

MATHEMATICAL RIGOR

BETWEEN THEORY AND PRACTICE

John P. Burgess & Silvia De Toffoli

with guest appearances by Benjamin Morison & Desmond Hogan

COURSE DESCRIPTION

Over the last century, the philosophy of mathematics has been primarily focused on the quest to establish logical foundations and to justify axioms. But there are also philosophical issues that have to do with how living, breathing human agents practice mathematics. Our seminar will explore the middle ground between the tidy abstract realm of mathematical theories and the messy concrete world of human mathematicians. While the correctness of a mathematical argument is nowadays often identified with its formalizability, traditional proofs frequently present gaps and involve various high-level inferences; they include diagrams as well as heterogeneous notations: in short, they are unlike formal proofs. What, then, is the relationship between formal and informal proofs? How can we explain our reliability in getting things right?

"Gray ... is all theory, and green the golden tree of life."

*34.42. $\vdash (z) . R^z z = P^z Q^z . \supset . R = P \mid Q$
Dem.
 $\vdash . *14.21 . \quad \supset \vdash . \text{Hp} . \supset (z) . E ! R^z z : (z) . E ! P^z Q^z z \quad (1)$
 $\vdash . (1) . *34.41 . \supset \vdash . \text{Hp} . \supset (z) . R^z z = (P \mid Q)^z z :$
 $[*30.42.(1)] \quad \supset . R = P \mid Q : . \supset \vdash . \text{Prop}$
 *34.5. $\vdash : x R^2 y . \equiv . (\exists z) . x R z . z R y \quad [*34.1 . (*34.02)]$
 *34.51. $\vdash : x R^2 y . \equiv . (\exists z, w) . x R z . z R w . w R y$
Dem.
 $\vdash . *34.1 . (*34.03) . \supset$
 $\vdash : . x R^2 y . \equiv : (\exists w) . x R^2 w . w R y :$
 $[*34.5] \quad \equiv : (\exists w) : (\exists z) . x R z . z R w : w R y :$
 $[*11.55] \quad \equiv : (\exists w, z) . x R z . z R w . w R y :$
 $[*11.2] \quad \equiv : (\exists z, w) . x R z . z R w . w R y : . \supset \vdash . \text{Prop}$
 *34.52. $\vdash . R^3 = R \mid R^2 \quad [*34.21]$
 *34.53. $\vdash : \exists ! R^2 . \equiv . \exists ! D^2 R \cap \text{Id}^2 R \quad [*34.3]$
 *34.531. $\vdash : D^2 R \cap \text{Id}^2 R = \Delta . \equiv . R^2 = \Delta \quad [*34.53 . \text{Transp}]$
 *34.54. $\vdash : x R x . \supset . x R^2 x$
Dem.
 $\vdash . *4.24 . \supset \vdash : x R x . \supset . x R x . x R x .$
 $[*10.24] \quad \supset . (\exists y) . x R y . y R x .$
 $[*34.5] \quad \supset . x R^2 x : \supset \vdash . \text{Prop}$
 *34.55. $\vdash : . R^2 \in S . \equiv : x R y . y R z . \supset_{x, y, z} . x S z \quad [*34.5 . *10.23]$
 *34.56. $\vdash . D^2 R^2 \subset D^2 R . \text{Id}^2 R^2 \subset \text{Id}^2 R . C^2 R^2 \subset C^2 R \quad [*34.36.38]$

from Russell & Whitehead *Principia Mathematica*



Fields Medalist Maryam Mirzakhani lecturing

TENTATIVE SCHEDULE
TUESDAYS 10:00-12:50 / 201 MARX HALL

WEEK 1 (SEP 07) POSING THE QUESTION

Burgess *Rigor & Structure*, chapters 1 & 2

Steiner *Mathematical Knowledge*, chapter 3, section 3

optional Burgess "Proofs about Proofs" and "Foundational Work"

optional Avigad "Does Philosophy Still Need Mathematics and Vice Versa?"

WEEK 2 (SEP 14) HISTORICAL BACKGROUND: EUCLID

Guest Appearance: **Benjamin Morison**

Beere/Morison "A Mathematical Form of Knowing How: the Nature of Problems in Euclid's Geometry"

WEEK 3 (SEP 21) HISTORICAL BACKGROUND: KANT

Guest Appearance: **Desmond Hogan**

Hogan "Kant and the Character of Mathematical Inference"

WEEK 4 (SEP 28) HISTORICAL BACKGROUND: EULER

Lakatos "Proofs and Refutations"

Steiner *Mathematical Knowledge*, chapter 3, section 4

WEEK 5 (OCT 05) FORMAL vs INFORMAL PROOFS: A DEBATE AMONG PHILOSOPHERS, PART I

Avigad "Reliability of Mathematical Inference"

Tanswell "A Problem with the Dependence of Informal Proofs on Formal Proofs"

optional Azzouni "The Derivation-Indicator View of Mathematical Practice"

WEEK 6 (OCT 12) FORMAL vs INFORMAL PROOFS: A DEBATE AMONG PHILOSOPHERS, PART II

Fallis "Intentional Gaps in Mathematical Proofs"

Andersen "Acceptable Gaps in Mathematical Proofs"

WEEK 7 (OCT 26) FORMAL vs INFORMAL PROOFS: A DEBATE AMONG MATHEMATICIANS

Jaffe/Quinn "Theoretical Mathematics: Towards a Cultural Synthesis..."

Thurston "On Proof and Progress in Mathematics"

optional Replies to Jaffe/Quinn

WEEK 8 (NOV 02) PROOFS AND JUSTIFICATION: A DEBATE AMONG PHILOSOPHERS

Paseau "What is the Point of Complete Rigour?"

De Toffoli "Groundwork for a Fallibilist Account of Mathematics"

Pelc "Why Do We Believe Theorems?"

optional Rav "Why Do We Prove Theorems?"

WEEK 9 (NOV 09) PROOFS AND JUSTIFICATION: A DEBATE AMONG MATHEMATICIANS

Dehn "The Mentality of the Mathematician. A Characterization"

Zeilberger "THEOREMS FOR A PRICE: Tomorrow's Semi-Rigorous Mathematical Culture"

Hales "Formal Proofs"

WEEK 10 (NOV 16) COMPUTER ASSISTED PROOFS: THE 4-COLOR THEOREM

Appel/Haken "The Four Color Proof Suffices"

Detlefsen and Luker "The Four-Color Theorem and Mathematical Proof"

optional Tymoczko "The Four-Color Problem and Its Philosophical Significance"

WEEK 11 (NOV 23) MATHEMATICS AND THE A PRIORI

Burge "Content Preservation" *extracts*

Burge "Computer Proofs and the A Priori"

De Toffoli "Diagrams and the a priori" (manuscript)

optional De Toffoli, "Who is Afraid of Mathematical Diagrams?"

WEEK 12 (NOV 30) NON-DEDUCTIVE METHODS IN MATHEMATICS

Fallis "The Epistemic Status of Probabilistic Proofs"

De Toffoli "Beliefs, Degrees of Beliefs, and 'Probabilistic Proofs'" (manuscript)

Baker "Enumerative Induction in Mathematics: The Lure of Small Numbers" (manuscript)

COURSE MECHANICS

GRADUATE STUDENTS	UNDERGRADUATE STUDENTS
<p>Princeton philosophy graduate students seeking to earn a unit in connection with the seminar may write three short (1500± words) papers each related to one week's session, containing a clear and concise account of the main questions raised and the most important points about them made in the readings and discussion, plus a tentative conclusion favoring one or another position, or indicating further issues that would need to be explored.</p> <p>Papers will be due by 5 pm Princeton time 08 OCT (last class day before undergrad midterms) 23 NOV (last day before Thanksgiving break) 14 DEC ("dean's date")</p> <p>For one paper a student may substitute undertaking the oral presentation of one main reading for a given week, and leading / moderating the ensuing discussion, as per the department's oral presentation requirement.</p> <p>If the student prefers, a single longer (3000± words) paper may be substituted for the last two papers, to be due 14 DEC with a brief preliminary draft due 23 NOV.</p>	<p>The Office of the Dean of the College prohibits undergraduates officially enrolling in Philosophy graduate courses, but the ban can be worked around as follows:</p> <p>The interest student should obtain from their academic dean a form petitioning for a reading course that just happens to have the same instructors, readings, and meeting times as the graduate seminar.</p> <p>The form will need to be signed by JPB (as instructor & director of undergraduate studies). The form also calls for the department chair's signature, but JPB can take care of that.</p> <p>The form must then be submitted to the student's dean together with this syllabus highlighting the following statement about grading:</p> <p>Undergraduates will graded based on the three papers as described under requirements for graduate students. Class participation will also be taken into account.</p>