

June 1984

③ GENERAL EQUILIBRIUM MODELS OF ECONOMIC FLUCTUATIONS:

A SURVEY OF THEORY

by

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During the last few months I've discussed issues related to this paper with almost every economist I met, but I owe a special debt to Robert Barro, Truman Bewley, J.M. Grandmont, Sanford Grossman, Lars Hansen, James Heckman, David Kreps, Robert Lucas and Larry Weiss. Paulo Leme was an able research assistant. Financial support from the NSF through grant SES-8308485 is gratefully acknowledged.

1. Introduction

In this survey I focus on general equilibrium models of aggregate economic fluctuations. The word equilibrium is used here in a particular restrictive way and in order to qualify a model must start from first principles, i.e., a description of the agents and how they interact; it must be explicitly dynamic and possess a notion of equilibrium that encompasses not only actions but also beliefs. On the other hand, I do not restrict myself to Walrasian, and certainly not to Arrow-Debreu types of economies, and had they existed I would gladly have included models with monopoly elements, rationing, etc. The increased interest in this type of formulation on the part of our profession stems not only from curiosity as to how macroeconomic phenomena is created but also from the realization that an equilibrium account of such phenomena is essential in order to evaluate hypothetical policy interventions. Further, the ability of some particular model to track some aspects of actual macro time series does not make it appropriate to evaluate possible policy changes. Thus, the fact that a complete markets model may explain well the comovements between output and hours worked does not make it adequate to study the impact of a compulsory social insurance program on employment.

Though I want to concentrate on models where markets are not complete, I start this survey by looking at some examples of complete market models. My intention in doing this is two-fold. On the one hand, I wish to show that there is much that the so-called representative consumer model can accommodate, and on the other hand, I wish to present some thoughts on how one could, in principle, reject it. The basic idea here is to explore the fact that in a

complete markets equilibrium, under reasonable assumptions, a lot of pooling of risks among consumers must occur.

In Section 3, I exhibit examples of how, even in the absence of uncertainty, a completely stationary model may exhibit cyclic behavior. Though they are not meant to be taken seriously as models of actual economic fluctuations--business cycles do not resemble deterministic motions--they serve to point out that some cyclic forces may not originate from exogenous shocks.

Models in which asymmetric information cause fluctuations are reviewed in Section 4. The model of Lucas [1972] is also the only one surveyed here where the money supply process plays a role in generating fluctuations. In Section 5, my paper with Weiss [1983] which suggests that the absence of some insurance markets may cause output fluctuations, is reviewed. An example is presented to show that under certain conditions the lack of such markets may have the opposite effect in the hope of clarifying the mechanism through which this assumption alters the amount of fluctuations present in an economy. Diamond and Fudenberg [1983] study a model where all centralized exchange is ruled out and where the level of economic activity may vary due to the beliefs of agents about trading opportunities. This, together with some other literature, is considered in Section 6.

The models in Sections 4, 5 and 6 are definitely particular examples as opposed to attempts at building a unified framework. There is a quantum leap in difficulty when we leave the Arrow-Debreu paradigm where the qualitative properties of solutions to a particular equilibrium model can be obtained by considering the associated maximum problem. In the incomplete markets case one must face the fact that agents will maximize taking parameters as given and that these parameters must equilibrate the model. Given the infinite

dimensional aspects inherent in these dynamic models one cannot hope to establish the properties of the equilibrium processes unless one is willing to make quite restrictive assumptions. Anyone who has attempted to do comparative statics of simple equilibrium systems with three goods must sympathize with this point of view. In fact, even establishing existence results in this type of framework is far from trivial. These models either involve an infinite number of agents or a finite number of agents, each one facing an infinite number of budget constraints. Bewley [1984] presents a proof of existence that covers many cases of interest.

There are two sets of "facts" about the cycle. One is that deviations of output from trend are persistent. The other one, less documented, consists of correlations of certain series, relations among their amplitudes, etc. The models discussed in Sections 4 to 6 make no attempt to explain simultaneously the diverse aspects of economic fluctuations. The Lucas [1972] model described in Section 4 is "rigged" towards producing a single one of these relationships: the Phillips Curve. The economies in Scheinkman-Weiss or in Diamond-Fudenberg are designed to produce examples of persistency even in the absence of such "easy" channels as inventories or capital goods. The fact that no model explains all the "known facts" is not all negative: These "facts" themselves tend to change (for a heterodox view see Litterman and Weiss [1984]).

In Section 7 we study some of the implications to asset valuation of the incomplete market models. Though this has no direct relationship to economic fluctuations--except perhaps through some correlations--there are natural scale economies in studying the two phenomena. Thus, for example, in Scheinkman-Weiss [1984] the equation that describes the equilibrium conditions is an asset pricing equation.

Except for some passing discussion in Section 6 there is no mention here of the popular "contract based" papers surveyed in Taylor [1983]. The reason is, of course, our stringent conditions regarding micro-foundations. Similarly, the requirement of an explicit dynamics eliminated from this survey a number of papers that discuss output variability in a static context as e.g., the optimal labor contracts literature. Finally, much of the work on Samuelson's consumption loan model is omitted since it is being presented in a companion lecture.

Some caveats are in order. This is not a formal survey but a set of notes for a lecture. If it sometimes reads as a research proposal it is because much is still to be done in the field. Further, even though I've worked in many aspects of dynamics there are areas covered here in which I am a beginner and where a specialist is bound to find fault. One hopes that one of these is in the audience to correct me. Worse than that: there must be papers not covered due to my ignorance. I do not want to apologize for the strict requirements I've placed on candidate models. To use an image that seems to be popular among Keynesians (cf., Tobin [1982], Hahn [1984]): I think it is better that we spend some time building some streetlights than looking for our keys in a large dark field.

2. "Brock and Mirman-Type Models"

Optimal growth models with or without the presence of uncertainty have long been used as examples of simple general equilibrium models. In fact, it follows from Debreu's [1954] theorem on the optimality of competitive equilibrium that if all consumers have time-separable utilities with a common rate of discount and if complete markets are present, then any competitive equilibrium must maximize a time-separable objective function with the same rate of discount subject to the technology constraints. By characterizing the solutions to this optimal growth problem one can then "read" the equilibrium prices, quantities, etc. Simple arbitrage arguments can then be used to "price" securities.

It is also clear that one may achieve optimality with considerably fewer markets. In fact all one needs is that at each "date-event" point the markets at all possible "date-events" next period be open and that agents correctly forecast future prices conditional on the states. I wish to use a simple example to clarify this discussion and, I believe, dispell some misunderstandings concerning such models. The example is basically the Brock-Mirman [1972] growth model. Consider a situation where at each time t a random variable $\theta_t \in S$, a finite set, is observed. We will write $\theta^t = (\theta_1 \dots \theta_t)$ for the histories up to t . There is a single capital good which is also the consumption good. Production is described by a neoclassical production function $f(\cdot, \theta^t)$ at date t for each given history θ^t . Agent i , $i = 1 \dots m$, has utility function $E_0 \sum_{t=1}^{\infty} \delta^t u^i(c_t(\theta^t), l_t(\theta^t), \theta^t)$ where $u^i(\cdot, \theta^t)$ is strictly concave, increasing and bounded function.

Let us denote by $\pi_t(\theta^t, \theta_{t+1})$ the "common" probability belief that event θ_{t+1} will occur given that θ^t occurred to t . At each date t "prices"

$p_t(\theta^t, \theta_{t+1})$, expressing the number of consumption goods at t that must be delivered in order to assure a unit of the consumption good at time $t+1$ if θ_{t+1} occurs, are quoted, as well as a wage rate $w_t(\theta^t)$.

The "firm" then solves

$$\text{Max}_{k, l, \theta_t} \{f(k, l, (\theta^{t-1}, \theta_t))p(\theta^{t-1}, \theta_t)\} - k - w_{t-1}(\theta^{t-1})l.$$

If we assume that f is strictly concave and with infinite marginal products at the boundary, then there exists a unique $k_{t-1}(\theta^{t-1})$, $l_{t-1}(\theta^{t-1})$ that solves this problem. Further, the maximized value that we will denote by $\gamma_{t-1}(\theta^{t-1})$ is strictly positive.

Shares to the profits of the firm are issued to be traded in a spot market. We will write $q_t(\theta^t)$ for the price at t given history θ^t .

Consumer i takes as given the random variables $q_t(\theta^t)$, $\gamma_t(\theta^t)$, $p_t(\theta^t, \theta_{t+1})$, $w_t(\theta^t)$, and solve:

$$\text{Max } E_{\theta_t} \sum_{t=1}^{\infty} \delta^t u^i(c_t^i(\theta^t), l_t^i(\theta^t), \theta^t) \quad \text{s.t.}$$

$$\begin{aligned} (P_t^i) \quad & c_t^i(\theta^t) + [s_t^i(\theta^t) - s_{t-1}^i(\theta^{t-1})]q_t(\theta^t) + \sum_{\theta_{t+1} \in S} x_{t+1}^i(\theta^t, \theta_{t+1})p_t(\theta^t, \theta_{t+1}) \\ & = x_{t-1}^i(\theta^t) + s_{t-1}^i(\theta^{t-1})\gamma_t(\theta^t) + l_t^i(\theta^t)w_t(\theta^t) \end{aligned}$$

$$x_0^i(\theta^1), \quad s_0^i \geq 0 \quad \text{given with} \quad \sum_1 s_0^i = 1 \quad \text{and}$$

$$\liminf_{t \rightarrow \infty} \delta^t E[s_t^i(\theta^t) q_t(\theta^t) + \sum_{\theta_{t+1} \in S} p_t(\theta^t, \theta_{t+1}) x_{t+1}^i(\theta^t, \theta_{t+1})] \geq 0.^1$$

Here, $x_t^i(\theta^t, \theta_{t+1})$ denotes the number of claims acquired by consumer i , at t , given history θ^t for the consumption good at $t+1$ if event θ_{t+1} occurs,

¹This last condition is needed to avoid "Ponzi" types of schemes.

and $x_{t-1}^i(\hat{\theta}^t) = x_{t-1}^i(\hat{\theta}^{t-1}, \hat{\theta}_t)$ where $(\hat{\theta}^{t-1}, \hat{\theta}_t) = \hat{\theta}^t$. An equilibrium is thus sequences $q_t(\theta^t)$, $\gamma_t(\theta^t)$ and $p_t(\theta^t, \theta_{t+1})$, $w_t(\theta^t)$ such that when each individual i solves P^i we obtain sequences of random variables $s_t^i(\theta^t)$, $l_t^i(\theta^t)$, $c_t^i(\theta^t)$ $i=1\dots m$ such that $k(\theta^t) + \sum_i c_t^i(\theta^t) = f(k_{t-1}(\theta^{t-1}), l_{t-1}(\theta^{t-1}), \theta^t)$, $\sum_i l_t^i(\theta^t) = l_t(\theta^t)$, and $\sum_i s_t^i(\theta^t) = 1$. Here $f(k_0, l_0, \theta^1) \equiv \sum_i x_0^i(\theta^1)$.

Such an equilibrium solves

$$\text{Max } E_0 \sum_{t=1}^{\infty} \delta^t v(c_t(\theta^t), l_t(\theta^t), \theta^t)$$

$$(Q) \quad f(k_{t-1}(\theta^{t-1}), l_{t-1}(\theta^{t-1}), \theta^t) = k_t(\theta^t) + c_t(\theta^t) \quad t > 1,$$

$$k_1 + c_1 = \sum_i x_0^i(\theta^1)$$

where $v(c, l, \theta) = \text{Max}_i \{ \sum_i \alpha^i u(c^i, l^i, \theta) \mid \sum_i c^i = c, \sum_i l^i = l \}$ where

$$\sum_i \alpha^i = 1, \alpha^i \geq 0 \text{ for some } \alpha.$$

The proof of this fact is immediate. On the one hand, if we simply require—a priori—that $q_t(\theta^t)$, $\gamma_t(\theta^t)$, $p_t(\theta^t, \theta_{t+1})$ and $w_t(\theta^t)$ belong to a suitable space (say L_1) then it is simply an implication of Debreu's [1954] theorem. If we adopt a more sequential view where these prices are not necessarily restricted to some pre-chosen space, one may use the necessity of the transversality condition at infinity for each P^i (cf., Weitzman [1973], Benveniste-Scheinkman [1972] or Ekeland and Scheinkman [1983]) to conclude that a transversality condition for Q applies. Since this, together with the first order conditions implies optimality, the result is proven. This type of

reasoning was first used by Scheinkman [1977] and generalized by Brock [1982]. This of course avoids what is known as the Hahn [1966] problem, but at a clear cost: consumers must perfectly forecast all future prices and take them into account at all times.

Further it is immediate from arbitrage considerations that

$$\theta_{t+1} \sum_{\theta \in S} [q_{t+1}(\theta^t, \theta_{t+1}) + \gamma_{t+1}(\theta^t, \theta_{t+1})] p_t(\theta^t, \theta_{t+1}) = q_t(\theta^t) .$$

This "prices" the shares and gives us Lucas' [1978] formula. Constantinides [1982] had already observed that this formula held for an economy with heterogeneous consumers provided complete markets were present and each consumer was like ours. In particular we know that trading in shares is superfluous to achieve this (optimal) allocation. But since $\gamma_t(\theta^t) > 0$ for each θ^t , we can in fact at each θ^{t-1} give up some market where prices $p_t(\theta^{t-1}, \theta_t)$ are quoted and put, in its place, the infinitely lived stock.

One could go further if "many" production processes were present with the corresponding shares. If the span of $q_{t+1}(\theta^t, \theta_{t+1}) + \gamma_{t+1}(\theta^t, \theta_{t+1})$ had dimensionality equal to $\#S$ (the cardinality of S), then trading in securities will be enough (see Kreps [1982]). This requires, of course, at least $\#S$ firms, or, if a risk-free security is introduced in the form of borrowing and lending on consumption goods, $\#S-1$. A reasonable conjecture is that the generic result of Kreps [1982] will hold in this context. This result was proven in the context of securities with exogenous payoffs at a terminal rate. He assumed that in our notation, $\#S$ securities existed. (One can do with less if not all $\theta \in S$ are achievable from all histories.) He then showed that for "almost all" exogenous payoffs at the terminal date the Pareto-optimum allocation could be achieved. The corresponding result here would probably go as follows: Suppose we have $\#S-1$ firms and that we

fix everything but a single (well chosen) parameter of the production function. Then, except for a closed set of values of this parameter with Lebesgue measure zero, trading in securities plus the risk-free bond will suffice. David Kreps guarantees that the modifications would be minor.

The model could be complicated either by introducing a non-linear transformation between capital and consumption goods to accommodate the changes in the relative prices of these goods through the cycle or by introducing adjustment costs. In the latter case, share prices are functions of the capital stock. Firms will then maximize the sum of tomorrow's profits and share prices which, in equilibrium would equal today's share price. In principle those agents would have to forecast correctly the potential price of a firm with an arbitrary capital stock even though such firm may never have existed or come to exist (since in equilibrium certain capital configurations will never appear). This seems to defy the plausibility of rational expectations equilibria as arising from a repetitive environment. We must thus fall back on the idea that rational agents "know" the model.

Nonetheless after such embellishments the equilibria of such an economy will generate stochastic processes that will mimic the ones obtained from the single consumer model. Note, however, that in such an economy asset trading could occur every period. If we define a set of assets such that at each instant no asset can be written as a linear combination of other existing assets, we may talk about the volume of trading in each asset. (Such a restriction is needed for making volume a meaningful concept in any case). Further, if we define the wealth of consumer i at time t given history θ_t as $w^i(\theta^t) \equiv x_{t-1}^i(\theta^t) + s_{t-1}^i(\theta^{t-1})[\gamma_t(\theta^t) + q_t(\theta^t)]$, i.e., the number of consumption goods that he could acquire at t , we see that such numbers may fluctuate over time so that the wealth distribution will change.

A theory of asset prices based on such economies was developed by Lucas [1978], Brock [1982], Constantinides [1982] and Prescott and Mehra [1980].

Kreps summarized it here in 1982. As he commented, restrictions on the state dependency of utilities are necessary if one is to obtain testable implications besides no-arbitrage of complete market models with additive utilities. (See Beja [1971] and Kreps [1981]). The basic idea behind this type of result is that if one prices certain contingent claims in such a way that non-negative and non-zero consumption streams are positively priced one can extend the pricing functional to all contingent claims preserving the positivity property. This extended pricing functional can then be assumed to be the (state dependent) additive utility functional for a single individual.

The same reasoning applies if besides pricing data one had an individual consumption data, since these individuals would be risk-neutral. Thus one could not reject a model of this type with any quantity and price data unless arbitrage conditions failed. I emphasize that this has nothing to do with aggregation results. The fact that the models always fit stems from looking at the data as a single price-quantity pair. Thus, in particular, based only on price-quantity data, one could not reject the fact that the observed fluctuations stem from a Pareto-optimum outcome if no a priori restrictions were placed on utilities.

"Explaining" the data this way would be, of course, utterly useless activity. Thus one must look first for reasonable restrictions that make the model testable. Though these are not minimal, one set of restrictions consists in assuming that individuals' utilities are common and nonstate dependent. The second requirement could be somewhat weakened by requiring some independence on shocks to individual's utilities and to output. The first one can also be probably weakened but if tastes are sufficiently diverse, Arrow-Debreu markets may not require any pooling of risks. Laurence Weiss' favorite example is one where

one agent only likes apples, another oranges, and there are two possible states of the world at each time: in one, only oranges are available, in the other only apples. Utility levels in each state of the world will be quite different across consumers. Under the above hypothesis, however, any individual's consumption (well defined, including leisure) should show positive correlation with aggregate consumption. In fact, though non-linearities may be hard to deal with, all variations in individual's consumption should be accounted for variations in aggregate consumption and none by variations on individual's income. Notice that this property aggregates across subgroups and thus it could, in principle, be tested using somewhat aggregated data. Thus, for instance, if one had measures of consumption and income across states one could, find out if the prediction of complete markets are violated. As far as I know, no tests of this kind have ever been carried out. Since consumption and income data for countries are easily available, Paulo Leme, a student at Chicago who is investigating capital market integration across countries, prepared the graphs that follow. In them, percentage deviations of consumption and income from trend are plotted for each of seven different countries and the U.S. Though they are only meant to be suggestive they seem to show that a lot of pooling is left out. Episodes where the consumption of a country goes up while the United States' consumption goes down (or vice-versa) are incompatible with complete markets and the above identifying assumptions. Nonetheless, in each of the graphs consumption of each country seems to follow its own income.

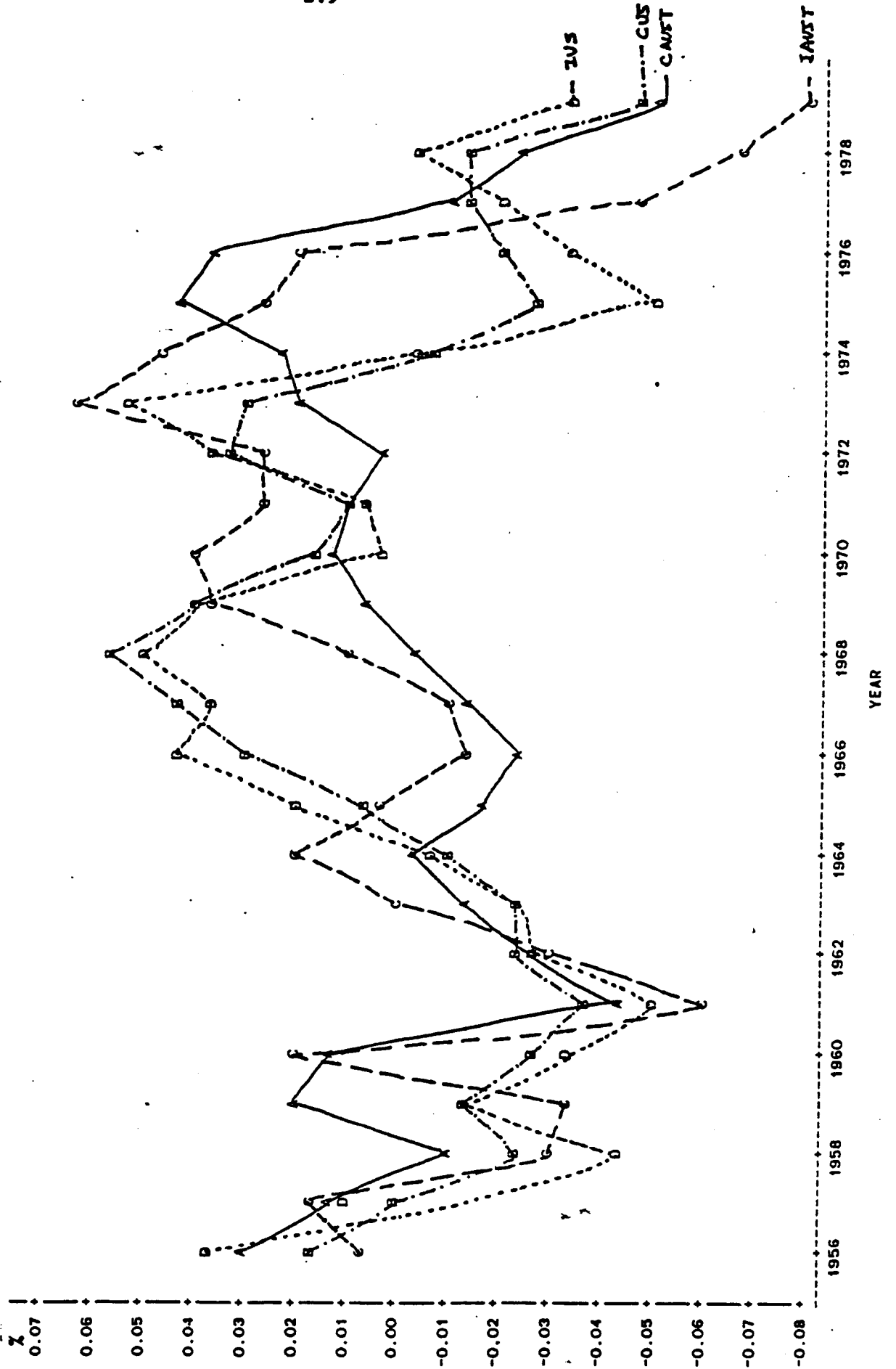
It seems to me that this type of testing is much more likely to shed light on the discussion on whether Walrasian equilibrium (really Arrow-Debreu equilibrium) models are adequate descriptions of actual economic fluctuations than arguments about fixed versus flexible prices or rational versus irrational expectations.

Variants of the stochastic model described above, that took into account inventories, labor supply, etc., has been confronted against post war aggregate U.S. data (cf. Prescott's IMSSS 1983 Survey). Though it seems to "fit" some of the data characteristics well, e.g., relative variability of consumption and investment, the comovements of output and hours worked, it did much worse in dealing with other aspects of the cycle, e.g., it does not deliver the pro-cyclical characteristics of inventories, and it predicts a much too high variability of average product of labor than is found on the data. Worse than that is the fact that no standard statistical testing of the model is provided and thus it is impossible to tell what constitutes success.

Aust - Australia

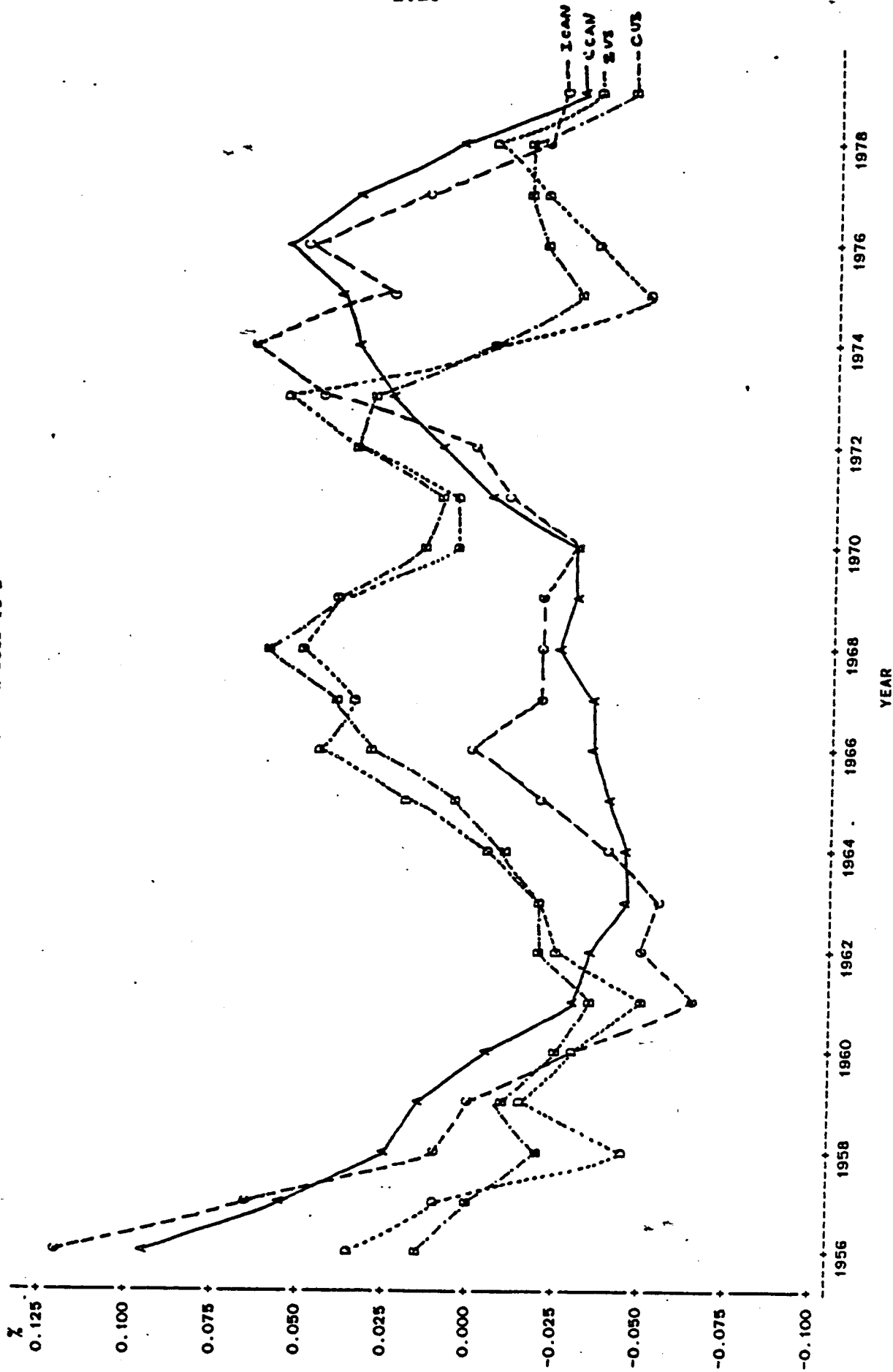
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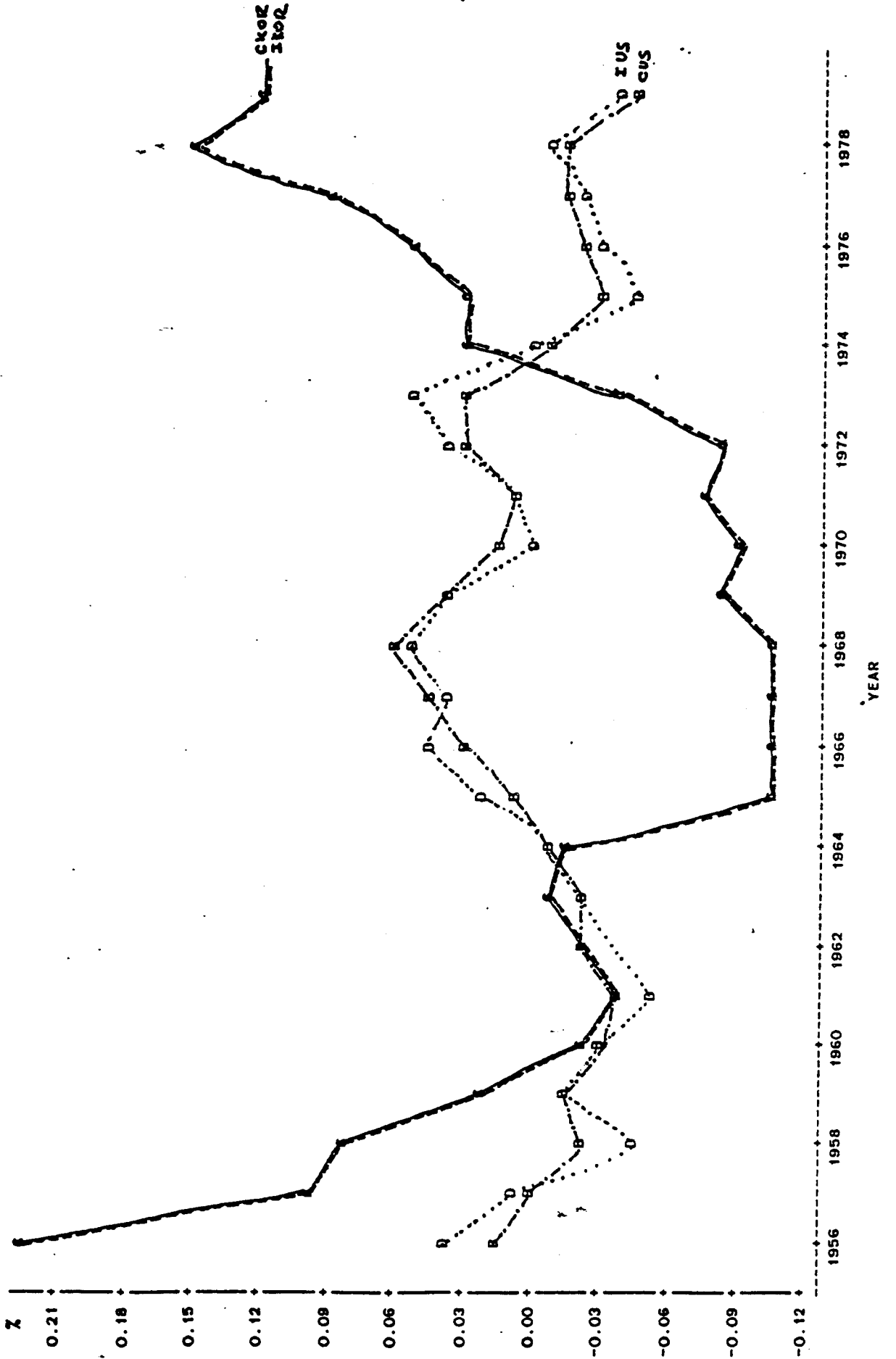


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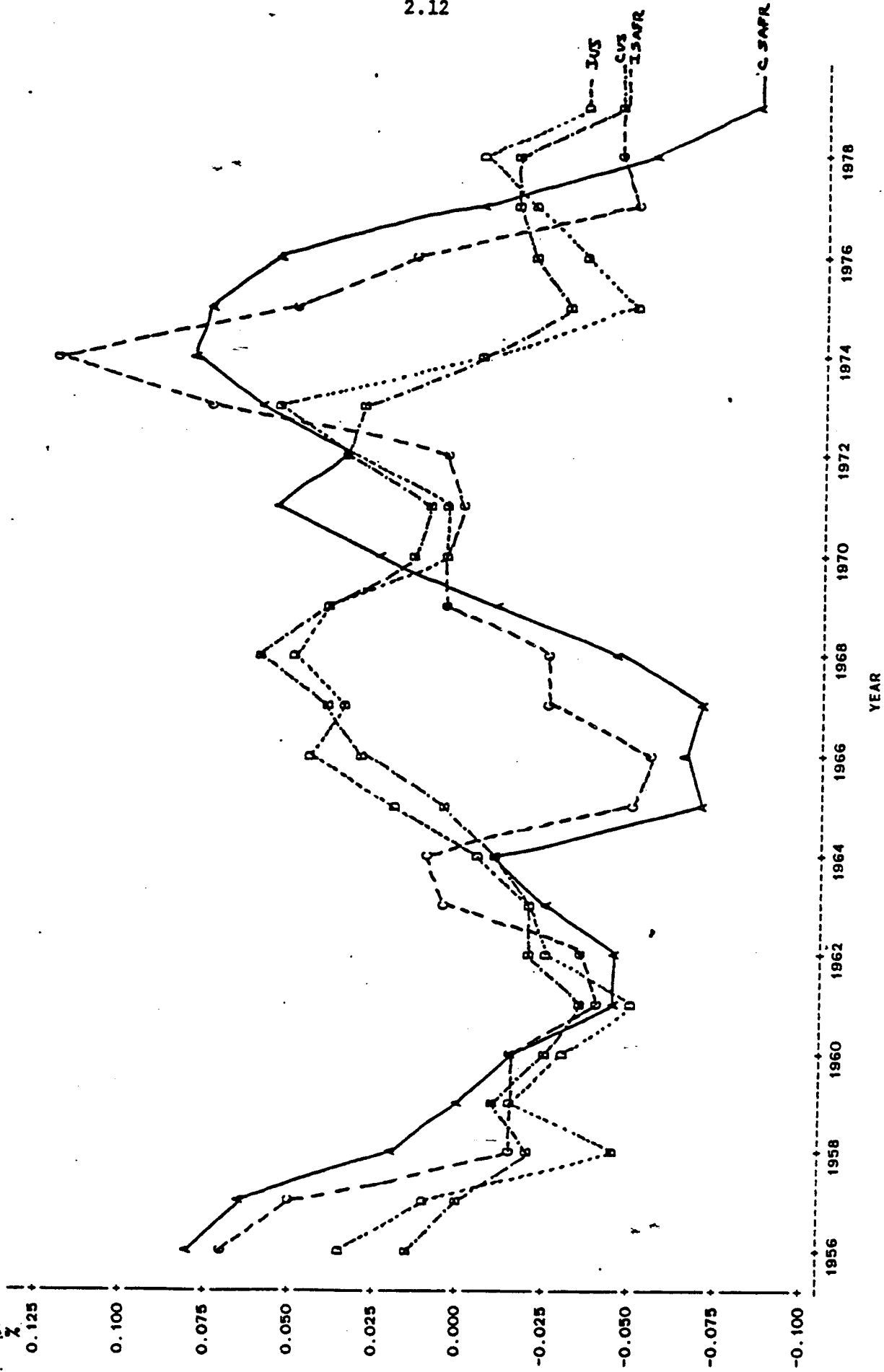


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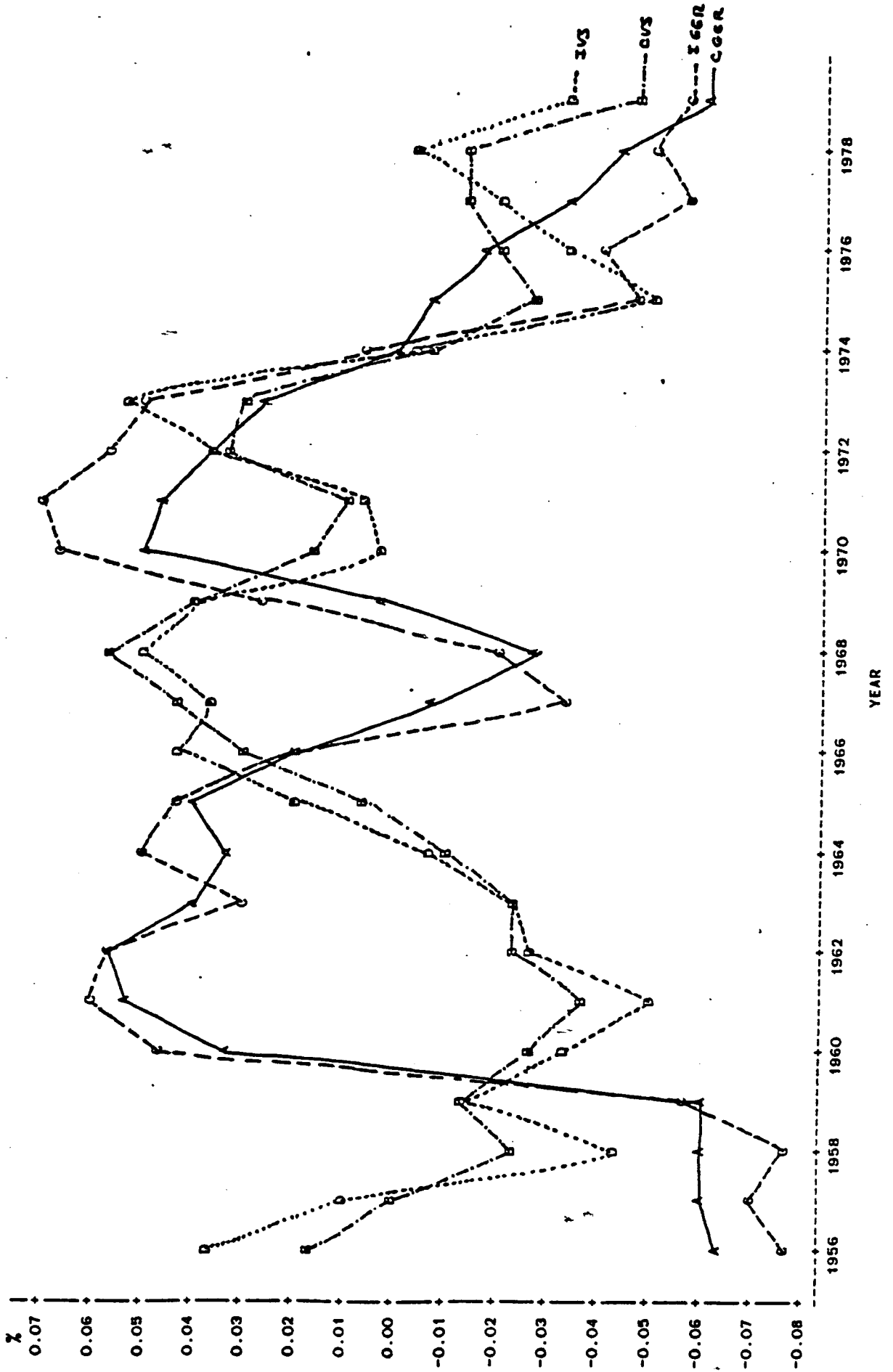
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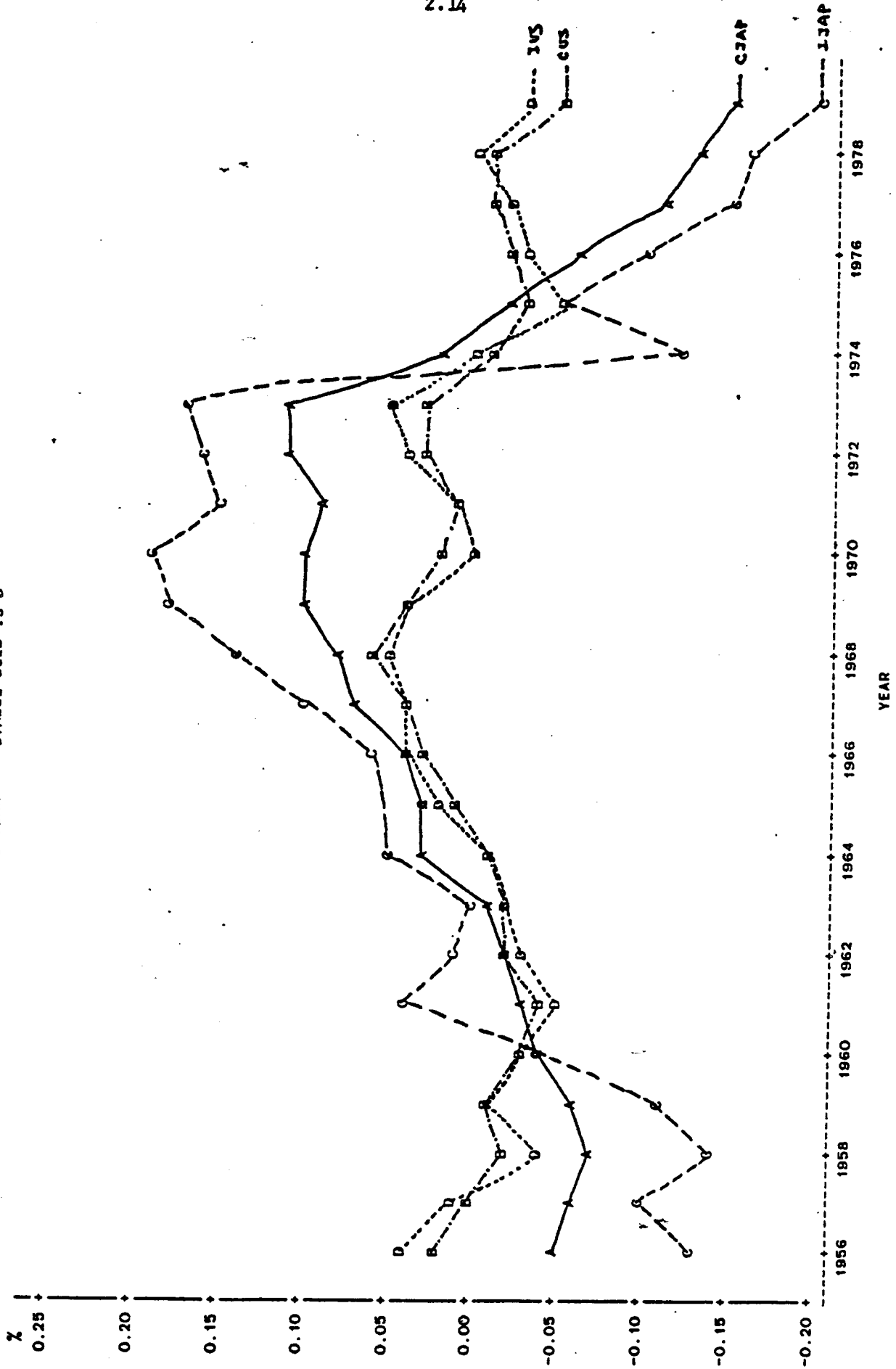
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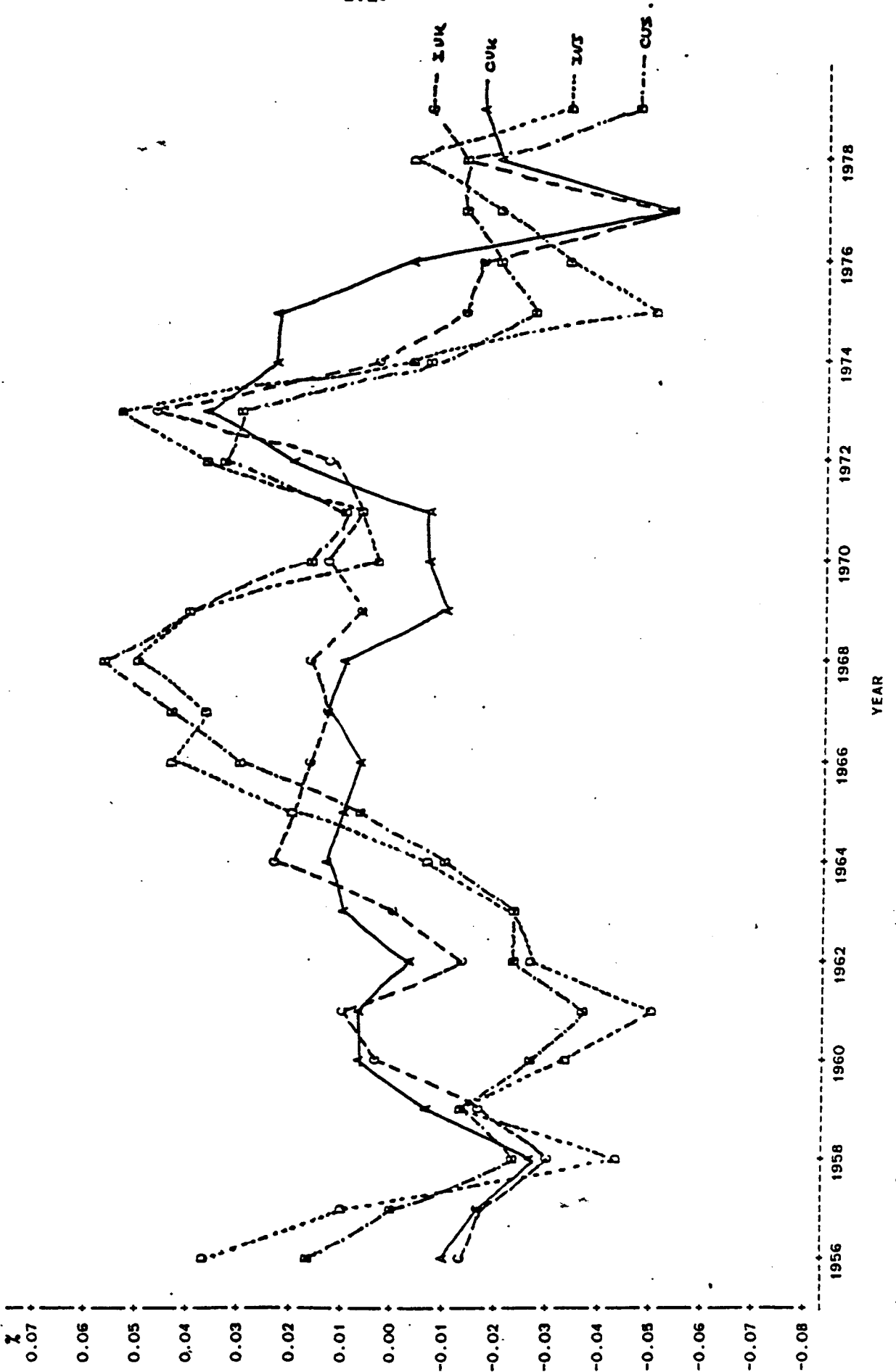


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3. Deterministic Fluctuations

Until recently there were no examples of deterministic cycles in optimizing models of infinitely lived agents that satisfied some natural economic restrictions, e.g., positivity of marginal products, non-satiation, etc. In fact, the results concerning the one sector growth model (cf. Cass [1965]), the undiscounted case with many goods, Gale [1967], McKenzie [1968] and the case of small rates of discount, Scheinkman [1976], all pointed in the direction of global asymptotic stability. The counterexamples in the literature (e.g., Kurz [1968]) still exhibited convergence to some stationary state. The work of Benhabib and Nishimura [1982] has given us a first example of an infinite horizon optimization problem involving discounted utility and a single capital good whose optimal solution exhibits a cycle. (They had previously shown that in a continuous time 2-sector model periodic solutions existed. See Benhabib and Nishimura [1979]). The intuition behind this type of example is that even though the concavity in the problem works against cycles, if the discount rate is large enough it may not pay to smooth the cycle. The decentralized version of such a model is not intended as a serious candidate to explain business fluctuations. The cycle is not the result of deterministic waves, at least with low periodicity. (It may be one with a high enough periodicity such that we have not yet seen a complete wave.) Nonetheless, it serves to illustrate the point that some cyclical behavior may have non-stochastic origin.

In order to understand what is needed for such examples, let us first write a general one-capital good model

$$\begin{aligned}
 (P) \quad & \text{Max } \sum_{t=0}^{\infty} \delta^t u(k_t, k_{t+1}) \\
 & \text{s.t.} \\
 & k_0 \text{ given}
 \end{aligned}$$

The function u is assumed strictly concave, increasing, etc. Now a problem as (P) yields, by dynamic programming a policy function $k_{t+1} = \phi_\delta(k_t)$ that describes the evolution of the system. Clearly, if ϕ_δ is nondecreasing (as in the Cass-Koopmans model) no cyclical behavior can appear. Thus the first thing that must go away is a monotone increasing policy function.

Let us thus look at the case $u(k_t, k_{t+1}) = k_t^\alpha (1 - k_{t+1})^\beta$. One way of thinking of this model is to consider an economy with a single consumer with one unit of labor and with a linear utility function on consumption and where the single consumption good is produced using the technology $k_t^\alpha l_t^\beta$ and capital is produced using only labor in constant returns. This example when $\alpha + \beta = 1$ was introduced by Weitzman (cf. Samuelson []) to generate cycles when $\delta = 1$. This is, however, not stable under perturbations since if $\alpha + \beta < 1$ and δ is equal or even close to one all optimal paths converge to the unique optimal stationary state (cf. Scheinkman [1976]). However, if we choose $\alpha + \beta < 1$, $\alpha > \frac{1}{2}$ and δ small enough, Benhabib and Nishimura showed that a cycle of order two obtains.

This is, however, the most complicated behavior one can get in this context, as can be seen by the following.

Remark 1: If in problem (P), $u(k_t, k_{t+1}) = k_t^\alpha (1 - k_{t+1})^\beta$ then ϕ_δ is monotone decreasing. To see this notice that $\phi_\delta: [0,1] \rightarrow [0,1]$, $\phi_\delta(0) = 1$ for each $\delta > 0$. Also if $k > 0$, $\phi_\delta(k) < 1$. By continuity (which is obvious), if ϕ_δ not monotone, there exists \bar{k} and \tilde{k} such that $\phi_\delta(\bar{k}) = \phi_\delta(\tilde{k})$ since ϕ_δ solves the Euler equation, i.e.,

$$-\beta k^\alpha (1 - \phi_\delta(k))^{\beta-1} + \delta \alpha (\phi_\delta(k))^{\alpha-1} [1 - \phi_\delta(\phi_\delta(k))]^\beta = 0$$

we get that either $\bar{k} = \bar{k}$ or, $\phi_\delta(\bar{k}) = 1$. But then $\bar{k} = \bar{k} = 0$.

The above remark guarantees us that ϕ_δ is monotone decreasing. But then ϕ_δ^2 being monotone increasing, it has no cycles and the only admissible cycles for ϕ_δ are of order two.

Thus in order to exhibit a cycle of higher order one needs to complicate further the example. Recently Ivar Ekeland and I have been looking at

$$u(k_t, k_{t+1}) = (k_t - \gamma k_{t+1})^\alpha (1 - k_{t+1})^\beta, \quad \gamma \text{ "small"}$$

The story is just as in the case above, except that to produce tomorrow's capital one needs one unit of labor and γ units of today's capital, in fixed proportions.

In this case, the policy function is no longer monotone (note $\phi_\delta(0) = 0$). We are not using approximation methods to "solve" the Euler equation and believe cycles of arbitrary order appear as $\delta \rightarrow 0$. Notice that a small " δ " could be a proxy for consumption complementarity over time though such connection is not very tight.

If this conjecture is confirmed, high order cycles will then be shown to result from a simple aggregative model. Notice that it would be easier to generate a cycle in a higher dimensional case but this would not guarantee that some aggregate measure of capital does cycle.

Deterministic cycles also have appeared in the context of overlapping generations models. The earliest example is from Gale [1973]. Benhabib and Day [1982] and Grandmont [1983] present further examples, the latter making an exhaustive study of the possibilities of cycles in such an economy.

4. Imperfect Information

The type of model described above is bound to be deemed unsatisfactory both to those that believe that we do not live in the best of all worlds and to those who think that we would provided the government kept constant the money supply (properly defined, of course).

Though one could, in principle, place money on individual's utility function or (why not after going this far?) on production functions and thus have monetary shocks causing fluctuations, serious monetarists are led to incomplete markets as the only setting where money holdings are justified and hence necessarily the framework in which to study monetary-caused fluctuations. As mentioned in the introduction, this forces one to stay within restricted classes of examples.

The leading example is, of course, Lucas'[1972]. No single paper has been more influential in the modern theory of economic fluctuations. By writing down a general equilibrium model in order to generate a relationship between the rate of inflation and the level of output, Lucas went against the then consensus of the economics profession that macromodelling was better done in a non-equilibrium fashion.¹ He considered an economy in which exchange took place in separate markets. Each agent lived for two periods, but produced only in the first one. The fraction of producers allocated to each market was random and thus relative prices fluctuated. Further, stochastic fluctuations on the money supply, through transfers to "old" agents, induced changes in the nominal price level. Agents seeing exclusively prices in their own market, could not separate price movements resulting from a relative demand shift from

¹Of course he had already written a partial equilibrium description of business cycles with Rapping (1969).

those resulting from a change in the money stock. Part of the change in prices caused by the money supply's unanticipated movement is considered by the agents to be a change in today's prices relative to tomorrow's and hence (given the linear production function) an equal change in today's wage rate versus tomorrow's prices. Thus unanticipated increases in the money supply cause increased output. A short run Phillips curve was delivered. The difficulty in analyzing even the relatively simple model chosen, probably led him to eliminate any source of persistence of fluctuations like capital accumulation or long lived agents. This is not a model that could fit the persistent character of actual macro-fluctuations. In Lucas [1975] correlated money shocks together with long lived agents and capital accumulation is introduced, but the model is no longer built from first principles.

All output variations in this model are the result of variations on the expected real interest rates. Notice that real wages are constant in contrast with the models of complete markets discussed above, where shocks to the technology cause both the real wage and the expected real interest rate to vary. Though output fluctuations would appear in this model even if the money stock was held constant, due to the non-linearity of labor supply responses and the finite number of islands, the emphasis is on the Phillips curve and hence on the mechanism by which money shocks cause output to vary. The role of the assumption of the impossibility of observing all prices in inducing the variations in output is not crucial, and other types of asymmetric information may be used. Grossman and Weiss (1982) analyzed a model where agents must invest labor today to obtain consumption goods tomorrow, and where they have incomplete information about other trader's productivity. There is a nominal bond market, and shocks to individual's money demand. At each time t agents

know the realization of their own shock to money demand at time $t+1$ but not the economy-wide average. A positive shock on tomorrow's money demand causes the nominal rate of interest to be lower and is partially attributed to low average productivity (i.e., a low real interest rate). This implies low output tomorrow and hence each agent invests more labor. Thus here it is exactly when the anticipated real rate of return is low that employment goes up. But the realized real rate of return on bonds is actually higher and thus in such episodes higher employment is associated with higher ex-post real rates of return on bonds.

The Grossman-Weiss model has money in the utility function. A true infinite horizon counterpart to the Lucas model should have an explicit motive for holding money. This could include a Clower constraint or a borrowing constraint of the type used by myself and Weiss in the model discussed below. No version of such models in which the money supply is not observable and consumers use prices to obtain information seems to exist.

The empirical evaluation of the informational type models is mixed. Barro [1980] cites Sargent [1976] and Fair [1979] to conclude that price surprises play a relatively minor role. This seems to support other channels for the influence of monetary shocks than price surprises. On the other hand, Barro also presents evidence that monetary shocks cause ex post real rates of return to fall—a result that is consistent with the Lucas model.

5. Borrowing Constraints

In my work with Weiss [1984] we explored the effects of a particular form of market incompleteness, that we called "borrowing constraints" on economic fluctuations. Since the paper is appended I will only sketch the main arguments in order to prepare for further discussion.

We start with an economy in which there are a large number of each of two types of agents referred to here as "agent 1" and "agent 2" each with the same utility function, namely:

$$E_0 \int_0^{\infty} e^{-\beta t} [u(c(t)) - l(t)] dt$$

Here $c(t)$ denotes the consumption at time t and $l(t)$ the amount worked at time t . The expected value refers to the uncertainty described below.

At each instant in time the economy is in one of two states. In state i , $i=1$ or 2 , agent i produces one unit of the single consumption good per unit of labor, and agent j produces zero. State changes are described by a Poisson counting process $N_t(\omega)$ with rate λ , i.e., if $N_t(\omega)$ is even state 1 occurs if $N_t(\omega)$ is odd, state 2 occurs. A symmetric Arrow-Debreu allocation would have constant output given by $2c$ where $u'(c) = 1$.

We then rule out insurance markets and introduce an asset in fixed amount that yields at every instant δ units of the consumption good. When $\delta = 0$ we call the asset "money" though it certainly does not have many of the uses one may want to attribute to money. Agents will now hold the asset in order to smooth out their consumption over time. We can now describe the main characteristics of the equilibrium. Suppose at time t we are in state 1. Agent 1 is accumulating assets, whereas agent 2 is doing the reverse. After a certain amount of time τ has elapsed agent 1 is richer

while agent 2 is poorer. However, if the state is still the same the Poisson aspect of the state changes guarantees that the distribution of productivities of agents at time $t+\tau$ is exactly the same as the one for time t was at t . Since agent 1 is richer he is less eager to sacrifice leisure for the asset. On the other hand, since agent 2 is poorer he is less willing to part with assets. In equilibrium it can be shown that the price of the asset falls and output also falls. This goes on until an epoch (state change) occurs. On the average, at such an occurrence the productive agent is "rich." Thus at such epochs it is as if the productive agent became poorer and hence output rises and the price of the asset goes up.

The demand for the asset is, of course, at each instant of time a function of each agent's wealth and of the distribution of rates of return. In equilibrium, however, the fraction of wealth held by the productive agent and the expected rate of return are positively related. The intuition behind this result is that if the productive agent holds little wealth, the probability that he may become unproductive leads him to acquire the asset even though the expected return is negative. However, if he has enough wealth he demands a positive rate of return in order to (as he must in equilibrium) be a net buyer of the asset. Since, as remarked above, output is inversely related to the fraction of total assets held by the productive agent we have that output is negatively related to the expected rate of return of the asset. Notice that this is precisely the reverse of Lucas [1972] though the conflict would be smaller if we compared realized rates of return. There, imperfect information plays an important role and the same money shock that raises the expected rates of return lowers the realized rate.

In a somewhat imprecise but suggestive manner one may think of supply and demand schedules for the consumption good as a function of expected rates of return. These schedules are being shifted by changes in wealth. The demand schedule can be decomposed in two parts. The demand on the part of the unproductive agents is shifted downward while (due to the linearity of the utility for leisure) the demand on the part of the productive agent stays constant. On the other hand the supply curve is also shifting downward.

Wealth redistribution is the main mechanism generating the fluctuations. The real wage is constant (in contrast with the Arrow-Debreu model described above) and the substitution effect caused by changes in the expected returns (which is the reason for fluctuations in Lucas [1972]) ends up as a brake to fluctuations. Of course, at such a level of abstraction it is hard to decide if anything in the "real world" causes the wealth redistributions that the shocks to productivity cause here. Incidentally, Grossman, Hart and Maskin [1983] study a model where because workers are less informed about their marginal products than firms, relative prices or productivity shocks induce, via the mechanism of optimal contracts, employment fluctuations. They argue further that unanticipated changes in the rate of inflation may, because much of the debt is nominal, cause changes in the distribution of wealth.

In the older macro literature mention is made of the role of wealth effects in causing economic fluctuations. This model has certainly some of this flavor and, in fact, it can be shown that the value of assets in equilibrium is a positive function of output. This corresponds to the procyclical character of wealth. It must be emphasized, however, that the presence of two types of agents is essential in our case. In fact, if a representative individual view is taken, as is frequent in macroeconomics, and if leisure is not an inferior good, a wealth-induced decline in employment must, if markets are clearing, be associated with increases in wealth, a point made earlier by Lucas [1981].

This seems to be contrary to the evidence.

Before proceeding further, it seems fair to ask whether borrowing constraints always lead to more fluctuations rather than the complete markets case. The next example shows that this is not necessarily so but also illustrates when the constraints do lead to excess fluctuations.

Example 1: There are two types of agents, and 3 states of the world. The following table relates output per unit of labor of agent $i=1,2$ in date of the world $j=1, 2,3$

$j \backslash i$	1	2
1	4	2
2	3	3
3	2	4

Output is perishable and the utility function of agent i is given by $E_0 \sum_{t=1}^{\infty} \beta^t (\log c_i^t - l_i^t)$. States are i.i.d. and $\pi_1 = \pi_3 = .25$ and $\pi_2 = .5$, where π_i denotes the probability of state i . The symmetric Arrow-Debreu allocation requires $l_1^t = 2, l_2^t = 0$ if $j = 1$; $l_1^t = 0, l_2^t = 2$ if $j = 3$; $l_1^t = l_2^t = 1$ if $j = 2$. In any case, $l_1^t + l_2^t \equiv 2$ and aggregate consumption varies between 8 and 6 each with probability .5.

Now if borrowing constraints are present and if there is no money, then $l_1^t \equiv l_2^t \equiv 1$ for any j and aggregate consumption is 6 in any state. Notice that the constraints impede the "specialization" that causes the aggregate fluctuations in the complete market case. By using a non-linear utility of leisure one could make a similar point about employment as opposed to output.

The above example illustrates the general principle (as opposed to theorem) that complete market allocations maximize individual output variations across states as they make use of the trading opportunities. Only if, to start with, the individual variations aggregate out in each state will the Arrow-Debreu allocation present little variability across states. As trading opportunities diminish individual output variability across states will also diminish. But, in return, the individual output variability may no longer aggregate out in each state and thus extra fluctuations are created. In the above example individual variations do not aggregate out in each state to start with. It should be emphasized that it is too much to hope for a general theorem: the result involves equilibrium prices and the economic notion of variability is not obvious. In simulations of my model with Weiss, described above, the variance of individual output falls when borrowing constraints are present, even though the variance of aggregate output goes up.

In the Lucas [1972] model the same mechanism is operative. If the money supply noise is absent, agents in islands where they are few young ones, work harder relative to those in islands where they are many, than if the noise is present. However, in the first case, if "many" islands exist aggregate output is a sum of many independent random variables and thus mean output does not vary. In the second case the random variables are positively correlated since they all take the higher nominal price as a signal that their island has few young agents, and aggregate noise results.

Several tests of "borrowing constraints" appear in the literature, usually associated with studies of the permanent income hypothesis, e.g., Hall [1980], Hall and Mishkin [1982]. Though the results are mixed, what is being tested is usually whether sure future income can be used for present consumption

(cf. Hall and Mishkin [1982], where the maximization problem facing consumers is made explicit). Though in the Scheinkman-Weiss model presented above these two concepts would coincide if, in the worst state productivity were non-zero, it would be natural to let consumers have negative non-human wealth provided it could be paid back with probability one, while still ruling out full insurance. Testing for such constraints could then be done as discussed above in the section on Brock-Mirman models.

The assumption of perfectly correlated productivity shocks across agents that underlies the Scheinkman-Weiss model should be contrasted with a case where many types have independent productivities. This was the case studied by Foley and Hellwig (1975). (See also Lucas [1980].) Each agent solved

$$\text{Max } E_0 \sum_{t=0}^{\infty} \alpha^t u^i(c_t^i, l_t^i)$$

s. t.

$$c_t^i \geq 0, \quad l_t^i \geq 0$$

$$m_{t+1}^i = m_t^i + (w_t^i l_t^i - c_t^i) p_t$$

$$m_t^i \geq 0, \quad m_0^i \text{ given}$$

where $\{w_t^i\}$ is a sequence of i.i.d. random variables with $\text{Prob}\{w_t^i = 1\} = q$, and $\text{Prob}\{w_t^i = 0\} = 1-q$, p_t is the price level prevailing at t , and m_t^i the asset holdings by the i -th agent, etc. Further, u is a neoclassical utility function $u_c > 0$, $u_l < 0$ with $\lim_{c \rightarrow 0} u_c(c, l) = \infty$, $\lim_{l \rightarrow 0} u_l(c, l) = 0$ for all c , $\lim_{l \rightarrow \bar{l}} u_l(c, l) = -\infty$ for some \bar{l} .

In Foley-Hellwig $p_t \equiv p$ is assumed and it is shown that a stationary distribution $F^*(m)$ exists. Of course, if initial money balances are distributed as $F^*(m)$, $p_t \equiv p$ is an equilibrium and the distribution remains invariant. In particular, aggregate consumption, labor supply, etc. remains constant. There is, however, no proof that the distribution is ergodic except when consumers starting from an arbitrary distribution assume (wrongly!) that $p_t \equiv p$ will prevail. Technically, the difficulty is that the state variable is actually the distribution of wealth among individuals and this is a function (whereas in Scheinkman and Weiss it is a real number). Thus, while it still seems reasonable to conjecture that uninsurable idiosyncratic shocks are not enough to generate fluctuations in the long run, this seems not to be a theorem.

Perhaps more promising is the case where aggregate productivity shocks are combined with idiosyncratic shocks. Suppose, for example, that we modify the Foley-Hellwig framework so that each unit of the asset yields γ_t units of the consumption good at time t where γ_t assumes one of two values $\underline{\gamma}$ and $\bar{\gamma}$, and that it switches from one value to the other in accordance to some symmetric Markov matrix. Assume now the w^i are independent across consumers and independent of γ_t . Larry Weiss and I have recently been studying examples of this type.

Finally, some comment must be made about the exogeneity of the market structure in this type of model. Whereas it is now well understood that informational barriers will limit the number of state contingent markets, it is also known that an optimal trading structure taking into account such barriers can be quite complicated. In the optimal contracts literature, for example, the fact that a firm is the only one to observe the productivity of a worker leads to contracts where the wage is a function of the number of workers employed. Though there exists a growing literature showing how

such optimal contracts could give rise to magnification of shocks in a static context (see Grossman, Hart and Maskin [1983] or Holmstrom and Weiss [1984]) we are not yet in possession of a dynamic theory that starts from informational constraints, derives the "contracts" structure, and studies the evolution of the economy over time.

6. Contracts, Coconuts, etc.

The belief that activist monetary policy must matter led to a large literature on the role of long term contracts. Fisher [1977], Phelps and Taylor [1977] and Taylor [1980] are early examples. Taylor [1983] summarizes the theoretical work and comments on its empirical success. The basic ingredient in this type of model is the assumption that some agents (usually workers and firms) contract in nominal terms for several periods. In all the above cited sources, however, the decision problem faced by agents is never made explicit and thus they fall outside the scope of this survey. An exception is in Prescott's [1983] survey where he writes down a model with a single consumer facing a "Clower constraint", and a single firm. Wages must be set one period in advance though employment may be state dependent. Not surprisingly, he finds that in such a framework "real" contracts would do better.

Peter Diamond and Drew Fudenberg [1983], building on early work by Diamond [1982], consider a model where individuals' production opportunities arrive as a Poisson process. If an opportunity occurs the individual can at a fixed utility cost produce one unit of a good. Individuals can consume any good but the one they produce. After they obtain one unit they cannot add to their stock until they trade it and consume it. At each instant in time a certain fraction e of the population is carrying inventories. It is assumed that each individual will meet a trading partner according to a Poisson process with arrival rate $b(e)$ where $b' > 0$, $b'' < 0$, $b(0) = 0$.

Now if everyone guesses that $e = 0$, then each individual will not undertake the fixed utility cost and thus $e = 0$ will result. On the other hand,

suppose an $\hat{e} > 0$ exists such that at such \hat{e} all opportunities are taken and, when all such opportunities are taken and individual's meet at rate $\hat{b}(e)$ then the fraction of the population carrying inventories does not change. Clearly such \hat{e} is another equilibrium. This establishes the multiplicity of stationary equilibria but not yet the fact that multiple equilibria may exist once one is given an initial level of e . In order to see how the latter is possible one must realize that at a given instant the decision of whether or not to take an opportunity to produce, an individual must know not only the present level of e but must also forecast the level of nearby future e 's that will determine their chances of selling the object in the next few instants. The evolution of e depends not only on the present e but on whether other individuals take their opportunity to produce or not. In the first case e will rise, in the second, e will fall. Thus, for certain intermediate levels of e it is quite likely that if an agent imagines that everyone intends to make use of their production opportunities (optimism) he will do so himself but if he imagines that all others will pass it up (pessimism) he will do the same. Fudenberg and Diamond establish this precisely and then proceed to create a cycle out of well chosen sequences of optimistic and pessimistic unanimities. This bears some resemblance to the "sun-spots" literature that is covered in Woodford's companion survey.

Finally, let me mention briefly the fixed price literature formalized in e.g., Barro and Grossman [1971], Benassy [1975] or Malinvaud [1977]. This literature has concentrated mostly on short-run considerations and has not dealt explicitly with the price formation mechanism. Green and Laffont [1981] discuss a dynamic model of this type where prices are endogenously determined, but they do not make explicit the maximization problems and constraints faced

by agents. I have no doubt, however, that one could solve simple dynamic equilibrium models built from first principles in which rates of exchange between goods had to be set in advance of the actual date of exchange.

7.1 Asset Values

The incomplete markets model described in Section 4 and 5 have different implications concerning the valuations of assets than the complete markets ones of Sections 2 and 3. The most obvious one is the fact that an asset that pays no dividends may have a positive price. Though economists had become adept at obtaining a positive price for such assets even in a complete markets situation (through devices like placing money in the utility function) it is now clear that incomplete markets are an essential ingredient if one wants to price such claims non-trivially. In the case where agents maximize (expected) discounted sum of utilities, if an asset is always held one must have

$$(7.1a) \quad u'(c(t))q_t = \beta E[u'(c_{t+1})(q_{t+1} + D_{t+1}) | \mathcal{F}_t]$$

where c_t denotes the consumption at time t , q_t the price of the asset at t , D_t its dividend and \mathcal{F}_t the σ -field generated by the random variables observed up to t , i.e., the "information" about the future that the agent has at t . In models in continuous time the endogenous formula is

$$(7.1b) \quad u'(c(t))q(t) = E \left[\int_0^\tau e^{-\beta h} u'(c(t+h)) D(t+h) dh + e^{-\beta \tau} u'(c(t+\tau)) q(t+\tau) | \mathcal{F}_t \right].$$

Normally one "iterates" (7.1) to obtain

$$(7.2a) \quad u'(c_t)q_t = E \left[\sum_{j=1}^{\infty} \beta^j u'(c_{t+j}) D_{t+j} | \mathcal{F}_t \right]$$

or

$$(7.2b) \quad u'(c(t))q(t) = E \left[\int_0^{\infty} e^{-\beta h} u'(c(t+h)) D(t+h) dh \middle| \mathcal{F}_t \right].$$

Implicit in such "iteration" is the statement that

$$(7.3a) \quad \lim_{h \rightarrow \infty} E[\beta^h u'(c_{t+h}) q_{t+h} | \mathcal{F}_t] = 0 \quad \text{or}$$

$$(7.3b) \quad \lim_{h \rightarrow \infty} E[e^{-\beta h} u'(c(t+h)) q(t+h) | \mathcal{F}_t] = 0$$

The equations in (7.3) are usually claimed to be true due to some "transversality condition." Clearly, if $D_{t+j} \equiv 0$ this would imply that $q_t \equiv 0$. In fact, the transversality condition at infinity (cf., Weitzman [1973], Benveniste-Scheinkman [1982], Ekeland-Scheinkman [1983]) for the consumer's maximization problem simply states that

$$(7.4) \quad \lim_{h \rightarrow \infty} E[e^{-\beta h} u'(c(t+h)) q(t+h) z(t+h) | \mathcal{F}_t] = 0$$

where $z(t)$ denotes the holdings of the asset at time t . If one can put a lower bound on $z(t)$ for some consumer, then (7.3) and consequently (7.2) follows from (7.1). This is clearly true in a single consumer model with a fixed supply of the asset but does not need to hold if heterogeneous consumers are present. Thus, in Scheinkman-Weiss (7.4) holds but not (7.3) when the dividend rate is zero. A consumer that at $t+h$ has a very high shadow price for the asset ($u'(c(t+h))q(t+h)$) would hold a small quantity of it, making (7.4) possible.

This situation can be contrasted with the case where $u'(0)$ is finite or where consumers receive positive endowment income with probability 1.

Then, (7.1) becomes, if z_t denotes the amount of the asset held,

$$(7.5) \quad u'(c_t)q_t = \beta E[u'(c_{t+1})(q_{t+1} + D_{t+1}) | \mathcal{F}_t] + \lambda_t$$

where $\lambda_t = 0$ unless $z_t = 0$.

Thus

$$(7.6) \quad u'(c_t)q_t = E \left[\sum_{j=1}^T \beta^j (u'(c_{t+j})D_{t+j} + \lambda_{t+j}) + \beta^T u'(c_{t+T})q_{t+T} | \mathcal{F}_t \right] + \lambda_t.$$

Hence, even if (7.3) holds, and $D_{t+j} \equiv 0$, $q_t > 0$ is possible. Notice that (7.5) in order to be non-trivial also requires heterogeneous consumers. After all, if the asset is in fixed supply the representative agent would hold a positive amount and thus (7.5) reduces to (7.1). Clearly, if the utility function is, say, logarithmic, there is only one asset and zero income in a period may occur (as in Scheinkman-Weiss [1983]) then 7.3 cannot hold since $\lambda_t \equiv 0$. Actually, the differences between the two situations can be reconciled when one considers the consumer's problem as one of choosing stochastic processes of consumption subject to an infinite number of constraints. The "shadow prices" are then elements of an appropriate dual space and it is not true in general that they are expressible as a sequence of prices for each

constraint. In cases analogous to the one treated in Scheinkman-Weiss [1983] one cannot express them in this way.

Though I know of no example where dividends are not always zero and (7.3) is violated, I conjecture that this will be true at least when with some positive probability dividends can be zero for long enough periods.

One may also obtain a positive price for an asset that yields no dividends using the device of a "Clower constraint." In this case multipliers for the constraint will appear in the formula analogous to (7.1) (cf. Townsend [1982]).

The presence of incomplete markets also has implications for asset pricing formulas. Though (7.1) holds in any case, the consumption that appears in such formulas refer to a single individual as opposed to the aggregate. Notice that even in Arrow-Debreu economies (7.1) may not aggregate across individuals, unless some particular form is assumed for u , but at least (7.1) would hold for some artificial utility function. Thus, while in the complete markets case the estimated parameters may not mean much in terms of individual's parameters, the model should not be rejected by the data. Further, though I am not aware of any results, it is reasonable to conjecture that for certain large classes of utility functions, the estimated parameters yield bounds to the individual's parameters, given that complete markets prevails.

The situation is different when markets are missing, since in this case no artificial consumer whose utility is being maximized exists.

Equation (7.1) is tested and the preference parameters estimated (discount factor and constant relative risk aversion coefficient) either by choosing an asset (really a combination of assets) and regressing its price on measures of aggregate consumption or by choosing a pair of assets, writing (7.1) for each and looking at differences in the rates of return.

A series of studies using aggregate data to test formulas like (7.1) and to estimate preference parameters has appeared (cf., Grossman and Shiller [1981], Hall [1981], or Hansen and Singleton [1982], [1983]). The results are at best mixed. The model is sometimes definitely rejected and the estimates of risk-aversion parameters are implausibly high and with large standard errors. Prescott and Mehra [1982] constructed a model economy with a single asset-claims to GNP. There is a representative consumer that maximizes a discounted sum of utilities. The growth rate of the asset's dividends is a stochastic process chosen such that the rate of growth, variance and intertemporal covariance of dividends are grossly comparable to those of the American GNP series. They then compute the risk-premium that such a claim should have over a risk-free asset with "reasonable" risk aversion coefficients (less than 4) and find that it is much smaller than the actual risk-premium that stocks command over short-term government debt. They went on to suggest that part of that discrepancy might be due to uninsured individual risk.

Bewley [1982] develops the idea of how uninsurable risks could cause these "excess returns to risk" to appear, and gives examples where ignoring the lack of insurance and estimating the risk premium from aggregate consumption (and known utility functions, etc.) leads to errors in either direction. Labadie [1984] implements the Prescott-Mehra strategy postulating a model of overlapping generations and shows that one can find risk-premia of the order of the actual one in the American economy with a mean risk-aversion coefficient that is less than one.

Even when one considers a single asset at a time there are certain clear biases introduced by ignoring the lack of complete markets. The following examples illustrate the point:

Example 7.1. In a Samuelson Consumption Loan model with fixed stock of money, fixed aggregate endowment and where utility function $u(c_1, c_2) = v(c_1) + \delta v(c_2)$, if one writes (7.1) in terms of aggregate consumption then one concludes that $\delta = 1$. The same is true in the Foley-Hellwig model described in Section 5.

Example 7.2. In the Scheinkman-Weiss model of Section 5 agents' consumption vary both due to variation on aggregate consumption and to movements in their share of consumption due to changes in their productivity. Thus, aggregate consumption variability underestimates a single consumer's risk. After simulating the model with logarithmic utility we tried to "estimate" preference parameters as if a single consumer with constant relative risk aversion was generating the data. The estimate of risk aversion was around three. We did not, however, formally "test" the single consumer model against our data.

The lesson of all these examples as well as others in Bewley [1982] is that the "price-quantity" tests of time-separable utility competitive models done with the aid of an Euler-equation are also testing the complete market hypothesis. If the complete market hypothesis fails the type of "quantity-quality" tests proposed in Section 2, there is no reason to expect the Euler-equation tests to do well or to attribute meaning to estimated preference parameters.

Finally, I would like to note the fact that with incomplete markets the firms' decision problems are not obvious, due to the unanimity question. This is the reason why, if production is treated at all, it is in the absence of firms.

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