Exchange Rate Disconnect in General Equilibrium

Oleg Itskhoki
itskhoki@Princeton.edu

Dmitry Mukhin
mukhin@Princeton.edu

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Motivation

- **Exchange Rate Disconnect (ERD)** is one of the most pervasive and challenging puzzles in macroeconomics
  - exchange rates are present in all international macro models
  - yet, we do not have a satisfactory theory of exchange rates
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  - yet, we do not have a satisfactory theory of exchange rates

- Broader ERD combines five exchange-rate-related puzzles:
  1. **Meese-Rogoff (1983) puzzle**
     - NER follows a volatile RW, uncorrelated with macro fundamentals
  2. **PPP puzzle** (Rogoff 1996)
     - RER is as volatile and persistent as NER, and the two are nearly indistinguishable at most horizons (also related Mussa puzzle)
     - LOP violations for tradables account for nearly all RER dynamics
     - ToT is three times less volatile than RER
  4. **Backus-Smith (1993) puzzle**
     - Consumption is high when prices are high (RER appreciated)
     - Consumption is five times less volatile than RER
  5. **Forward-premium puzzle** (Fama 1984)
     - High interest rates predict nominal appreciations (UIP violations)
**Motivation**

GBP/USD (GBPUSD=X) 1.3304 -0.0047 (-0.3499%) As of 10:16 AM EDT. CCY Delayed Price. Market open.
Our Approach

• The literature has tried to address one puzzle at a time, often at the expense of aggravating the other puzzles

• We provide a unifying theory of exchange rates, capturing simultaneously all stylized facts about their properties
Our Approach

• The literature has tried to address one puzzle at a time, often at the expense of aggravating the other puzzles.

• We provide a unifying theory of exchange rates, capturing simultaneously all stylized facts about their properties.

• A theory of exchange rate (disconnect) must specify:
  1. The **exogenous shock process** driving the exchange rate
     — little empirical guidance here
     — we prove theoretically that only the **financial shock** is a likely candidate and then show its quantitative performance.
  2. The **transmission mechanism** muting the response of the macro variables to exchange rate movements relies on:
     a) **strategic complementarities** in price setting resulting in PTM
     b) **weak substitutability** between home and foreign goods
     c) **home bias** in consumption
     d) **monetary policy rule** stabilizing domestic inflation
     — all admitting tight empirical discipline
     → **nominal rigidities** are not essential.
Contributions

• A dynamic general equilibrium model of exchange rate
  — fully analytically tractable, yet quantitative

• Four new mechanisms:
  1. Equilibrium exchange rate determination and dynamics
     (cf. Engel and West 2005)
  2. PPP puzzle and related puzzles
     (Rogoff ’96, CKM ’02, Kehoe and Midrigan ’08, Monacelli ’04)
  3. Backus-Smith puzzle
     (cf. Corsetti, Dedola and Leduc 2008)
  4. Forward premium puzzle
     (Engel 2016)
MODELING FRAMEWORK
Model setup

- Two countries: home (Europe) and foreign (US, denoted w/\(\ast\))
- Nominal wages \(W_t\) in euros and \(W_t^\ast\) in dollars, the numeraires
- \(E_t\) is the nominal exchange rate (price of one dollar in euros)
- Baseline model:
  - representative households
  - representative firms
  - one internationally-traded foreign-bond
- We allow for all possible shocks/CKM-style wedges:
  \[\Omega_t = (w_t, \chi_t, \kappa_t, a_t, g_t, \mu_t, \eta_t, \xi_t, \psi_t)\]
  and foreign counterparts
Equilibrium conditions

① Households:
   (i) labor supply and asset demand
   (ii) expenditure on home and foreign good
       — $\gamma$ expenditure share on foreign goods
       — $\theta$ elasticity of substitution between home and foreign goods

② Firms:
   (i) production and profits
   (ii) price setting
       — $\alpha$ strategic complementarity elasticity in price setting

③ Government: balanced budget

④ Foreign: symmetric

⑤ GE: market clearing and country budget constraint
DISCONNECT IN THE LIMIT
Disconnect in the Autarky Limit

- Consider an economy in **autarky** = complete ER disconnect
  1. NER is not determined and can take any value
  2. this has no effect on domestic quantities, prices or interest rates
  3. as price levels are determined independently from NER, RER moves one-to-one with NER

+ the further from autarky, the less likely the disconnect
Disconnect in the Autarky Limit

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- **Definition**: Exchange rate disconnect in the autarky limit

\[ \lim_{\gamma \to 0} \frac{dZ_{t+j}}{d\varepsilon_t} = 0 \quad \forall j \quad \text{and} \quad \lim_{\gamma \to 0} \frac{d\varepsilon_t}{d\varepsilon_t} \neq 0. \]
Disconnect in the Autarky Limit

- Consider an economy in \textit{autarky} = complete ER disconnect
  
  (i) NER is not determined and can take any value
  
  (ii) this has no effect on domestic quantities, prices or interest rates
  
  (iii) as price levels are determined independently from NER, RER moves one-to-one with NER

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\textbf{Definition}: Exchange rate disconnect in the autarky limit

\[
\lim_{\gamma \to 0} \frac{dZ_{t+j}}{d\varepsilon_t} = 0 \quad \forall j \quad \text{and} \quad \lim_{\gamma \to 0} \frac{d\varepsilon_t}{d\varepsilon_t} \neq 0.
\]

- \textbf{Proposition 1}: The model cannot exhibit exchange rate disconnect in the limit with zero weight on:
  
  (i) LOP deviation shocks: \( \eta_t \)
  
  (ii) Foreign-good demand shocks: \( \xi_t \)
  
  (iii) Financial (international asset demand) shocks: \( \psi_t \)

- A pessimistic result for IRBC and NOEM models
Admissible Shocks

- Intuition: two international conditions
  - risk sharing:
    \[ \mathbb{E}_t \left\{ R_{t+1}^* \left[ \Theta_{t+1}^* - \Theta_{t+1} \frac{\xi_{t+1}}{\xi_t} e^{\psi_t} \right] \right\} = 0 \]
  - budget constraint:
    \[ B_{t+1}^* - R^* B_t^* = NX^* (Q_t; \eta_t, \xi_t) \]

- In the limit, shocks to these conditions have a vanishingly small effect, while other shocks still have a direct effect.
Admissible Shocks

- Intuition: two international conditions
  - Risk sharing: $\mathbb{E}_t \left\{ R_{t+1}^* \left[ \Theta_{t+1}^* - \Theta_{t+1} \frac{\xi_{t+1}}{\xi_t} e^{\psi_t} \right] \right\} = 0$
  - Budget constraint: $B_{t+1}^* - R_t^* B_t^* = NX^*(Q_t; \eta_t, \xi_t)$

- In the limit, shocks to these conditions have a vanishingly small effect, while other shocks still have a direct effect

- **Proposition 2**: In the autarky limit, $\psi_t$ is the only shock that simultaneously and robustly produces:
  
  (i) positively correlated ToT and RER (Obstfeld-Rogoff moment)
  (ii) negatively correlated relative consumption growth and real exchange rate depreciations (Backus-Smith correlation)
  (iii) deviations from the UIP (negative Fama coefficient).

$\Rightarrow$ $\psi_t$ is the prime candidate shock for a **quantitative** model of ER disconnect
BASELINE MODEL
OF EXCHANGE RATE DISCONNECT
1 Financial exchange rate shock $\psi_t$ only:

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \psi_t$$

— persistent ($\rho \lesssim 1$, e.g. $\rho = 0.97$) w/small innovations ($\sigma_\varepsilon \gtrsim 0$):

$$\psi_t = \rho \psi_{t-1} + \varepsilon_t, \quad \beta \rho < 1$$

— important limiting case: $\beta \rho \to 1$
Ingredients

1 Financial exchange rate shock $\psi_t$ only:

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \psi_t$$

— persistent ($\rho \lesssim 1$, e.g. $\rho = 0.97$) w/small innovations ($\sigma_{\epsilon} \gtrsim 0$):

$$\psi_t = \rho \psi_{t-1} + \varepsilon_t, \quad \beta \rho < 1$$

— important limiting case: $\beta \rho \to 1$

2 Transmission mechanism

(i) Strategic complementarities: $\alpha = 0.4$ (AIK 2015)

(ii) Elasticity of substitution: $\theta = 1.5$ (FLOR 2014)

(iii) Home bias: $\gamma = 0.07 = \frac{1}{2} \frac{\text{Imp+Exp}}{\text{GDP}} \times \frac{\text{GDP}}{\text{Prod-n}}$ (for US, EU, Japan)

- Monetary regime: $W_t \equiv 1$ and $W_t^* \equiv 1$

- Other parameters:

  $$\beta = 0.99, \quad \sigma = 2, \quad \nu = 1, \quad \phi = 0.5, \quad \zeta = 1 - \phi$$
Microfoundations for \( \psi_t \) shock

Risk premium shock: \( \psi_t = i_t - i_t^* - E_t \Delta e_{t+1} \)

1. International asset demand shocks (in the utility function)
   — e.g., Dekle, Jeong and Kiyotaki (2014)

2. Noise trader shocks and limits to arbitrage
   — e.g., Jeanne and Rose (2002)
   - noise traders can be liquidity/safety traders
   - arbitrageurs with downward sloping demand
   - multiple equilibria \( \rightarrow \) Mussa puzzle

3. Heterogenous beliefs or expectation shocks
   — e.g., Bacchetta and van Wincoop (2006)
   - huge volumes of currency trades (also order flows)
   - \( \psi_t \) are disagreement or expectation shocks

4. Financial frictions (e.g., Gabaix and Maggiori 2015)

5. Risk premia models
   (rare disasters, long-run risk, habits, segmented markets)
Roadmap

1. Equilibrium exchange rate dynamics
2. Real and nominal exchange rates
3. Exchange rate and prices
4. Exchange rate and quantities
5. Exchange rate and interest rates
Exchange Rate Dynamics

1. International risk sharing (financial market):

\[
\begin{align*}
  \left( i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \psi_t \right) \ \Rightarrow \ \mathbb{E}_t \Delta e_{t+1} &= - \frac{1}{1 + \gamma \lambda_1} \psi_t \\
  \propto \gamma \psi_t
\end{align*}
\]

2. Flow budget constraint (goods market):

\[
\beta b_{t+1}^* - b_t^* = nx_t, \quad nx_t = \gamma \lambda_2 \cdot e_t
\]
Exchange Rate Dynamics

1. International risk sharing (financial market):

\[ i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \psi_t \quad \Rightarrow \quad \mathbb{E}_t \Delta e_{t+1} = -\frac{1}{1 + \gamma \lambda_1} \psi_t \]

2. Flow budget constraint (goods market):

\[ \beta b_{t+1}^* - b_t^* = n x_t, \quad n x_t = \gamma \lambda_2 \cdot e_t \]

- Solving forward risk sharing (cf. Engel and West 2005):

\[ e_t = \lim_{T \to \infty} \mathbb{E}_t e_{t+T} + \frac{1}{1 + \gamma \lambda_1} \sum_{j=0}^{\infty} \mathbb{E}_t \psi_{t+j} \]

\[ \equiv \mathbb{E}_t e_\infty \]

\[ = \frac{1}{1 - \rho} \psi_t \]

- Intertemporal budget constraint:

\[ b_t^* + \sum_{j=0}^{\infty} \beta^j \cdot \gamma \lambda_2 e_{t+j} = 0 \]
Exchange Rate Dynamics

1. International risk sharing (financial market):
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i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \psi_t \quad \Rightarrow \quad \mathbb{E}_t \Delta e_{t+1} = -\frac{1}{1 + \gamma \lambda_1} \psi_t
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2. Flow budget constraint (goods market):
\[
\beta b_{t+1}^* - b_t^* = n x_t, \quad n x_t = \gamma \lambda_2 \cdot e_t
\]

Proposition

When \( \psi_t \sim AR(1) \), the equilibrium exchange rate follows ARIMA:
\[
\Delta e_t = \rho \Delta e_{t-1} + \frac{1}{1 + \gamma \lambda_1} \frac{\beta}{1 - \beta \rho} \left( \varepsilon_t - \frac{1}{\beta} \varepsilon_{t-1} \right).
\]

This process becomes arbitrary close to a random walk as \( \beta \rho \to 1 \).

— This is the unique equilibrium solution, bubble solutions do not exist
Exchange Rate Dynamics

1. International risk sharing (financial market):

\[
(i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}) \propto \gamma \psi_t \Rightarrow \mathbb{E}_t \Delta e_{t+1} = -\frac{1}{1 + \gamma \lambda_1} \psi_t
\]

2. Flow budget constraint (goods market):

\[
\beta b_{t+1}^* - b_t^* = nx_t, \quad nx_t = \gamma \lambda_2 \cdot e_t
\]

Proposition

When \( \psi_t \sim \text{AR}(1) \), the equilibrium exchange rate follows ARIMA:

\[
(1 - \rho L) \Delta e_t = \frac{1}{1 + \gamma \lambda_1} \frac{\beta}{1 - \beta \rho} \left( 1 - \frac{1}{\beta} L \right) \varepsilon_t.
\]

This process becomes arbitrary close to a random walk as \( \beta \rho \to 1 \).

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Exchange Rate Dynamics

1. International risk sharing (financial market):

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\underbrace{i_t - i_t^\ast - \mathbb{E}_t \Delta e_{t+1}} \propto \gamma \psi_t \Rightarrow \mathbb{E}_t \Delta e_{t+1} = - \frac{1}{1 + \gamma \lambda_1} \psi_t
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\]

This process becomes arbitrary close to a random walk as \( \beta \rho \rightarrow 1 \).

- This is the unique equilibrium solution, bubble solutions do not exist
- NFA \( \Delta b_{t+1}^\ast \sim AR(1) \): \( \Delta b_{t+1}^\ast = \frac{\gamma \lambda_2}{1 + \gamma \lambda_1} \frac{1}{1 - \beta \rho} \psi_t \)
Properties of the Exchange Rate

- **Near-random-walk behavior** (as $\beta \rho \to 1$):
  1. $\text{corr}(\Delta e_{t+1}, \Delta e_t) \to 0$
  2. $\frac{\text{var}(\Delta_k e_{t+k} - \mathbb{E}_t \Delta_k e_{t+k})}{\text{var}(\Delta_k e_{t+k})} \to 1$
  3. $\frac{\text{std}(\Delta e_t)}{\text{std}(\psi_t)} \to \infty$

### Impulse Response

![Impulse Response Graph](image1)

### Variance Decomposition

![Variance Decomposition Graph](image2)
Proposition

*RER and NER are tied together by the following relationship:*

\[ q_t = \frac{1}{1 + \frac{1}{1 - \phi \frac{2\gamma}{1 - 2\gamma}} e_t}. \]

- \((q_t - e_t) \xrightarrow{\gamma \to 0} 0\)
- Relative volatility: \[ \frac{\text{std}(\Delta q_t)}{\text{std}(\Delta e_t)} = \frac{1}{1 + \frac{1}{1 - \phi \frac{2\gamma}{1 - 2\gamma}}} = 0.75 \]
- Heterogenous firms and/or LCP sticky prices further increase volatility of RER
PPP Puzzle

Intuition

• Real exchange rate:

\[ Q_t = \frac{P^*_t \varepsilon_t}{P_t} \]

1. either \( P_t \) and \( P^*_t \) are very sticky (+ monetary shocks); or
2. or economies are very closed, \( \gamma \approx 0 \) (+ \( \psi_t \) shocks)
PPP Puzzle

Intuition

• Real exchange rate:

\[ Q_t = \frac{P^*_t E_t}{P_t} \]

1. either \( P_t \) and \( P^*_t \) are very sticky \((+ \text{ monetary shocks})\); or
2. or economies are very closed, \( \gamma \approx 0 \) \((+ \psi_t \text{ shocks})\)

• Intuition (failure of IRBC and NOEM models):

\[
\begin{align*}
p_t &= (w_t - a_t) + \frac{1}{1-\phi} \frac{\gamma}{1-2\gamma} q_t \\
p^*_t &= (w^*_t - a^*_t) - \frac{1}{1-\phi} \frac{\gamma}{1-2\gamma} q_t
\end{align*}
\]

\[\Rightarrow \quad \left[1 + \frac{1}{1-\phi} \frac{2\gamma}{1-2\gamma}\right] q_t = e_t + (w^*_t - a^*_t) - (w_t - a_t)\]
Exchange Rates and Prices

• Three closely related variables:

\[ Q_t = \frac{P_t^* E_t}{P_t} \quad Q_t^P = \frac{P_F^* E_t}{P_H^*} \quad S_t = \frac{P_F^*}{P_H^* E_t} \]

• Two relationships:

\[ q_t = (1 - \gamma) q_t^P - \gamma s_t \]
\[ s_t = q_t^P - 2\alpha q_t \]

• In the data: \( q_t^P \approx q_t, \text{ std}(\Delta q_t) \gg \text{ std}(\Delta s_t), \text{ corr}(\Delta s_t, \Delta q_t) > 0 \)

• Proposition:

\[ q_t^P = \frac{1 - 2\alpha \gamma}{1 - 2\gamma} q_t \quad \text{and} \quad s_t = \frac{1 - 2\alpha (1 - \gamma)}{1 - 2\gamma} q_t \]

— conventional models with \( \alpha = 0 \) cannot do the trick
— \( \alpha \) needs to be positive, but not too large
Exchange Rates and Prices

\[ \text{var}(s_t) > \text{var}(q_t) \]

\[ \text{corr}(s_t, q_t) < 0 \]

**Figure:** Terms of trade and Real exchange rate
Exchange Rate and Quantities

Backus-Smith puzzle

• “Dismiss” asset market (Backus-Smith) condition:
  \[ \sigma (c_t - c_t^*) = q_t \quad \text{vs.} \quad \mathbb{E}_t \Delta (c_{t+1} - c_{t+1}^* - q_{t+1}) = \psi_t \]
Exchange Rate and Quantities

Backus-Smith puzzle

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• Static relationship between consumption and RER:
  
  (i) labor market clearing: \[ \sigma \tilde{c}_t + \frac{1}{\nu} \tilde{y}_t = -\gamma q_t \]

  (ii) goods market clearing: \[ \tilde{y}_t = (1 - 2\gamma) \tilde{c}_t + 2\theta(1 - \alpha)\gamma q_t \]

• Three alternatives in the literature to get BS puzzle:
  1. Super-persistent (news-like) shocks (CC 2013)
  2. Low elasticity of substitution \( \theta < 1 \) (CDL 2008)
  3. Non-tradable productivity shocks (BT 2008)
Exchange Rate and Quantities

Backus-Smith puzzle

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• Proposition: Static expenditure switching implies:

\[ c_t - c_t^* = -\frac{2\theta(1 - \alpha)(1 - \gamma) + \nu}{(1 - 2\gamma) + \sigma \nu} \frac{2\gamma}{1 - 2\gamma} q_t \]
Exchange Rate and Quantities

Backus-Smith puzzle

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- Static relationship between consumption and RER:
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- Proposition: Static expenditure switching implies:
  \[ c_t - c_t^* = -\frac{2\theta(1 - \alpha)(1 - \gamma)}{(1 - 2\gamma) + \sigma \nu} \frac{2\gamma}{1 - 2\gamma} q_t + \kappa(a_t - a_t^*) \]

- Three alternatives in the literature to get BS puzzle:
  1. Super-persistent (news-like) shocks (CC 2013)
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Exchange Rate and Quantities

Figure: Exchange rate disconnect: relative consumption volatility
Exchange Rate and Interest rates

• Two equilibrium conditions:

\[ \psi_t = (i_t - i_t^*) - \mathbb{E}_t \Delta e_{t+1} \quad \text{and} \quad i_t - i_t^* = -\gamma \lambda_1 \mathbb{E}_t \Delta e_{t+1} \]

Proposition

Fama-regression coefficient:

\[ \mathbb{E}\{\Delta e_{t+1}|i_{t+1} - i_{t+1}^*\} = \beta_F (i_{t+1} - i_{t+1}^*), \quad \beta_F \equiv -\frac{1}{\gamma \lambda_1} < 0. \]

In the limit \( \beta \rho \to 1 \):

(i) Fama-regression \( R^2 \to 0 \)

(ii) \( \text{var}(i_t - i_t^*)/\text{var}(\Delta e_{t+1}) \to 0 \)

(iii) \( \rho(\Delta e_t) \to 0, \text{ while } \rho(i_t - i_t^*) \to 1 \)

(iv) the Sharpe ratio of the carry trade: \( SR_C \to 0 \)

* carry trade return: \( r_{t+1}^C = x_t \cdot (i_t - i_t^* - \Delta e_{t+1}) \) with \( x_t = i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} \)
EXTENSIONS
Extensions

1. Monetary model with nominal rigidities and a Taylor rule
   - different transmission mechanism
   - similar quantitative conclusions for $\psi_t$ shock
   - Mussa puzzle

2. Multiple shocks:
   - productivity, monetary, foreign good and asset demand
   - variance decomposition: contribution of $\psi_t \approx 70\%$
   - international business cycle (BKK) moments

3. Financial model with noise traders and limits to arbitrage
   (De Long et al 1990, Jeanne and Rose 2002)
   - A model of upward slopping supply in asset markets with endogenous equilibrium volatility of $\psi_t$ and $\Delta e_{t+1}$
   - Stationary model with similar small sample properties
   - Additional moments: the Engel (2016) “risk premium” puzzle

4. Robustness to parameters
Monetary model

• Standard New Keynesian Open Economy model

• Baseline: sticky wages and LCP sticky prices

• Taylor rule: \( i_t = \rho_i i_{t-1} + (1 - \rho_i) \delta_{\pi} \pi_t + \varepsilon^m_t \)

• New transmission: \( i_t \) does not respond directly to the \( \psi_t \) shock, but instead through inflation it generates

• Results:
  1. monetary shock alone results in numerous ER puzzles
  2. financial shock \( \psi_t \) has quantitative similar properties, with two exceptions:
     + makes RER more volatile and NER closer to a random walk
     – RER is negatively correlated with ToT (see Gopinath et al)

## Model comparison

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>A: Single-shock models</th>
<th>B: Multi-shock models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Fin. shock $\psi$</td>
<td>NOEM</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$\rho(\Delta e)$</td>
<td>0.00</td>
<td>-0.02</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.09)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>$\rho(q)$</td>
<td>0.95</td>
<td>0.93</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.04)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>$\sigma(\Delta q)/\sigma(\Delta e)$</td>
<td>0.99</td>
<td>0.79</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.98</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma(\Delta c - \Delta c^*)/\sigma(\Delta q)$</td>
<td>0.20</td>
<td>0.31</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.04)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>$\text{corr}(\Delta c - \Delta c^*, \Delta q)$</td>
<td>-0.20</td>
<td>-1</td>
<td>-0.95</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.09)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>$\sigma(\Delta nx)/\sigma(\Delta q)$</td>
<td>0.10</td>
<td>0.26</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.04)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>$\text{corr}(\Delta nx, \Delta q)$</td>
<td>$\approx$ 0</td>
<td>1</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.09)</td>
<td>(0.09)</td>
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<tr>
<td>$\sigma(\Delta s)/\sigma(\Delta e)$</td>
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<tr>
<td>$\text{corr}(\Delta s, \Delta e)$</td>
<td>0.60</td>
<td>1</td>
<td>-0.93</td>
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<td></td>
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<tr>
<td><strong>Fama $\beta$</strong></td>
<td>$\lessapprox$ 0</td>
<td>-2.4</td>
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<tr>
<td></td>
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<td>(1.7)</td>
<td>(2.6)</td>
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<tr>
<td><strong>Fama $R^2$</strong></td>
<td>0.02</td>
<td>0.03</td>
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<tr>
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<tr>
<td><strong>$\sigma(i - i^*)/\sigma(\Delta e)$</strong></td>
<td>0.06</td>
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<td><strong>$\rho(i - i^*)$</strong></td>
<td>0.90</td>
<td>0.93</td>
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<td></td>
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<td>(0.04)</td>
<td>(0.01)</td>
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<tr>
<td><strong>Carry SR</strong></td>
<td>0.20</td>
<td>0.21</td>
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<td>(0.04)</td>
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## Variance decomposition

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<th>NOEM</th>
<th>IRBC</th>
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<tr>
<td></td>
<td>(\text{var}(\Delta e_t))</td>
<td>(\text{var}(\Delta q_t))</td>
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<tr>
<td>Monetary (Taylor rule)</td>
<td>(\varepsilon_t^m) 10%</td>
<td>10%</td>
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<tr>
<td>Productivity</td>
<td>(a_t)  —</td>
<td>—</td>
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<tr>
<td>Foreign-good demand</td>
<td>(\xi_t) 19%</td>
<td>20%</td>
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<tr>
<td>Financial</td>
<td>(\psi_t) 71%</td>
<td>70%</td>
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<td>Moment</td>
<td>Data</td>
<td>Model (1)</td>
</tr>
<tr>
<td>------------------------------------------</td>
<td>-------</td>
<td>-----------</td>
</tr>
<tr>
<td>(\text{std}(\Delta e_t))</td>
<td>0.13</td>
<td>0.13</td>
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<tr>
<td>(\text{std}(\Delta q_t))</td>
<td>0.26</td>
<td>0.18</td>
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<tr>
<td>(\text{corr}(\Delta q_t, \Delta e_t))</td>
<td>0.66</td>
<td>0.79</td>
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<tr>
<td>(\text{std}(\Delta c_t - \Delta c^*_t))</td>
<td>(\approx 1)</td>
<td>2.63</td>
</tr>
<tr>
<td>(\text{corr}(\Delta c_t - \Delta c^*_t, \Delta q_t))</td>
<td>(&gt;0)</td>
<td>(-0.63)</td>
</tr>
<tr>
<td>Fama (\beta)</td>
<td>(&gt;0)</td>
<td>(-0.1)</td>
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**International business cycle (BKK) calibration**

<table>
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<th>Data Original</th>
<th>Replication</th>
<th>Model with $\psi_t$ Multi-shock</th>
<th>$\psi_t$ only</th>
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<td><strong>Panel A: Exchange rate disconnect moments</strong></td>
<td></td>
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<td>$\rho (\triangle e)$</td>
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<td></td>
<td>(0.09)</td>
<td>(0.09)</td>
<td>(0.09)</td>
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</tr>
<tr>
<td>$\rho (\triangle q)$</td>
<td>0.95</td>
<td>0.97</td>
<td>0.93</td>
<td>0.93</td>
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<tr>
<td></td>
<td>(0.02)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>$\text{corr} (\triangle e, \triangle q)$</td>
<td>0.98</td>
<td>-0.96</td>
<td>0.99</td>
<td>1</td>
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<tr>
<td></td>
<td>(0.02)</td>
<td>(0.00)</td>
<td>(0.00)</td>
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<td>$\sigma (\triangle c - \triangle c^*) / \sigma (\triangle q)$</td>
<td>0.20</td>
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<td>(0.01)</td>
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<tr>
<td>$\text{corr} (\triangle c - \triangle c^*, \triangle q)$</td>
<td>-0.20</td>
<td>1.00</td>
<td>-0.20</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.09)</td>
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</tr>
<tr>
<td>$\text{corr} (\triangle nx, \triangle q)$</td>
<td>$\approx 0$</td>
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<td></td>
<td>(0.08)</td>
<td>(0.09)</td>
<td>(0.09)</td>
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<td><strong>Fama $\beta$</strong></td>
<td>$\lesssim 0$</td>
<td>1.3</td>
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<td>(0.6)</td>
<td>(3.2)</td>
<td>(4.4)</td>
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<tr>
<td><strong>Fama $R^2$</strong></td>
<td>0.02</td>
<td>0.04</td>
<td>0.00</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.02)</td>
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<tr>
<td><strong>Panel B: International busyness cycle moments</strong></td>
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<tr>
<td>$\sigma (\triangle c) / \sigma (\triangle gdp)$</td>
<td>0.49</td>
<td>0.47</td>
<td>0.35</td>
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<td>(0.01)</td>
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<td>$\sigma (\triangle z) / \sigma (\triangle gdp)$</td>
<td>3.15</td>
<td>3.48</td>
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<td>(0.16)</td>
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<td>$\text{corr} (\triangle c, \triangle gdp)$</td>
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<td>(0.00)</td>
<td>(0.05)</td>
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<tr>
<td>$\text{corr} (\triangle z, \triangle gdp)$</td>
<td>0.90</td>
<td>0.93</td>
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<tr>
<td></td>
<td>(0.00)</td>
<td>(0.03)</td>
<td>(0.03)</td>
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<tr>
<td>$\text{corr} (\triangle nx, \triangle gdp)$</td>
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<td>-0.64</td>
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<td></td>
<td>(0.07)</td>
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<td>(0.09)</td>
<td>1</td>
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<td>$\text{corr} (\triangle gdp, \triangle gdp^*)$</td>
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<td>(0.08)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td></td>
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<tr>
<td>$\text{corr} (\triangle c, \triangle c^*)$</td>
<td>0.46</td>
<td>0.77</td>
<td>0.37</td>
<td>-1</td>
</tr>
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<td>(0.08)</td>
<td>(0.07)</td>
<td>(0.07)</td>
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</tr>
<tr>
<td>$\text{corr} (\triangle z, \triangle z^*)$</td>
<td>0.33</td>
<td>0.18</td>
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<tr>
<td></td>
<td>(0.09)</td>
<td>(0.06)</td>
<td>(0.06)</td>
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International business cycle (BKK) calibration

<table>
<thead>
<tr>
<th></th>
<th>BKK with $(a_t, a_t^*)$ only</th>
<th>Model with $\psi_t$</th>
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<tbody>
<tr>
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<td>Original</td>
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<tr>
<td>Nominal exchange rate, $\text{var}(\Delta e)$:</td>
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<tr>
<td>Productivity shocks, $(a_t, a_t^*)$</td>
<td>100%</td>
<td>1%</td>
</tr>
<tr>
<td>Foreign-good demand shocks, $\bar{\xi}_t$</td>
<td>—</td>
<td>40%</td>
</tr>
<tr>
<td>Financial shock, $\psi_t$</td>
<td>—</td>
<td>59%</td>
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<tr>
<td>Consumption, $\text{var}(\Delta c)$:</td>
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<tr>
<td>Productivity shocks, $(a_t, a_t^*)$</td>
<td>100%</td>
<td>77%</td>
</tr>
<tr>
<td>Foreign-good demand shocks, $\bar{\xi}_t$</td>
<td>—</td>
<td>7%</td>
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<tr>
<td>Financial shock, $\psi_t$</td>
<td>—</td>
<td>16%</td>
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Financial model

- Symmetric countries with international bond holding intermediated by a financial sector

- Three type of agents: $B_{t+1}^* + N_{t+1}^* + D_{t+1}^* = 0$

- Noise traders: $N_{t+1}^* = n(e^{\psi_t} - 1)$

- Arbitrageurs: $\max_d \left\{ d \mathbb{E}_t \tilde{R}_{t+1} - \frac{\omega}{2} \text{var}_t(\tilde{R}_{t+1})d^2 \right\}$, $\tilde{R}_{t+1}^* \equiv R_{t}^* - R_t \frac{\mathcal{E}_t}{\tilde{e}_{t+1}}$
  results in bond supply:

  $$D_{t+1}^* = m \frac{\mathbb{E}_t \tilde{R}_{t+1}}{\omega \text{var}_t(\tilde{R}_{t+1})}$$

- Generalized UIP condition:

  $$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \chi_1 \psi_t - \chi_2 b_{t+1}, \quad \chi_1 \equiv \frac{n/\beta}{m/(\omega \sigma_e^2)}, \quad \chi_2 \equiv \frac{\bar{Y}}{m/(\omega \sigma_e^2)}$$

- Proposition: $e_t$ and $q_t$ follow an ARMA(2,1), but with the same near-random-walk properties.
Financial model

Equilibrium exchange rate volatility

- Three equilibria exist when \( d = \frac{1}{\beta(1+\gamma \lambda_1)} \frac{n \omega \sigma_e}{m} > \hat{d} \)
- When \( d < \hat{d} \), the only equilibrium is \( \sigma_e = 0 \)
Engel (2016) “risk premium” puzzle

Figure: Response of $e_{t+j}$ to innovation in $i_t - i_t^*$
Engel (2016) “risk premium” puzzle

Figure: Projections on $i_t - i_t^*$

(a) Risk premium, $\mathbb{E}_t \rho_{t+j}$

(b) Exchange rate, $e_{t+j} - e_t$

where $\rho_t = i_t - i_t^* - \Delta e_{t+1}$
Conclusion

- Exchange rates have been very puzzling for macroeconomists
- We offer a unifying quantitative GE theory of exchange rates
- Which international macro results are robust?
  - Monetary policy transmission and spillovers: likely yes
  - Welfare analysis and optimal exchange rate regimes: likely no
- Our tractable macro GE environment can be useful for both:
  1. empirical/quantitative studies of ER and transmission
  2. financial models of exchange rates
APPENDIX
<table>
<thead>
<tr>
<th>Puzzle</th>
<th>Ingredients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meese-Rogoff, UIP</td>
<td>${ \bullet$ persistent financial shock $\psi_t$</td>
</tr>
<tr>
<td></td>
<td>$\bullet$ conventional Taylor rule</td>
</tr>
<tr>
<td>PPP</td>
<td>$+$</td>
</tr>
<tr>
<td></td>
<td>$\bullet$ home bias $\gamma$</td>
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<td>Terms-of-trade</td>
<td>$+$</td>
</tr>
<tr>
<td></td>
<td>$\bullet$ strategic complementarities $\alpha$</td>
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<tr>
<td>Backus-Smith</td>
<td>$+$</td>
</tr>
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<td>$\bullet$ weak substitutability $\theta$</td>
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</table>
# Puzzle Resolution Mechanism

<table>
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<tr>
<th>Puzzle</th>
<th>Ingredients</th>
<th>Parameters</th>
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</thead>
<tbody>
<tr>
<td>Meese-Rogoff, UIP</td>
<td>${ \bullet \text{ persistent financial shock, } \psi_t }$</td>
<td>$\psi_t$</td>
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<td>$\bullet \text{ conventional Taylor rule}$</td>
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<td>PPP</td>
<td>$\bullet \text{ home bias, } \gamma$</td>
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<td>Terms-of-trade</td>
<td>$\bullet \text{ strategic complementarities, } \alpha$</td>
<td>$\alpha$</td>
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<tr>
<td>Backus-Smith</td>
<td>$\bullet \text{ weak substitutability, } \theta$</td>
<td>$\theta$</td>
</tr>
</tbody>
</table>

- **Parameter restrictions:**
  1. Marshall-Lerner condition: $\theta > 1/2$
  2. Nominal UIP: $\theta > IES$
New Mechanisms

1. Exchange rate dynamics:
   - near random-walk behavior emerging from the intertemporal budget constraint under incomplete markets
   - small but persistent expected appreciations require a large unexpected devaluation on impact

2. PPP puzzle
   - no wedge between nominal and real exchange rates, unlike IRBC and NOEM models

3. Violation of the Backus-Smith condition:
   - we demote the dynamic risk-sharing condition from determining consumption allocation
   - instead static market clearing determination of consumption

4. Violation of UIP and Forward premium puzzle:
   - small persistent interest rate movements support consumption allocation, disconnected from volatile exchange rate
   - negative Fama coefficient, yet small Sharpe ratio on carry trade
• Representative home household solves:

\[
\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t e^{\chi_t} \left( \frac{1}{1 - \sigma} C_t^{1-\sigma} - \frac{e^{\kappa_t}}{1 + 1/\nu} L_t^{1+1/\nu} \right)
\]

s.t. \[ P_t C_t + \frac{B_{t+1}}{R_t} + \frac{B_{t+1}^* \mathcal{E}_t}{e^{\psi_t} R_t^*} \leq B_t + B_t^* \mathcal{E}_t + W_t L_t + \Pi_t + T_t \]

• Household optimality (labor supply and demand for bonds):

\[ e^{\kappa_t} C_t^{\sigma} L_t^{1/\nu} = \frac{W_t}{P_t}, \]

\[ R_t \mathbb{E}_t \{ \Theta_{t+1} \} = 1, \]

\[ e^{\psi_t} R_t^* \mathbb{E}_t \left\{ \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \Theta_{t+1} \right\} = 1, \]

where the home nominal SDF is given by:

\[ \Theta_{t+1} \equiv \beta e^{\Delta \chi_{t+1}} \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \]
• Consumption expenditure on home and foreign goods:

\[ P_t C_t = P_{Ht} C_{Ht} + P_{Ft} C_{Ft} \]

arises from a homothetic consumption aggregator:

\[ C_{Ht} = (1 - \gamma)e^{-\gamma \xi_t} h\left(\frac{P_{Ht}}{P_t}\right) C_t, \]

\[ C_{Ft} = \gamma e^{(1-\gamma)\xi_t} h\left(\frac{P_{Ft}}{P_t}\right) C_t \]

• The foreign share and the elasticity of substitution:

\[ \gamma_t \equiv \frac{P_{Ft} C_{Ft}}{P_t C_t} \bigg|_{P_{Ht}=P_{Ft}=P_t = \gamma} \]

\[ \theta_t \equiv -\frac{\partial \log h(x_t)}{\partial \log x_t} \bigg|_{x_t=1} = \theta \]
Production and profits

- Production function with intermediates:
  \[ Y_t = e^{at} L_t^{1-\phi} X_t^\phi \]
  \[ MC_t = e^{-at} \left( \frac{W_t}{1-\phi} \right)^{1-\phi} \left( \frac{P_t}{\phi} \right)^\phi \]

- Profits:
  \[ \Pi_t = (P_{Ht} - MC_t) Y_{Ht} + (P_{Ht}^* E_t - MC_t) Y_{Ht}^* \]
  where \( Y_t = Y_{Ht} + Y_{Ht}^* \)

- Labor and intermediate goods demand:
  \[ W_t L_t = (1 - \phi) MC_t Y_t \]
  \[ P_t X_t = \phi MC_t Y_t \]

and fraction \( \gamma_t \) of \( P_t X_t \) is allocated to foreign intermediates.
• We postulate the following price setting rule:

\[ P_{Ht} = e^{\mu t} MC_t^{1-\alpha} P_t^\alpha \]

\[ P_{Ht}^* = e^{\mu t + \eta t} (MC_t/E_t)^{1-\alpha} P_t^{*\alpha} \]

• LOP violations:

\[ Q_{Ht} \equiv \frac{P_{Ht}^* E_t}{P_{Ht}} = e^{\eta t} Q_t^\alpha \]

where the real exchange rate is given by:

\[ Q_t \equiv \frac{P_t^* E_t}{P_t} \]
• Government runs a balanced budget, using lump-sum taxes to finance expenditure:

\[ P_t G_t = P_t e^{g_t}, \]

where fraction \( \gamma_t \) of \( P_t G_t \) is allocated to foreign goods.

• The transfers to the households are given by:

\[ T_t = (e^{-\psi_t} - 1) \frac{B^*_t}{R^*_t} - P_t e^{g_t} \]
• **Foreign households and firms** are symmetric, subject to:

\[
\{\chi^*_t, \kappa^*_t, \xi^*_t, a^*_t, \mu^*_t, \eta^*_t, g^*_t\}
\]

• Foreign households only differ in that they do not have access to the home bond, which is not internationally traded.

As a result, their only Euler equation is for foreign bonds:

\[
R^*_t \mathbb{E}_t \{\Theta^*_{t+1}\} = 1, \quad \Theta^*_{t+1} \equiv \beta e^{\Delta x^*_t} \left( \frac{C^*_{t+1}}{C^*_t} \right)^{-\sigma} \frac{P^*_t}{P^*_{t+1}}
\]
1. Labor market clearing

2. Goods market clearing, e.g.:

   \[ Y_{Ht}^* = \gamma e^{(1-\gamma)\xi_t^*} h \left( \frac{P_{Ht}^*}{P_t^*} \right) [C_t^* + X_t^* + G_t^*] \]

3. Bond market clearing:

   \[ B_t = 0 \quad \text{and} \quad B_t^* + B_t^{*F} = 0 \]

4. Country budget constraint:

   \[ \frac{B_{t+1}^* \xi_t}{R_t^*} - B_t^* \xi_t = NX_t, \quad NX_t = P_{Ht}^* \xi_t Y_{Ht}^* - P_{Ft} Y_{Ft}, \]

   and we define the terms of trade:

   \[ S_t \equiv \frac{P_{Ft}}{P_{Ht}^* \xi_t} \]
The figure plots $\frac{\partial z_t}{\partial \varepsilon_t}$ for different values of $\gamma$, where $z \in \{p, c, y\}$ are different macro variables and $\varepsilon \in \Omega$ are different shocks.
Properties of the Exchange Rate

- Near-random-walk behavior (as $\beta \rho \rightarrow 1$)

\[
\begin{align*}
\text{corr}(\Delta e_{t+1}, \Delta e_t) & \rightarrow 0 \\
\frac{\text{var}(\Delta_k e_{t+k} - \mathbb{E}_t \Delta_k e_{t+k})}{\text{var}(\Delta_k e_{t+k})} & \rightarrow 1 \\
\frac{\text{std}(\Delta e_t)}{\text{std}(\psi_t)} & \rightarrow \infty
\end{align*}
\]

$\rho = 0.96$ and $\beta = 0.99$

$\rho = 0.99$ and $\beta = 0.995$

Figure: Impulse response of the exchange rate $\Delta e_t$ to $\psi_t$
Properties of the Exchange Rate

- Near-random-walk behavior (as $\beta \rho \to 1$)

$$\text{corr}(\Delta e_{t+1}, \Delta e_t) \to 0 \quad \frac{\text{var}(\Delta_k e_{t+k} - \mathbb{E}_t \Delta_k e_{t+k})}{\text{var}(\Delta_k e_{t+k})} \to 1 \quad \frac{\text{std}(\Delta e_t)}{\text{std}(\psi_t)} \to \infty$$

$$\rho = 0.96 \text{ and } \beta = 0.99$$

Figure: Contribution of the unexpected component (in small sample)
Figure: Persistence of the real exchange rate $q_t$ in small samples
Backus-Smith illustration

\[ \tilde{c}_t = \frac{(c_t - c_t^*)}{2} \]

\[ \tilde{y}_t = \frac{(1 - \phi)(1 - 2\gamma)}{1 - \phi(1 - 2\gamma)} \tilde{c}_t + \gamma \kappa_2 q_t \]

Goods market clearing

\[ \sigma \nu \tilde{c}_t + \tilde{y}_t = -\gamma \kappa_1 q_t \]

Labor market clearing

\[ \tilde{y}_t = \frac{(y_t - y_t^*)}{2} \]
Labor Supply:

\[ \sigma \tilde{c}_t + \frac{1}{\nu} \tilde{l}_t = -\frac{1}{1 - \phi \frac{1}{1 - 2\gamma}} q_t \]

Recall that:

\[ p_t = w_t + \frac{1}{1 - \phi \frac{1}{1 - 2\gamma}} q_t \]

Labor Demand:

\[ \tilde{l}_t = \tilde{y}_t + \frac{\phi}{1 - \phi \frac{1}{1 - 2\gamma}} q_t. \]

Goods market clearing:

\[ \tilde{y}_t = \frac{\zeta}{\zeta + \frac{2\gamma}{1 - 2\gamma}} \tilde{c}_t + \frac{2\theta(1 - \alpha)}{\zeta + \frac{2\gamma}{1 - 2\gamma}} - \frac{(1 - \zeta)}{\zeta + \frac{2\gamma}{1 - 2\gamma}} \frac{\gamma}{1 - 2\gamma} q_t \]
Exchange Rate and Interest Rate

Figure: Deviations from UIP (in small samples)
## ER Disconnect: Robustness

### Table: Robustness Analysis

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Baseline</th>
<th>Robustness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho(\Delta e)$</td>
<td>0.00</td>
<td>$-0.02$ (0.09)</td>
</tr>
<tr>
<td>1.</td>
<td>$\rho(q)$</td>
<td>0.94</td>
<td>0.93* (0.04)</td>
</tr>
<tr>
<td>2.</td>
<td>$HL(q)$</td>
<td>12.0</td>
<td>9.9* (6.4)</td>
</tr>
<tr>
<td></td>
<td>$\sigma(\Delta q)/\sigma(\Delta e)$</td>
<td>0.98</td>
<td>0.75</td>
</tr>
<tr>
<td>3.</td>
<td>$\sigma(\Delta s)/\sigma(\Delta q)$</td>
<td>0.34</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>$\sigma(\Delta q^p)/\sigma(\Delta q)$</td>
<td>0.98</td>
<td>1.10</td>
</tr>
<tr>
<td>4.</td>
<td>$\sigma(\Delta c - \Delta c^*)/\sigma(\Delta q)$</td>
<td>$-0.25$</td>
<td>$-0.31$</td>
</tr>
<tr>
<td></td>
<td>Fama $\beta_F$</td>
<td>$\leq 0$</td>
<td>$-8.1^*$ (4.7)</td>
</tr>
<tr>
<td>5.</td>
<td>Fama $R^2$</td>
<td>0.02</td>
<td>0.04 (0.02)</td>
</tr>
<tr>
<td></td>
<td>$\sigma(i - i^*)/\sigma(\Delta e)$</td>
<td>0.06</td>
<td>0.03 (0.01)</td>
</tr>
<tr>
<td></td>
<td>Carry $SR$</td>
<td>0.20</td>
<td>0.21 (0.04)</td>
</tr>
</tbody>
</table>

**Note:** Baseline parameters: $\gamma = 0.07$, $\alpha = 0.4$, $\theta = 1.5$, $\rho = 0.97$, $\sigma = 2$, $\nu = 1$, $\phi = 0.5$, $\mu = 0$, $\beta = 0.99$. Results are robust to changing $\nu$, $\phi$, $\mu$ and $\beta$. * Asymptotic values: $\rho(q) = 1$, $HL(q) = \infty$, $\beta_F = -4.6$. 
Mechanism

1. An international asset demand shock $\varepsilon_t > 0$ results in an immediate sharp ER depreciation to balance the asset market.
2. Exchange rate then gradually appreciates (as the $\psi_t$ shock wears out) to ensure the intertemporal budget constraint.
3. Nominal and real devaluations happen together, and the real wage declines.
4. Devaluation is associated with a dampened deterioration of the terms of trade and the resulting expenditure switching towards home goods.
5. Consumption falls to ensure equilibrium in labor and goods markets.
6. Consumption fall is supported by an increase in the interest rate, which balances out the fall in demand for domestic assets.