MODEL THEORY: WHAT IT IS AND ISN'T

INTRODUCTION

Model theory is the branch of logic concerned with relations between formal languages and extralinguistic structures. The most basic technical notion is that of a formula being *true* in a given structure, and the subject originated with Tarski and his famous definition of truth for classical first-order languages. On a common usage followed here, a *model* is just a structure of the kind sentences are true or false in.[1] Model theory contrasts with proof theory, which is concerned only with formal deducibility relations among sentences, regardless of any connection with extralinguistic structures.[2] Important post-Tarski contributors to the model theory of classical first-order languages have included Abraham Robinson, who gave the subject an orientation towards applications to abstract algebra, where the pertinent structures are groups and rings and fields, and Saharon Shelah, who introduced methods of great technical sophistication. Many mathematical logicians today use 'model theory' to refer only to technically-sophisticated, algebra-oriented, first-order model theory, a subject with little bearing on philosophy. Model theory has been developed also, however, for non-classical formal languages (tense, modal, and other), especially by Saul Kripke and his successors. This branch of the subject claims applications to philosophy of language and theoretical computer science.

In the usage of computer science and philosophical logic, model-theoretic concepts are often called 'semantic', and proof-theoretic concepts 'syntactic'. This usage can be traced back to Tarski, who noted that the word 'true' was used in several senses, and to distinguish the one that concerned him drew on vocabulary used in Morris (1938), where *syntax* is concerned with relations among words, *semantics* with relations between words and things, and *pragmatics* with relations among words and things and people. Tarski was concerned with the notion of truth according to which 'Snow is white' is true iff snow is white. William James, by contrast, took 'Snow is white' to be true iff it is useful to believe that snow is white. (Here 'iff' abbreviates 'if and only if'.) Tarski's notion is 'semantic' in involving words and a thing (snow), while James's is 'pragmatic' in involving also people (believers). Tarski called his development of a rigorous definition of truth as he conceived it 'the establishment of scientific semantics'. Linguists, by contrast, apply the term 'semantics' to the branch of their subject concerned with meaning in natural language. That there is a tension between this usage and the Morris-Tarski usage is evident if one considers the contrast between (i) denotation or reference or extension and (ii) connotation or sense or intension. Morris's words-and-things conception of semantics is more suggestive of (i), while the linguists' meaning conception of semantics is more suggestive of (ii).

It is hard to keep the two notions of 'semantics' apart in theoretical computer science, since there one is concerned with artificial languages, and the specification of the conditions under which a sentence of such a language is to count as true in a given structure is about all one gets by way of a specification of what such a sentence is supposed to mean. Elsewhere it is crucial to distinguish, since fallacies of equivocation between the two senses of 'semantics' can be damaging to the understanding of scope, limits, and status both of model theory and of meaning theory. For instance, confusion of 'formal semantics' or model theory, to which Tarski's definition of truth is central, with 'linguistic semantics' or meaning theory can lead straight to *Davidsonianism*, the contentious claim that specification of truth conditions is central to giving a theory of meaning for natural language; but to arrive at Davidsonianism *that* way would be to prejudge a controversy that can only be fairly judged after an examination of rival research programs and empirical linguistic data.[3]

In logic, the nature of any relationship between models and meaning varies with the formalism under consideration (classical first-order, tense, modal, or other) and must be considered separately in each case. The methods of model theory are too flexible for it to be safe to assume that, just because there is a rigorously defined notion of model for a given formalism, it must have some intuitively intelligible meaning. And they are often too contrived for it to be safe to assume that, just because some feature is present in the models used for a given formal language, an analogous feature must be present also in whatever that formal language may be intended to represent. What follows here is an elaboration and illustration of such warnings.

CLASSICAL FIRST-ORDER LOGIC

On one traditional conception, logic's central concern is with *consequence* or *implication*, where premises have as a consequence or imply a conclusion iff form alone guarantees that the premises cannot be true without the conclusion being so too. The notion of *validity*, or truth by virtue of form, is the degenerate, zero-premise case. The general case can be reduced to it, since B is a consequence of or implied by $A_1, \ldots, A_n$ iff

(1) It is not the case that A_1 and … and A_n and not B

is valid. In modern logic, forms are represented by or identified with formulas; validity of a formula is identified with truth of all *instances*, all results of substituting specific sentences, predicates, or whatever for the sentence letters, predicate letters, or whatever in the formula; and validity of a conclusion is identified with being an instance of a valid formula.[4]

One can show that a given conclusion is *not* a consequence or implication of given premises by replacing the nonlogical words in premises and conclusions by letters to give a formula, and then presenting an instantiation in which other nonlogical words to replace the letters in which the premises become true and the conclusion false. This is not essentially different from the method traditionally used in mathematics to show that, for instance, the parallel postulate does not follow from other axioms of Euclidean geometry, except that traditionally the form was not explicitly exhibited, and one simply presented a substitution of new nonlogical words for the original ones.

The most basic goals of model theory for pure logic are to define rigorously: first, a technical notion of model *capturing all and only the features of an instance that are relevant to truth*; and second, a technical notion of truth in a model such that *truth in the intuitive sense of an instance will coincide with truth in the technical sense in a corresponding model*. It then follows that the intuitive notion of validity as truth in all instances will coincide with the rigorously-defined notion of validity as truth in all models. For this coincidence to obtain and be useful, the class of models must be (a) broad enough that there is a model corresponding to any instance; and (b) narrow enough to be more tractable than the class of all instances.

At the level of classical sentential logic, instantiation will simply be substitution of sentences $\pi, \pi', \ldots$ for sentence letters $p, p', \ldots$, and since all that matters for the truth of the result is the truth-values of the sentences, and not, for instance, their meaning, for a *model* we may simply take an assignment of truth-values to sentence letters. We can then define the notion $\mathbf{M} \vDash A$ of the truth in a model $\mathbf{M}$ of a formula A built up from sentence letters using negation and conjunction and disjunction ~ and ∧ and ∨, inductively as follows:

(2a)	$\mathbf{M} \vDash p_i$	iff	$\mathbf{M}(p_i) = \text{true}$
(2b)	$\mathbf{M} \vDash {\sim}A$	iff	not $\mathbf{M} \vDash A$
(2c)	$\mathbf{M} \vDash A \wedge B$	iff	$\mathbf{M} \vDash A$ and $\mathbf{M} \vDash B$
(2d)	$\mathbf{M} \vDash A \vee B$	iff	$\mathbf{M} \vDash A$ or $\mathbf{M} \vDash B$

(Similarly for the conditional → and biconditional ↔.) A valid formula is then one that is true in all such models. The desiderata (a) and (b) are achieved, the former obviously, the latter outstandingly, since for a formula containing only specific sentence letters (for example, the three p, q, r) there will be only finitely many (in the example, just eight) models or combinations of truth values, whereas there are an infinity of instances (triples of sentences that might be put in for p, q, r). This reduction from infinite to finite is responsible for the decidability of classical sentential logic.

At the level of classical predicate logic, the notion of model becomes more complicated. What matters for the truth of an instance is, first of all, which things are being spoken of, or in jargon, the *quantifier domain*; and further, for each k-place predicate, which k-tuples of elements of that universe it is true of. Differences in *meaning* between the predicates in different instantiations will not matter, so long as the predicates are *true* of the same things, so the notion of model may ignore them. The official definition of a model **M** accordingly takes one to consist of a nonempty set *M* as quantifier domain, and an assignment to each *k*-place predicate letter *P* of a set $P^{\mathbf{M}}$ of *k*-tuples of elements thereof, called **M**'s *interpretation* of *P*. The notion of truth in a model also becomes more complicated, involving a detour through the notion of an *open* formula's *satisfaction* by elements in a model; but Tarski's definition is given in all standard logic textbooks today, need not be repeated here.

As for desideratum (b), the class of models is vast, but basic results of model theory imply that truth in all models is equivalent to truth in all models of a restricted kind. To begin with, the *Löwenheim-Skolem* theorem states that a formula will be true in all models iff it is true in all models with a countable quantifier domain.[5]

As for desideratum (a), there is a problem. We want to apply classical logic to evaluate arguments in, among other fields, set theory. We need therefore to consider formulas with a single two-place predicate letter and the instantiation in which the quantifier domain consists of all sets, and what is put in for the predicate letter is 'is an element of'. But there is no model corresponding to this instantiation, since our official definition of model requires the quantifier domain to be a set, and there is no set of all sets. In Kreisel (1967), where the problem was first noted, a solution is provided, based on comparison of the notions of: (i) formal provability, (ii) validity in the intuitive sense (truth in all instances, even if the domain is not a set), (iii) validity in the official sense (truth in all models, where the domain must be a set). Intuitively, (i) is found to imply (ii) on inspecting the axioms and rules of formal proof; and obviously (ii) implies (iii); but the Gödel completeness theorems tells us (iii) implies (i), so the intuitive notion (ii) is caught in a scissors between the technical notions (i) and (iii), and all three notions agree.[6]

TENSE LOGIC

The simplest time-distinctions are expressed in English not through overt quantification over 'times' but by verb inflections or auxiliary verbs. There is nothing comparable to these verb modifications in classical logic, which was designed for application to pure mathematics, where nothing ever has been or ever will be other than as it is. Tense logic as developed in Prior (1967) and elsewhere introduces operators F and P, pronounced '(sometime) will be' and '(sometime) was'. In terms these, other operators $\mathsf{G} = {\sim}\mathsf{F}{\sim}$ and $\mathsf{H} = {\sim}\mathsf{P}{\sim}$, pronounced '*always* will be' and '*always* was' can be defined. For simplicity we consider only the future pair F, G in examples. The form of

(3) If he ever goes, she will go later

might be represented using these operators thus

(4) $\mathsf{G}(p \rightarrow \mathsf{F}q)$

with p/q for 'he/she goes'.

Though nothing ever changes in mathematics, mathematics nonetheless can be and is applied to physics, where everything changes. The application requires some departure from our usual ways of speaking and thinking, adding a phrase 'at time t' to each verb, and replacing the present-tense 'is' that contrasts with 'was' and 'will be' with a tenseless '/is/' that is short for 'was or is or will be'. Taking this approach, the form of the premise (3) might be represented classically as

(4') $\forall t(t_0 < t \wedge Pt \rightarrow \exists t'(t < t' \wedge Qt'))$

with t_0 standing for the present time, $<$ for the relative-futurity or earlier-later relation, and Pt/Qt for 'he/she /goes/ at time t'. Comparison of (4) and (4') suggests how in general tense-logical formulas can be transcriptions into classical-logical formulas. Each G or F will correspond to an $\forall$ or $\exists$, each sentence letter p_i will correspond to a one-place predicate letter P_i, with P_it to be thought of as meaning something like 'p_i /is/ true at time t'. Classical first-order logic counts premises as implying conclusion in very few tense-logical arguments as they stand, though the conclusion is often a consequence of the stated premises plus some further unstated premise about the structure of time, such as the transitivity of the earlier-later relation:

(5) $\forall t \forall t' \forall t''(t < t' \wedge t' < t'' \rightarrow t < t'')$

It is the job of the physicist to say which such assumptions are true, and of the logician to say what follows from each.

Tense logic seeks to develop systems of axioms and rules for deriving tense-logical theorems working entirely within the tense-logical language. The tense logician would like to be able to tell the physicist which axioms hold on which assumptions about time, but there is this difficulty, that the physicist's assumptions about time are formulated in the mathematical style (5) as principles about a timeless relation of earlier to later among entities called times, and not in the ordinary tensed language that tense logic seeks to represent. It is in order to connect the two languages that tense logicians introduce a notion of model that is a direct adaptation of the classical notion of model for the first-order language with one two-place predicate $<$ and many one-place predicates P_i.

A model **M** consists of (i) a set M of elements representing 'times', (ii) a two-place relation $<^{\mathbf{M}}$ on the set representing 'earlier-later', and (iii) a component telling us for each sentence letter p_i and each time t whether p_i is true or not at t. Here (i) and (ii) are just are the classical quantifier domain and interpretation of <. Classically (iii) would be represented by an assignment to each P_i of an interpretation $P_i^{\mathbf{M}}$, to be thought of as the set of times when p_i is true; tense-logically, (iii) is more conveniently represented by a function assigning each time t a function $\mathbf{M}_t$ assigning each sentence letter a truth value; the two representations are mathematically equivalent.

The notion of the truth of a formula A at a time t in a model **M**, written $\mathbf{M}, t \vDash A$ can then be defined by recursion on the complexity of the formula, adapting the classical definition thus:

(6a)	$\mathbf{M}, t \vDash p_i$	iff	$\mathbf{M}_t(p_i) = \text{true}$
(6b)	$\mathbf{M}, t \vDash \sim A$	iff	not $\mathbf{M}, t \vDash A$
(6c)	$\mathbf{M}, t \vDash A \wedge B$	iff	$\mathbf{M}, t \vDash A$ and $\mathbf{M}, t \vDash B$
(6d)	$\mathbf{M}, t \vDash A \vee B$	iff	$\mathbf{M}, t \vDash A$ or $\mathbf{M}, t \vDash B$
(6e)	$\mathbf{M}, t \vDash \mathsf{F}A$	iff	$\mathbf{M}, t' \vDash A$ for some t' with $t <^{\mathbf{M}} t'$
(6f)	$\mathbf{M}, t \vDash \mathsf{G}A$	iff	$\mathbf{M}, t' \vDash A$ for all t' with $t <^{\mathbf{M}} t'$

A model may be required to have also (iv) a designated element t_0 representing the 'present', in which case truth in the model as a whole is defined as truth at t_0, and validity as truth in all models; more usually, validity is simply defined as truth at all times in all models.

A certain minimal axiom system has been identified, which gives as theorems all formulas that are valid without special assumptions about the structure of time, and various additional axioms corresponding to various such special assumptions have also been identified. For instance, corresponding to transitivity (5) we have the axiom

(7) $\mathsf{FF}p \rightarrow \mathsf{F}p$

intuitively meaning roughly

(8) Whatever *will be* going to be *is* going to be.

Something like (8) represents an attempt to give the 'cash value' in ordinary tensed language of what is asserted using tenseless mathematical language by something like (5). The 'cash values' of quite sophisticated physical theories of time, as in relativity theory, have been worked out by tense logicians.

If one confused 'semantics' in the sense of model theory with 'semantics' in the sense of meaning theory, one would have to conclude that ordinary tensed speech, despite superficially appearances, deep down really means the same as is classical tenseless transcription (and in particular, that ordinary tensed speech covertly quantifies over and is 'ontologically committed' to such entities as 'times'). But such a conclusion is an empirical linguistic hypothesis, requiring empirical linguistic evidence. The fact that tense logicians have found it convenient to adopt the kind of model-theory just sketched when trying to relate their ordinary tensed language to the mathematically-formulated hypotheses of physicists is hardly the sort of evidence required. There may well be good evidence of the right kind for the hypothesis, but if so, that only makes it the more important to emphasize that such genuine empirical linguistic evidence must be sharply distinguished from the spurious support that would be provided by an equivocating argument sliding fallaciously from the presence of some feature in models to the presence of that feature in 'semantics' to the presence of that feature in meaning.

MODAL LOGIC

Modal logic is concerned with the distinction between what actually is and what is in one sense or another necessary or possible. The simplest modal distinctions are expressed in English not through modal auxiliary verbs such as 'must' and 'may'. There is nothing comparable in classical logic, because in pure mathematics, for which that logic was designed, nothing could have been other than as it is. Modal logic as developed by C. I. Lewis (1918) and his successors introduces operators $\Box$ and $\Diamond$, pronounced 'necessarily' and 'possibly', the two being interdefinable since $\Box = {\sim}\Diamond{\sim}$ and $\Diamond = {\sim}\Box{\sim}$, and develops systems of axioms and rules for deriving modal theorems.

Historically, systems quickly proliferated without much regard to intuitive interpretation, differing in their axioms regarding iterated modalities, such as

(9a) $\Box p \rightarrow \Box\Box p$

(9b) $\Diamond p \rightarrow \Box\Diamond p$

For instance, the systems known as **S4** and **S5** both have (9a) as a theorem, while the former lacks but the latter has (9b) as well. Modal logic was pursued for half a century, and dozens of systems were developed, before logicians gave much attention even to so basic a distinction as that between *metaphysical* possibility (concerning what potentially might have been if the world had been otherwise than it actually is) and *logical* possibility (concerning what can without contradiction be supposed about how the world actually is).

The fact that formal development ran ahead of intuitive interpretation was in some way a good thing, since it left the formalism open to unanticipated interpretations and applications, which have included the *epistemic interpretation of intuitionistic logic*, as in Shapiro (1985), the provability interpretation of modal logic, or *provability logic* for short, as in Boolos (1995), as well as what is called *dynamic logic* in theoretical computer science. But it does leave the logician with the question which of the many systems in existence is appropriate for which of the various senses of modality that have been distinguished.

During the period before logical and metaphysical modality were clearly distinguished, a model theory was developed for many modal systems, primarily Kripke (1963a), in order to address such issues as decidability. Considering successively such formulas as

(10a) $\sim(p \wedge q) \wedge \Diamond(p \wedge q)$

(10b) $\sim(p \wedge q) \wedge \sim\Diamond(p \wedge q) \wedge \Diamond\Diamond(p \wedge q)$

(10c) $\Diamond(p \wedge q \wedge \Diamond(\sim p \wedge q)) \wedge \Diamond(p \wedge \sim q \wedge \sim\Diamond(\sim p \wedge q))$

(10d) $\Diamond(p \wedge q \wedge \Diamond(\sim p \wedge q)) \wedge \Diamond(p \wedge q \wedge \sim\Diamond(\sim p \wedge q))$

will lead to concluding successively that a model will have to (a) represent not only what combination of truth values the sentences used to instantiate sentence letters actually have, but also what combinations they could possibly have had, and (b) represent possibilities 'of different orders', not only actual possibilities but also possible possibilities, and (c) represent not only possibilities of various orders but also which higher-order possibilities are possible relative to which lower-order possibilities, (d) allow distinct possibilities for which the sentences involved have the same combination of truth values.

All these features are present in a *Kripke model* **M**, which consists of (i) a set M of elements representing possibilities of various orders, (ii) a two-place relation $<^{\mathbf{M}}$ on the set representing relative possibility, and (iii) a function assigning each element t of the set a function $\mathbf{M}_t$ assigning each sentence letter a truth value. This is, of course, exactly what a model for tense logic amounts to, and the same definition of truth in a model is used in both logics, $\Box$ and $\Diamond$ replacing G and F, so that clauses (6abcd) are retained, and (6ef) replaced by

(11e) $\mathbf{M}, t \models \Diamond A$ iff $\mathbf{M}, t' \models A$ for some t' with $t <^{\mathbf{M}} t'$

(11f) $\mathbf{M}, t \models \Box A$ iff $\mathbf{M}, t' \models A$ for all t' with $t <^{\mathbf{M}} t'$

(Historically, the models for modal logic came first, and those for tense logic later.) Traditionally, the elements of the set M are called 'worlds' and the relation $<^{\mathbf{M}}$ 'accessibility'.

A certain minimal axiom system **K** has been identified, which gives as theorems all formulas that are valid without special assumptions about the accessibility relation among worlds, and various additional axioms corresponding to various such special assumptions have also been identified. Notably

(12a) $\Box p \to p$

(12b) $\Box p \to \Box\Box p$

(12c) $p \to \Box\Diamond p$

correspond respectively to the assumptions of reflexivity and transitivity and symmetry of accessibility.

But does this tell us anything about which of the many modal axioms systems is appropriate for which of the various notions of necessity that have been distinguished? Here the contrast with tense logic is striking. Physicists' assumptions about the structure of time do naturally present themselves as assumptions about the earlier-later relation among times. Different notions of necessity do *not* naturally present themselves as assumptions about the 'accessibility' relation among 'worlds'.[7]

This is not to say that models are useless, or useful only for technical purposes. About a decade elapsed between the time when the model theory became well known through Kripke (1963b) and the time when the distinction between metaphysical and logical modality became well known through Kripke (1972), but during in that decade the model theory already had an important influence even on intuitive as opposed to technical issues. It motivated the general rejection of the so-called *converse Barcan formula*

(13) $\Box\forall x Px \rightarrow \forall x \Box Px$

by confirming pre-existing suspicions that this formula somehow was saying that whatever exists necessarily exists, and also by suggesting an axiom system that avoids (13) as a theorem.[8] Moreover, even in connection with the question of which axiom system is appropriate for which notion of modality, the model theory has played an auxiliary role.

If one wishes to argue that **S5** is *sound* — that every theorem of the system is true in all instances — when the box is read as 'it is true by virtue of logical form that', there is no reason to consider models. One may simply argue intuitively that whatever sentence π is put in for p in (12c), the result will be true, and similarly for every other axiom.[9]

It is when one wishes to argue that **S5** is *complete* for logical necessity in the indicated sense — that every formula true in all instances is a theorem, or equivalently, that every formula whose negation is not a theorem is true in some instance — that the model theory becomes useful. A formula whose negation is not a theorem, such as

(14) $\Diamond(p_1 \wedge p_2) \wedge \Diamond(p_2 \wedge p_3) \wedge \Diamond(p_3 \wedge p_1) \wedge {\sim}\Diamond(p_1 \wedge p_2 \wedge p_3)$

will have a model, in this case, one with three worlds w_1, w_2, w_3, all accessible from each other, with each p_i true at precisely the w_j for $j \neq i$. The model can be used to find an instantiation for (14) that will be true. We first want, corresponding to the w_i, three sentences ω_i whose logical form will guarantee that exactly one of them is true. These can be obtained starting from any two logically independent sentences σ and τ, as follows:

(15a) σ = 'Snow is white'
(15b) τ = 'Grass is green'
(15c) ω_1 = σ or τ
(15d) ω_2 = σ or not τ
(15e) ω_3 = not σ

Then to each p_i, associate the disjunction π_i of the ω_j corresponding to w_j where it is true. The results amount to

(15f) π_1 = 'Snow is not white or grass is not green'
(15g) π_2 = 'Snow is not white or grass is green'
(15h) π_3 = 'Snow is white'

Logical form does not preclude any two of the π_i being true, but does preclude all three being true, as required to give a true instance of (14). The method generalizes.

The correct logic for the provability interpretation has also been determined, and again models play an auxiliary role, though most of the hard work is getting from a model for a formula to a true instance of that formula. There is not space to enter into details here, nor to enter into the situation in other non-classical logics, such as conditional or intuitionistic or relevance/relevant logic. In every case, mathematical results about model theory can be useful for philosophical purposes when properly employed. In every case, the model theory does not come with directions for its proper employment, which rather can only be discovered by philosophical analysis and reflection.

NOTES

[1] On a conflicting usage avoided here, a model *of given sentences* is a structure *in which those sentences are true*.

[2] Of the trio of basic metatheorems treated in standard textbooks of classical first-order logic, the *Gödel completeness* and *compactness* and *Löwenheim-Skolem* theorems, the first connects model theory and proof theory, while the other two belong to pure model theory.

[3] It would also be to arrive at a crude version of Davidsonianism, directly identifying meaning with truth-conditions, rather than the sort of refined version found in Lepore & Ludwig (2007), that makes the connection much less direct.

[4] These identifications calls for two glosses that for simplicity are ignored in the text. First, the 'premises' and 'conclusions' with which we are concerned must be taken to be sentence *types*, since otherwise, owing to the presumed nonexistence of more than finitely many tokens, most formulas would count under our definition as vacuously valid simply for lack of any instantiations; but then, since outside mathematics indexicality is ubiquitous and prevents sentence types from having truth values except relative to a context, truth in all instances must be tacitly understood to require truth for all instances *relative to all contexts*. Second, the instantiation of a sentence letter may be by an *open* sentence, containing free variables, and analogously for the instantiation of predicate letters, and truth in all instances must be tacitly

understood to require truth for all instances *for all values of such free variables*.

[5] Further, the *isomorphism lemma* states that models have the same formulas true in them, so we only really need to consider mathematical models in which the quantifier domain is either one of the form $\{0, 1, \ldots , n\}$ for some n or the set $\{0, 1, 2, \ldots\}$. Finally, the *arithmetical Löwenheim-Skolem theorem* states that if there is any model of the latter kind in which a sentence is true, there is one where the interpretation $P^{\mathbf{M}}$ of each predicate letter P is arithmetically definable.

[6] For second-order logic, by contrast, and with it plural logic, there is no completeness theorem and no scissors, and the assumption that every formula that is valid in the official sense is valid in the intuitive sense is known to be unprovable from the usual axioms of set theory, leaving the ultimate significance of the model theory in some doubt.

[7] Use of the colorful terminology of 'worlds' rather than, say, the plainer terminology of 'states', has if anything had the opposite of a clarifying effect.

[8] On the other hand, the mere existence of a formal model theory, which in itself shows nothing more than that the axioms systems are formally consistent, encouraged complacency in the face of Quine's questioning whether 'quantifying in' to modal contexts made any intuitive sense. If metaphysical modality is meant, it does; if logical modality is meant, it does not; or so it is argued in Burgess (1988).

[9] What is to be shown is that if π is true, so is 'it is true by virtue of logical form that it is not false by virtue of logical form that π', which is to say, that anything of the same logical form as 'it is not false by virtue of logical form that π' is true. But any such thing will be 'it is not false by virtue of logical form that ρ' for some sentence ρ of the same logical form as π, and the truth of the required sentence is equivalent to the existence of some sentence of the same logical form as ρ that is true, and π is such a sentence.

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