

NOTES ON ASSET TRADING IN AN OVERLAPPING  
GENERATIONS MODEL

by

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I wish to thank W. A. Brock, J. Fraenkel, S. Rosen, A. Weiss, and especially R. Lucas for comments. A conversation with Neil Wallace in the winter of 1978 made me aware that he had found independently that the introduction of a real asset (that he called land) in the overlapping generations models was sufficient to bring optimality. This of course is similar to the result in Section 2 below. Financial support from NSF by a grant to the University of Chicago is acknowledged.

January 1978 - Revised July 1980

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Samuelson's (1958) overlapping generations model is perhaps the simplest general equilibrium model in which, even in the absence of externalities, a competitive equilibrium may fail to be efficient. Samuelson himself noticed that if fiat money was introduced and if it had a constant positive price then efficiency could be attained. This solution turned out not to be satisfactory since a model with overlapping generations and fiat money exhibits in general a continuum of equilibria of which only one had a constant price for money. All the other ones were, in general, inefficient. Gale (1973) contains a thorough discussion of this subject. Diamond (1965) showed that the inefficiency result still holds in an economy with production, provided certain types of constant returns are present.

On the other hand, the determination of the value of a share of a firm in a competitive economy had been studied mostly in terms of arbitrage relationships between prices today, tomorrow and profits. One outstanding question was if in competitive equilibrium the value of a share at time zero equaled the discounted value of the maximum profit stream from time zero to infinity or equivalently if the present value as of time zero of the shares at time  $t$  goes to zero as  $t$  goes to infinity. For economies with a representative infinitely lived individual the necessity of the transversality condition at infinity (cf. Benveniste-Scheinkman [1976]) that roughly says that individuals do not want to hold "much" wealth at infinity was used by

myself (Scheinkman [1977]) to show that this result must be true. Later Brock [1978] used the same reasoning to price shares in an economy with uncertainty. In an economy with overlapping generations there is no transversality condition at infinity. However, in such economies the fact that young individuals inherit nothing from the past, puts a natural bound in the current value of shares since those shares must be acquired by such individuals with funds obtained only from the sale of their initial endowment. Notice that this will be sufficient to prove the desired results provided the interest rate sequence behaves nicely; more precisely, if the discounted value at time zero, of one unit of numeraire at time  $t$  goes to zero as  $t$  goes to infinity, which in particular would imply production efficiency. (cf., Cass [1972]). In order to show this last result we will strongly use the fact that the firm may produce positive profits each period if zero interest rates prevail. If interest rates get too low the discounted value of profits gets too large and the current value of the shares, for which the discounted value of future profits acts as a floor, becomes again too large to be compatible with equilibria.

One fact that complicates the analysis of asset prices is that, in the presence of adjustment costs, the value of a firm is a non-linear function of its capital stock. It seems easier to first expose our results in the absence of this complication. For this reason we start, in Section 2, by introducing a firm which rents out capital and that is capable of generating at zero interest rates positive profits. We then show that the equilibrium is production efficient. The logic behind this result is easy to explain. If the present value of a unit of consumption at time  $t$  as of time zero does not go to zero "quickly enough" as  $t$  goes to infinity the discounted

profits stream and hence the value of the shares at time zero would get arbitrarily large. This is incompatible with competitive equilibria since the only demands for shares at time zero are of the young generation born at zero and their wealth is independent of the value of the shares held by the old generation. If the reader wants to imagine a dynamics he may think that if the discounted profits stream gets too close to their initial endowment, young individuals' borrowing gets too large to be compatible with equilibrium. An easy corollary is that the present value as of time zero of a share at time  $t$  goes to zero as  $t$  goes to infinity.

The model in Section 2 does not allow for asset prices that depend on the firm's capital stock in use and hence the question of share pricing is much simplified. In Section 3 we introduce a firm facing adjustment costs and show that the results of the preceding section still hold. In particular if the firm maximizes shareholders' welfare at each time  $t$ , taking into account the asset pricing function, it acts as if it is maximizing discounted profits.

The result proved in Section 3 assumes that prices for the shares of firms with any capital structure are quoted. This is an assumption that corresponds to the existence of prices for goods that are perhaps non-produced in equilibrium in the standard general equilibrium model. Although this requirement seems rather innocuous in the standard model it is essential in proving optimality even there. For if consumers optimize subject to a budget constraint and a given set of available goods, some good may not be produced in an equilibrium even though it would be optimal to do so. Hart (1980) analyses this issue in the context of non-cooperative equilibria with many agents. Khilstrom and Laffont (1980) face the same issue in the context of share pricing in a two-period model under uncertainty. In their

paper, the absence of adjustment cost requires, for the firms which are active in equilibria, that the share price be linear on the capital stock. In order to obtain optimality of equilibria they assume that agents take the same linear function as the expected share price for a firm with any capital stock. They could equivalently have assumed that individuals may present demand for shares of firms with an arbitrary capital stock and require that in equilibrium the total demand for shares of inexistent firms be zero. Since we want to prove restrictions on share prices and in our context there is no linearity present, we take here this last alternative. The reader should, however, take note of the tremendous informational requirements that are present in our equilibria.

Finally we present an example that serves to illustrate the effect of sharetrading. Without sharetrading the C.E. is inefficient but with it, it turns out to be Pareto Optimum. This example is, we believe, close in spirit to those available in the literature (as in Diamond [1965]), although it does not exhibit constant returns since in this case our hypothesis would fail.

The results in this paper complement our previous results (Scheinkman [1977]). There we proved similar theorems for a model in which consumers are infinitely lived. The two sets of theorems taken together suggests that the results may be pretty general. In Section 5 we comment on the relationship between the results where a "real asset" is present with those obtained when "fiat-money" is present.

## Section 2. The Simple Model

A typical individual born at  $t$  lives for two periods. When young he receives endowment  $w_y$ , when old,  $w_o$ . When young he may borrow or lend at an interest rate  $R_t$  and also buy shares of the firm. Hence he wishes to maximize his utility function

$$u(c_t^y, c_t^o) \text{ among } c_t^y \geq 0, c_t^o \geq 0$$

subject to the budget constraint

$$(2.1) \quad (w_y - c_t^y - \gamma_t v_t) \phi_t = c_t^o - w_o - \gamma_t v_{t+1} - \gamma_t \pi_{t+1}$$

where  $\gamma_t$  is the number of shares he buys at  $t$ ,  $v_t$  the price of a share at  $t$ ,  $\phi_t = 1 + R_t$  and  $\pi_t$  the profit realized by the firm at  $t$ . We assume that  $u$  is continuously differentiable, strictly concave, and that for any  $\hat{x} > 0$ ,  $\hat{w} > 0$

$$\lim_{x \rightarrow 0} u_1(\hat{x}, w) = \lim_{w \rightarrow 0} u_2(\hat{x}, w) = \infty.$$

Maximization of utility subject to (2.1) implies that if we assume that there exists a strictly positive feasible consumption vector,

$$(2.2) \quad u_1(c_t^y, c_t^o) - u_2(c_t^y, c_t^o) \phi_t = 0$$

$$(2.3) \quad u_2(c_t^y, c_t^o) (-\phi_t v_t + v_{t+1} + \pi_{t+1}) \leq 0 \quad (\text{with equality if } \gamma_t > 0) .^1$$

Each firm at time  $t$  rents capital at rate  $R_t$  and produces an output  $f(k_t)$  in the beginning of period  $t+1$ . Hence  $\pi_{t+1} = f(k_t) - \phi_t k_t$ . (Notice that we assume that depreciation is included in  $f'$ .) We will assume that  $f$  is concave, with  $f(0) = 0$  that  $\lim_{k \rightarrow 0} f'(k) > 1$  and that

$\lim_{k \rightarrow \infty} f'(k) \leq 0$ . Notice that this rules out pure storage technologies.

Each firm's demand for capital at  $t$ , thus solves  $\max_{y > 0} \{f(y) - \phi_t y\}$ . We

assume a large number of firms or equivalently a continuum of firms.

For given  $\phi_t$  we define the firm's demand  $\Gamma(\phi_t)$  as a pair  $(K(\phi_t), \pi(\phi_t))$  where  $K(\phi_t) = \{k/f(k) - \phi_t k \geq f(y) - \phi_t y \text{ for all } y \geq 0\}$  and  $\pi(\phi_t) \equiv f(k) - \phi_t k$  for all  $k \in K(\phi_t)$ . Notice that  $K(\phi_t)$  is a convex set. Thus average demand for capital is in turn an element of  $K(\phi_t)$ .

Our definition of competitive equilibrium (henceforth C.E.) would be the usual one under perfect foresight.

Formally, a C.E. is a pair of non-negative sequences  $\hat{\phi} = \{\hat{\phi}_t\}_{t=1}^{\infty}$  and  $\hat{v} = \{\hat{v}_t\}_{t=1}^{\infty}$  such that if  $\hat{k}_t \in K(\hat{\phi}_t)$  for each  $t = 1, 2, \dots$  and

$\hat{\pi} = \{\hat{\pi}_t\}_{t=1}^{\infty}$  with  $\hat{\pi}_t = \pi(\hat{\phi}_t)$  and if the triple of sequences

$[\{\hat{\gamma}_t\}_{t=1}^{\infty}, \{\hat{c}_t^y\}_{t=1}^{\infty}, \{\hat{c}_t^o\}_{t=1}^{\infty}]$  solves the consumer's maximization problems

when the sequences  $\hat{\phi}$ ,  $\hat{v}$  and  $\hat{\pi}$  are taken as given then

$$(2.4) \text{ (a)} \quad \hat{\gamma}_t \leq 1 \quad \text{for } t = 1, 2, \dots \quad \text{with equality whenever } \hat{v}_t > 0$$

$$(2.4) \text{ (b)} \quad \hat{c}_t^y + \hat{c}_{t-1}^o = \omega_y + \omega_o - k_t + f(k_{t-1}) \quad \text{for } t = 1, 2, \dots$$

Equations (2.4)(a) and (b) state that the share market (with the normalization that there exists one share per individual of each generation) and the goods market are cleared.

Now we first notice that at an equilibrium we get

$$(2.5) \quad \hat{v}_t = \phi_t^{-1} (\hat{v}_{t+1} + \hat{\pi}_{t+1})$$

To see this just notice that when  $\hat{\gamma}_t > 0$  this equation holds by (2.3). On the other hand if  $\hat{\gamma}_t = 0$  then  $\hat{v}_t = 0$  and since  $\hat{v}_{t+1} \geq 0$  and  $\hat{\pi}_{t+1} = \max_y \{f(y) - \phi_t y\} \geq 0$  equality must hold in (2.3) in equilibria.

The existence of an inverse for  $\phi_t$  is guaranteed by (2.2). In particular if  $\hat{v}_t = 0$  then from (2.5),  $\hat{v}_{t+j} = 0$ , for any  $j \geq 0$  and  $\hat{\pi}_{t+j} = 0$  for  $j \geq 1$ .

Equation (2.5) is a simple arbitrage relationship that states that the value of a share today equals the discounted profits that the ownership of such a share entitles, plus the discounted value of the share tomorrow. Again, writing  $\psi_t = \prod_{s=1}^t \phi_s^{-1}$  we have

$$(2.6) \quad \hat{v}_1 = \sum_{t=1}^T \hat{\psi}_t \hat{\pi}_{t+1} + \hat{\psi}_T \hat{v}_{T+1} \quad \text{for each } T = 1, 2, \dots$$

Our objective is to establish that  $\lim_{T \rightarrow \infty} \hat{\psi}_T \hat{v}_{T+1} = 0$ , i.e., that the present value of a share at  $T$  as of time zero goes to zero as  $T$  goes to infinity. First observe that since  $\hat{c}_t^y \geq 0$  and  $\hat{k}_t \geq 0$  and  $\hat{c}_{t-1}^o \geq R_t \hat{k}_{t-1} + \hat{\pi}_{t-1} f(\hat{k}_{t-1})$  we must have  $\hat{v}_{T+1} < w_y$ . This expresses

the fact that the young must buy the shares from their initial endowment.

The result will be thus established if we can show that  $\lim_{T \rightarrow \infty} \hat{\psi}_T = 0$ .

We will actually prove a stronger result, namely that  $\lim_{T \rightarrow \infty} \sum_{t=0}^T \hat{\psi}_t$  exists.

To show this, first notice that

$$w_y > \hat{v}_1 \geq \sum_{t=1}^T \hat{\psi}_t \hat{\pi}_{t+1} \geq \sum_{t=1}^T \hat{\psi}_t [f(\bar{k}) - \hat{\phi}_t \bar{k}]$$

where  $\bar{k} > 0$ , where  $f(\bar{k}) > \bar{k} + \epsilon$  for some  $\epsilon > 0$ . One such  $\bar{k}$  must exist since  $f'(0) > 1$ .

Thus,

$$w_y \geq \hat{v}_1 \geq \sum_{t=1}^T (\hat{\psi}_t - \hat{\psi}_{t-1}) \bar{k} + \epsilon \sum_{t=1}^T \hat{\psi}_t = (\hat{\psi}_T - 1) \bar{k} + \sum_{t=1}^T \hat{\psi}_t$$

Thus  $\sum_{t=1}^T \hat{\psi}_t \leq \frac{w_y + \bar{k}}{\epsilon}$  and since  $\hat{\psi}_t \geq 0$  we have that  $\lim_{T \rightarrow \infty} \sum_{t=1}^T \hat{\psi}_t$  exists.

This result says in particular that equilibrium in this model cannot be production inefficient since the interest rate sequence satisfies the production efficiency criteria of Cass (1972).

A further implication of the above reasoning is that since  $\hat{v}_T \leq w_y$ ,  $\lim_{T \rightarrow \infty} \hat{\psi}_T \hat{v}_T = 0$ , i.e., the present value as of time zero of the shares of the

firm at time  $T$  goes to zero as  $T$  goes to infinity or equivalently from

$$(2.6) \quad \hat{v}_1 = \sum_{t=1}^{\infty} \hat{\psi}_t \hat{\pi}_{t+1}, \text{ i.e., the value of the firm is equal to the (maximum)}$$

discounted profit stream.

### Section 3. A Model with Adjustment Costs

In the model of the previous section the firm was simply a production process. It was for this reason that we chose the convention that all capital was borrowed. We could have allowed the firm to own capital but this would have merely increased  $v_t$  by  $k_t$  and could not have altered any of the results. The situation is, however, much more complicated when adjustment costs are present. In this case the value of the firm is in general a non-linear function of its capital stock. Hence the firm's objective function in time  $t$  should be to maximize the sum of profits plus the value of left-over capital stock. Our aim in this section is not only to show that the efficiency results proven above still hold but also that in competitive equilibria the value of the firm equals the maximum discounted profits from the initial capital stocks. This is another way of saying that the share valuation function gives the "right" signals towards capital accumulation. As we noted in the introduction the set of required signals is indeed large since not only capital stocks that appear in equilibrium must be evaluated, but also those which are absent.

The consumer's side of our economy is as in the previous section. Each firm's output at time  $t+1$  depends, however, not only on the capital stock it hires at time  $t$ ,  $k_t$  but also on the capital stock it used in production at time  $t$ ,  $k_{t-1}$ . Formally, output at  $t$  is given by a concave function  $g(k_{t-1}, k_t)$  on non-negative pairs of real numbers. In order to rule out negative asset prices we assume that  $g(x, 0) > 0$  for any  $x > 0$ . We next need an assumption to substitute for the hypothesis made in Section 2 that  $\lim_{k \rightarrow \infty} f'(k) > 1$ , i.e., that positive profits are possible at zero interest rates. We will thus assume that there exists  $\bar{x} > 0$

such that if  $0 < x < \bar{x}$ ,  $g(x, x) > x$ . This says that by keeping a low enough constant capital stock each firm may produce profits if interest rates are zero. The firm's management faces a sequence of non-negative asset pricing functions  $v_t(k_{t-1})$ ,  $t = 1, 2, \dots$ . Notice that we make no assumption on stationarity of this asset pricing function, except that each of the large number of firms we assume to exist is indistinguishable except for the capital stock it held in the previous period. In order to avoid technical issues relating to the measurability of certain functions we will assume that we have  $n$  firms where  $n$  is large but finite.

We will also assume that at each time  $t$  the  $i$ -th firm's management maximizes time  $t$  shareholder's welfare, i.e., choose  $k_t^i$  such that  $f(k_{t-1}^i, k_t^i) + v_{t+1}(k_t^i) - \phi_t k_t^i \geq f(k_{t-1}^i, y) + v_{t+1}(y) - \phi_t y$  for each  $y \geq 0$ . By hypothesis since  $f(k_{t-1}^i, 0) + v_{t+1}(0) \geq 0$  it always attains a non-negative value.

For a given function  $v$  and  $\phi \geq 0$ ,  $x \geq 0$  let  $K(v, \phi, x) = \{k \geq 0 \mid f(x, k) - \phi k + v(k) \geq f(x, y) - \phi y - v(y) \text{ for each } y > 0\}$ . Thus  $k$  is the capital stock chosen by a firm who had capital stock  $x$ , facing an interest rate  $1 - \phi$  and asset pricing function  $v$ . Since no a priori hypothesis is set on  $v$ ,  $K$  may very well contain many or no points. However, if  $K$  is non-empty,  $\eta(v, \phi, x) \equiv f(x, k) - \phi k + v(k)$  for each  $k \in K(v, \phi, x)$  is well defined. For our consumers, what matters is the sum of profits per share paid at  $t + 1$  plus the value of the share at  $t + 1$ . Thus, let us define  $\eta(v, \phi, x)$  as the total return of a share of a firm who had capital stock  $x$ , facing an interest rate  $1 - \phi$ , when the asset pricing function for next period is  $v$ . Consumer's will take as given the sequences of interest factors  $\phi_t$  and return functions  $\eta_t(x)$ . Obviously  $\eta_{t+1}(x) = \eta(v_{t+1}, \phi_t, x)$ .

Our definition of C. E. now parallels the one of Section 2.

An equilibrium with an initial capital structure  $\{k_{-1}^i\}_{i=0}^n$  is a pair of sequences  $\{\hat{\phi}_t\}_{t=1}^\infty$  and  $\{\hat{v}_t\}_{t=1}^\infty$  where each  $\phi_t$  is a non-negative number and  $\hat{v}_t$  are non-negative functions on the real line such that there exists sequences  $\{\hat{k}_t^i\}_{t=0}^\infty$   $i = 1, \dots, n$  with  $\hat{k}_t^i \in K(v_{t+1}, \phi_t, \hat{k}_{t-1}^i)$  and if  $v_{t+1}(x) = \eta(\gamma_{t+1}, \phi_t, x)$  and if the consumer's maximization problem given the sequences  $\hat{\phi}$ ,  $\hat{v}$  and  $\hat{\eta}$  is solved by  $\hat{\gamma} = \{\hat{\gamma}_t(k)\}$ ,  $\{\hat{c}_t^y\}_{t=1}^\infty$ ,  $\{\hat{c}_t^o\}_{t=1}^\infty$  then

$$(3.1) \text{ (a)} \quad \hat{\gamma}_t(\hat{k}_t^i) \leq 1 \text{ for } t = 1, 2, \dots \text{ with equality if } \hat{v}_t(\hat{k}_t^i) > 0$$

$$\text{and } \hat{\gamma}_t(k) = 0 \text{ if } k \in \{\hat{k}_t^1, \dots, \hat{k}_t^n\}$$

$$(3.1) \text{ (b)} \quad \hat{c}_t^y + \hat{c}_t^o = \sum_{i=1}^n f(\hat{k}_{t-2}^i, \hat{k}_{t-1}^i) - \sum_{i=1}^n \hat{k}_t^i + w_y + w_o$$

Again (3.1a) expresses the clearing of the market for shares, while (3.1b) the clearing of the output market. In (3.1a) we require, in particular, that the demand for shares of firms with capital structure that do not appear in equilibria be zero.

As before, we first establish an arbitrage equation, i.e., we will derive the analogue of equation (2.5). Let us first rewrite (2.3) as,

$$(3.2) \quad u_2(c_t^y, c_t^o)(-\phi_t v_t(k) + \eta_{t+1}(k)) \leq 0$$

with  $=$  if  $\gamma_t(k) > 0$

In equilibrium since  $\hat{\gamma}_t(k) > 0$  (free disposal), and  $\hat{\eta}_{t+1}(k) \geq 0$  one must in fact, as before, have equality in (3.2) if  $k \in \{\hat{k}_{t-1}^1 \dots \hat{k}_{t-1}^n\}$ . If  $k \notin \{\hat{k}_{t-1}^1 \dots \hat{k}_{t-1}^n\}$ , then

$$(3.3) \quad \hat{v}_t(k) > \hat{\phi}_t^{-1} \hat{\eta}_{t+1}(k)$$

As in the previous section it is sufficient to show that if  $\hat{\psi}_t = \prod_{s=1}^t \hat{\phi}_s^{-1}$  then  $\sum_{t=0}^{\infty} \hat{\psi}_t < \infty$ . This is so since  $\hat{v}_t(k_t^i) \leq w_y$ ,  $i=1 \dots n$  and thus in equilibrium the present value of a share of a firm at time  $t$  as of time zero, will go to zero as  $t$  goes to infinity provided  $\psi_t$  is summable. From here it is pretty easy to show, using that equality must hold in (3.2) for capital stocks that actually appear in equilibrium, that, in particular, a firm's value equals the maximum discounted profit. To show the desired result we will show that if  $\hat{\psi}_t$  is not summable a firm's share value at zero would exceed  $w_y$  by exploring the feasible path  $(\hat{k}_{t-1}, \bar{x}, \bar{x} \dots)$  where  $\bar{x}$  satisfies  $f(\bar{x}, \bar{x}) > \bar{x} + \epsilon$ .

Now from (3.2) we must have

$$(3.4) \quad w_y \geq \hat{v}_1(\hat{k}_0^i) = \hat{\phi}_1^{-1} \hat{\eta}_2(\hat{k}_0^i) = \hat{\phi}_1^{-1} \max_k \{f(\hat{k}_0^i, k_1^i) - \hat{\phi}_1 k_1^i + \hat{v}_2(k_1^i)\} \geq f(\hat{k}_0^i, \bar{x}) - \bar{x} + \psi_1 \hat{v}_2(\bar{x})$$

Furthermore, from (3.3) we obtain,

$$(3.5) \quad \hat{\psi}_{t-1} \hat{v}_t(\bar{x}) \geq \hat{\psi}_t \hat{\eta}_{t+1}(\bar{x}) \geq \hat{\psi}_t(\bar{x} + \epsilon) - \hat{\psi}_{t-1} \bar{x} + \hat{\psi}_t \hat{v}_t(\bar{x})$$

Thus, applying successfully (3.5) to (3.4) we obtain,

$$w_y \geq \epsilon \sum_{t=2}^T \hat{\psi}_t + \psi_T \bar{x} - \bar{x} + f(k_0^1, \bar{x})$$

and thus  $\hat{\psi}_t$  is summable.

Section 4. An Example

We present here an example of how tradeable shares move an economy from an inefficient equilibrium to an efficient one.

Let us consider an economy in which

$$f(k) = 2k \quad 0 \leq k \leq \frac{1}{10}$$

$$f(k) = 1/5 + 1/2(k - 1/10) \quad k > 1/10$$

Let the consumer's endowment be  $(w_y, w_o) = (\frac{12}{11}, \frac{6}{55})$  and the utility function  $u(c^y, c^o) = \log c^y + \log c^o$ . If the profits are distributed to the old, and there is no share trading the C.E.<sup>2</sup> is given by  $\phi_t \equiv \frac{1}{2}$ ,

$c_t^y \equiv \frac{177}{220}$ ,  $c_{-1}^o = 6/55$ ,  $c_t^o = \frac{177}{440}$ ;  $t = 0, 1, \dots$ ,  $k_t \equiv 63/220$ . Now if assets

are traded and bought by the young from the old in order to enjoy profits next period then the C.E. is  $\phi_6 \equiv 6/5$ ,  $c_t^y \equiv 12/22$ ,  $c_t^o \equiv 39/55$ ,  $t \geq 0$ ,

$c_{-1}^o = 28/55$ ,  $k_t \equiv \frac{1}{10}$  and  $v_t \equiv 2/5$ .

Notice that while the just allocation presents capital overaccumulation the second one is Pareto optimum. The existence of tradeable shares makes the total value of the assets larger than the economy-wide capital stock. In this sense it acts like government bonds or money do in the more traditional overlapping generations model.

## Section 5. Money x "Real" Assets

As we mentioned in the introduction the presence of fiat-money in overlapping generations models is not sufficient to guarantee the optimality of equilibria. This is not, however, a special feature of overlapping generations models. Scheinkman [1980] showed that in a model where agents live infinitely and trade using money or a "barter" technology perfect foresight equilibria with a constant amount of money may yield an infinite price level in the limit. The technical reason behind this is that the transversality condition at infinity places an upper bound in the value of assets and thus avoids overaccumulation of total assets but does not guarantee a lower bound on the value of assets. Notice that in the example of Section 4, total assets are larger in the case where tradeable shares exist (they equal  $v_t + k_t = \frac{1}{2}$ ). It is only the value of accumulated capital that goes down. If instead one were to use "fiat" money as an alternative asset there would be equilibria in which the value of monetary holdings would go to zero and thus agents would accumulate more capital (though less assets). The distinguishing property of our asset, as opposed to fiat money, is that at zero real interest rates it provides claims to an amount of consumption goods which is strictly positive. In this sense it is a "real" asset. The results presented here seem to further indicate that it is this distinction and not whether individuals live forever that matters.

### Footnotes

<sup>1</sup>Notice that we rule out short-sales. If those were allowed equality would hold in (2.3) and nothing would be changed.

<sup>2</sup>In this example we assume that at time zero there are old people from the previous generation and young people.

## Bibliography

- Benveniste, L. M. and Scheinkman, J. A. 1976. Duality theory for dynamic optimization models of economics: the continuous time case. Manuscript. University of Chicago
- Brock, W. A. 1978. Asset prices in a production economy. Manuscript. University of Chicago.
- Cass, D. 1972. Capital Overaccumulation in the aggregative, neoclassical model of economic growth: a complete characterization. Journal of Economic Theory, Vol. 4, No. 2 (April): 200-223.
- Diamond, P. A. 1965. National Debt in a neoclassical growth model. American Economic Review, Vol. LV, No. 5, Part 1 (December): 1126-1150.
- Gale, D. 1973. Pure exchange equilibrium of dynamic economic models. Journal of Economic Theory, Vol. 6 (February): 12-36.
- Hart, O. D. 1980. Perfect competition and optimal product differentiation. Journal of Economic Theory, Vol. 22, No. 2 (April): 279-312.
- Kilstrom, R. and Laffont, J.J. 1980. Competitive Entrepreneurial Model of a Stock Market, unpublished.
- Samuelson, P. A. 1958. An exact consumption-loan model of interest with or without the social contrivance of money. Journal of Political Economy, Vol. 66 (December): 467-482.
- Scheinkman, J. A. 1977. Notes on Asset Prices. Manuscript. University of Chicago.
- \_\_\_\_\_. 1979. Discussion of Overlapping Generations Model of Fiat Money (Wallace). Models of Monetary Economies, J. H. Kareken and N. Wallace, eds. Federal Reserve Bank of Minneapolis. pp. 91-96.