_{Ft},$$

as follows from (A26)–(A29). Intuitively, ToT equals PPI-RER adjusted for LOP deviations; and CPI-RER equals PPI-RER adjusted for ToT. Using these relationships, we solve for s_t and q^P_t as a function of q_t:

$$s_t = \tfrac{1-2\alpha(1-\gamma)}{1-2\gamma} q_t, \tag{A34}$$
$$q^P_t = \left[1 + (1-\alpha)\tfrac{2\gamma}{1-2\gamma}\right] q_t. \tag{A35}$$

Finally, we combine (A24)–(A29) to derive the relationship between q^W_t and q_t:

$$q^W_t = \left[1 + \tfrac{1}{1-\phi}\tfrac{2\gamma}{1-2\gamma}\right] q_t - (a_t - a^*_t), \tag{A36}$$

and in addition we have the expressions for the equilibrium real wages:

$$w_t - p_t = a_t - \tfrac{1}{1-\phi}\tfrac{\gamma}{1-2\gamma} q_t, \tag{A37}$$
$$w^*_t - p^*_t = a^*_t + \tfrac{1}{1-\phi}\tfrac{\gamma}{1-2\gamma} q_t. \tag{A38}$$

Intuitively, the real wage reflects the country productivity level adjusted by the international purchasing power of the country, which is proportional to the strength of its RER.

Real exchange rate and quantities The labor supply (3) and labor demand (14) equations (together with the marginal cost (13)) can be written as:

$$\sigma c_t + \tfrac{1}{\nu}\ell_t = w_t - p_t,$$
$$\ell_t = -(1-\phi)a_t - \phi(w_t - p_t) + y_t.$$

Combining the two to solve out ℓ_t, and using (A37) to solve out $(w_t - p_t)$, we obtain:

$$\nu\sigma c_t + y_t = (1+\nu)a_t - \tfrac{\nu+\phi}{1-\phi}\tfrac{\gamma}{1-2\gamma}q_t.$$

Subtracting a symmetric equation for foreign yields:

$$\nu\sigma\tilde{c}_t + \tilde{y}_t = (1+\nu)\tilde{a}_t - \tfrac{\nu+\phi}{1-\phi}\tfrac{2\gamma}{1-2\gamma}q_t, \tag{A39}$$

where $\tilde{x}_t \equiv x_t - x_t^*$ for any pair of variables (x_t, x_t^*). This characterizes the supply side.

The demand side is the goods market clearing (A17) together with (18)–(19), which log-linearize as:

$$\begin{aligned}
y_t &= (1-\gamma)y_{Ht} + \gamma y_{Ht}^*, \\
y_{Ht} &= -\theta(p_{Ht} - p_t) + (1-\phi)c_t + \phi\big[(1-\phi)(w_t - p_t - a_t) + y_t\big], \\
y_{Ht}^* &= -\theta(p_{Ht}^* - p_t^*) + (1-\phi)c_t^* + \phi\big[(1-\phi)(w_t^* - p_t^* - a_t^*) + y_t^*\big],
\end{aligned}$$

where $\phi = 1-\zeta \equiv \bar{X}/\bar{Y}$, and we used expression (A20) and (13) to substitute for X_t (and correspondingly for X_t^*). Combining together, we derive:

$$y_t = \phi[y_t - \gamma\tilde{y}_t] + (1-\phi)[c_t - \gamma\tilde{c}_t] + \gamma\left[\theta(1-\alpha)\tfrac{2(1-\gamma)}{1-2\gamma} - \phi\right]q_t$$

where we have solved out $(p_{Ht} - p_t)$ and $(p_{Ht}^* - p_t^*)$ using (A26)–(A29) and $(w_t - p_t - a_t)$ and $(w_t^* - p_t^* - a_t^*)$ using (A37)–(A38). Adding and subtracting the foreign counterpart, we obtain:

$$[1-(1-2\gamma)\phi]\tilde{y}_t = (1-2\gamma)(1-\phi)\tilde{c}_t + 2\gamma\left[\theta(1-\alpha)\tfrac{2(1-\gamma)}{1-2\gamma} - \phi\right]q_t. \tag{A40}$$

Combining (A39) and (A40) we can solve for $\tilde{y}_t$ and $\tilde{c}_t$. For example, the expression for $\tilde{c}_t$ is:

$$\tilde{c}_t = \kappa_a\tilde{a}_t - \gamma\kappa_q q_t, \qquad \text{where} \tag{A41}$$

$$\kappa_a \equiv \frac{(1+\nu)(1+\varkappa)}{1+\nu\sigma(1+\varkappa)} \quad \text{and} \quad \kappa_q \equiv \frac{\varkappa}{\gamma}\frac{2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} + \nu(1+\varkappa) + \phi\varkappa}{1+\nu\sigma(1+\varkappa)}, \quad \text{with} \quad \varkappa \equiv \frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}.$$

Note that $\kappa_a, \kappa_q > 0$ independently of the values of the parameters, and in the autarky limit as $\gamma \to 0$ we have $\kappa_a \to \frac{1+\nu}{1+\nu\sigma}$ and $\kappa_q \to \frac{2}{1-\phi}\frac{2\theta(1-\alpha)+\nu}{1+\nu\sigma}$, since $\frac{\varkappa}{\gamma} \to \frac{2}{1-\phi}$. Therefore, (κ_a, κ_q) are positive derived parameters separated from zero even as $\gamma \to 0$.

Lastly, we log-linearize the flow budget constraint (20) as:

$$\beta b_{t+1} - b_t = nx_t = \gamma\Big(y_{Ht}^* - y_{Ft} - s_t\Big), \tag{A42}$$

where $\beta = 1/\bar{R}$ and since $\bar{B} = \bar{NX} = 0$ in a symmetric steady state, we define $b_{t+1} = B_{t+1}/\bar{Y}$ and $nx_t = NX_t/\bar{Y}$, so that γ represents the steady-state share of imports (and also exports) in output, which is the relevant coefficient in the log-linearization (A42). Next we use the expression for export quantity y_{Ht}^* above and a symmetric counterpart for y_{Ft}, together with the solution for prices and quantities, to derive:[47]

$$nx_t = \gamma\big[\lambda_q q_t - \lambda_a\tilde{a}_t\big], \tag{A43}$$

[47] This is a rather tedious deviations, which relies on the previous equilibrium relationships. We start with:

$$\begin{aligned}
y_{Ht}^* - y_{Ft} - s_t &= -\theta(p_{Ht}^* - p^*) + \theta(p_{Ft} - p) - \phi(1-\phi)(\tilde{w}_t - \tilde{p}_t - \tilde{a}_t) - [\phi\tilde{y}_t + (1-\phi)\tilde{c}_t] - s_t \\
&= -[\theta(1-\alpha)+\phi](1-\phi)(\tilde{w}_t - \tilde{p}_t - \tilde{a}_t) + 2\theta(1-\alpha)q_t - \tfrac{1-2\alpha(1-\gamma)}{1-2\gamma}q_t - [\phi\tilde{y}_t + (1-\phi)\tilde{c}_t] \\
&= \left([\theta(1-\alpha)+\phi]\frac{2\gamma}{1-2\gamma} + 2\theta(1-\alpha) - \tfrac{1-2\alpha(1-\gamma)}{1-2\gamma}\right)q_t - [\phi\tilde{y}_t + (1-\phi)\tilde{c}_t],
\end{aligned}$$

where the first line substitutes the expressions for y_{Ht}^* and y_{Ft}, the second equality uses (A27), (A29) and (A34), and the third where the second equality substitutes in the solution for $(\tilde{y}_t, \tilde{c}_t)$ from (A39)–(A40), and we define:

$$\lambda_a \equiv \frac{1}{1-2\gamma}\frac{1+\nu}{1+\nu\sigma(1+\varkappa)},$$
$$\lambda_q \equiv \frac{1}{1-2\gamma}\left(\frac{1+\nu\sigma}{1+\nu\sigma(1+\varkappa)}\left[2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma}+\phi\varkappa+\frac{\nu\varkappa}{1+\nu\sigma}\right]-[1-2\alpha(1-\gamma)]\right).$$

Note that $\lambda_a \equiv \frac{1}{1-2\gamma}\frac{\kappa_a}{1+\varkappa} > 0$, and as $\gamma \to 0$ we have $\lambda_a - \kappa_a \to 0$. Furthermore, $\lambda_q > 0$ is equivalent to the generalized *Marshall-Lerner condition* in our general-equilibrium model, and $\theta > 1/2$ is sufficient to ensure this independently of the values of other parameters $(\gamma, \alpha, \phi, \nu, \sigma)$. To see this, note that the case of $\alpha \in [1/2, 1)$ is trivially satisfied; for $\alpha \in [0, 1/2)$, the sufficiency of $\theta > 1/2$ follows from the fact that $\frac{1-2\alpha(1-\gamma)}{(1-\alpha)(1-\gamma)} \le 1$ and $\frac{1+\nu\sigma}{1+\nu\sigma(1+\varkappa)}\left[\frac{1}{1-2\gamma}+\phi\varkappa\right] \ge 1$. Recall that $\theta > 1$ by assumption of the model. In the limit $\gamma \to 0$, we have $\lambda_q \to 1 + 2(\theta-1)(1-\alpha)$, which is in general different from κ_q.

Exchange rate and interest rates We log-linearize the household Euler equation (4) and its foreign counterpart:

$$i_t = \mathbb{E}_t\left\{\sigma\Delta c_{t+1} + \Delta p_{t+1}\right\} \quad \text{and} \quad i_t^* = \mathbb{E}_t\left\{\sigma\Delta c_{t+1}^* + \Delta p_{t+1}^*\right\},$$

where $i_t \equiv \log R_t - \log \bar{R}$ and similarly for i_t^*. Taking the difference, we can express the interest rate differential as:

$$i_t - i_t^* = \mathbb{E}_t\left\{\sigma\Delta\tilde{c}_{t+1} + \Delta\tilde{p}_{t+1}\right\}, \tag{A44}$$

Subtracting the expected inflation differential, $\mathbb{E}_t\Delta\tilde{p}_{t+1}$, on both sides allows to characterize the equilibrium real interest rate differential as

$$i_t - i_t^* - \mathbb{E}_t\Delta\tilde{p}_{t+1} = \sigma\mathbb{E}_t\Delta\tilde{c}_{t+1} = \sigma\kappa_a\mathbb{E}_t\Delta\tilde{a}_{t+1} - \gamma\sigma\kappa_q\mathbb{E}_t\Delta q_{t+1}, \tag{A45}$$

where we substituted the solution for $\Delta\tilde{c}_{t+1}$ from (A41). To solve for the equilibrium nominal interest rate differential we combine (A44) with the Taylor rule (22) and its foreign counterpart. In the limiting case (R2), we have $\Delta\tilde{p}_{t+1} \equiv 0$, and the nominal interest rate (differential) tracks the real interest rate (differential), equal σ times the expected (relative) consumption growth.

Next, subtracting $\mathbb{E}_t\Delta e_{t+1}$ on both sides of (A44) and combining with the modified UIP equation (28), we have:

$$\mathbb{E}_t\left\{\sigma\Delta\tilde{c}_{t+1} - \Delta q_{t+1}\right\} = i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} = \chi_1\psi_t - \chi_2 b_{t+1}, \tag{A46}$$

which amounts to the international risk-sharing condition in this economy. Combining (A46) with the solution for the equilibrium consumption differential (A41), we arrive at the condition for the expected change in the real exchange rate:

$$(1+\gamma\sigma\kappa_q)\mathbb{E}_t\Delta q_{t+1} = \chi_2 b_{t+1} - \chi_1\psi_t + \sigma\kappa_a\mathbb{E}_t\Delta\tilde{a}_{t+1}. \tag{A47}$$

Substituting this into (A45) yields the solution for the interest rate differential:

$$i_t - i_t^* - \mathbb{E}_t\Delta\tilde{p}_{t+1} = -\frac{\gamma\sigma\kappa_q}{1+\gamma\sigma\kappa_q}[\chi_2 b_{t+1} - \chi_1\psi_t] + \frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}\mathbb{E}_t\Delta\tilde{a}_{t+1}. \tag{A48}$$

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equality uses (A37) and (A38). Next we use (A40) to solve for:

$$\phi\tilde{y}_t + (1-\phi)\tilde{c}_t = \frac{1}{1-2\gamma}\tilde{y}_t - \frac{2\gamma}{1-2\gamma}\left[\theta(1-\alpha)\tfrac{2(1-\gamma)}{1-2\gamma} - \phi\right]q_t$$
$$= \frac{1}{1-2\gamma}\frac{1+\nu}{1+\nu\sigma(1+\varkappa)}\tilde{a}_t - \tfrac{2\gamma}{1-2\gamma}\left[\theta(1-\alpha)\tfrac{2(1-\gamma)}{1-2\gamma} - \phi + \tfrac{1}{1-2\gamma}\tfrac{\nu+\phi}{1-\phi} - \frac{\nu\sigma\kappa_q}{2}\right]q_t,$$

where the second line uses (A39) to solve out $\tilde{y}_t$ and then (A41) to solve out $\tilde{c}_t$. Finally, substituting in the expression for κ_q from (A41) and rearranging terms yields the resulting (A43).

A.3.2 Proofs of the results for Section 3

Equilibrium exchange rate dynamics We first characterize the generalized version of the exchange rate dynamics in (33) corresponding to the modified UIP condition (28) with endogenous coefficients χ_1 and χ_2. We combine (A47) with (A42)–(A43) and the exogenous shock processes (12) and (24) assuming for concreteness $\rho_a = \rho_\psi = \rho$ (and analogous derivations apply in the general case with $\rho_a \neq \rho_\psi$). This system corresponds to the generalized versions of (29)—(31) in the text.[48] We write the equilibrium dynamic system in matrix form:

$$\begin{pmatrix} 1 & -\hat{\chi}_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbb{E}_t q_{t+1} \\ \hat{b}_{t+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1/\beta \end{pmatrix} \begin{pmatrix} q_t \\ \hat{b}_t \end{pmatrix} - \begin{pmatrix} \hat{\chi}_1 & (1-\rho)k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \psi_t \\ \hat{a}_t \end{pmatrix},$$

where for brevity we made the following substitution of variables:

$$\hat{b}_t \equiv \frac{\beta}{\gamma\lambda_q} b_t, \quad \hat{a}_t \equiv \frac{\lambda_a}{\lambda_q}\tilde{a}_t, \quad \hat{\chi}_1 \equiv \frac{\chi_1}{1+\gamma\sigma\kappa_q}, \quad \hat{\chi}_2 \equiv \frac{\gamma\lambda_q/\beta}{1+\gamma\sigma\kappa_q}\chi_2, \quad k \equiv \frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}\frac{\lambda_q}{\lambda_a}. \tag{A49}$$

Diagonalizing the dynamic system, we have:

$$\mathbb{E}_t z_{t+1} = B z_t - C \begin{pmatrix} \psi_t \\ \hat{a}_t \end{pmatrix}, \quad \text{where} \quad B \equiv \begin{pmatrix} 1+\hat{\chi}_2 & \hat{\chi}_2/\beta \\ 1 & 1/\beta \end{pmatrix}, \quad C \equiv \begin{pmatrix} \hat{\chi}_1 & (1-\rho)k + \hat{\chi}_2 \\ 0 & 1 \end{pmatrix},$$

and we denoted $z_t \equiv (q_t, \hat{b}_t)'$. The eigenvalues of B are:

$$\mu_{1,2} = \frac{(1+\hat{\chi}_2+1/\beta) \mp \sqrt{(1+\hat{\chi}_2+1/\beta)^2 - 4/\beta}}{2/\beta} \qquad \text{such that} \qquad 0 < \mu_1 \leq 1 < \frac{1}{\beta} \leq \mu_2,$$

and $\mu_1 + \mu_2 = 1 + \hat{\chi}_2 + 1/\beta$ and $\mu_1 \cdot \mu_2 = 1/\beta$. Note that when $\chi_2 = 0$, and hence $\hat{\chi}_2 = 0$, the two roots are simply $\mu_1 = 1$ and $\mu_2 = 1/\beta$.

The left eigenvalue associated with $\mu_2 > 1$ is $v = (1, 1/\beta - \mu_1)$, such that $vB = \mu_2 v$. Therefore, we can pre-multiply the dynamic system by v and rearrange to obtain:

$$vz_t = \frac{1}{\mu_2}\mathbb{E}_t\{vz_{t+1}\} + \frac{1}{\mu_2}\hat{\chi}_1\psi_t + \left[\frac{(1-\rho)k + \hat{\chi}_2}{\mu_2} + \frac{1/\beta - \mu_1}{\mu_2}\right]\tilde{a}_t.$$

Using the facts that $\hat{\chi}_2 + 1/\beta - \mu_1 = \mu_2 - 1$ and $1/\mu_2 = \beta\mu_1$, we solve this dynamic equation forward to obtain the equilibrium cointegration relationship:

$$vz_t = q_t + (1/\beta - \mu_1)\hat{b}_t = \frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{1-\beta\mu_1+\beta(1-\rho)k\mu_1}{1-\beta\rho\mu_1}\hat{a}_t. \tag{A50}$$

Combining this with the second dynamic equation for $\hat{b}_{t+1}$, we solve for:

$$\hat{b}_{t+1} - \mu_1\hat{b}_t = \overbrace{q_t + \big(\tfrac{1}{\beta} - \mu_1\big)\hat{b}_t}^{=vz_t} - \hat{a}_t = \frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{\beta(1-\rho)(k-1)\mu_1}{1-\beta\rho\mu_1}\hat{a}_t, \tag{A51}$$

Note that $\hat{b}_{t+1}$ in (A51) follows a stationary AR(2) with roots ρ and μ_1. Recall that as $\chi_2 \to 0$, $\mu_1 \to 1$, and the process for $\hat{b}_{t+1}$ becomes an ARIMA(1,1,0), which corresponds to the solution in footnote 25 in the text (after reverse substitution of variables).

[48] Note that, while monetary policy results in $e_t = q_t$, it does not affect the equilibrium system for RER q_t in the flexible-price version of the model we consider here.

Finally, we apply lag operator $(1-\mu_1 L)$ to (A50) and use (A51) to solve for:

$$\begin{aligned}(1-\mu_1 L)q_t &= (1-\beta^{-1}L)\left[\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{\beta(1-\rho)(k-1)\mu_1}{1-\beta\rho\mu_1}\hat{a}_t\right] + (1-\mu_1 L)\hat{a}_t \\ &= (1-\beta^{-1}L)\left[\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{\beta(1-\rho)\mu_1}{1-\beta\rho\mu_1}k\hat{a}_t\right] + \frac{1-\beta\mu_1}{1-\beta\rho\mu_1}(1-\rho\mu_1 L)\hat{a}_t,\end{aligned} \tag{A52}$$

where L is the lag operator such that $Lq_t = q_{t-1}$. Therefore, equilibrium RER q_t follows a stationary ARMA(2,1) with autoregressive roots μ_1 and ρ. Again, in the limit $\chi_2 \to 0$, $\mu_1 \to 1$, and this process becomes an ARIMA(1,1,1), which corresponds to (33) in the text.[49] ■

Equilibrium variance of the exchange rate Solution (A52) characterizes the behavior of q_t for given values of χ_1 and χ_2 (and hence μ_1, μ_2), which from (28) themselves depend on $\sigma_e^2 = \mathrm{var}_t(\Delta e_{t+1})$. Since the monetary policy stabilizes inflation, ensuring $e_t = q_t$, we also have $\sigma_e^2 = \mathrm{var}_t(\Delta q_{t+1})$, and we now solve for the equilibrium value of σ_e^2, and hence $(\chi_1, \chi_2, \mu_1, \mu_2)$.

Using (A52), we calculate $\sigma_e^2 = \mathrm{var}_t(\Delta q_{t+1})$ for given χ_1 and χ_2:

$$\sigma_e^2 = \mathrm{var}_t(\Delta q_{t+1}) = \left(\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\right)^2\sigma_\psi^2 + \left(\frac{\beta(1-\rho)\mu_1 k + (1-\beta\mu_1)}{1-\beta\rho\mu_1}\right)^2\sigma_a^2 = \frac{\hat{\chi}_1^2\sigma_\psi^2 + \big((1-\rho)k + (\mu_2-1)\big)^2\sigma_a^2}{(\mu_2-\rho)^2},$$

where the second line used the fact that $\beta\mu_1 = 1/\mu_2$. In addition, recall that:

$$\hat{\chi}_1 = \frac{n/\beta}{1+\gamma\sigma\kappa_q}\frac{\omega\sigma_e^2}{m}, \qquad \hat{\chi}_2 \equiv \frac{\gamma\lambda_q\bar{Y}/\beta}{1+\gamma\sigma\kappa_q}\frac{\omega\sigma_e^2}{m} \quad \text{and} \quad \mu_2 = \frac{(1+\beta\hat{\chi}_2+\beta) + \sqrt{(1+\beta\hat{\chi}_2+\beta)^2 - 4\beta}}{2}.$$

We therefore can rewrite the fixed point equation for $\sigma_e^2 > 0$ as follows:

$$F(x,\tilde{\omega}) = \big(\mu_2(\tilde{\omega}x) - \rho\big)^2 x - b(\tilde{\omega}x)^2 - c = 0, \tag{A53}$$

where we used the following notation:

$$x \equiv \sigma_e^2 \geq 0, \qquad \tilde{\omega} = \frac{\omega}{m}, \qquad b \equiv \left(\frac{n/\beta}{1+\gamma\sigma\kappa_q}\right)^2\sigma_\psi^2, \qquad c \equiv \big((1-\rho)k + (\mu_2-1)\big)^2\sigma_a^2 \geq 0,$$

and $\mu_2(\cdot)$ is a function which gives the equilibrium values of μ_2 defined above as a function of $\tilde{\omega}\sigma_e^2$ for given values of the model parameters. Note that for any given $\tilde{\omega} > 0$:

$$\begin{aligned}&\lim_{x\to 0} F(x,\tilde{\omega}) = -c \leq 0, \\ &\lim_{x\to\infty}\frac{F(x,\tilde{\omega})}{x^3} = \lim_{x\to\infty}\left(\frac{\mu_2(\tilde{\omega}x)}{x}\right)^2 = \left(\frac{\beta\hat{\chi}_2^2}{\sigma_e^2}\right) = \left(\frac{\gamma\lambda_q\bar{Y}}{1+\gamma\sigma\kappa_q}\tilde{\omega}\right)^2 > 0.\end{aligned}$$

Therefore, by continuity at least one fixed-point $F(\sigma_e^2, \omega) = 0$ with $\sigma_e^2 \geq 0$ exists, and all such $\sigma_e^2 > 0$ whenever $c > 0$ (that is, when $\sigma_a > 0$). One can further show that when σ_a/σ_ψ is not too small, this equilibrium is unique, which is in particular the case under our calibration.[50]

Finally, we consider the limit of log-linearization in Lemma 3, where $(\sigma_a, \sigma_\psi) = \sqrt{\xi}\cdot(\bar{\sigma}_a, \bar{\sigma}_\psi) = \mathcal{O}(\sqrt{\xi})$ as $\xi \to 0$, where $(\bar{\sigma}_a, \bar{\sigma}_\psi)$ are some fixed numbers. Then in (A53), $(b, c) = \mathcal{O}(\xi)$, as (b, c) are linear in $(\sigma_a^2, \sigma_\psi^2)$. This implies that for any given fixed point $(\bar{\sigma}_e^2, \bar{\omega})$, with $F(\bar{\sigma}_e^2, \bar{\omega}; \bar{\sigma}_a^2, \bar{\sigma}_\psi^2) = 0$, there exists a sequence of fixed

[49]Note that we prefer the specification in the second line of (A52) since it separates the exchange rate effects of $\hat{a}_t$ via the budget constraint (latter term) and via the modified UIP condition (the former term with factor k in front of $\hat{a}_t$). In the limit of $\rho \to 1$, the effect through the UIP condition vanishes, while the effect through the budget constraint results in a random walk response of q_t to $\hat{a}_t$.

[50]For $\sigma_a/\sigma_\psi \approx 0$, there typically exist three equilibria. In particular, when $\sigma_a = 0$, there always exists an equilibrium with $\sigma_e^2 = \chi_1 = 0$, in addition to two other potential equilibria with $\sigma_e^2 > 0$, which exist when σ_ψ is not too small (see Itskhoki and Mukhin 2017).

points $F(\xi\bar{\sigma}_e^2, \bar{\omega}/\xi; \xi\bar{\sigma}_a^2, \xi\bar{\sigma}_\psi^2) = 0$ as $\xi \to 0$, for which $\sigma_e^2 = \xi\bar{\sigma}_e^2 = \mathcal{O}(\xi)$, $\tilde{\omega} = \bar{\omega}/\xi = \mathcal{O}(1/\xi)$ and $\tilde{\omega}\sigma_e^2 = \bar{\omega}\bar{\sigma}_e^2 = const$. To verify this, one can simply divide (A53) by ξ and note that, for a given $\tilde{\omega}x$, $F(x, \tilde{\omega})$ is linear in (x, b, c), which means that the fixed point x scales with (b, c) provided that $\tilde{\omega}x$ stays constant. This confirms the conjecture used in the proof of Lemma 3. ■

Proofs of Propositions In the remainder of the proofs, we specialize to the case of $\chi_2 = 0$ and normalization $\chi_1 = 1$, which we adopted in the text of Section 3. The results, however, obtain under the general case with endogenous nonzero χ_1 and χ_2, as we considered until now. Throughout the proofs we use the notation for $\hat{\chi}_1$, k and $\hat{a}_t$ introduced in (A49) above.

Proof of Proposition 1 The equilibrium process for the nominal exchange rate (33) follows directly from the solution (A52), as $\mu_1 = 1$ when $\chi_2 = 0$, and considering the fact that the assumed monetary policy ensures $e_t = q_t$ (see (29)). This is the unique path of the exchange rate that simultaneously satisfies the modified UIP condition (30) and the country budget constraint (31). Indeed, (30) determines e_t and $\mathbb{E}_t e_{t+j}$ for all $j > 0$ up to a long-run expectation $\mathbb{E}_t e_\infty \equiv \lim_{j\to\infty} \mathbb{E}_t e_{t+j}$, which in turn is uniquely pinned down by the budget constraint, as any departures from $\mathbb{E}_t e_\infty$ result in expected violations of the intertemporal budget of the country.

As $\beta, \rho \to 1$, we have $\big[(1-\beta^{-1}L) - (1-\rho L)\big]x_t \to 0$ for any stationary x_t, and therefore we can use (33) to show that in this limit:

$$\lim_{\beta,\rho\to 1}\Big\{\Delta e_t - \Big[\overbrace{\tfrac{\beta\hat{\chi}_1}{1-\beta\rho}\sigma_\psi\varepsilon_t^\psi + \big(\tfrac{\beta(1-\rho)}{1-\beta\rho}k + \tfrac{1-\beta}{1-\beta\rho}\big)\hat{\sigma}_a\hat{\varepsilon}_t^a}^{\sim iid}\Big]\Big\}$$
$$= \lim_{\beta,\rho\to 1}\Big\{\big[(1-\beta^{-1}L) - (1-\rho L)\big]\Big(\tfrac{\beta\hat{\chi}_1}{1-\beta\rho}\psi_t + \tfrac{\beta(1-\rho)}{1-\beta\rho}k\hat{a}_t\Big)\Big\} = 0,$$

where we used the notation $\hat{\chi}_1$, k and $\hat{a}_t$ from (A49) and denoted with $\hat{\sigma}_a\hat{\varepsilon}_t^a$ the innovation of the $\hat{a}_t$ shock process, so that $\hat{\sigma}_a\hat{\varepsilon}_t^a = (1-\rho L)\hat{a}_t$ and similarly $\sigma_\psi\varepsilon_t^\psi = (1-\rho L)\psi_t$. Therefore, in the limit Δe_t is iid, and hence e_t follows a random walk.[51]

We next prove related limiting results characterizing the second moments of the exchange rate process, by rewriting (33) as follows:

$$\Delta e_t = \tfrac{\beta\hat{\chi}_1}{1-\beta\rho}\Big[\sigma_\psi\varepsilon_t^\psi - \tfrac{1-\beta\rho}{\beta}\psi_{t-1}\Big] + \tfrac{\beta(1-\rho)k}{1-\beta\rho}\Big[\hat{\sigma}_a\hat{\varepsilon}_t^a - \tfrac{1-\beta\rho}{\beta}\hat{a}_{t-1}\Big] + \tfrac{1-\beta}{1-\beta\rho}\hat{\sigma}_a\hat{\varepsilon}_t^a,$$

where again we use notation $\hat{\chi}_1$, k and $\hat{a}_t$ from (A49). We can then calculate:

$$\begin{aligned}
\mathrm{var}(\Delta e_t) &= \tfrac{1+\beta^2-2\beta\rho}{(1-\beta\rho)^2(1-\rho^2)}\big[\hat{\chi}_1^2\sigma_\psi^2 + (1-\rho)^2k^2\hat{\sigma}_a^2\big] + \Big(\tfrac{1-\beta}{1-\beta\rho}\Big)^2\Big[1 + \tfrac{2\beta(1-\rho)k}{1-\beta}\Big]\hat{\sigma}_a^2,\\
\mathrm{cov}(\Delta e_t, \Delta e_{t-1}) &= -\tfrac{\beta-\rho}{(1-\beta\rho)(1-\rho^2)}\big[\hat{\chi}_1^2\sigma_\psi^2 + (1-\rho)^2k^2\hat{\sigma}_a^2\big] - \tfrac{(1-\rho)(1-\beta)}{1-\beta\rho}k\hat{\sigma}_a^2,\\
\mathrm{var}_t(\Delta e_{t+1}) &= \Big(\tfrac{\beta}{1-\beta\rho}\Big)^2\hat{\chi}_1^2\sigma_\psi^2 + \Big(\tfrac{\beta(1-\rho)k}{1-\beta\rho} + \tfrac{1-\beta}{1-\beta\rho}\Big)^2\hat{\sigma}_a^2.
\end{aligned}$$

Next, taking first the $\beta \to 1$ limit, we can characterize the following ratios for $\rho < 1$ and as $\rho \to 1$:[52]

$$\mathrm{corr}(\Delta e_t, \Delta e_{t-1}) = \frac{\mathrm{cov}(\Delta e_t, \Delta e_{t-1})}{\mathrm{var}(\Delta e_t)} = -\frac{1-\rho}{2} \to 0,$$

[51] Two remarks are in order. First, as $\beta\rho \to 1$, we should ensure that $\psi_t/(1-\beta\rho)$ does not explode in order to keep the volatility of Δe_t finite; a natural approach is to require that $\sigma_\psi/(1-\beta\rho)$ remains finite in this limit. Second, note that the limiting process for Δe_t depends on the relative speed of convergence of β and ρ to 1, as depending on it $\frac{1-\rho}{1-\beta\rho}, \frac{1-\beta}{1-\beta\rho} \in [0,1]$. For example, if we first take $\beta \to 1$, as will be our approach below, then $\Delta e_t \to \frac{\hat{\chi}_1}{1-\rho}\Delta\psi_t + k\Delta\hat{a}_t$, as the last term in (33) disappears already in this first limit, prior to taking $\rho \to 1$.

[52] We focus on this sequence of taking limits as the variance of Δe_t is not well-defined when $\rho = 1$, as this results in a double-integrated process for the exchange rate. In contrast, all second moments are well-defined for $\beta = 1$, and we can then study their properties as ρ increases towards 1. In addition, this sequence of limits provides a better approximation to our calibrated model, in which we have $\rho < \beta < 1$, namely $\rho = 0.97$ and $\beta = 0.99$.

$$\frac{\mathrm{var}_t(\Delta e_{t+1})}{\mathrm{var}(\Delta e_{t+1})} = \frac{\mathrm{var}(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})}{\mathrm{var}(\Delta e_{t+1})} = \frac{1+\rho}{2} \to 1,$$

$$\frac{\mathrm{var}(\Delta e_t)}{\mathrm{var}(\hat{\chi}_1 \psi_t)} = \frac{2}{1-\rho} \frac{\hat{\chi}_1^2 \sigma_\psi^2 + (1-\rho)^2 k^2 \hat{\sigma}_a^2}{\hat{\chi}_1^2 \sigma_\psi^2} \to \infty,$$

$$\frac{\mathrm{var}(\Delta e_t | \psi_t)}{\mathrm{var}(\Delta e_t)} = \frac{\hat{\chi}_1^2 \sigma_\psi^2}{\hat{\chi}_1^2 \sigma_\psi^2 + k^2 \hat{\sigma}_a^2 (1-\rho)^2} \to 1,$$

where we used the fact that $\mathrm{var}(\psi_t) = \sigma_\psi^2/(1-\rho^2)$. These are the results summarized in Proposition 1 and in the text following it. The last result in particular confirms that as $\beta\rho \to 1$, the contribution of the ψ_t shock to the variance of Δe_t fully dominates the contribution of the productivity shock. The second-to-last result confirms that as $\beta\rho \to 1$, an arbitrarily small volatility of the UIP shock $\chi_1 \psi_t$ results in an arbitrarily large volatility of Δe_t. The first two results are the confirmation of the limiting random-walk behavior of the exchange rate as $\beta\rho \to 1$.

Lastly, we prove that as γ *decreases* or $\beta\rho$ *increases*, the volatility of the exchange rate response to the UIP shock $\chi_1\psi_t$ *increases.* We denote $\sigma_{e|\psi}^2 \equiv \mathrm{var}(\Delta e_t|\psi_t) = \frac{1+\beta^2-2\beta\rho}{(1-\beta\rho)^2(1-\rho^2)} \hat{\chi}_1^2 \sigma_\psi^2$ the variance of the exchange rate conditional on the financial shock ψ_t, or equivalently when we switch off the other shock, $\sigma_a = 0$. We characterize:

$$\frac{\mathrm{var}(\Delta e_t | \psi_t)}{\mathrm{var}(\chi_1 \psi_t)} = \frac{1+\beta^2 - 2\beta\rho}{(1-\beta\rho)^2} \frac{1}{(1+\gamma\sigma\kappa_q)^2},$$

where we used the notation for $\hat{\chi}_1$ in (A49) and the fact that $\mathrm{var}(\psi_t) = \sigma_\psi^2/(1-\rho^2)$, which already adjusts for the persistence of the shock.[53] It is immediate to see that the volatility of the exchange rate response always increases in β and increases in ρ iff $\rho < \beta$, which we take as the empirically relevant case. Note that the overall unscaled volatility of the exchange rate, $\sigma_{e|\psi}^2 \equiv \mathrm{var}(\Delta e_t|\psi_t)$, always increases in both β and ρ. Finally, we establish that $\gamma\kappa_q$ is increasing in γ, where κ_q is defined in (A41), reducing the volatility of the exchange rate response. We have:

$$\gamma\kappa_q = \varkappa \frac{2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} + \nu(1+\varkappa) + \phi\varkappa}{1+\nu\sigma(1+\varkappa)}, \qquad \text{where} \qquad \varkappa \equiv \frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}.$$

Since $\varkappa$ is increasing in γ on its range $[0, 1/2)$, we only need to establish that the right-hand side is increasing in $\varkappa$, which can be immediately verified by differentiation. This completes the proof of Proposition 1. ∎

Proofs of Propositions 2 and 3 The relationships between nominal exchange rate, real exchange rates and terms of trade in Propositions 2 and 3 are derived above — see (A34)–(A36). The results follow immediately from these equations. In particular, in response to ψ_t, all real exchange rates — q_t, q_t^P and q_t^W — comove perfectly with the nominal exchange rate e_t, which under the consumer-price-stabilizing monetary policy equals RER, $e_t = q_t$. Smaller γ and larger α ensure that the ratios of volatility of q_t^W and q_t^P to that of q_t are closer to 1. In contrast, higher α reduces the relative volatility of the terms of trade s_t relative to q_t.

The proof of Proposition 1 already establishes that, as $\beta\rho \to 1$, the process for $q_t = e_t$ converges to a random walk with an arbitrarily large volatility of $\Delta q_t = \Delta e_t$ (when scaled by the exogenous volatility of the UIP shock ψ_t, that is arbitrarily small volatility of ψ_t can result in arbitrarily large volatility of Δq_t). We now show that the small-sample persistence of the real exchange also increases without bound, and hence so does the measured half-live of the RER process. Specifically, we calculate the finite-sample autocorrelation of the real exchange rate in levels, that is the coefficient from a regression of q_t on q_{t-1} (with a constant) in a sample with $T+1$ observations:

$$\hat{\rho}_q(T) = \frac{\frac{1}{T}\sum_{t=1}^T (q_t - \bar{q})(q_{t-1} - \bar{q})}{\frac{1}{T}\sum_{t=1}^T (q_{t-1} - \bar{q})^2} = 1 + \frac{\frac{1}{T}\sum_{t=1}^T \Delta q_t q_{t-1}}{\frac{1}{T}\sum_{t=1}^T (q_{t-1} - \bar{q})^2}.$$

[53] We could alternatively characterize the response to the primitive noise-trader shock ψ_t with an endogenous coefficient χ_1, rather than the overall endogenous UIP shock $\chi_1\psi_t$; this however simply introduces an amplification loop, by which an initial increase in $\sigma_{e|\psi}$ gets amplified by an endogenous increase in χ_1, leaving the qualitative result unchanged.

Note that the denominator is positive and finite for any finite T, but diverges as $T \to \infty$, since q_t is an integrated process. The numerator, however, has a finite limit (conditional on a given initial value q_0, and due to stationarity of Δq_t):

$$p\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \Delta q_t q_{t-1} = \text{cov}(\Delta q_t, q_{t-1}) = \sum_{j=1}^{\infty} \text{cov}(\Delta q_t, \Delta q_{t-j}) = -\frac{\beta-\rho}{(1-\rho)(1-\beta\rho)} \frac{\chi_1^2\sigma_\psi^2}{(1+\gamma\sigma\kappa_q)^2},$$

where we used the fact that for process (33) $\text{cov}(\Delta q_t, \Delta q_{t-j}) = \rho^{j-1}\text{cov}(\Delta q_t, \Delta q_{t-1})$ for $j \geq 1$, and the expression for $\text{cov}(\Delta q_t, \Delta q_{t-1}) = \text{cov}(\Delta e_t, \Delta e_{t-1})$ calculated above (conditional on ψ_t shocks, i.e. when $\sigma_a = 0$). This implies that the finite sample autocorrelation of q_t: (a) tends to 1 asymptotically as samples size increases (and hence the associated half-life tends to infinity); and (b) is smaller than 1 in large but finite samples, provided that $\rho < \beta$. The associated half-live of q_t is given by $\log(0.5)/\log\hat{\rho}_q(T)$, is finite in finite samples, and increases unboundedly with T. This completes the proof of Proposition 2. ■

Proposition 3 follows directly from (A34)–(A35). Specifically, requiring that the proportionality coefficient between s_t and q_t in (A34), $0 < \frac{1-2\alpha(1-\gamma)}{1-2\gamma} \ll 1$, implies the condition in Proposition 3. Having larger α or smaller γ also brings the proportionality coefficient between q_t and q_t^P in (A35) closer to 1. ■

Proof of Proposition 4 The equilibrium condition (40) and the coefficients κ_a, κ_q are derived above: see (A41).

(i) Consider a ψ_t shock first. From (33) and Proposition 1, it results in exchange rate depreciation, $\Delta q_t = \Delta e_t > 0$. From (40), this depreciation is associated with a reduction in consumption, since $\gamma\kappa_q > 0$. The Backus-Smith correlation in response to ψ_t derives from:

$$\text{cov}(\Delta c_t - \Delta c_t^*, \Delta q_t|\psi_t) = -\gamma\kappa_q \text{var}(\Delta q_t|\psi_t) < 0,$$

which follows directly from (40). The relative volatility of consumption in response to ψ_t is:

$$\frac{\text{var}(\Delta\tilde{c}_t|\psi_t)}{\text{var}(\Delta q_t|\psi_t)} = (\gamma\kappa_q)^2,$$

which decreases as γ decreases (see the Proof of Proposition 1), and converges to zero in the limit $\gamma \to 0$ (as $\lim_{\gamma\to 0}\kappa_q = \frac{2}{1-\phi}\frac{2\theta(1-\alpha)+\nu}{1+\nu\sigma}$ if finite; see (A41)).

(ii) Next consider a $\tilde{a}_t$ shock. From (33) and Proposition 1, it also results in exchange rate depreciation, $\Delta q_t = \Delta e_t > 0$. We now combine (33) with (40) to evaluate the relative consumption response to $\tilde{a}_t$:

$$\Delta\tilde{c}_t = \kappa_a\Delta\tilde{a}_t - \gamma\kappa_q\left[\frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}\frac{\beta(1-\rho)}{1-\beta\rho}\left(\tilde{a}_t - \beta^{-1}\tilde{a}_{t-1}\right) + \frac{\lambda_a}{\lambda_q}\frac{1-\beta}{1-\beta\rho}\left(\tilde{a}_t - \rho\tilde{a}_{t-1}\right)\right],$$

so that the impulse response:

$$\begin{aligned}
\frac{\partial\tilde{c}_t}{\partial\tilde{\varepsilon}_t^a} &= \kappa_a - \gamma\kappa_q\left[\frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}\frac{\beta(1-\rho)}{1-\beta\rho} + \frac{\lambda_a}{\lambda_q}\frac{1-\beta}{1-\beta\rho}\right] \\
&= \kappa_a\left[1 - \frac{\gamma\sigma\kappa_q}{1+\gamma\sigma\kappa_q}\frac{\beta(1-\rho)}{1-\beta\rho} - \frac{\gamma\kappa_q/\lambda_q}{(1-2\gamma)(1+\varkappa)}\frac{1-\beta}{1-\beta\rho}\right] \\
&= \kappa_a\left[\frac{\beta(1-\rho)}{1-\beta\rho}\frac{1}{1+\gamma\sigma\kappa_q} + \frac{1-\beta}{1-\beta\rho}\frac{2(1-\gamma)\big[\theta(1-\alpha)+\big(1+2\gamma\frac{\phi}{1-\phi}\big)\alpha\big]-1}{(1+\varkappa)(1-2\gamma)^2\lambda_q}\right] > 0,
\end{aligned}$$

where the second equality uses the expression for λ_a in (A43), and the third equality uses the expressions for κ_q and λ_q in (A41) and (A43) respectively. The result follows from the fact that $\theta > 1$ and $\gamma < 1/2$, and hence $2(1-\gamma)\big[\theta(1-\alpha)+\big(1+2\gamma\frac{\phi}{1-\phi}\big)\alpha\big] > 1$, independently of the values of β, ρ, γ, α, θ and ϕ.

Further, note from (33) that:

$$\frac{\partial q_t}{\partial\tilde{\varepsilon}_t^a} = \frac{\partial e_t}{\partial\tilde{\varepsilon}_t^a} = \frac{\beta(1-\rho)}{1-\beta\rho}\frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q} + \frac{1-\beta}{1-\beta\rho}\frac{\lambda_a}{\lambda_q},$$

and therefore:

$$\frac{\partial\tilde{c}_t/\partial\tilde{\varepsilon}^a_t}{\partial q_t/\partial\tilde{\varepsilon}^a_t}=\frac{1+\frac{1-\beta}{\beta(1-\rho)}\frac{2(1-\gamma)\left[\theta(1-\alpha)+(1+2\gamma\frac{\phi}{1-\phi})\alpha\right]-1}{(1+\varkappa)(1-2\gamma)^2\lambda_q}}{\sigma+\frac{1-\beta}{\beta(1-\rho)}\frac{\lambda_a/\kappa_a}{\lambda_q}(1+\gamma\sigma\kappa_q)}.$$

Note that if $\frac{1-\beta}{1-\rho}\approx 0$ (relevant when $\rho<\beta\approx 1$), then $\frac{\partial\tilde{c}_t/\partial\tilde{\varepsilon}^a_t}{\partial q_t/\partial\tilde{\varepsilon}^a_t}\approx\frac{1}{\sigma}$ independently of γ and other parameters. For simplicity, we evaluate this expression for general β and ρ, but as $\gamma\to 0$:

$$\lim_{\gamma\to 0}\frac{\partial\tilde{c}_t/\partial\tilde{\varepsilon}^a_t}{\partial q_t/\partial\tilde{\varepsilon}^a_t}=\frac{1+\frac{1-\beta}{\beta(1-\rho)}}{\sigma+\frac{1-\beta}{\beta(1-\rho)}\frac{1}{1+2(1-\alpha)(\theta-1)}}>\min\{1/\sigma,1\}.$$

Therefore, even as the economy becomes closed to international trade, the effect of productivity shocks on consumption relative to the real exchange rate is at least $1/\sigma$ (assuming $\sigma>1$), independently of their persistence (i.e., even in the limit of $\rho\to 1$, in which case the relative consumption response necessarily becomes larger than that of the real exchange, as $1+2(1-\alpha)(\theta-1)>1$).

In addition, we evaluate the Backus-Smith correlation conditional on productivity shocks:

$$\frac{\mathrm{cov}(\Delta\tilde{c}_t,\Delta q_t|\tilde{a}_t)}{\mathrm{var}(\Delta q_t|\tilde{a}_t)}=\kappa_a\frac{\mathrm{cov}(\Delta\tilde{a}_t,\Delta q_t|\tilde{a}_t)}{\mathrm{var}(\Delta q_t|\tilde{a}_t)}-\gamma\kappa_q=\kappa_a\frac{\lambda_q}{\lambda_a}\frac{\frac{1-\beta\rho}{1-\beta}\left[1+\frac{1-\rho}{1-\beta}\frac{1+\beta}{1+\rho}k\right]}{1+\frac{2\beta(1-\rho)}{1-\beta}k+\frac{1+\beta^2-2\beta\rho}{(1-\beta)^2}\frac{1-\rho}{1+\rho}k^2}-\gamma\kappa_q,$$

where we followed a similar approach in calculating the covariance term as in the proof of Proposition 1, and k was defined in (A49). To evaluate this expression, we note that k can be expanded as:

$$k=k_1\cdot k_2,\qquad\text{with}\quad k_1\equiv\frac{\gamma\sigma\kappa_q}{1+\gamma\sigma\kappa_q}\le 1\quad\text{and}\quad k_2\equiv\frac{\lambda_q}{\lambda_a}\frac{\kappa_a}{\gamma\kappa_q}>1,$$

where the latter inequality can be verified using the definitions of the coefficients in (A41) and (A43): indeed after some manipulation $k_2>1$ is equivalent to $\left[2(1-\gamma)\right]\left[\theta(1-\alpha)+\left(1+2\gamma\frac{\phi}{1-\phi}\right)\alpha\right]>1$, which always holds as both brackets on the left-hand side are greater than 1 given that $\gamma<1/2$ and $\theta>1$. We can thus rewrite the condition for the Backus-Smith correlation to be positive as follows:

$$1+\frac{1-\rho}{1-\beta}\frac{1+\beta}{1+\rho}k_1\Big[\underbrace{1-\frac{1+\beta^2-2\beta\rho}{(1+\beta)(1-\beta\rho)}k_1}_{>0\text{ since }k_1\le 1}\Big]k_2>\frac{1}{k_2}\frac{1-\beta}{1-\beta\rho}+\frac{2\beta(1-\rho)}{1-\beta\rho}k_1,$$

where we substituted k_2 for $\frac{\lambda_q}{\lambda_a}\frac{\kappa_a}{\gamma\kappa_q}$ are rearranged the terms. The left-hand side is increasing in k_2 and the right-hand side is decreasing in k_2; therefore, it is sufficient to evaluate the inequality as $k_2\to 1$, since $k_2>1$. Evaluating the inequality at $k_2=1$ and rearranging the terms we have:

$$\left[\beta(1-\beta)(1+\rho)+(1+\beta^2-2\beta\rho)k_1\right](1-k_1)\ge 0,$$

which indeed always holds since $k_1\le 1$. Since $k_2>1$, the original inequality is in fact strict on the range of parameter values, and therefore we conclude that the Backus-Smith correlation is always positive for productivity shocks, $\frac{\mathrm{cov}(\Delta\tilde{c}_t,\Delta q_t|\tilde{a}_t)}{\mathrm{var}(\Delta q_t|\tilde{a}_t)}>0$. ■

Proof of Proposition 5 The solution for interest rate differential (41) is derived in Appendix A.2: see (A48); we use it in the proof.

(i) The proof of this part follows from the equilibrium relationship (A45) between expected (real) devaluation and the interest rate differential, which combines the household Euler equations with the market-clearing relationship between consumption and the real exchange rate (40). In view of perfect price level stabilization by the monetary authority, implying in particular $e_t=q_t$, we rewrite (A45) as:

$$i_t-i^*_t=-(1-\rho)\sigma\kappa_a\tilde{a}_t-\gamma\sigma\kappa_q\mathbb{E}_t\Delta e_{t+1},$$

where we also made use of the AR(1) assumption for $\tilde{a}_t$ to simplify the expression.

The Fama coefficient, β_F, is a projection coefficient of Δe_{t+1} on $i_t - i_t^*$, and since $i_t - i_t^*$ is known at time t, it is equivalent to the projection of $\mathrm{E}_t\Delta e_{t+1}$ instead of Δe_{t+1}. Using the expression above, we have:

$$\beta_F \equiv \frac{\mathrm{cov}(\mathbb{E}_t\Delta e_{t+1}, i_t - i_t^*)}{\mathrm{var}(i_t - i_t^*)} = -\frac{1}{\gamma\sigma\kappa_q} - (1-\rho)\frac{\sigma\kappa_a}{\gamma\sigma\kappa_q}\frac{\mathrm{cov}(\tilde{a}_t, i_t - i_t^*)}{\mathrm{var}(i_t - i_t^*)}.$$

In the absence of productivity shocks, $\sigma_a = 0$, the second term is zero, and $\beta_F = -1/(\gamma\sigma\kappa_q) < 0$. Note that this result does not rely on (R3), namely that $\chi_2 = 0$ in (28), since it derives from equilibrium conditions other than (28) (akin to our resolution of the Backus-Smith puzzle in Proposition 4).

Furthermore, under the restriction (R3) that $\chi_2 = 0$, we can use the solution for the interest rate differential (41) to calculate:

$$\frac{\mathrm{cov}(\tilde{a}_t, i_t - i_t^*)}{\mathrm{var}(i_t - i_t^*)} = -\frac{1+\gamma\sigma\kappa_q}{(1-\rho)\sigma\kappa_a}\frac{(1-\rho)^2(\sigma\kappa_a)^2\sigma_a^2}{(\gamma\sigma\kappa_q)^2\chi_1^2\sigma_\psi^2 + (1-\rho)^2(\sigma\kappa_a)^2\sigma_a^2},$$

where the second term in the product on the right-hand side is the share of the productivity shock in the variance decomposition of $\mathrm{var}(i_t - i_t^*)$. When $\sigma_\psi = 0$, this share is one, and it follows that $\frac{\mathrm{cov}(\tilde{a}_t, i_t - i_t^*)}{\mathrm{var}(i_t - i_t^*)} = -\frac{1+\gamma\sigma\kappa_q}{(1-\rho)\sigma\kappa_a}$, resulting in $\beta_F = -\frac{1}{\gamma\sigma\kappa_q} + (1-\rho)\frac{\sigma\kappa_a}{\gamma\sigma\kappa_q}\cdot\frac{1+\gamma\sigma\kappa_q}{(1-\rho)\sigma\kappa_a} = 1$. $\beta_F = 1$ could also be obtained directly from (28) after imposing $\chi_2 = 0$ and $\sigma_\psi = 0$. This completes the proof of part (i).

We note, in addition, that the qualitative result still applies when $\chi_2 > 0$; in this case one can calculate β_F using (A48) and the equilibrium solution for the dynamics of b_{t+1} in (A51). As already shown above, this does not affect the value of $\beta_F = -1/(\gamma\sigma\kappa_q)$ conditional on ψ_t shocks. It does, however, affect the value of β_F conditional on productivity shocks, which can be below or above 1 depending on whether k defined in (A49) is above or below 1. Nevertheless, the departure of β_F from 1 in this case is quantitatively small, as it is of the order of γ^2. We omit the derivation for brevity.

(ii) The proof here relies on the modified UIP condition (28) and the results in Proposition 1. For simplicity, we make use of (R3) that $\chi_2 = 0$. Note from the solutions (30) and (41) that as $\rho \to 1$, the variance contribution of $\tilde{a}_t$ to both $\mathrm{var}(\mathbb{E}_t\Delta e_{t+1})$ and $\mathrm{var}(i_t - i_t^*)$ goes to zero, and the variance of both is fully shaped by the ψ_t shock, independently of the value of $\sigma_a, \sigma_\psi > 0$. Therefore, from our characterization in the proof of part (i), $\beta_F \to -1/(\gamma\sigma\kappa_q) < 0$ in this case. In order to characterize R^2 in the Fama regression, we additionally use the solution (33), which we rewrite as follows:

$$\Delta e_{t+1} = \mathbb{E}_t\Delta e_{t+1} + \frac{1}{1+\gamma\sigma\kappa_q}\frac{\beta\chi_1}{1-\beta\rho}\sigma_\psi\varepsilon_t^\psi + \left[\frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}\frac{\beta(1-\rho)}{1-\beta\rho} + \frac{\lambda_a}{\lambda_q}\frac{1-\beta}{1-\beta\rho}\right]\tilde{\varepsilon}_t^a.$$

Since $i_t - i_t^*$ is orthogonal to the innovation $\Delta e_{t+1} - \mathbb{E}_t\Delta e_{t+1}$, we can establish an upper bound:

$$R^2 \le \frac{\mathrm{var}(\mathbb{E}_t\Delta e_{t+1})}{\mathrm{var}(\Delta e_{t+1})} = 1 - \frac{\mathrm{var}_t(\Delta e_{t+1})}{\mathrm{var}(\Delta e_{t+1})} \to 0$$

as $\beta\rho \to 1$, since the proof of Proposition 1 establishes that in this case $\frac{\mathrm{var}_t(\Delta e_{t+1})}{\mathrm{var}(\Delta e_{t+1})} \to 1$.

By the same token, using the expression for $\Delta e_{t+1} - \mathbb{E}_t\Delta e_{t+1}$ above and (41) , we can evaluate $\frac{\mathrm{var}(i_t - i_t^*)}{\mathrm{var}_t(\Delta e_{t+1})}$. As in Proposition 1, we interpret the $\beta\rho \to 1$ limit sequentially with $\beta \to 1$ first and $\rho \to 1$ next (see the argument in footnote 52), and the results apply more generally with β and ρ converging to 1 simultaneously with $\rho \le \beta < 1$ along the limit sequence. We have:

$$\begin{aligned}\lim_{\beta\to 1}\frac{\mathrm{var}(i_t - i_t^*)}{\mathrm{var}_t(\Delta e_{t+1})} &= \lim_{\beta\to 1}\frac{\left(\frac{\gamma\sigma\kappa_q}{1+\gamma\sigma\kappa_q}\right)^2\frac{\chi_1^2\sigma_\psi^2}{1-\rho^2} + (1-\rho)^2\left(\frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}\right)^2\frac{\sigma_a^2}{1-\rho^2}}{\left(\frac{1}{1+\gamma\sigma\kappa_q}\frac{\beta}{1-\beta\rho}\right)^2\chi_1^2\sigma_\psi^2 + \left[\frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}\frac{\beta(1-\rho)}{1-\beta\rho} + \frac{\lambda_a}{\lambda_q}\frac{1-\beta}{1-\beta\rho}\right]^2\sigma_a^2} \\ &= \frac{1-\rho}{1+\rho}\frac{(\gamma\sigma\kappa_q)^2\chi_1^2\sigma_\psi^2 + (1-\rho)^2(\sigma\kappa_a)^2\sigma_a^2}{\chi_1^2\sigma_\psi^2 + (1-\rho)^2(\sigma\kappa_a)^2\sigma_a^2} \to 0,\end{aligned}$$

as $\rho \to 1$. Since $\text{var}_t(\Delta e_{t+1}) \leq \text{var}(\Delta e_{t+1})$, the variance of $i_t - i_t^*$ is indeed arbitrarily smaller than that of Δe_{t+1} in the limit as $\beta\rho \to 1$.

Proposition 1 has already established that the autocorrelation of Δe_{t+1} becomes arbitrarily close to zero as $\beta\rho \to 1$. At the same time, from (41), the autocorrelation of $i_t - i_t^*$ is equal to ρ, and hence it becomes arbitrarily close to 1 as $\beta\rho \to 1$.

Lastly, we characterize the Carry trade return and its Sharpe ratio. Consider a zero-capital strategy of buying a (temporarily) high-interest bond and selling short low-interest bond, with return $R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R_t^*$ per dollar of gross investment, and the size of the gross position determined by the expected return, $\mathbb{E}_t\big\{R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R_t^*\big\}$, which can be positive or negative.[54] Therefore, a log approximation to the return on this trade is:

$$r_{t+1}^C = (i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1})(i_t - i_t^* - \Delta e_{t+1}) = \chi_1 \psi_t \big[\chi_1 \psi_t - (\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})\big], \tag{A54}$$

where the second equality uses (28) under (R3) that $\chi_2 = 0$. The (unconditional) Sharp ratio associated with this trade is given by $SR^C = \frac{\mathbb{E}\, r_{t+1}^C}{\text{std}(r_{t+1}^C)}$. We calculate:

$$\mathbb{E}\, r_{t+1}^C = \mathbb{E}\big\{\chi_1 \psi_t\, \mathbb{E}_t\{\chi_1 \psi_t - (\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})\}\big\} = \chi_1^2 \mathbb{E}\psi_t^2 = \chi_1^2 \text{var}(\psi_t),$$

$$\begin{aligned}
\text{var}(r_{t+1}^C) = \mathbb{E}(r_{t+1}^C)^2 - (\mathbb{E}r_{t+1}^C)^2 &= \mathbb{E}\big\{\chi_1^2 \psi_t^2 \mathbb{E}_t\{\chi_1\psi_t - (\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})\}^2\big\} - \big[\chi_1^2 \text{var}(\psi_t)\big]^2 \\
&= \chi_1^4 \mathbb{E}\psi_t^4 + \mathbb{E}\left\{\chi_1^2 \psi_t^2 \text{var}_t(\Delta e_{t+1})\right\} - \big[\chi_1^2 \text{var}(\psi_t)\big]^2 = 2\big[\chi_1^2 \text{var}(\psi_t)\big]^2 + \sigma_e^2 \chi_1^2 \text{var}(\psi_t),
\end{aligned}$$

where the last line uses the fact that $\sigma_e^2 = \text{var}_t(\Delta e_{t+1})$ does not depend on t (and in particular on the realization of ψ_t, that is the unexpected component of Δe_{t+1} is homoskedastic, as we proof in Proposition 1), as well as the facts that $\mathbb{E}\{\psi_t^3(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})\} = \mathbb{E}\{\psi_t^3 \mathbb{E}_t\{\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1}\}\} = 0$ and $\mathbb{E}\psi_t^4 = 3(\mathbb{E}\psi_t^2)^2$ due to the normality of the shocks. With this, we calculate:

$$SR^C = \frac{\chi_1^2 \text{var}(\psi_t)}{\sqrt{2[\chi_1^2 \text{var}(\psi_t)]^2 + \sigma_e^2 \chi_1^2 \text{var}(\psi_t)}} = \left(2 + \frac{\sigma_e^2}{\chi_1^2 \text{var}(\psi_t)}\right)^{-1/2},$$

which declines to zero as $\beta\rho \to 1$, since in this limit $\frac{\sigma_e^2}{\chi_1^2 \text{var}(\psi_t)} \to \infty$, as we proved in Proposition 1. Indeed, our derivations show that the expected Carry trade return is proportional to the volatility of the UIP shock, $\chi_1 \psi_t$, while the standard deviation of the return increases additionally with the volatility of the exchange rate, which in the limit becomes arbitrarily larger than the volatility of the UIP shock. ∎

A.3.3 Engel decomposition and Balassa-Samuelson effect

Engel (1999) decomposes $q_t = q_t^T + q_t^N$ into the tradable RER q_t^T and the relative price of non-tradables to tradables in the two countries q_t^N, and shows that q_t^T dominates the variance decomposition of q_t at all horizons. Our model reproduces this pattern. Since our model does not incorporate any asymmetry between domestically produced tradables and non-tradables (i.e., no Balassa-Samuelson force), the price index for non-tradables at home is $p_{Nt} = p_{Ht}$, i.e. the same as the price of domestically produced tradables shipped to the domestic market. Assuming ω is the share of tradable sectors in expenditure, we have:

$$p_t = \omega p_{Tt} + (1-\omega) p_{Nt} \qquad \Rightarrow \qquad p_{Tt} = \tfrac{1}{\omega}[p_t - (1-\omega)p_{Ht}] = \tfrac{\gamma}{\omega} p_{Ft} + \big(1 - \tfrac{\gamma}{\omega}\big) p_{Ht},$$

where we used the definition of p_t in (8). Note that our model does not need to take a stand whether γ is due to home bias in tradables ($\omega = 1$), or due to non-tradables ($\omega = \gamma$), or a combination of the two ($\gamma < \omega < 1$, where $1-\omega$ is the expenditure share of non-tradables and γ/ω is the home bias in the tradable sector). For concreteness, and in line with the empirical patterns in Engel (1999), we assume that $\omega \approx 1/2$, so that $\gamma \ll \omega$,

[54] Note that in a symmetric steady state $\bar{R} = \bar{R}^* = 1/\beta$, and there is no low or high interest rate bond in the long run, yet over time the relative interest rates on bonds fluctuate according to (41), allowing for a temporary Carry trade (or 'forward premium trade' in the terminology of Hassan and Mano 2014).

and there exists a considerable home bias in tradables (on the role of the local distribution margin in tradable prices, see e.g. Burstein, Eichenbaum, and Rebelo 2005).

The tradable RER is defined as $q_t^T = p_{Tt}^* + e_t - p_{Tt}$. We then calculate the non-tradable RER:[55]

$$
\begin{aligned}
q_t^N \equiv q_t - q_t^T &= (1-\omega)\big[(p_{Nt}^* - p_{Tt}^*) - (p_{Nt} - p_{Tt})\big] \\
&= \gamma\tfrac{1-\omega}{\omega}\big[(p_{Ft}^* - p_{Ht}^*) - (p_{Ht} - p_{Ft})\big] = \gamma\tfrac{1-\omega}{\omega}\big[q_t^P + s_t\big] = \tfrac{1-\omega}{\omega}(1-\alpha)\tfrac{2\gamma}{1-2\gamma}q_t
\end{aligned}
$$

Therefore, the contribution of the non-tradable component to the volatility of RER is:

$$
\frac{\operatorname{cov}(\Delta q_t^N, \Delta q_t)}{\operatorname{var}(\Delta q_t)} = \frac{1-\omega}{\omega}(1-\alpha)\frac{2\gamma}{1-2\gamma},
$$

which is small whenever γ is small, even if $\alpha = 0$. LOP deviations, due to PTM ($\alpha > 0$) or LCP, further reduce the contribution of the non-tradable component, but are conceptually not necessary to replicate the patterns documented in Engel (1999) and Betts and Kehoe (2008), as home bias in tradables is sufficient.

It is straightforward to extend the model and allow for the relative tradable-nontradable productivity shocks in order to address the evidence on the Balassa-Samuelson effect (see Rogoff 1996). In the data, Balassa-Samuelson forces are difficult to detect under a floating exchange rate regime, yet they become considerably more pronounced under a peg (see Mendoza 2005, Berka, Devereux, and Engel 2018). This is exactly in line with the prediction of the disconnect model, as we explore in Itskhoki and Mukhin (2019), where we use the model to study a switch from a peg to a float. Intuitively, to the extent that relative productivity shocks are considerably less volatile than the nominal exchange rate under a float, Balassa-Samuelson effect would be difficult to detect, yet when nominal exchange rate volatility is switched off under a peg, the relative productivity shocks become important drivers of the (much less volatile) real exchange rate. To summarize, the seminal evidence in Engel (1999) should not be interpreted as against the importance of nontradables in understanding the real exchange rate, but rather against the importance of (relative) productivity shocks as the key driver of the real exchange rate under a floating regime.

A.4 Nominal rigidities

This section outlines the details of the monetary model with nominal wage and price rigidities. As before, we focus on Home and symmetric relationships hold in Foreign. We consider a standard New Keynesian two-country model in a cashless limit, as described in Galí (2008).

Wages The aggregate labor input is a CES aggregate of individual varieties with elasticity of substitution ϵ, which results in labor demand:

$$
L_{it} = \left(\frac{W_{it}}{W_t}\right)^{-\epsilon} L_t, \quad \text{where} \quad L_t = \left(\int L_{it}^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i\right)^{\frac{\epsilon}{\epsilon-1}} \quad \text{and} \quad W_t = \left(\int W_{it}^{1-\epsilon} \mathrm{d}i\right)^{\frac{1}{1-\epsilon}},
$$

and the rest of the model production structure is unchanged. Households set wages à la Calvo and supply as much labor as demanded at a given wage rate. The probability of changing wage in the next period is $1 - \lambda_w$. The first-order condition for wage setting is:

$$
\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\lambda_w)^{s-t} \frac{C_s^{-\sigma}}{P_s} W_s^{\epsilon} L_s \left(\bar{W}_t^{1+\epsilon/\nu} - \frac{\kappa\epsilon}{\epsilon-1} P_s C_s^{\sigma} L_s^{1/\nu} W_s^{\epsilon/\nu}\right) = 0.
$$

[55] Similarly, we can evaluate the tradable component:

$$
q_t^T = p_{Tt}^* + e_t - p_{Tt} = \tfrac{\gamma}{\omega}p_{Ht}^* + \big(1-\tfrac{\gamma}{\omega}\big)p_{Ft}^* + e_t - \tfrac{\gamma}{\omega}p_{Ft} - \big(1-\tfrac{\gamma}{\omega}\big)p_{Ht} = \big(1-\tfrac{\gamma}{\omega}\big)q_t^P - \tfrac{\gamma}{\omega}s_t.
$$

Substituting in labor demand and log-linearizing, we obtain:

$$\hat{w}_t = \frac{1-\beta\lambda_w}{1+\epsilon/\nu}\left(\sigma c_t + \frac{1}{\nu}\ell_t + p_t + \frac{\epsilon}{\nu}w_t\right) + \beta\lambda_w \mathbb{E}_t \hat{w}_{t+1},$$

where $\hat{w}_t$ denotes the log deviation from the steady state of the wage rate reset at t. Note that the wage inflation can be expressed as $\pi_t^w \equiv \Delta w_t = (1-\lambda_w)(\hat{w}_t - w_{t-1})$. Aggregate wages using these equalities and express the wage process in terms of cross-country differences to obtain the NKPC for wages:

$$\tilde{\pi}_t^w = k_w\left[\sigma\tilde{c}_t + \frac{1}{\nu}\tilde{\ell}_t + \tilde{p}_t - \tilde{w}_t\right] + \beta\mathbb{E}_t\tilde{\pi}_{t+1}^w,$$

where $k_w = \frac{(1-\beta\lambda_w)(1-\lambda_w)}{\lambda_w(1+\epsilon/\nu)}$.

Prices We assume that firms set prices à la Calvo with a probability of changing price next period equal to $1-\lambda_p$. There are two Phillips curves, one for domestic sales and one for exports. The first-order conditions for reset prices in log-linearized form are

$$\hat{p}_{Ht} = (1-\beta\lambda_p)\,\mathbb{E}_t \sum\nolimits_{j=t}^{\infty} (\beta\lambda_p)^{j-t}\left[(1-\alpha)\,mc_j + \alpha p_j\right],$$
$$\hat{p}_{Ht}^* = (1-\beta\lambda_p)\,\mathbb{E}_t \sum\nolimits_{j=t}^{\infty} (\beta\lambda_p)^{j-t}\left[(1-\alpha)\left(mc_j - (1-\iota)e_j\right) + \alpha(p_j^* + \iota e_j)\right],$$

where $mc_t = (1-\phi)\left(\vartheta r_t^K + (1-\vartheta)w_t - a_t\right) + \phi p_t$ are the marginal costs of Home firms, and $\iota \in \{0,1\}$ with $\iota = 1$ corresponding to the case of PCP (producer currency pricing) and $\iota = 0$ to the case of LCP (local currency pricing). The optimal reset prices are equal to an expected discounted sum of future optimal static prices, which in turn are a weighted average of marginal costs and competitor prices. Given the law of motion for home prices, $\pi_{Ht} = (1-\lambda_p)(\hat{p}_{Ht} - p_{Ht-1}) = \frac{1-\lambda_p}{\lambda_p}(\hat{p}_{Ht} - p_{Ht})$, the resulting NKPC is:

$$\pi_{Ht} = k_p\Big[(1-\alpha)mc_t + \alpha p_t - p_{Ht}\Big] + \beta\mathbb{E}_t\pi_{Ht+1}, \quad \text{where} \quad k_p = \frac{(1-\beta\lambda_p)(1-\lambda_p)}{\lambda_p}.$$

The law of motion for export prices depends on the currency of invoicing. Under LCP, the law of motion for prices is, $\pi_{Ht}^* = \frac{1-\lambda_p}{\lambda_p}(\hat{p}_{Ht}^* - p_{Ht}^*)$, and the export NKPC can be expressed as:

$$\pi_{Ht}^* = k_p\Big[(1-\alpha)(mc_t - e_t) + \alpha p_t^* - p_{Ht}^*\Big] + \beta\mathbb{E}_t\pi_{Ht+1}^*.$$

Under PCP, the law of motion for the export price index, $\pi_{Ht}^* = \frac{1-\lambda_p}{\lambda_p}(\hat{p}_{Ht}^* - p_{Ht}^*) - \Delta e_t$, implies that the NKPC is

$$(\pi_{Ht}^* + \Delta e_t) = k_p\Big[(1-\alpha)mc_t + \alpha(p_t^* + e_t) - (p_{Ht}^* + e_t)\Big] + \beta\mathbb{E}_t\left(\pi_{Ht+1}^* + \Delta e_{t+1}\right).$$

Finally, notice that the DCP case with all international trade invoiced in Foreign currency can be expressed as a mix of the two other regimes — Home exporters use LCP and Foreign exporters use PCP. The rest of the model is as described in Section 2.1.

A.5 Data appendix

As explained in Section 4, for comparability, we estimate most empirical moments in Tables 2 and 4 following CKM. In particular, we use their quarterly data from 1973–1994 available on Ellen McGrattan's website and estimate the moments for the U.S. against the PPP-weighted sum of France, Germany, Italy and the U.K. While we use the first differences of the variables rather than the HP-filtered series, our estimates are very close to the ones in CKM.

As a robustness exercise, we also collected quarterly data for a longer time period from 1981–2017 from FRED database. In particular, we use seasonally-adjusted GDP, consumption and gross capital formation in constant prices and aggregate them across the same countries using PPP-adjusted nominal GDP in 2000 (from OECD database). Table A2 shows that the estimates for the two periods are very similar. The only exception are the moments for net exports in Table 4: in contrast to CKM, who focus on bilateral trade between the U.S. and the European countries, we use total imports and exports of the U.S. Indeed, the former includes only a part of international trade of the U.S. and underestimates the openness of the U.S. economy.

The moments in Table 1 are calculated without Italy given that its quarterly PPI starts only in 1991. We use seasonally-adjusted hourly earnings in manufacturing as a proxy for nominal wages to compute the wage-based real exchange rate. Given the limited availability of the terms-of-trade data at the quarterly frequency, we use the estimates for terms-of-trade moments from Obstfeld and Rogoff (2001) and Gopinath et al. (2018). We also compute the same moments using FRED annual data on import and export price indices for the U.S. For our baseline period from 1973–1994, we obtain $\sigma(\Delta s)/\sigma(\Delta e) = 0.29$ and $\mathrm{corr}(\Delta s, \Delta e) = 0.20$, in line with the estimates in the literature (see Tables 1 and 4).

Finally, we borrow several financial moments from the previous literature. The slope coefficient β and R^2 in the Fama regression are from the survey by Engel (1996) and recent estimates by Burnside, Han, Hirshleifer, and Wang (2011, Table 1) and Valchev (2016, Table B.1). The estimates for the Sharpe ratio correspond to the forward premium trade from Hassan and Mano (2014, Table 2). We estimate the volatility and persistence of the interest rates using quarterly data for the U.S. versus the U.K., France, Germany and Japan from 1979–2009.

References

Adrian, T., E. Etula, and H. S. Shin (2015): "Risk appetite and exchange rates," Staff Reports 750, Federal Reserve Bank of New York.

Aguiar, M., and G. Gopinath (2007): "Emerging Market Business Cycles: The Cycle Is the Trend," *Journal of Political Economy*, 115, 69–102.

Alvarez, F., A. Atkeson, and P. J. Kehoe (2007): "If Exchange Rates are Random Walks, Then Almost Everything We Say About Monetary Policy is Wrong," *American Economic Review*, 97(2), 339–345.

——— (2009): "Time-Varying Risk, Interest Rates, and Exchange Rates in General Equilibrium," *Review of Economic Studies*, 76(3), 851–878.

Amiti, M., O. Itskhoki, and J. Konings (2018): "Dominant Currencies: How firms choose currency and why it matters," https://scholar.princeton.edu/itskhoki/pages/papers.

——— (2019): "International Shocks, Variable Markups and Domestic Prices," *Review of Economic Studies*, forthcoming.

Atkeson, A., and A. T. Burstein (2008): "Trade Costs, Pricing-to-Market, and International Relative Prices," *American Economic Review*, 98(5), 1998–2031.

Auer, R., A. T. Burstein, and S. Lein (2018): "Exchange Rates and Prices: Evidence from the 2015 Swiss Franc Appreciation," http://www.econ.ucla.edu/arielb/.

Ayres, J., C. Hevia, and J. P. Nicolini (2019): "Real Exchange Rates and Primary Commodity Prices," Federal Reserve Bank of Minneapolis Staff Report No. 584.

Bacchetta, P., and E. van Wincoop (2006): "Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?," *American Economic Review*, 96(3), 552–576.

——— (2010): "Infrequent Portfolio Decisions: A Solution to the Forward Discount Puzzle," *American Economic Review*, 100(3), 870–904.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1992): "International Real Business Cycles," *Journal of Political Economy*, 100(4), 745–75.

——— (1994): "Dynamics of the Trade Balance and the Terms of Trade: The J-Curve?," *American Economic Review*, 84(1), 84–103.

Backus, D. K., and G. W. Smith (1993): "Consumption and real exchange rates in dynamic economies with non-traded goods," *Journal of International Economics*, 35(3–4), 297–316.

Bansal, R., and M. Dahlquist (2000): "The forward premium puzzle: different tales from developed and emerging economies," *Journal of International Economics*, 51(1), 115–144.

Benigno, G., and C. Thoenissen (2008): "Consumption and real exchange rates with incomplete markets and non-traded goods," *Journal of International Money and Finance*, 27(6), 926–948.

Berka, M., M. B. Devereux, and C. Engel (2018): "Real Exchange Rates and Sectoral Productivity in the Eurozone," *American Economic Review*, 108(6), 1543–81.

Bernard, A. B., J. B. Jensen, S. J. Redding, and P. K. Schott (2009): "The Margins of US Trade," *American Economic Review*, 99(2), 487–93.

Betts, C. M., and T. J. Kehoe (2008): "Real Exchange Rate Movements and the Relative Price of Non-traded Goods," NBER Working Papers No. 14437.

Blanchard, O. J., and C. M. Kahn (1980): "The Solution of Linear Difference Models under Rational Expectations," *Econometrica*, 48(5), 1305–1311.

Blanco, A., and J. Cravino (2018): "Price Rigidities and Relative PPP," NBER Working Papers No. 24655.

Burnside, C., B. Han, D. Hirshleifer, and T. Y. Wang (2011): "Investor Overconfidence and the Forward Premium Puzzle," *Review of Economic Studies*, 78(2), 523–558.

Burstein, A. T., M. Eichenbaum, and S. Rebelo (2005): "Large Devaluations and the Real Exchange Rate," *Journal of Political Economy*, 113(4), 742–784.

Burstein, A. T., and G. Gopinath (2012): "International Prices and Exchange Rates," in *Handbook of International Economics*, ed. by G. Gopinath, E. Helpman, and K. Rogoff, vol. IV.

Caballero, R. J., E. Farhi, and P.-O. Gourinchas (2008): "An Equilibrium Model of "Global Imbalances" and Low Interest Rates," *American Economic Review*, 98(1), 358–393.

Camanho, N., H. Hau, and H. Rey (2018): "Global Portfolio Rebalancing and Exchange Rates," http://www.helenerey.eu/.

Campbell, J. Y., and L. M. Viceira (2002): *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors.* Oxford University Press.

Carvalho, C., and F. Nechio (2011): "Aggregation and the PPP Puzzle in a Sticky-Price Model," *American Economic Review*, 101(6), 2391–2424.

Casas, C., F. J. Díez, G. Gopinath, and P.-O. Gourinchas (2016): "Dominant Currency Paradigm: A New Model for Small Open Economies," https://scholar.harvard.edu/gopinath.

Chari, V., P. J. Kehoe, and E. R. McGrattan (2002): "Can Sticky Price Models Generate Volatile and Persistent Exchange Rates?," *Review of Economic Studies*, 69(3), 533–63.

Chen, Y.-c., and K. Rogoff (2003): "Commodity currencies," *Journal of International Economics*, 60(1), 133–160.

Clarida, R., J. Galí, and M. Gertler (2000): "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics*, 115(1), 147–180.

——— (2002): "A simple framework for international monetary policy analysis," *Journal of Monetary Economics*, 49(5), 879–904.

Colacito, R., and M. M. Croce (2013): "International Asset Pricing with Recursive Preferences," *Journal of Finance*, 68(6), 2651–2686.

Corsetti, G., L. Dedola, and S. Leduc (2008): "International Risk Sharing and the Transmission of Productivity Shocks," *Review of Economic Studies*, 75(2), 443–473.

——— (2010): "Optimal Monetary Policy in Open Economies," in *Handbook of Monetary Economics*, ed. by B. M. Friedman, and M. Woodford, vol. 3, chap. 16, pp. 861–933. Elsevier.

De Long, J., A. Shleifer, L. H. Summers, and R. J. Waldmann (1990): "Noise Trader Risk in Financial Markets," *Journal of Political Economy*, 98(4), 703–738.

Devereux, M. B., and C. Engel (2002): "Exchange rate pass-through, exchange rate volatility, and exchange rate disconnect," *Journal of Monetary Economics*, 49(5), 913–940.

——— (2003): "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange Rate Flexibility," *Review of Economic Studies*, 70, 765–84.

Devereux, M. B., and A. Sutherland (2011): "Country Portfolios In Open Economy Macro?Models," *Journal of the European Economic Association*, 9(2), 337–369.

Dornbusch, R. (1976): "Expectations and Exchange Rate Dynamics," *Journal of Political Economy*, 84(6), 1161–76.

Du, W., A. Tepper, and A. Verdelhan (2018): "Deviations from Covered Interest Rate Parity," *The Journal of Finance*, 73(3), 915–957.

Eaton, J., S. S. Kortum, and B. Neiman (2015): "Obstfeld and Rogoff's International Macro Puzzles: A Quantitative Assessment," NBER Working Paper No. 21774.

Egorov, K., and D. Mukhin (2019): "Optimal Monetary Policy under Dollar Pricing," https://sites.google.com/site/dmitry0mukhin/research.

Eichenbaum, M., and C. L. Evans (1995): "Some Empirical Evidence on the Effects of Shocks to Monetary Policy on Exchange Rates," *The Quarterly Journal of Economics*, 110(4), 975–1009.

Eichenbaum, M., B. K. Johannsen, and S. Rebelo (2018): "Monetary Policy and the Predictability of Nominal Exchange Rates," http://www.kellogg.northwestern.edu/faculty/rebelo/htm/papers.html.

Engel, C. (1996): "The forward discount anomaly and the risk premium: A survey of recent evidence," *Journal of Empirical Finance*, 3(2), 123–192.

——— (1999): "Accounting for U.S. Real Exchange Rate Changes," *Journal of Political Economy*, 107(3), 507–538.

——— (2016): "Exchange Rates, Interest Rates, and the Risk Premium," *American Economic Review*, 106(2), 436–474.

Engel, C., and K. D. West (2005): "Exchange Rates and Fundamentals," *Journal of Political Economy*, 113(3), 485–517.

Engel, C., and S. P. Y. Wu (2019): "Liquidity and Exchange Rates: An Empirical Investigation," https://www.ssc.wisc.edu/~cengel/working_papers.htm.

Evans, M. D. D., and R. K. Lyons (2002): "Order Flow and Exchange Rate Dynamics," *Journal of Political Economy*, 110(1), 170–180.

Fama, E. F. (1984): "Forward and spot exchange rates," *Journal of Monetary Economics*, 14(3), 319–338.

Farhi, E., and X. Gabaix (2016): "Rare Disasters and Exchange Rates," *Quarterly Journal of Economics*, 131(1), 1–52.

Farhi, E., and I. Werning (2012): "Dealing with the Trilemma: Optimal Capital Controls with Fixed Exchange Rates," http://scholar.harvard.edu/farhi/publications.

Feenstra, R. C., P. Luck, M. Obstfeld, and K. N. Russ (2018): "In Search of the Armington Elasticity," *The Review of Economics and Statistics*, 100(1), 135–150.

Fitzgerald, D., Y. Yedid-Levi, and S. Haller (2019): "Can Sticky Quantities Explain Export Insensitivity to Exchange Rates?," https://doireann.com/papers/.

Friedman, M. (1953): "The Case for Flexible Exchange Rates," *Essays in Positive Economics.*

Gabaix, X., and M. Maggiori (2015): "International Liquidity and Exchange Rate Dynamics," *The Quarterly Journal of Economics*, 130(3), 1369–1420.

Galí, J. (2008): *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework.* Princeton University Press.

Gopinath, G. (2016): "The International Price System," *Jackson Hole Symposium Proceedings.*

Gopinath, G., E. Boz, C. Casas, F. J. Díez, P.-O. Gourinchas, and M. Plagborg-Møller (2018): "Dominant Currency Paradigm," https://scholar.princeton.edu/mikkelpm/publications.

Gopinath, G., and O. Itskhoki (2010): "Frequency of Price Adjustment and Pass-through," *Quarterly Journal of Economics*, 125(2), 675–727.

——— (2011): "In Search of Real Rigdities," in *NBER Macroeconomics Annual 2010*, vol. 25, pp. 261–310.

Gourinchas, P.-O., and A. Tornell (2004): "Exchange rate puzzles and distorted beliefs," *Journal of International Economics*, 64(2), 303–333.

Guillén, D. A. (2013): "Financial Wedges, Frictions and Open Economy Puzzles," Ph.D. thesis, Princeton University.

Hansen, L. P., and J. Miao (2018): "Aversion to ambiguity and model misspecification in dynamic stochastic environments," *Proceedings of the National Academy of Sciences*, 115(37), 9163–9168.

Hansen, L. P., and T. J. Sargent (2011): "Robustness and ambiguity in continuous time," *Journal of Economic Theory*, 146(3), 1195–1223.

Hassan, T. A., and R. C. Mano (2014): "Forward and Spot Exchange Rates in a Multi-currency World," NBER Working Paper No. 20294.

Hau, H. (2002): "Real Exchange Rate Volatility and Economic Openness: Theory and Evidence," *Journal of Money, Credit and Banking*, 34(3), 611–630.

Heathcote, J., and F. Perri (2013): "The International Diversification Puzzle Is Not as Bad as You Think," *Journal of Political Economy*, 121(6), 1108–159.

Imbs, J., H. Mumtaz, M. O. Ravn, and H. Rey (2005): "PPP Strikes Back: Aggregation And the Real Exchange Rate," *The Quarterly Journal of Economics*, 120(1), 1–43.

Itskhoki, O., and D. Mukhin (2017): "Exchange Rate Disconnect in General Equilibrium," NBER Working Paper No. 23401.

——— (2019): "Mussa Puzzle Redux," https://scholar.princeton.edu/itskhoki/pages/papers.

Jeanne, O., and A. K. Rose (2002): "Noise Trading and Exchange Rate Regimes," *The Quarterly Journal of Economics*, 117(2), 537–569.

Jiang, Z., A. Krishnamurthy, and H. N. Lustig (2018): "Foreign Safe Asset Demand for U.S. Treasurys and the Dollar," https://people.stanford.edu/hlustig/.

Karabarbounis, L. (2014): "Home production, labor wedges, and international business cycles," *Journal of Monetary Economics*, 64(C), 68–84.

Kehoe, P. J., and V. Midrigan (2008): "Sticky Prices and Real Exchange Rates in the Cross-Section," http://www.virgiliumidrigan.com/.

Kehoe, P. J., and F. Perri (2002): "International Business Cycles with Endogenous Incomplete Markets," *Econometrica*, 70(3), 907–928.

Kim, S., and N. Roubini (2000): "Exchange rate anomalies in the industrial countries: A solution with a structural VAR approach," *Journal of Monetary economics*, 45(3), 561–586.

Kimball, M. (1995): "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of Money, Credit and Banking*, 27, 1241–77.

Klenow, P. J., and J. L. Willis (2016): "Real Rigidities and Nominal Price Changes," *Economica*, 83(331), 443–472.

Kocherlakota, N. R., and L. Pistaferri (2007): "Household Heterogeneity and Real Exchange Rates," *Economic Journal*, 117(519), C1–C25.

Koijen, R., and M. Yogo (2019): "Exchange Rates and Asset Prices in a Global Demand System," https://sites.google.com/site/motohiroyogo/research.

Kollmann, R. (1995): "Consumption, real exchange rates and the structure of international asset markets," *Journal of International Money and Finance*, 14(2), 191–211.

——— (2001): "The exchange rate in a dynamic-optimizing business cycle model with nominal rigidities: a quantitative investigation," *Journal of International Economics*, 55(2), 243–262.

——— (2005): "Macroeconomic effects of nominal exchange rate regimes: new insights into the role of price dynamics," *Journal of International Money and Finance*, 24(2), 275–292.

Koopman, R., Z. Wang, and S.-J. Wei (2014): "Tracing Value-Added and Double Counting in Gross Exports," *American Economic Review*, 104(2), 459–94.

Kouri, P. (1976): "Capital Flows and the Dynamics of the Exchange Rate," Stockholm, Institute for International Economic Studies, Seminar Paper 67.

——— (1983): "Balance of Payments and the Foreign Exchange Market: A Dynamic Partial Equilibrium Model," in *Economic Interdependence and Flexible Exchange Rates*, ed. by J. S. Bhandari, and B. H. Putnam. MIT Press.

Lustig, H. N., A. Stathopoulos, and A. Verdelhan (2016): "Nominal Exchange Rate Stationarity and Long-Term Bond Returns," https://people.stanford.edu/hlustig/.

Lustig, H. N., and A. Verdelhan (2011): "The Cross-Section of Foreign Currency Risk Premia and Consumption Growth Risk: Reply," *American Economic Review*, 101(7), 3477–3500.

——— (2018): "Does Incomplete Spanning in International Financial Markets Help to Explain Exchange Rates?," *American Economic Review*, forthcoming.

Meese, R., and K. Rogoff (1983): "Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?," *Journal of International Economics*, 14(1), 3–24.

Mendoza, E. G. (2005): "Real Exchange Rate Volatility and the Price of Nontradable Goods in Economies Prone to Sudden Stops," *Economía*, 6(1), 103–48.

Mukhin, D. (2017): "An Equilibrium Model of the International Price System," https://sites.google.com/site/dmitry0mukhin/research.

Müller, U. K., and M. W. Watson (2018): "Long-Run Covariability," *Econometrica*, 86(3), 775–804.

——— (2019): "Low-Frequency Analysis of Economic Time Series," work in progress.

Mussa, M. L. (1986): "Nominal exchange rate regimes and the behavior of real exchange rates: Evidence and implications," *Carnegie-Rochester Conference Series on Public Policy*, 25(1), 117–214.

Nakamura, E., and J. Steinsson (2018): "Identification in macroeconomics," *Journal of Economic Perspectives*, 32(3), 59–86.

Obstfeld, M., and K. Rogoff (1995): "Exchange Rate Dynamics Redux," *Journal of Political Economy*, 103, 624–60.

——— (2000): "New Directions for Stochastic Open Economy Models," *Journal of International Economics*, 50, 117–153.

——— (2001): "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?," in *NBER Macroeconomics Annual 2000*, vol. 15, pp. 339–390.

Pavlova, A., and R. Rigobon (2008): "The Role of Portfolio Constraints in the International Propagation of Shocks," *Review of Economic Studies*, 75(4), 1215–1256.

Reinhart, C. M., and K. S. Rogoff (2004): "The Modern History of Exchange Rate Arrangements: A Reinterpretation," *The Quarterly Journal of Economics*, 119(1), 1–48.

Rey, H. (2013): "Dilemma not trilemma: the global financial cycle and monetary policy independence," *Jackson Hole Symposium Proceedings*.

Rogoff, K. (1996): "The Purchasing Power Parity Puzzle," *Journal of Economic Literature*, 34, 647–668.

Schmitt-Grohé, S., and M. Uribe (2003): "Closing small open economy models," *Journal of International Economics*, 61(1), 163–185.

——— (2018): "Exchange rates and uncovered interest differentials: The role of permanent monetary shocks," NBER Working Paper No. 25380.

Scholl, A., and H. Uhlig (2008): "New evidence on the puzzles: Results from agnostic identification on monetary policy and exchange rates," *Journal of International Economics*, 76(1), 1–13.

Stavrakeva, V., and J. Tang (2015): "Exchange Rates and Monetary Policy," https://sites.google.com/site/vstavrakeva/research.

Tille, C., and E. van Wincoop (2010): "International capital flows," *Journal of International Economics*, 80(2), 157–175.

Valchev, R. (2016): "Bond Convenience Yields and Exchange Rate Dynamics," https://sites.google.com/site/valchevr/research.

Verdelhan, A. (2010): "A Habit-Based Explanation of the Exchange Rate Risk Premium," *Journal of Finance*, 65(1), 123–146.