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A Macroeconomic Model with Financial Panics

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This article incorporates banks and banking panics within a conventional macroeconomic framework to analyse the dynamics of a financial crisis of the kind recently experienced. We are particularly interested in characterizing the sudden and discrete nature of banking panics as well as the circumstances that make an economy vulnerable to such panics in some instances but not in others. Having a conventional macroeconomic model allows us to study the channels by which the crisis affects real activity both qualitatively and quantitatively. In addition to modelling the financial collapse, we also introduce a belief driven credit boom that increases the susceptibility of the economy to a disruptive banking panic.

Key words: Financial panic, Great recession, Credit boom.

JEL Codes: E23, E32, E44, G01, G21, G33

1. INTRODUCTION

As both Bernanke (2010, 2018) and Gorton (2010) argue, at the heart of the recent financial crisis was a series of bank runs that culminated in the precipitous demise of a number of major financial institutions. During the period where the panics were most intense in October 2008, all the major investment banks effectively failed, the commercial paper market froze, and the Reserve Primary Fund (a major money market fund) experienced a run. The distress quickly spilled over to the real sector. Credit spreads rose to Great Depression era levels. There was an immediate sharp contraction in economic activity: from 2008:Q4 through 2009:Q1 real output dropped at an 8% annual rate, driven mainly by a nearly 40% drop in investment expenditures. Also relevant is that this sudden discrete contraction in financial and real economic activity occurred in the absence of any apparent large exogenous disturbance to the economy.

In this article, we incorporate banks and banking panics within a conventional macroeconomic framework—a New Keynesian model with investment. Our goal is to develop a model where it

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is possible to analyse both qualitatively and quantitatively the dynamics of a financial crisis of the kind recently experienced. We are particularly interested in characterizing the sudden and discrete nature of banking panics as well as the circumstances that make the economy vulnerable to such panics in some instances but not in others. Having a conventional macroeconomic model allows us to study the channels by which the crisis affects aggregate economic activity and the effects of various policies in containing crises.

While our baseline framework focuses on capturing a financial collapse, we then extend the analysis to capture a credit boom that leads to a crisis. As Schularick and Taylor (2012) and many others have emphasized, financial crises are typically preceded by credit booms that lay the seeds for the subsequent collapse. In the spirit of Geanakoplos (2010) and Bordalo Gennaioli and Schleifer (2017), we model the boom by introducing optimistic beliefs. We then characterize how the boom increases the susceptibility of the economy to a disruptive banking panic.

Our article fits into a lengthy literature aimed at adapting core macroeconomic models to account for financial crises.[1] Much of this literature emphasizes the role of balance sheets in constraining borrowers from spending when financial markets are imperfect. Because balance sheets tend to strengthen in booms and weaken in recessions, financial conditions work to amplify fluctuations in real activity. Many authors have stressed that this kind of balance sheet mechanism played a central role in the crisis, particularly for banks and households, but at the height of the crisis also for non-financial firms. Nonetheless, as Mendoza (2010), He and Krishnamurthy (2017) and Brunnermeier and Sannikov (2014) have emphasized, these models do not capture the highly non-linear aspect of the crisis. Although the financial mechanisms in these papers tend to amplify the effects of disturbances, they do not easily capture sudden discrete collapses. Nor can they capture the run-like behaviour associated with financial panics.

Conversely, beginning with Diamond and Dybvig (1983), there is a large literature on banking panics. An important common theme of this literature is how liquidity mismatch, *i.e.* partially illiquid long-term assets funded by short-term debt, opens up the possibility of runs. Most of the models in this literature, though, are partial equilibrium and highly stylized (*e.g.* three periods). They are thus limited for analysing the interaction between financial and real sectors.

Our article builds on our earlier work—Gertler and Kiyotaki (GK, 2015) and Gertler Kiyotaki and Prestipino (GKP, 2016)—which analysed bank runs in an infinite horizon endowment economy. These papers characterize runs as self-fulfilling rollover crises, following the Calvo (1988) and Cole and Kehoe (2000) models of sovereign debt crises. Both GK and GKP emphasize the complementary nature of balance sheet conditions and bank runs. Balance sheet conditions affect not only borrower access to credit but also whether the banking system is vulnerable to a run. In this way, the model is able to capture the highly non-linear nature of a collapse: when bank balance sheets are strong, negative shocks do not push the financial system to the verge of collapse. When they are weak, the same size shock leads the economy into a crisis zone in which a bank run equilibrium exists.[2] While our earlier work restricted attention to a simple endowment economy, here we extend the analysis to a conventional macroeconomic model. By doing so, we can explicitly capture both qualitatively and quantitatively the effect of the financial collapse on investment, output, and employment. In particular, we proceed to show that a calibrated version of our model is capable of capturing the dynamics of key financial and

1. See Gertler and Kiyotaki (2011) and Brunnermeier et al. (2013) for recent surveys.

2. Some recent examples where self-fulfilling financial crises can emerge depending on the state of the economy include Benhabib and Wang (2013), Bocola and Lorenzoni (2017), Farhi and Maggiori (2018), and Perri and Quadrini (2018). For further attempts to incorporate bank runs in macro models, see Angeloni and Faia (2013), Cooper and Ross (1998), Martin Skeie and Von Thadden (2014), Robatto (2019), and Uhlig (2010) for example.

real variables over the course of the recent crisis. Another important difference from our earlier work is that we also model the credit boom preceding the crisis.

There are several other strands of literature relevant to our article. The first is the recent work on occasionally binding borrowing constraints as a source of non-linearity in financial crises such as Mendoza (2010), He and Krishnamurthy (2017), and Brunnermeier and Sannikov (2014). Our approach also allows for occasionally binding financial constraints and precautionary saving. However, in quantitative terms, bank runs provide the major source of non-linearity. Second, there is a literature on credit booms and busts, including Boissay Collard and Smets (2016) and Bordalo Gennaioli and Schleifer (2017). We differ mainly by explicitly modelling banking panics and by integrating the analysis within a reasonably conventional quantitative macroeconomic framework.

Section 2 presents the behaviour of bankers and workers, the sectors where the novel features of the model are introduced. Section 3 describes the features that are standard in the New Keynesian model: the behaviour of firms, price setting, investment, and monetary policy. Section 4 describes the calibration and presents a variety of numerical exercises designed to illustrate the main features of the model, including how the model can capture the dynamics of the recent financial crisis. We also show how it is possible to generate a credit boom that increases the financial fragility of the economy.

2. MODEL: OUTLINE, HOUSEHOLDS, AND BANKERS

The baseline framework is a standard New Keynesian model with investment.[3] In contrast to the conventional model, each household consists of bankers and workers. Bankers specialize in making loans and thus intermediate funds between households and productive capital. Households may also make these loans directly, but they are less efficient in doing so than bankers.[4] On the other hand, bankers may be constrained in their ability to raise external funds and also may be subject to runs. The net effect is that the cost of capital will depend on the endogenously determined flow of funds between intermediated and direct finance.

We distinguish between capital at the beginning of period t, K_t, and capital at the end of the period, S_t. Capital at the beginning of the period is used in conjunction with labour to produce output at t. Capital at the end of period is the sum of newly produced capital and the amount of capital left after production:

$$S_t = \Gamma(I_t) + (1-\delta)K_t, \tag{2.1}$$

where δ is the rate of depreciation. The quantity of newly produced capital, $\Gamma(I_t)$, is an increasing and concave function of investment expenditure, I_t, to capture convex adjustment costs.

A firm wishing to finance new investment as well as old capital, issues a state-contingent claim on the earnings generated by the capital. Let S_t be the total number of claims (effectively equity) outstanding at the end of period t (one claim per unit of capital), S_t^b be the quantity intermediated by bankers, and S_t^h be the quantity directly held by households. Then we have:

$$S_t^b + S_t^h = S_t. \tag{2.2}$$

Both the total capital stock and the composition of financing are determined in equilibrium.

3. For simplicity, we do not distinguish between non-residential and residential investment, though the latter clearly played an important role in the crisis. It should become obvious how to extend the model to make this distinction.

4. As Section 2.2 makes clear, technically it is the workers within the household that are left to manage any direct finance. But since these workers collectively decide consumption, labour, and portfolio choice on of behalf the household, we simply refer to them as the "household" going forward.

The capital stock entering the next period, K_{t+1}, differs from S_t due to a multiplicative "capital quality" shock, ξ_{t+1}, that randomly transforms the units of capital available at $t+1$:

$$K_{t+1}=\xi_{t+1}S_t. \tag{2.3}$$

The shock ξ_{t+1} provides an exogenous source of variation in the return to capital.[5]

To capture that households are less efficient than bankers in handling investments, we assume that they suffer a management cost that depends on the share of capital they hold, S_t^h/S_t. The management cost reflects their disadvantage relative to bankers in evaluating and monitoring capital projects. The cost is in utility terms and takes the following piece-wise form:

$$\varsigma(S_t^h,S_t)=\begin{cases} \frac{\chi}{2}\left(\frac{S_t^h}{S_t}-\gamma\right)^2 S_t, \text{ if } \frac{S_t^h}{S_t}>\gamma>0 \\ 0, \text{ otherwise} \end{cases} \tag{2.4}$$

with $\chi>0$.[6]

For $S_t^h/S_t \le \gamma$ there is no efficiency cost: households are able to manage a limited fraction of capital as well as bankers. As the share of direct finance exceeds γ, the efficiency cost $\varsigma(\cdot)$ is increasing and convex in S_t^h/S_t. In this region, limitations on the household's ability to manage capital become relevant. The convex form implies that the marginal efficiency losses rise with the size of the household's direct capital holdings, capturing limits on its capacity to handle investments.

We assume that the efficiency cost is homogenous in S_t^h and S_t to simplify the computation. Making the marginal efficiency cost linear in the share S_t^h/S_t reduces the non-linearity in the model. An informal motivation is that, as the capital stock S_t increases, the household has more options from which to select investments that it is better able to manage, which works to dampen the marginal efficiency cost.

Given the efficiency costs of direct household finance, absent financial frictions banks will intermediate at least the fraction $1-\gamma$ of the capital stock. However, when banks are constrained in their ability to obtain external funds, households will directly hold more than the share γ of the capital stock. As the constraints tighten in a recession, as will happen in our model, the share of capital held by households will expand. The reallocation of capital holdings from banks to less efficient households raises the cost of capital, reducing investment and output in equilibrium. In the extreme event of a systemic bank run, banks liquidate all their holdings, and the firesale of assets from banks to households will lead to a sharp rise in the cost of capital, leading to a deep contraction in investment and output.

In the rest of this section, we characterize the behaviour of households and bankers which are the non-standard parts of the model.

5. The advantage of the capital quality shock is that it can introduce an exogenous source of variation in the market value of bank assets, which will play a role in making banks vulnerable to panics. The disadvantage is that the shock has a direct supply effect on output by affecting the quantity of capital. However, we show that this direct effect is negligible for the size of shocks we consider. In addition, we also consider how beliefs about capital quality affect market values, which eliminates the direct supply effect altogether.

6. For a deeper model of the costs that non-experts face in financial markets see Kurlat (2016). Our assumption that households intermediation costs are non-pecuniary is made for simplicity only. All of our results go through if we assume that households' intermediated capital is less productive, as in *e.g.* Brunnermeier and Sannikov (2014), as long as productivity losses increase with the quantity of capital intermediated by households.

2.1. *Households*

We formulate this sector in a way that allows for financial intermediation yet preserves the tractability of the representative household setup. In particular, each household is a family that consists of a continuum of members with measure unity. Within the household there are $1-f$ workers and f bankers. Workers supply labour and earn wages for the household. Each banker manages a bank and transfers (non-negative) dividends back to the household. Within the family there is perfect consumption sharing.

In order to preclude a banker from retaining sufficient earnings to permanently relax any financial constraint, we assume the following: in each period, with i.i.d. probability $1-\sigma$, a banker exits. Upon exit, it gives all its accumulated earnings to the household. This stochastic exit in conjunction with the payment to the household upon exit is in effect a simple way to model dividend payouts.[7]

After exiting, a banker returns to being a worker. To keep the population of each occupation constant, each period, $(1-\sigma)f$ workers become bankers. At this time, the household provides each new banker with an exogenously given initial equity stake in the form of a wealth transfer, e_t. The banker receives no further transfers from the household and instead operates at arms length.

Households save in the form of deposits at banks and direct claims on capital. Bank deposits at t are one period bonds that promise to pay a non-contingent gross real rate of return $\overline{R}_{t+1}$ in the absence of default. In the event of default at $t+1$, depositors receive the fraction x_{t+1} of the promised return, where the recovery rate $x_{t+1} \in [0,1)$ is the value of bank assets per unit of promised deposit obligations.

There are two reasons the bank may default: first, a sufficiently negative return on its portfolio may make it insolvent. Second, even if the bank is solvent at normal market prices, the bank's creditors may "run" forcing the bank to liquidate assets at firesale prices. We describe each of these possibilities in detail in the next section. Let p_t be the probability that the bank defaults in period $t+1$. Given p_t and x_{t+1}, we can express the gross rate of return on the deposit contract R_{t+1} as

$$R_{t+1}=\begin{cases} \overline{R}_{t+1} \text{ with probability } 1-p_t \\ x_{t+1}\overline{R}_{t+1} \text{ with probability } p_t \end{cases}. \tag{2.5}$$

Similar to the Cole and Kehoe (2000) model of sovereign default, a run in our model will correspond to a panic failure of households to roll over deposits. This contrasts with the "early withdrawal" mechanism in the classic Diamond and Dybvig (1983) model. For this reason, we do not need to impose a "sequential service constraint" which is necessary to generate runs in Diamond and Dybvig. Instead, we make the weaker assumption that all households receive the same pro rata share of output in the event of default, whether it be due to insolvency or a run.

Let C_t be consumption, L_t labour supply, and $\beta \in (0,1)$ the household's subjective discount factor. As mentioned before, $\varsigma(S_t^h, S_t)$ is the household utility cost of direct capital holding S_t^h, where the household takes the aggregate quantity of claims S_t as given. Then household utility U_t is given by

$$U_t=E_t\left\{\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left[\frac{(C_\tau)^{1-\gamma_h}}{1-\gamma_h}-\frac{(L_\tau)^{1+\varphi}}{1+\varphi}-\varsigma(S_\tau^h,S_\tau)\right]\right\}.$$

Let Q_t be the relative price of capital, Z_t the rental rate on capital, w_t the real wage rate, T_t lump-sum taxes, and Π_t dividend distributions net transfers to new bankers, all of which the

7. As Section 2.2 makes clear, because of the financial constraint, it will always be optimal for a bank to retain earnings until exit.

household takes as given. Then the household chooses C_t, L_t, S_t^h, and deposits D_t to maximize expected utility subject to the budget constraint

$$C_t + D_t + Q_t S_t^h = w_t L_t - T_t + \Pi_t + R_t D_{t-1} + \xi_t [Z_t + (1-\delta) Q_t] S_{t-1}^h. \tag{2.6}$$

The first order condition for labour supply is given by:

$$w_t \lambda_t = (L_t)^{\varphi}, \tag{2.7}$$

where $\lambda_t \equiv (C_t)^{-\gamma_h}$ denotes the marginal utility of consumption.

The first order condition for bank deposits takes into account the possibility of default and is given by

$$1 = [(1-p_t) E_t \left(\Lambda_{t+1} | no\ def \right) + p_t E_t \left(\Lambda_{t+1} x_{t+1} | def \right)] \cdot \overline{R}_{t+1}, \tag{2.8}$$

where $E_t(\cdot \,|\, no\ def)$ (and $E_t(\cdot \,|\, def)$) are expected value of $\cdot$ conditional on no default (and default) at date $t+1$. The stochastic discount factor Λ_{t+1} satisfies

$$\Lambda_{t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t}. \tag{2.9}$$

Observe that the promised deposit rate $\overline{R}_{t+1}$ that satisfies equation (2.8) depends on the default probability p_t as well as the recovery rate x_{t+1}.[8]

Finally, the first-order condition for capital holdings is given by

$$E_t \left[\Lambda_{t+1} \xi_{t+1} \frac{Z_{t+1} + (1-\delta) Q_{t+1}}{Q_t + \frac{\partial \varsigma(S_t^h, S_t)}{\partial S_t^h} / \lambda_t} \right] = 1, \tag{2.10}$$

where

$$\frac{\partial \varsigma(S_t^h, S_t)}{\partial S_t^h} / \lambda_t = \text{Max} \left[\chi \left(\frac{S_t^h}{S_t} - \gamma \right) / \lambda_t,\ 0 \right] \tag{2.11}$$

is the household's marginal cost of direct capital holding.

The first-order condition given by (2.10) will be key in determining the market price of capital. Observe that the market price of capital will tend to be decreasing in the share of capital held by households above the threshold γ since the efficiency cost $\varsigma(S_t^h, S_t)$ is increasing and convex. As will become clear, in a panic run banks will sell all their securities to households, leading to a sharp contraction in asset prices. The severity of the drop will depend on the curvature of the efficiency cost function given by (2.4), which controls asset market liquidity in the model.

2.2. *Bankers*

The banking sector we characterize corresponds best to the shadow banking system which was at the epicenter of the financial instability during the Great Recession. In particular, banks in the model are completely unregulated, hold long-term securities, issue short-term debt, and as a consequence are potentially subject to runs.

8. Notice that we are already using the fact that in equilibrium all banks will choose the same leverage so that all deposits have the same probability of default.

2.2.1. Bankers optimization problem. Each banker manages a financial intermediary with the objective of maximizing the expected utility of the household. Bankers fund capital investments by issuing short-term deposits d_t to households as well as by using their own equity, or net worth, n_t. Due to financial market frictions described later, bankers may be constrained in their ability to obtain deposits.

So long as there is a positive probability that the banker may be financially constrained at some point in the future, it will be optimal for the banker to delay dividend payments until exit (as we will verify later). At this point, the dividend payout will simply be the accumulated net worth. Accordingly, we can take the banker's objective as to maximize the discounted expected value of net worth upon exit. Given that σ is the survival probability and given that the banker uses the household's intertemporal marginal rate of substitution $\widetilde{\Lambda}_{t,\tau} = \beta^{\tau-t}\lambda_\tau/\lambda_t$ to discount future payouts, we can express the objective of a continuing banker at the end of period t as

$$\begin{aligned} V_t &= E_t\left[\sum_{\tau=t+1}^{\infty} \widetilde{\Lambda}_{t,\tau}(1-\sigma)\sigma^{\tau-t-1} n_\tau\right] \\ &= E_t\left\{\Lambda_{t+1}[(1-\sigma)n_{t+1}+\sigma V_{t+1}]\right\}, \end{aligned} \tag{2.12}$$

where $(1-\sigma)\sigma^{\tau-t-1}$ is probability of exiting at date τ, and n_τ is terminal net worth if the banker exits at τ.

During each period t, a continuing bank (either new or surviving) finances asset holdings $Q_t s_t^b$ with newly issued deposits and net worth:

$$Q_t s_t^b = d_t + n_t. \tag{2.13}$$

We assume that banks can only accumulate net worth by retained earnings and do not issue new equity. While this assumption is a reasonable approximation of reality, we do not explicitly model the agency frictions that underpin it.

The net worth of surviving bankers, accordingly, is the gross return on assets net the cost of deposits, as follows:

$$n_t = R_t^b Q_{t-1} s_{t-1}^b - R_t d_{t-1}, \tag{2.14}$$

where R_t^b is the gross rate of return on capital intermediated by banks, given by:

$$R_t^b = \xi_t \frac{Z_t + (1-\delta)Q_t}{Q_{t-1}}. \tag{2.15}$$

So long as n_t is strictly positive the bank does not default. In this instance, it pays its creditors the promised rate $\overline{R}_t$. If the value of assets, $R_t^b Q_{t-1} s_{t-1}^b$, is below the promised repayments to depositors $\bar{R}_t d_{t-1}$ (due to either a run or simply a bad realization of returns), n_t goes to zero and the bank defaults. It then pays creditors the product of recovery rate x_t and $\overline{R}_t$, where x_t is given by:

$$x_t = \frac{R_t^b Q_{t-1} s_{t-1}^b}{\overline{R}_t d_{t-1}} < 1. \tag{2.16}$$

For each new banker at t, net worth simply equals the start-up equity e_t it receives from the household:

$$n_t = e_t. \tag{2.17}$$

To motivate a limit on a bank's ability to issue deposits, we introduce the following moral hazard problem: after accepting deposits and buying assets at the beginning of t, but still during the period, the banker decides whether to operate "honestly" or to divert assets for personal use. To operate honestly means holding assets until the payoffs are realized in period $t+1$ and then meeting deposit obligations. To divert means selling a fraction θ of assets secretly on a secondary market in order to obtain funds for personal use. We assume that the process of diverting assets takes time: the banker cannot quickly liquidate a large amount of assets without the transaction being noticed. Accordingly, the banker must decide whether to divert at t, prior to the realization of uncertainty at $t+1$. Further, to remain undetected, he can only sell up to a fraction θ of the assets. The cost to the banker of the diversion is that the depositors force the intermediary into bankruptcy at the beginning of the next period.[9]

The banker's decision on whether or not to divert funds at t boils down to comparing the franchise value of the bank V_t, which measures the present discounted value of future payouts from operating honestly, with the gain from diverting funds, $\theta Q_t s_t^b$. In this regard, rational depositors will not lend to the banker if he has an incentive to cheat. Accordingly, any financial arrangement between the bank and its depositors must satisfy the incentive constraint:

$$\theta Q_t s_t^b \leq V_t. \tag{2.18}$$

To characterize the banker's optimization problem it is useful to let ϕ_t denote the bank's ratio of assets to net worth, $Q_t s_t^b / n_t$, which we will call the "leverage multiple". Then, combining the balance sheet constraint (2.13) and the flow of funds constraint (2.14) yields the expression for the evolution of net worth for a surviving bank that does not default as:

$$n_{t+1} = [(R^b_{t+1} - \overline{R}_{t+1})\phi_t + \overline{R}_{t+1}]n_t, \tag{2.19}$$

where we used (2.5) to substitute the promised rate $\bar{R}_{t+1}$ for the deposit rate R_{t+1} in the case of no default.

Using the evolution of net worth equation (2.19), we can write the franchise value of the bank (2.12) as

$$V_t = (\mu_t \phi_t + \nu_t) n_t, \tag{2.20}$$

where

$$\mu_t = (1-p_t)E_t\{\Omega_{t+1}(R^b_{t+1} - \overline{R}_{t+1}) \,|\, no\ def\} \tag{2.21}$$

$$\nu_t = (1-p_t)E_t\{\Omega_{t+1}\overline{R}_{t+1} \,|\, no\ def\} \tag{2.22}$$

with

$$\Omega_{t+1} = \Lambda_{t+1}(1-\sigma+\sigma\psi_{t+1}), \text{ and}$$

$$\psi_{t+1} \equiv \frac{V_{t+1}}{n_{t+1}} \geq 1.$$

The variable μ_t is the expected discounted excess return on banks assets relative to deposits, and ν_t is the expected discounted cost of a unit of deposits. Intuitively, $\mu_t \phi_t$ is the excess return the bank receives from having an additional unit of net worth (taking into account the bank's ability to use leverage), while ν_t is the cost saving from substituting equity finance for deposit finance.

9. We assume households deposit funds in banks other than the ones they own. Hence, diverting involves stealing funds from families other than the one to which the banker belongs.

Notice that the bank uses the stochastic discount factor Ω_{t+1} to value returns in $t+1$. Ω_{t+1} is the bank's discounted shadow value of a unit of net worth at $t+1$, averaged across the likelihood of exit and the likelihood of survival. If the bank exits, which occurs with probability $1-\sigma$, it returns its net worth to the household. Accordingly, in this case the value of net worth is simply given by the household's marginal utility of consumption. If instead the bank survives, which occurs with probability σ, the unit value of bank net worth is larger than the household's marginal utility of consumption, *i.e.* ψ_{t+1} exceeds unity. This is because a surviving bank can use the additional unit of net worth to expand its capital holdings rather than to pay dividends to the households. As will become clear, the bank will always prefer to do so since financial market frictions prevent perfect arbitrage by banks and thus capital investment is always strictly profitable at the margin. Therefore, we can think of ψ_{t+1} in the expression for Ω_{t+1} as the bank's "Tobin's Q ratio", *i.e.* the shadow value of a unit of net worth within the bank.

The banker's optimization problem is then to choose the leverage multiple ϕ_t to solve

$$\psi_t = \max_{\phi_t} \ (\mu_t \phi_t + \nu_t), \tag{2.23}$$

subject to the incentive constraint (obtained from equations (2.18) and (2.20)):

$$\theta \phi_t \leq \mu_t \phi_t + \nu_t, \tag{2.24}$$

and the deposit rate constraint (obtained from equations (2.8) and (2.16)):

$$\overline{R}_{t+1} = \left[(1-p_t)E_t(\Lambda_{t+1} \,|\, no\ def) + p_t E_t(\Lambda_{t+1} x_{t+1} \,|\, def)\right]^{-1}, \tag{2.25}$$

where x_{t+1} is the following function of ϕ_t:

$$x_{t+1} = \frac{\phi_t}{\phi_t - 1} \frac{R^b_{t+1}}{\overline{R}_{t+1}},$$

and μ_t and ν_t are given by (2.21) and (2.22).

Note that in addition to the direct effect of the choice of leverage ϕ_t on the objective and the constraints, there is an indirect effect that could arise via the impact of leverage on the default probability p_t. However, this indirect effect on the banks franchise value is zero on net and hence does not show up in the bank's first order conditions.[10] Therefore, for ease of exposition, we relegate to the Appendix the explicit description of how an individual banker's choice of leverage influences its own run and insolvency thresholds.

Notice also that since individual bank net worth does not appear in the bank optimization problem, the optimal choice of ϕ_t is independent of n_t. This implies that the default probability, p_t, the promised rate on deposits, $\bar{R}_{t+1}$, and the bank's Tobin's Q ratio are all independent from bank's specific characteristics.

Since the franchise value of the bank V_t is proportionate to n_t by a factor that only depends on the aggregate state of the economy (see equation (2.20)), a bank cannot operate with zero net worth. In this instance V_t falls to zero, implying that the incentive constraint (2.18) would always be violated if the bank tried to issue deposits. That banks require positive equity to operate is vital to the possibility of bank runs. In fact, as we show below, a necessary condition for a bank run equilibrium to exist is that banks cannot operate with zero net worth.

10. This is because at the default threshold, the bank's assets are exactly equal to its liabilities, so that $n_{t+1}=0$ and $V_{t+1}=0$. Thus, a small shift in the probability mass from the no-default to the default state has no impact on V_t. Similarly, the indirect effect of ϕ_t on the promised deposit rate $\overline{R}_t$ through p_t is zero, since the recovery rate x_t is unity at the borderline of default. Important to the argument is the absence of deadweight losses associated with default.

2.2.2. Banker's decision rules. We derive the optimal portfolio choice of banks by restricting attention to a symmetric equilibrium in which all banks choose the same leverage.[11]

Let μ_t^r be the expected discounted *marginal return to* increasing the leverage multiple ϕ_t

$$\mu_t^r = \frac{d\psi_t}{d\phi_t} = \mu_t - (\phi_t - 1)\frac{\nu_t}{\overline{R}_{t+1}}\frac{d\overline{R}_{t+1}(\phi_t)}{d\phi_t} < \mu_t. \tag{2.26}$$

The second term on the right of equation (2.26) reflects the effect of the increase in $\overline{R}_{t+1}$ that arises as the bank increases ϕ_t. An increase in ϕ_t reduces the recovery rate, forcing $\overline{R}_{t+1}$ up to compensate depositors, as equation (2.25) suggests. The term $(\phi_t - 1)\nu_t/\overline{R}_{t+1}$ then reflects the reduction in the bank franchise value that results from a unit increase in $\overline{R}_{t+1}$. Due to the effect on $\overline{R}_{t+1}$ from expanding ϕ_t, the marginal return μ_t^r is below the average excess return μ_t.

The solution for ϕ_t depends on whether the incentive constraint binds. For the incentive constraint to bind μ_t^r must be positive at the optimum. With μ_t^r positive, the bank will want to borrow to acquire more assets but the incentive constraint will keep it from doing so by placing a limit on its leverage. Thus, if $\mu_t^r > 0$ at the optimum, (2.24) holds with equality, implying the following solution for ϕ_t:

$$\phi_t = \frac{\nu_t}{\theta - \mu_t}, \quad \text{if } \mu_t^r > 0. \tag{2.27}$$

The constraint (2.27) limits the leverage multiple to the point where the bank's gain from diverting funds per unit of net worth $\theta\phi_t$ is exactly balanced by the cost per unit of net worth of losing the franchise value, which is measured by $\psi_t = \mu_t\phi_t + \nu_t$. Note that μ_t (as given by equation (2.21)) tends to move countercyclically since the excess return on bank capital $E_t R_{t+1}^b - \overline{R}_{t+1}$ widens as the borrowing constraint tightens in recessions. As a result, ϕ_t tends to move countercyclically. As we show later, the countercyclical movement in ϕ_t contributes to making bank runs more likely in bad economic times.[12]

Conversely, if the marginal return to increasing the leverage multiple becomes zero before the incentive constraint binds, the bank chooses leverage as follows:

$$\mu_t^r = 0, \quad \text{if } \phi_t < \frac{\nu_t}{\theta - \mu_t}. \tag{2.28}$$

Even though the constraint does not bind, in this case, the bank will still choose to limit the leverage so long as there is a possibility that the incentive constraint could bind in the future. In this instance, as in Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2017), banks have a precautionary motive for scaling back their respective leverage multiples.[13] The

11. In this section, we describe the leverage choice of banks as determined by the first-order conditions of the banks' optimization problem. The Appendix discusses the assumptions under which first order conditions actually select a global optimum for the bank's problem, ensuring that a symmetric strategy equilibrium exists.

12. In the data, net worth of our model corresponds to the mark-to-market difference between assets and liabilities of the bank balance sheet. It is different from the book value often used in the official report, which is slow in reacting to market conditions. Also bank assets here are securities and loans to the non-financial sector, which exclude those to other financial intermediaries. In the data, the net mark-to-market leverage multiple of the financial intermediation sector—the ratio of securities and loans to the non-financial sector to the net worth of the aggregate financial intermediaries—tends to move countercyclically, even though the gross leverage multiple—the ratio of book value total assets (including securities and loans to the other intermediaries) to the net worth of some individual intermediaries may move procyclically. While Adrian and Shin (2010) show book leverage is procyclical for investment bankers, He Khang and Krishnamurthy (2010) and He Kelly and Manela (2017) show market leverage is countercyclical.

13. One difference of our model from these papers is that, because default occurs in equilibrium, the bank's leverage affects the promised deposit rate and the cost of funds. This effect provides an additional motive for the bank to reduce its leverage multiple as implied by the fact that when the constraint is not binding $\mu_t > \mu_t^r = 0$.

precautionary motive is reflected by the presence of the discount factor Ω_{t+1} in the measure of the discounted excess return μ_t. The discount factor Ω_{t+1}, which reflects the shadow value of net worth, tends to vary countercyclically given that borrowing constraints tighten in downturns. By reducing their leverage multiples, banks reduce the risk of taking losses when the shadow value of net worth is high.

2.2.3. Aggregation of the financial sector absent default. We now characterize the aggregate financial sector during periods where banks do not default. Given that the optimal leverage multiple ϕ_t is independent of bank-specific factors, individual bank portfolio decisions, s_t^b and d_t, are homogenous in net worth. Accordingly, we can sum across banks to obtain the following relation between aggregate bank asset holdings $Q_t S_t^b$ and the aggregate quantity of net worth N_t in the banking sector:

$$Q_t S_t^b = \phi_t N_t. \tag{2.29}$$

The evolution of N_t depends on both the retained earnings of bankers that survived from the previous period and the injection of equity to new bankers. For technical convenience again related to computational considerations, we suppose that the household transfer e_t to a each new banker is proportionate to the stock of capital at the end of the previous period, S_{t-1}, with $e_t = \frac{\zeta}{(1-\sigma)f} S_{t-1}$.[14] Aggregating across both surviving and entering bankers yields the following expression for the evolution of net worth

$$N_t = \sigma[(R_t^b - \overline{R}_t)\phi_{t-1} + \overline{R}_t]N_{t-1} + \zeta S_{t-1}. \tag{2.30}$$

The first term is the total net worth of bankers that operated at $t-1$ and survived until t. The second, ζS_{t-1}, is the total start-up equity of entering bankers.

2.3. *Runs, insolvency, and the default probability*

We now turn to the case of default due to either runs or insolvency. After describing bank runs and the condition for a bank run equilibrium to exist, we characterize the overall default probability.

2.3.1. Conditions for a bank run equilibrium. As in Diamond and Dybvig (1983), the runs we consider are runs on the entire banking system and not an individual bank. A run on an individual bank will not have aggregate effects as depositors simply shuffle their funds from one bank to the others in the system.[15] As we noted earlier, though, we differ from Diamond and Dybvig in that runs reflect a panic failure to roll over deposits as opposed to early withdrawal.

Consider the behaviour of a household that acquired deposits at $t-1$. Suppose further that the banking system is solvent at the beginning of time t: assets valued at normal market prices exceed liabilities. The household must then decide whether to roll over deposits at t. A self-fulfilling "run" equilibrium exists if and only if the household correctly believes that in the event all other depositors run, thus forcing the banking system into liquidation, the household will lose money if it rolls over its deposits individually. Note that this condition is satisfied if and only if the liquidation reduces the value of bank assets below promised obligations to depositors, driving

14. Here, we value capital at the steady state price $Q=1$. If we use the market price instead, the financial accelerator would be enhanced but not significantly.

15. Key to this argument is that banks are atomistic so that a run on a single bank has no effect on the price of capital.

aggregate bank net worth to zero. A household that deposits funds in a zero net worth bank will simply lose its money as the bank will divert the money for personal use.[16] If instead bank net worth is positive even at liquidation prices, banks would be able to offer a profitable deposit contract to an individual household deciding to roll over. In this instance, a panic equilibrium would not exist.

The condition for a bank run equilibrium to exist at t, accordingly, is that in the event of liquidation following a run, bank net worth goes to zero. Recall that earlier we defined the depositor recovery rate, x_t, as the ratio of the value of bank assets to promised obligations to depositors. Therefore, a bank run equilibrium exists at t if and only if the recovery rate conditional on a run, x_t^*, is less than unity:

$$\begin{aligned} x_t^* &= \frac{\xi_t[(1-\delta)Q_t^* + Z_t^*]S_{t-1}^b}{\overline{R}_t D_{t-1}} \\ &= \frac{R_t^{b*}}{\overline{R}_t} \cdot \frac{\phi_{t-1}}{\phi_{t-1}-1} < 1, \end{aligned} \tag{2.31}$$

where Q_t^* is the asset liquidation price, Z_t^* is rental rate, and R_t^{b*} is the return on bank assets, all conditional on a run. Since the liquidation price Q_t^* is below the normal market price Q_t, a run may occur even if the bank is solvent at normal market prices. Moreover, as we will show below, when either deteriorating economic conditions or bankers optimistic beliefs cause bank leverage ϕ_{t-1} to increase substantially, even relatively small new disturbances which decrease $\frac{R_t^{b*}}{\overline{R}_t}$ can open up the possibility of a banking panic.

2.3.2. The liquidation price. Key to the condition for a bank run equilibrium is the behaviour of the liquidation price Q_t^*. A depositor panic at t induces all the existing banks to liquidate their assets by selling them to households. Accordingly in the wake of the run:

$$S_t^h = S_t. \tag{2.32}$$

To obtain Q_t^*, we iterate the household Euler equation:

$$\begin{aligned} Q_t^* = E_t &\left\{ \sum_{\tau=t+1}^{\infty} \widetilde{\Lambda}_{t,\tau}(1-\delta)^{\tau-t-1} \left(\prod_{j=t+1}^{\tau} \xi_j \right) \cdot \left[Z_\tau - (1-\delta)\chi \left(\frac{S_\tau^h}{S_\tau} - \gamma \right) / \lambda_\tau \right] \right\} \\ &- \chi(1-\gamma)/\lambda_t, \end{aligned} \tag{2.33}$$

where the term $\chi(1-\gamma)/\lambda_t$ is the period t marginal efficiency cost following a run at t (given $S_t^h/S_t = 1$ in this instance).[17] The liquidation price is thus equal to the expected discounted stream of dividends net the marginal efficiency losses from household portfolio management. Since marginal efficiency losses are at a maximum when S_t^h equal S_t, Q_t^* is at a minimum, given the

16. As we mention below, we assume that new entrant banks during a run do not setup their banking operations until a period after the run. Thus, an individual depositor who does not run would be forced to save in a bank with zero net worth instead of in a new bank.

17. We are imposing that $\frac{S_t^h}{S_t} - \gamma \geq 0$ as is the case in all of our numerical simulations.

expected future path of S_τ^h. Further, the longer it takes the banking system to recover (so S_τ^h falls back to its steady state value) the lower will be Q_t^*. Finally, note that Q_t^* will vary positively with the expected paths of ξ_τ, Z_τ and the stochastic discount factor $\widetilde{\Lambda}_{t,\tau}$.

The banking system then recapitalizes as follows: we suppose that during a run at t new banks cannot set up business. However, they can store their initial equity and enter in the following period along with the new banks scheduled to begin at $t+1$

$$N_{t+1}=\zeta S_t+\sigma\zeta S_{t-1}. \tag{2.34}$$

Given that banks can operate in $t+1$ (the period after the run), it follows that starting from $t+2$ the evolution of bank equity follows the standard process: for all $\tau\geq t+2$

$$N_\tau=\begin{cases}\sigma[(R_\tau^b-R_\tau)\phi_{\tau-1}+R_\tau]N_{\tau-1}+\zeta S_{\tau-1} & \text{if there is no run at } \tau\\ 0 & \text{if there is a run at } \tau\end{cases}.$$

2.3.3. The default probability: illiquidity versus insolvency. In the run equilibrium, banks default even though they are solvent at normal market prices. It is the forced liquidation at firesale prices during a run that pushes these banks into bankruptcy. Thus, in the context of our model, a bank run can be viewed as a situation of illiquidity. In contrast, default is also possible if banks enter period t insolvent at normal market prices.

Accordingly, the total probability of default in the subsequent period, p_t, is the sum of the probability of a run p_t^R and the probability of insolvency p_t^I:

$$p_t=p_t^R+p_t^I. \tag{2.35}$$

We begin with p_t^I. By definition, banks are insolvent if the ratio of assets valued at normal market prices is less than liabilities. In our economy, the only exogenous shock to the aggregate economy is a shock to the quality of capital ξ_t. Let us define ξ_{t+1}^I to be the value of capital quality, ξ_{t+1}, that makes the depositor recovery rate at normal market prices, $x(\xi_{t+1}^I)$, equal to unity:

$$x(\xi_{t+1}^I)=\frac{\xi_{t+1}^I[Z_{t+1}(\xi_{t+1}^I)+(1-\delta)Q_{t+1}(\xi_{t+1}^I)]S_t^b}{\overline{R}_tD_t}=1. \tag{2.36}$$

For values of ξ_{t+1} below ξ_{t+1}^I, the bank will be insolvent and must default. The probability of default due to insolvency is then given by

$$p_t^I=prob_t\left(\xi_{t+1}<\xi_{t+1}^I\right), \tag{2.37}$$

where $prob_t(\cdot)$ is the probability of satisfying $\cdot$ conditional on date t information.

We next turn to the determination of the run probability. In general, the time t probability of a run at $t+1$ is the product of the probability a run equilibrium exists at $t+1$ times the probability a run will occur when it is feasible. We suppose the latter depends on the realization of a sunspot. Let ι_{t+1} be a binary sunspot variable that takes on a value of 1 with probability $\varkappa$ and a value of 0 with probability $1-\varkappa$. In the event of $\iota_{t+1}=1$, depositors coordinate on a run if a bank run equilibrium exists. Note that we make the sunspot probability $\varkappa$ constant so as not to build in exogenous cyclicality in the movement of the overall bank run probability p_t^R.

A bank run arises at $t+1$ iff (1) a bank run equilibrium exists at $t+1$ and (2) $\iota_{t+1}=1$. Let ω_t be the probability at t that a bank run equilibrium exists at $t+1$. Then the probability p_t^R of a run at $t+1$ is given by

$$p_t^R = \omega_t \cdot \varkappa. \tag{2.38}$$

To find the value of ω_t, let us define ξ_{t+1}^R as the value of ξ_{t+1} that makes the recovery rate conditional on a run x_{t+1}^* unity when evaluated at the firesale liquidation price Q_{t+1}^* and rental rate during run Z_{t+1}^*:

$$x^*(\xi_{t+1}^R) = \frac{\xi_{t+1}^R[(1-\delta)Q^*(\xi_{t+1}^R)+Z^*(\xi_{t+1}^R)]S_t^b}{\overline{R}_t D_t} = 1. \tag{2.39}$$

For values of ξ_{t+1} below ξ_{t+1}^R, x_{t+1}^* is below unity and a bank run equilibrium is feasible. Therefore, the probability that a bank run equilibrium exists is given by the probability that ξ_{t+1} lies in the interval below ξ_{t+1}^R but above the threshold for insolvency ξ_{t+1}^I. In particular,

$$\omega_t = prob_t\left(\xi_{t+1}^I \leq \xi_{t+1} < \xi_{t+1}^R\right). \tag{2.40}$$

Given equation (2.40), we can distinguish regions of ξ_{t+1} where insolvency emerges ($\xi_{t+1} < \xi_{t+1}^I$) from regions where an illiquidity problem may emerge ($\xi_{t+1}^I \leq \xi_{t+1} < \xi_{t+1}^R$).

Overall, the probability of a run varies inversely with the expected recovery rate $E_t x_{t+1}$. The lower the forecast of the depositor recovery rate, the higher ω_t and thus the higher p_t. In this way, the model captures that an expected weakening of the bank balance sheets raises the likelihood of a run. Further, because the leverage multiple is countercyclical (see Section 2.2.2), $E_t x_{t+1}$ is procyclical, implying that the run probability is endogenously countercyclical: runs are more likely in bad times than in good.

Finally, comparing equations (2.37) and (2.40) makes clear that the possibility of a run equilibrium significantly expands the chances for a banking collapse, beyond the probability that would arise simply from default due to insolvency. In this way, the possibility of runs makes the system more fragile. Indeed, within the numerical exercises we present the probability of a fundamental shock that induces an insolvent banking system is negligible. However, the probability of a shock that induces a bank run equilibrium is not negligible.

3. PRODUCTION, MARKET CLEARING, AND POLICY

The rest of the model is fairly standard. There is a production sector consisting of producers of final goods, intermediate goods, and capital goods. Prices are sticky in the intermediate goods sector. In addition, there is a central bank that conducts monetary policy.

3.1. *Final and intermediate goods firms*

There is a continuum of measure unity of final goods producers and intermediate goods producers. Final goods firms make a homogenous good Y_t that may be consumed or used as input to produce new capital goods. Each intermediate goods firm $f \in [0,1]$ makes a differentiated good $Y_t(f)$ that is used in the production of final goods.

Final goods firms transform intermediate goods into final output according to the following CES production function:

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3.41}$$

where $\varepsilon > 1$ is the elasticity of substitution between intermediate goods.

Let $P_t(f)$ be the nominal price of intermediate good f. Then cost minimization of final goods firms yields the following demand function for each intermediate good f (after integrating across the demands of by all final goods firms):

$$Y_t(f) = \left[\frac{P_t(f)}{P_t} \right]^{-\varepsilon} Y_t, \tag{3.42}$$

where P_t is the price index as

$$P_t = \left[\int_0^1 P_t(f)^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}}.$$

There is a continuum of intermediate good firms owned by consumers, indexed by $f \in [0,1]$. Each produces a differentiated good and is a monopolistic competitor. Intermediate goods firm f uses both labour $L_t(f)$ and capital $K_t(f)$ to produce output according to:

$$Y_t(f) = A_t K_t(f)^{\alpha} L_t(f)^{1-\alpha}, \tag{3.43}$$

where A_t is a technology parameter and $0 > \alpha > 1$ is the capital share.

Both labour and capital are freely mobile across firms. Firms rent capital from owners of claims to capital (*i.e.* banks and households) in a competitive market on a period by period basis. Then from cost minimization, all firms choose the same capital labour ratio, as follows

$$\frac{K_t(f)}{L_t(f)} = \frac{\alpha}{1-\alpha} \frac{w_t}{Z_t} = \frac{K_t}{L_t}, \tag{3.44}$$

where, as noted earlier, w_t is the real wage rate and Z_t is the rental rate of capital. The first-order conditions from the cost minimization problem imply that marginal cost is given by

$$MC_t = \frac{1}{A_t} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{Z_t}{\alpha} \right)^{\alpha}. \tag{3.45}$$

Observe that marginal cost is independent of firm-specific factors.

Following Rotemberg (1982), each monopolistically competitive firm f faces quadratic costs of adjusting prices. Let ρ^r ("r" for Rotemberg) be the parameter governing price adjustment costs. Then each period, it chooses $P_t(f)$ and $Y_t(f)$ to maximize the expected discounted value of profit:

$$E_t \left\{ \sum_{\tau=t}^{\infty} \widetilde{\Lambda}_{t,\tau} \left[\left(\frac{P_\tau(f)}{P_\tau} - MC_\tau \right) Y_\tau(f) - \frac{\rho^r}{2} Y_\tau \left(\frac{P_\tau(f)}{P_{\tau-1}(f)} - 1 \right)^2 \right] \right\}, \tag{3.46}$$

subject to the demand curve (3.42). Here, we assume that the adjustment cost is proportional to the aggregate demand Y_t.

Taking the firm's first-order condition for price adjustment and imposing symmetry implies the following forward looking Phillip's curve:

$$(\pi_t - 1)\pi_t = \frac{\varepsilon}{\rho^r}\left(MC_t - \frac{\varepsilon - 1}{\varepsilon}\right) + E_t\left[\Lambda_{t+1}\frac{Y_{t+1}}{Y_t}\left(\pi_{t+1} - 1\right)\pi_{t+1}\right], \tag{3.47}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ is the realized gross inflation rate at date t.

3.2. *Capital goods producers*

There is a continuum of measure unity of competitive capital goods firms, owned by households. Each produces new investment goods that it sells at the competitive market price Q_t. By investing $I_t(j)$ units of final goods output, firm j can produce $\Gamma(I_t(j))$ new capital goods, with $\Gamma' > 0$ and $\Gamma'' < 0$.

The decision problem for capital producer j is accordingly

$$\max_{I_t(j)} Q_t \Gamma\left(I_t(j)\right) - I_t(j). \tag{3.48}$$

Given symmetry for capital producers $(I_t(j) = I_t)$, we can express the first-order condition as the following "Q" relation for investment:

$$Q_t = \left[\Gamma'\left(I_t\right)\right]^{-1} \tag{3.49}$$

which yields a positive relation between Q_t and investment.

3.3. *Monetary policy*

Let Θ_t be a measure of cyclical resource utilization, *i.e.* resource utilization relative to the flexible-price equilibrium. Next, let $R = \beta^{-1}$ denotes the real interest rate in the deterministic steady state with zero inflation. We suppose that the central bank sets the nominal rate on the riskless bond R_t^n according to the following Taylor rule:

$$R_t^n = \frac{1}{\beta}\left(\pi_t\right)^{\kappa_\pi}\left(\Theta_t\right)^{\kappa_y} \tag{3.50}$$

with $\kappa_\pi > 1$. Unfortunately, due to the high degree of non-linearity, we are unable to impose the zero lower bound on the net nominal rate. We instead start with a high equilibrium real rate of 4% so that under a standard Taylor rule the nominal interest does not drop significantly below the ZLB.

A standard way to measure Θ_t is to use the ratio of actual output to a hypothetical flexible-price equilibrium value of output. Computational considerations lead us to use a measure which similarly captures the cyclical efficiency of resource utilization but is much easier to handle numerically. Specifically, we take as our measure of cyclical resource utilization the ratio of the desired markup, $1 + \mu = \varepsilon/(\varepsilon - 1)$ to the current markup $1 + \mu_t$.[18]

$$\Theta_t = \frac{1 + \mu}{1 + \mu_t} \tag{3.51}$$

18. In the case of consumption goods only, our markup measure of efficiency corresponds exactly to the output gap.

with

$$1+\mu_t = MC_t^{-1} = \frac{(1-\alpha)(Y_t/L_t)}{L_t^{\varphi}C_t^{\gamma_h}}. \tag{3.52}$$

The markup corresponds to the ratio of the marginal product of labour to the marginal rate of substitution between consumption and leisure, which corresponds to the labour market wedge. The inverse markup ratio Θ_t thus isolates the cyclical movement in the efficiency of the labour market, specifically the component that is due to nominal rigidities.

Finally, one period bonds which have a riskless nominal return have zero net supply. (Bank deposits have default risk.) Nonetheless, we can use the following household Euler equation to price the nominal interest rate of these bonds R_t^n as

$$E_t\left(\Lambda_{t+1}\frac{R_t^n}{\pi_{t+1}}\right)=1. \tag{3.53}$$

3.4. *Resource constraints and equilibrium*

Total output is divided between consumption, investment, the adjustment cost of nominal prices, and a fixed value of government consumption G:

$$Y_t = C_t + I_t + \frac{\rho^r}{2}(\pi_t - 1)^2 Y_t + G. \tag{3.54}$$

Given a symmetric equilibrium, we can express total output as the following function of aggregate capital and labour:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}. \tag{3.55}$$

Although we consider a limiting case in which supply of government bonds and money is zero, the government adjusts lump sum taxes to satisfy the budget constraint. Finally, the labour market must clear, which implies that aggregate labour demand of producers equals aggregate labour supply of households.

This completes the description of the model. See the Appendix for details.[19]

4. NUMERICAL EXERCISES

4.1. *Calibration*

Table 1 lists the choice of parameter values for our model. Overall there are 21 parameters. Thirteen are conventional as they appear in standard New Keynesian DSGE models. The other eight parameters govern the behaviour of the financial sector, and hence are specific to our model.

We begin with the conventional parameters. For the discount rate β, the risk aversion parameter γ_h, the inverse Frisch elasticity φ, the elasticity of substitution between goods ε, the depreciation rate δ, and the capital share α we use standard values in the literature. Three additional parameters (η, a, b) involve the investment technology, which we express as follows:

$$\Gamma(I_t) = a(I_t)^{1-\eta} + b.$$

We set η, which corresponds to the elasticity of the price of capital with respect to investment rate, equal to 0.25, a value within the range of estimates from panel data as in, for instance,

19. We focus throughout on the two equilibria described in the text. There is also a third equilibrium that we rule out because it is not *tatonnement* stable.

TABLE 1

Parameter	Description	Value	Target
	Standard parameters		
β	Impatience	0.99	Risk Free Rate
γ_h	Risk Aversion	2	Literature
φ	Inverse Frisch Elasticity	0.5	Literature
ϵ	Elasticity of Subst. Across Varieties	11	Markup 10%
α	Capital Share	0.33	Capital Share
δ	Capital Depreciation	0.33	$\frac{I}{K}=0.025$
η	Elasticity of Q w.r.t. I	0.25	Literature
a	Inv. Technology	0.53	Q=1
b	Inv. Technology	−0.83%	$\Gamma(I)=I$
G	Government Expenditure	0.45	$\frac{G}{Y}=0.2$
ρ^{jr}	Price Adj. Costs	1000	Slope of Phillips curve = 0.01
κ_π	Policy Response to Inflation	1.5	Literature
κ_y	Policy Response to Output	0.125	Literature
	Financial intermediation parameters		
σ	Banker Survival Rate	0.93	Leverage = 10
ζ	New Banker Endowment	0.1%	Investment Drop in Crisis = 35%
θ	Share of Divertible Assets	0.22	Spread Increase in Crisis = 1.5%
γ	Threshold for HH Intermediation Costs	0.61	$\frac{S^b}{S}=0.33$
χ	Marginal HH Intermediation Costs	0.105	$ER^b-R=2\%$ Annual
$\varkappa$	Sunspot Probability	0.15	Run Probability= 4% Annual
$\sigma(\epsilon^\xi)$	Standard Deviation of Innovation to Capital Quality	0.5%	Standard Deviation of Output (C+I)
ρ^ξ	Serial Correlation of Capital Quality	0.7	Standard Deviation of Investment

Gilchrist and Himmelberg (1995), Eberly (1997), and Hall (2004). We then choose a and b to hit two targets: first, a ratio of quarterly investment to the capital stock of 2.5% and, second, a value of the price of capital Q equal to unity in the risk-adjusted steady state. We set the value of government expenditures G to 20% of steady state output. Next, we choose the cost of price adjustment parameter ρ^{jr} to generate an elasticity of inflation with respect to marginal cost equal to 1%, which is roughly in line with the estimates.[20] Finally, we set the feedback parameters in the Taylor rule, κ_π and κ_y, to their conventional values of 1.5 and 0.5, respectively. As we noted earlier, due to the high degree on non-linearity in the model, we are unable to impose the zero lower bound. We verify, however, that the violation of the ZLB that occurs during a crisis is not large.

We now turn to the financial sector parameters. There are six parameters that directly affect the evolution of bank net worth and credit spreads: the banker's survival probability σ; the initial equity injection to entering bankers as a share of capital ζ; the asset diversion parameter θ; the threshold share for costless direct household financing of capital, γ; the parameter governing the convexity of the efficiency cost of direct financing χ; and the probability of observing a sunspot $\varkappa$.

We choose the values of these parameter to hit the following six targets: (i) the average arrival rate of a systemic bank run equals 4% annually, corresponding to a frequency of banking panics of once every 25 years, which is in line with the evidence for advanced economies;[21] (ii) the average bank leverage multiple equals 10;[22] (iii) the average excess rate of return on bank assets

20. See, for example, Del Negro et al. (2015).

21. See, for example, Bordo et al. (2001), Reinhart and Rogoff (2009), and Schularick and Taylor (2012).

22. We think of the banking sector in our model as including both investment banks and some large commercial banks that operated off balance sheet vehicles without explicit guarantees. Ten is on the high side for commercial banks and on the low side for investment banks. See Gertler Kiyotaki and Prestipino (2016).

TABLE 2

	Data 1983–2007 Q3	Model No runs happen
Y	1.9	1.4
C+I	2.7	2.9
I	7.2	7.1
C	1.3	2
L	3.1	2.6

Notes: For output, investment, consumption, and government expenditure, we compute real per capital values by dividing by population and adjusting by the GDP deflator. For labour, we compute per capita hours worked by dividing total hours by population. We then report standard deviations of the logged variables in deviations from a linear trend starting in 1983 Q1 and ending in 2007 Q3.
Source: Output, investment (gross domestic investment plus durable good consumption), consumption (personal consumption expenditure less durables), government expenditure, and GDP deflator are from the Bureau of Economic Analysis. Total labour hours (aggregate hours, non-farm payrolls) and population (civilian non-institutional, 16 years and over) are from the Bureau of Labor Statistics.

over deposits equals 2%, based on Philippon (2015); (iv) the average share of bank intermediated assets equals 0.33, which is a reasonable estimate of the share of intermediation performed by shadow banks;[23] (v) and (vi) the increase in excess returns (measured by credit spreads) and the drop in investment following a bank run to match the evidence from the recent crisis.

The remaining two parameters determine the serial correlation of the capital quality ρ_ξ and the standard deviation of the innovations σ_ξ. That is we assume that the capital quality shock obeys the following first-order process:

$$\xi_{t+1} = 1 - \rho_\xi + \rho_\xi \xi_t + \epsilon_{t+1}$$

with $0 < \rho_\xi < 1$ and where ϵ_{t+1} is a (truncated) normally distributed i.i.d. random variable with mean zero and standard deviation σ_ξ.[24] We choose ρ_ξ and σ_ξ so that the unconditional standard deviations of investment and aggregate domestic demand are consistent with the evidence observed over 1983Q1-2008Q3.

Given that our policy functions are non-linear, we obtain model implied moments by simulating our economy for one hundred thousand periods.[25] Table 2 shows unconditional standard deviations for some key macroeconomic variables in the model and in the data. The volatilities of output, investment, and labour are reasonably in line with the data. Consumption is too volatile, but the variability of the sum of consumption and investment matches the evidence.[26]

4.2. *Experiments*

In this section, we perform several experiments that are meant to illustrate how our model economy behaves and compares with the data. We show the response of the economy to a capital quality

23. The bulk of shadow bank intermediation was performed by investment banks and special purpose vehicles of large commercial banks. Begenau and Landvoight (2017) use a share of shadow bank intermediation of 33% based on work by Gallin (2015). Pozsar *et al.* (2010) also have similar estimates for the share of shadow bank intermediation.

24. In practice, we assume that ϵ_{t+1} is a truncated normal with support $(-10\sigma_\xi, 10\sigma_\xi)$. Given our calibration for σ_ζ and ρ_ξ, the probability that ξ_t goes below zero is computationally zero.

25. As we are excluding the financial crisis from the sample, the sunspot variable is set to zero in these simulations.

26. One way to reduce consumption volatility would be to introduce habit formation though this would come at the cost of an additional state variable. Also some of the excess volatility in consumption arises because we are keeping the output share of government spending fixed.

shock first without and then with runs to illustrate how the model generates a financial panic with significant real effects on the economy. We also illustrate the role that the New Keynesian features play. We then show how the model can replicate salient features of the recent financial crisis. Finally, we model a credit boom by introducing optimistic beliefs and show how the boom can help precipitate the subsequent financial collapse.

4.2.1. Response to a capital quality shock: no bank run case. We suppose the economy is initially in a risk-adjusted steady state. Figure 1 shows the response of the economy to a negative one standard deviation (0.50%) shock to the quality of capital without runs.[27] The solid line is our baseline model and the dotted line is the case where there are no financial frictions. For both cases the shock reduces the expected return to capital, reducing investment and in turn aggregate demand. In addition, for the baseline economy with financial frictions, the weakening of bank balance sheets amplifies the contraction in demand through the financial accelerator/credit cycle mechanism of Bernanke Gertler and Gilchrist (1999) and Kiyotaki and Moore (1997). Poor asset returns following the shock cause bank net worth to decrease by about 13%. As bank net worth declines, incentive constraints tighten and banks decrease their demand for assets causing the price of capital to drop. The drop in asset prices feeds back into lower bank net worth, an effect that is magnified by the extent of bank leverage. As financial constraints tighten and asset prices decline, excess returns rise by 50 basis points which allows banks to increase their leverage multiple by about 9%. Overall, a 0.5% decline in the quality of capital results in a drop in investment by 4% and a drop in output by slightly less than 1%. The drop in investment is roughly double the amount in the case absent financial frictions, while the drop in output is about 30% greater.

In the experiment of Figure1, the economy is always ex post in a "safe zone", where a bank run equilibrium does not exist. Under our parametrization, a bank run cannot happen in the risk-adjusted steady state because bank leverage is too low. The dashed line in the first panel of Figure 1 shows the size of the shock needed in the subsequent period to push the economy into the run region. In our example, a two standard deviation shock is needed to open up the possibility of runs starting from the risk-adjusted steady state, which is double the size of the shock considered in Figure 1.

Even though in this case the economy is always in a safe region ex post, it is possible ex ante that a run equilibrium could occur in the subsequent period. In particular, the increase in leverage following the shock raises the probability that a sufficiently bad shock in the subsequent period pushes the economy into the run region. As the top middle panel of Figure 1 shows, the overall probability of a run increases following the shock.

4.2.2. Bank runs. In the previous experiment, the economy was well within a safe zone. A one standard deviation shock did not and could not produce a financial panic. We now consider a case where the economy starts in the safe zone but is gradually pushed to the edge of the crisis zone, where a run equilibrium exists. We then show how an arrival of a sunspot induces a panic with damaging effects on the real economy.

To implement this experiment, we assume that the economy is hit by a sequence of three modest and equally sized negative shocks that push the economy to the run threshold. That is, we find a shock ϵ^* that satisfies:

$$\xi_3^R = 1+\epsilon^*\left(1+\rho_\xi+\rho_\xi^2\right),$$

27. In all of the experiments, we trace the response of the economy to the shocks considered assuming that after these shocks capital quality is exactly equal to its conditional expectations, *i.e.* setting future ε_t to 0. See Appendix Section 6.7 for a formal definition of the risk-adjusted steady state and our impulse response functions.

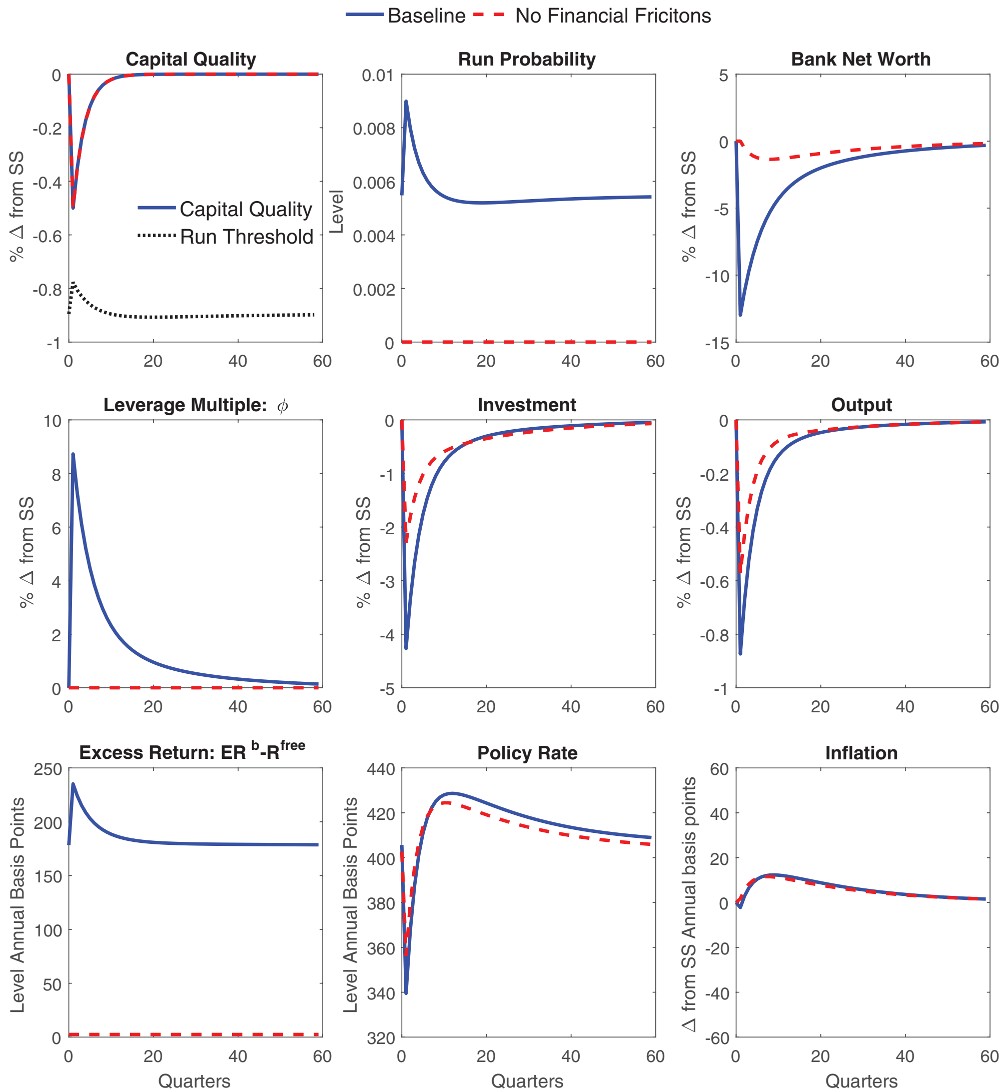


FIGURE 1

Response to a capital quality shock (1 std): no run case

where ξ_3^R is the threshold level for the capital quality below which a run is possible in period 3, given that the economy is in steady state in period 0 and is hit by two equally sized shocks in periods 1 and 2, *i.e.* $\epsilon_1 = \epsilon_2 = \epsilon^*$. The first two shocks push the economy to the edge of the crisis zone. The third pushes it just in.

The solid line in Figure 2 shows the response of the economy starting from period two onwards under the assumption that the economy experiences a run with arrival of a sunspot in period 3. For comparison, the dashed line shows the response of the economy to the same exact capital quality shocks but assuming that no sunspot is observed and so no run happens.

As shown in panel 1, the size of the threshold shock to the quality of capital needed to push the economy into a crisis zone turns out to be roughly the same size as the one standard deviation

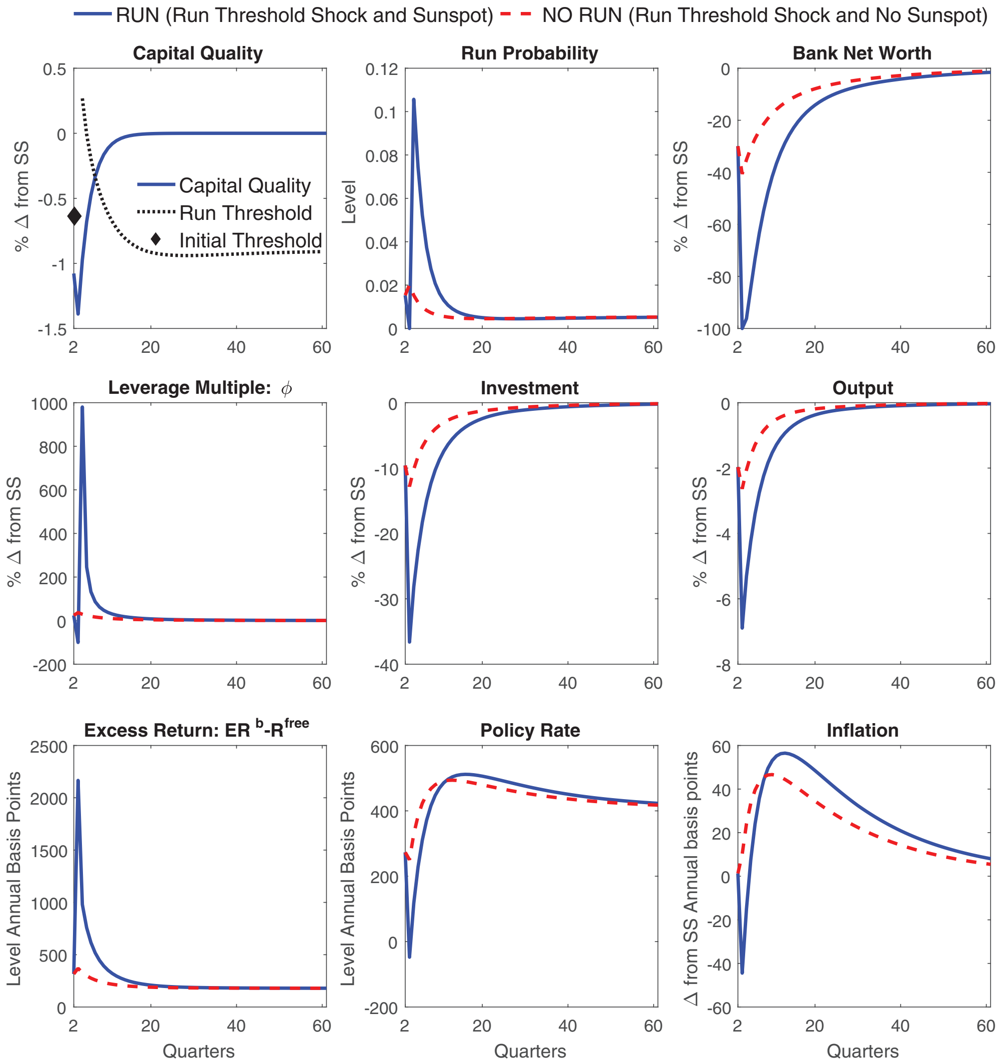


FIGURE 2

Response to a sequence of shocks: run versus no run

shock used in Figure 1, *i.e.* the threshold is $\epsilon^* = -0.63\%$ while $\sigma(\epsilon^\xi) = 0.5$. After the first two innovations, the capital quality shock is 1.1% below average and the run probability is about 2% quarterly. The last innovation pushes the economy into the run region. When the sunspot is observed and the run occurs, bank net worth is wiped out which forces banks to liquidate assets. In turn, households absorb the entire capital stock. Households however are only willing to increase their portfolio holdings of capital at a discount, which leads excess returns to spike and investment to collapse. When the run occurs, investment drops an additional 25% resulting in an overall drop of 35%. Comparing with the case of no run clarifies that almost none of this additional drop is due to the capital quality shock itself: the additional drop in investment absent a run is only 2.5%. The collapse in investment demand causes inflation to decrease and induces monetary policy to ease by reducing the policy rate to slightly below zero.[28] However, reducing the nominal interest rate to roughly zero is not sufficient to insulate output which drops by 7%.

As new bankers enter the economy, bank net worth is slowly rebuilt and the economy returns to the steady state. This recovery is slowed down by a persistent increase in the run probability following the banking panic. The increase in the run probability induces banks to reduce the rate at which they increase borrowing to expand asset holdings during the recovery, which in turn suppresses the capital price, investment, and output.

4.2.3. Banking panics: the role of nominal rigidities.

To understand the role that the New Keynesian features of the model play in the banking crisis, Figure 3 describes the effect of bank runs in the economy with flexible prices. For comparison with the analogous experiment in our baseline (in Figure 2), we hit the flex-price economy with the same sequence of shocks that would take the baseline economy to the run threshold.[29] The run still produces a significant disruption of the economy but the output drop in the flexible-price case is only about half that in our benchmark sticky price case. Because the zero lower bound on nominal rates is not a consideration in the flexible-price economy, the real interest rate drops roughly 800 basis points below zero in this setting, leading to a temporary expansion in consumption demand. While banking crises induce a large drop in investment also in the flexible-price model, the overall drop of about 22% is much smaller than the 35% drop in our benchmark model.

To dig deeper into how the New Keynesian features affect the banking crisis dynamics, it is useful to consider the behaviour of the market value of capital Q_t in the wake of a run: given the Q relation in equation (3.49), the market value of capital governs investment dynamics.

Let H_τ denote the rental rate at time τ for a unit of capital in place at time $t+1$, adjusted for depreciation and the capital quality shock as follows,

$$H_\tau = (1-\delta)^{\tau-t-1}\left(\prod_{j=t+1}^{\tau} \xi_j\right) Z_\tau,$$

where Z_τ is the rental rate per unit of capital at time τ. Then, we can express the market value of capital (equation (2.33)) as follows:

$$Q_t = E_t \sum_{\tau=t+1}^{\infty} \widetilde{\Lambda}^X_{t,\tau} \cdot H_\tau, \tag{4.56}$$

where $\widetilde{\Lambda}^X_{t,\tau}$ is the rate at which the future rental stream is discounted and is given by the household's stochastic discount factor, $L_{t,\tau}$, adjusted for the required excess return on capital due to financial market frictions, X_s:

$$\widetilde{\Lambda}^X_{t,\tau} = \frac{\widetilde{\Lambda}_{t,\tau}}{\prod_{s=t}^{\tau-1}(1+X_s)}. \tag{4.57}$$

28. Had we been able to impose the ZLB, the drop in output would have been somewhat larger.

29. However, since in the flex-price economy there is much less amplification, the ex post run that we consider is actually not an equilibrium. As the first panel in the figure shows, even after the first two shocks the shock that is needed to push the economy to the threshold is still very large in the flex price economy, *i.e.* around −4%.

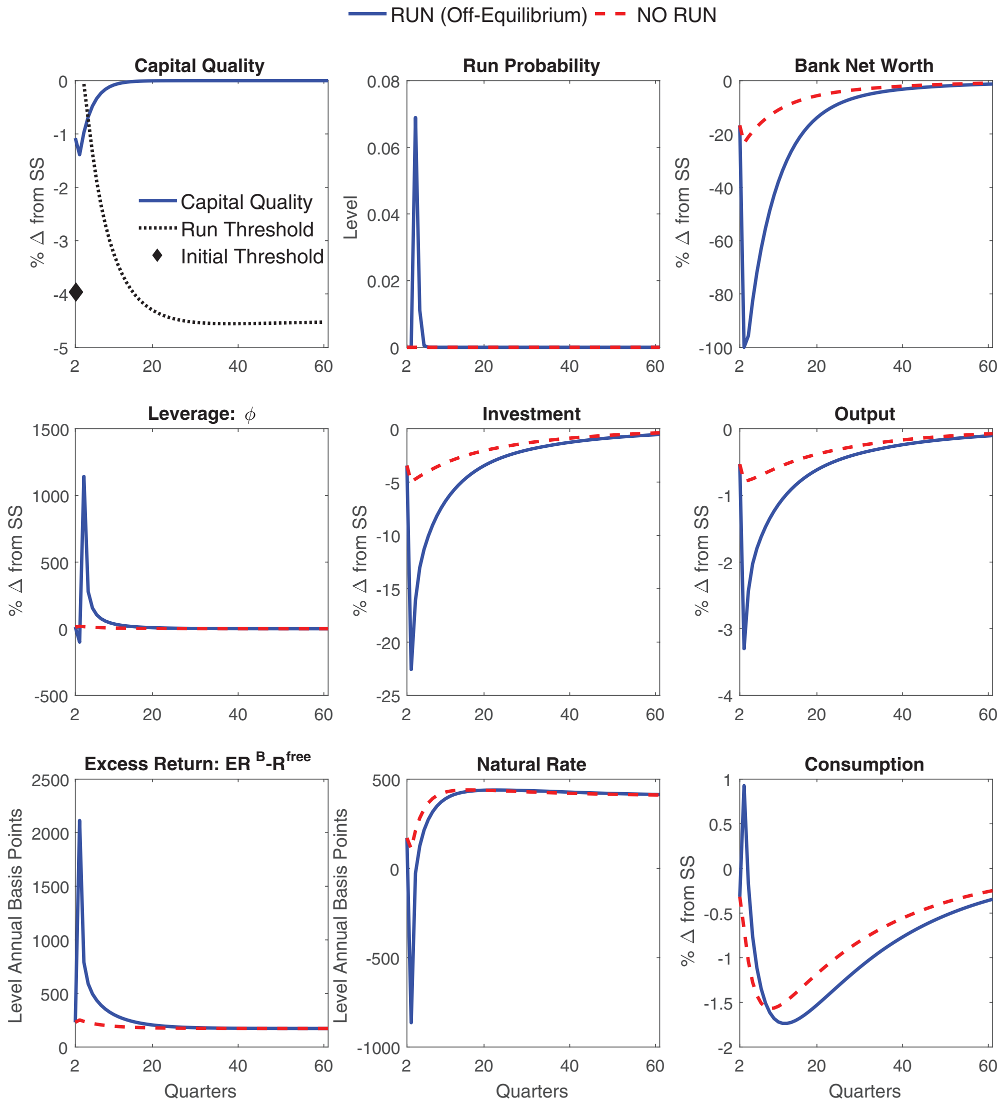


FIGURE 3

Response to the same sequence of shocks in flex-price economy: run versus no run

The excess return X_t is the discounted spread between the rate of return on capital to banks, R^b_{t+1}, and the risk free rate, $R^f_{t+1} \equiv \frac{R^n_t}{\pi_{t+1}}$. Given equations (2.10) and (3.53), we can express X_t as follows:

$$X_t \equiv E_t[\Lambda_{t+1}(R^b_{t+1} - R^f_{t+1})] = \frac{\chi\left(\frac{S^h_t}{S_t} - \gamma\right)/\lambda_t}{Q_t}. \tag{4.58}$$

Equations (4.56)–(4.58) show that Q_t depends on the future paths of three main factors: (1) dividend payouts, $\{Z_\tau\}_{\tau \geq t}$; (2) the risk free rate, which in turn is a function of the household discount factor $\left\{\widetilde{\Lambda}_{t,\tau}\right\}_{\tau \geq t}$; and (3) the excess returns stemming from market frictions $\{X_\tau\}_{\tau \geq t}$. To see how the New Keynesian features matter, we can assess how they influence each of these three factors in the wake of a run. In particular, we can decompose the present value expression for Q_t, equation (4.56), into three components with each representing one of the factors, as follows:

$$Q_t = \Xi_t + \mathcal{R}_t + \Upsilon_t \tag{4.59}$$

with

$$\Xi_t = E_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \cdot H_\tau; \;\; \mathcal{R}_t = E_t \sum_{\tau=t+1}^{\infty} \left(\widetilde{\Lambda}_{t,\tau} - \beta^{\tau-t}\right) \cdot H_\tau; \;\; \Upsilon_t = E_t \sum_{\tau=t+1}^{\infty} \left(\widetilde{\Lambda}^X_{t,\tau} - \widetilde{\Lambda}_{t,\tau}\right) \cdot H_\tau$$

Ξ_t captures the effect on Q_t of the future rental stream. It is the present discounted value of future dividends at a fixed interest rate β^{-1}. $\mathcal{R}_t$ captures the effect of variation in the riskless interest rate on the present value stream.[30] Finally, Υ_t captures the effect of excess returns in discounting future cash flows.

We now compare the response of Q_t and its three components in the wake of a run in the New Keynesian economy versus the flex-price economy. Figure 4 shows the response of these three components during a panic in our baseline economy (solid line) and in the flex-price economy (dashed line). We start each economy from the same initial state (K_t, ξ_t) of the run in our baseline experiment above.

The drop in Q in the baseline New Keynesian model is 10%. What mainly accounts for this drop is a 12% decline in the "excess returns" component of the price, Υ_t, that reflects the tightening of financial constraints following the run. Working in the other direction is a modest 0.5% increase in the "dividend stream" component Ξ_t, due to higher future rental rates on capital (that stem from the investment decline), and also a small boost of 1.5% from the "real interest rate" component, $\mathcal{R}_t$. The rise in $\mathcal{R}_t$ reflects that the central bank easing of monetary policy causes real rates to decline and hence boosts asset valuations.

In contrast, the drop in Q in the flex-price economy is 5%, half the drop in the baseline. Each of the three components of Q contributes to the difference. Two of the five percentage points are explained by a smaller increase in the excess returns component: Υ_t contributes to a 10% decline in Q_t in the flex-price model versus 12% in the New Keynesian case. Intuitively, given the extra amplification that stems from nominal rigidities, in the New Keynesian framework there is greater likelihood the economy can be pushed in the crisis zone for a given distribution of shocks. Hence, as the bottom left panel in Figure 4 shows, following the panic, the probability of another run is elevated in the New Keynesian model relative to the flexible-price model, and remains elevated for a period of time. The higher run probability, in turn, leads to a tighter leverage constraint that limits banks ability to expand assets, leading to a slower recovery of the banking system. The net effect is that excess returns are both higher and slower to return to normal in the New Keynesian case.

The dividend stream component, Ξ_t, is roughly a percentage point higher in the flex-price case as the smaller employment contraction keeps the path of dividends higher.[31] Finally, the real interest rate component adds another 2% more in value than in the New Keynesian case reflecting the much sharper drop in real interest rates in the flex-price economy.

Overall, while the banking panic is disruptive even within the flexible-price model, the New Keynesian features help the framework account better for the depth and persistence of the banking crisis. We elaborate this point within the next two subsections.

30. $\mathcal{R}_t$ will also capture the effects of variations in the covariance between consumption and the rental rate of capital. These effects however are quantitatively small.

31. We thank one of the referees for pointing out this channel.

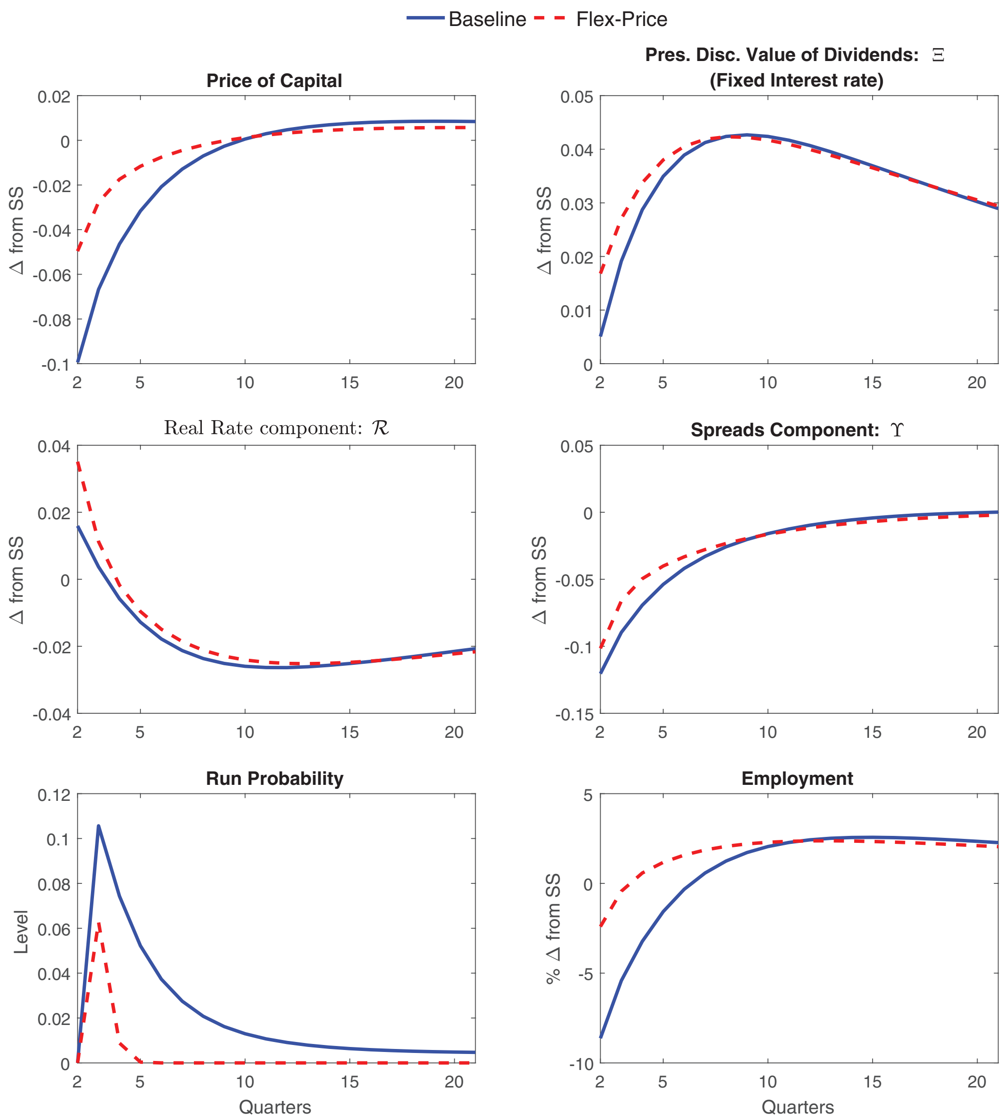
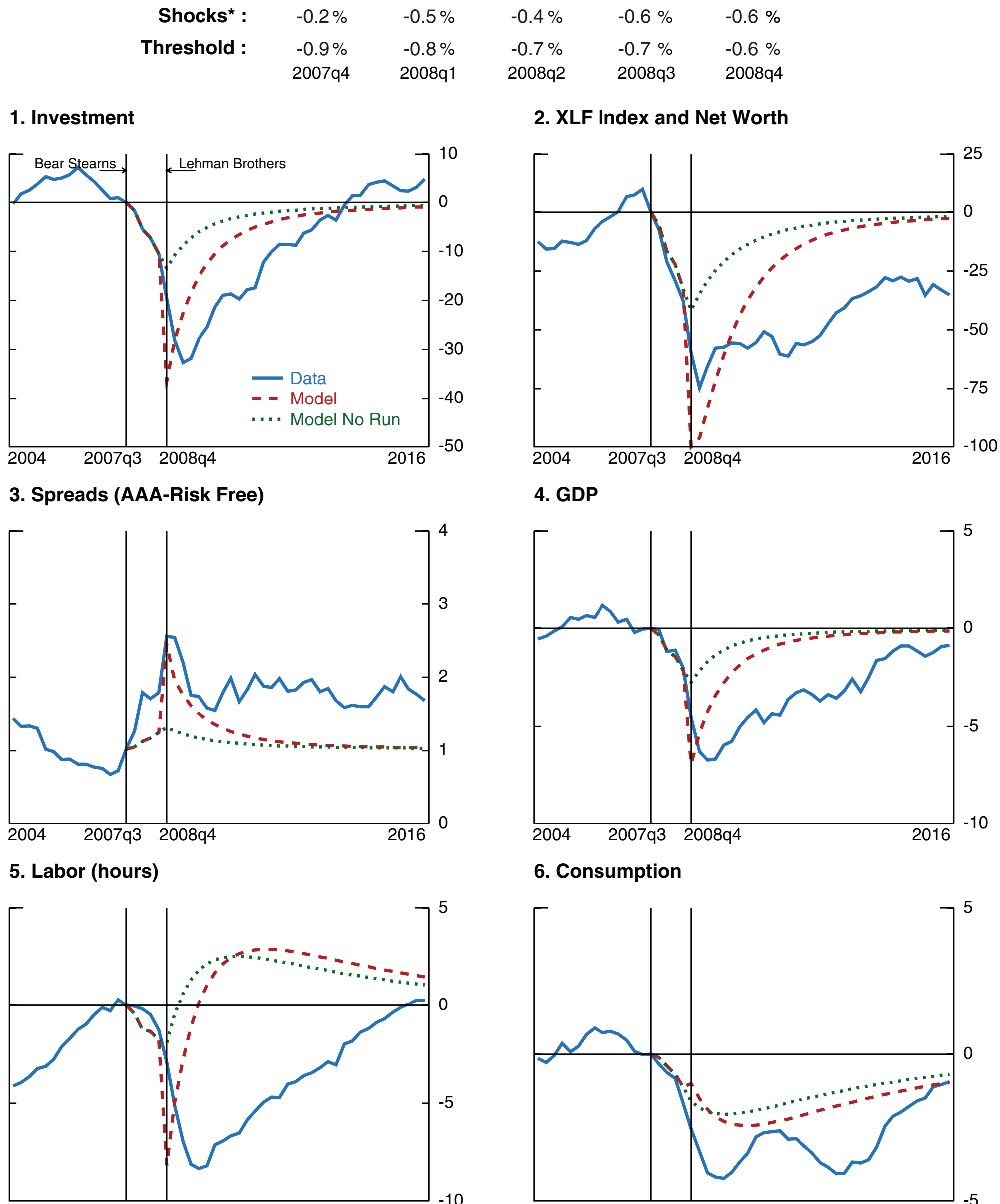


FIGURE 4

Banking panics: the role of nominal rigidities

4.2.4. Crisis experiment: model versus data. Figure 5 illustrates how the model can replicate some salient features of the recent financial crisis. We hit the economy with a series of modest-sized capital quality shocks over the period 2007Q4 until 2008Q3. The starting point is the beginning of the recession, which roughly coincides with the time credit markets first came under stress following Bear Stearns' losses on its MBS portfolios. We pick the size of the capital quality shocks to match the observed decline in investment during this period, in panel 1. We then hit the economy with another small shock in 2008Q4 that is just sufficient to move the economy into the region where a run equilibrium exists. We then assume that a run happens in 2008Q4, the quarter in which Lehman failed and the shadow banking system collapsed.



NOTE: The data for GDP, Investment, and Consumption are computed as logged deviations from trend where the trend is the CBO potential GDP. Labor data is computed as logged deviations from trend where the trend is the CBO potential hours worked. The XLF Index data is computed as the percent deviation from its 2007q3 level.

FIGURE 5

Banking panics: the role of nominal rigidities

The solid line in Figure 5 shows the observed response of some key macroeconomic variables. For output, investment, and consumption, we show the deviation from a trend computed by using CBO estimates of potential output and similarly for hours worked we let the CBO estimate of potential labour represent the trend. To measure bank equity we use the XLF index, which is an index of S&P 500 financial stocks. For excess returns, we use the spread between the rates on AAA corporate bonds and 10-year government bonds. An advantage of the AAA spread is that private sector forecasts of this variable are available, which we can subsequently make use of in our analysis of belief driven credit booms in Section 4.3. We use the AAA rather than the BAA spread, because the latter varies considerably with costly default risk, something our model does not fully address.[32]

The dashed line in Figure 5 shows the response of the economy when a run occurs in 2008Q4 and the dotted line shows the response under the assumption that a run does not happen. As indicated in panel 2, the sequence of negative surprises in the quality of capital needed to match the observed contraction in investment leads to a gradual decline in banks net worth that matches closely the observed decline in financial sector equity as measured by the XLF index, which is an index of S&P 500 financial stocks. Given that banks net worth is already depleted by poor asset returns, a modest innovation in 2008Q4 pushes the economy into the run region. When the run occurs, the model economy generates a sudden spike in excess returns and a drop in investment, output, consumption, and employment of similar magnitudes as those observed during the crisis, as panels 3 to 6 show.[33] The dotted line shows how, absent a run, the same shocks would generate a much less severe downturn.

The model economy also predicts a rather slow recovery following the financial crisis, although faster than what we observed in the data. It is important however to note that in the experiment we are abstracting from any disturbances after 2008Q4. In practice, there were a series of contractionary disturbances that followed the banking crisis, including the tightening of fiscal policy, the European debt crisis, and the tightening of bank regulations. Another factor we have abstracted from is the deleveraging of household balance sheets, which likely also contributed to the prolonged stagnation. Incorporating these factors could help the model account for the slow recovery. However, we leave this extension for future research.

4.3. *A news driven credit boom that leads to a bust*

In the baseline experiment depicted in Figure 5, we assumed for simplicity that the economy is in steady state in 2007q3. We then fed in a sequence of modest fundamental shocks that moved the economy steadily from a safe zone to a crisis zone where a banking panic is possible. This exercise is useful for illustrating both the non-linear dimension of the banking crisis and how the crisis can spill over into real activity. However, it is silent on the pre-crisis buildup of vulnerability of the banking system. As many have argued, the pre-crisis credit boom laid the seeds of the crisis by increasing banks' fragility.

In this section, we extend the model to capture the pre-crisis credit boom. Here, we follow the lead of several authors by introducing optimistic beliefs about the return on capital that are eventually disappointed. That investors were overly optimistic pre-crisis is a widespread theme in the literature. As emphasized by Bordalo Gennaioli and Schleifer (2017), survey data on investor expectations confirm this view. For our purposes, the advantage of appealing to investor optimism is 2-fold: first, we can generate a pre-crisis leverage buildup without having a counterfactually large increase in real activity. Second, having a leverage buildup that is not eventually supported by fundamentals can raise the probability the economy enters into a crisis zone. It is then possible,

32. To make a proper comparison between the model and the data, we construct a model measure of bank excess returns that has a maturity similar to that of the AAA spread. For simplicity, we use the expectations hypothesis of the term structure to compute long rates. Details are in the Appendix Section 6.8.

33. The behaviour of the nominal interest rate is very similar to that portrayed during the bank run illustrated by Figure 2: it drops slightly below zero during the panic before recovering.

as we demonstrate, that a banking panic can arise within a given period, absent even modest new aggregate disturbances.

To model optimistic beliefs, we consider a variant of a "news" shock.[34] Under the standard formulation, at time t, individuals suddenly learn that a fundamental disturbance of a given size will occur j periods in the future. We make two modifications. First, we assume that there is a probability the shock may not occur. Second, we assume that rather than having a single date in the future when the shock can occur, there is a probability distribution over a number of possible dates. As time passes without the occurrence of the shock, individuals update their priors on these various possibilities. We also assume that only bankers, who are the experts at managing assets, have optimistic beliefs. In fact, it is the relative optimism of bankers with respect to households that generates the vulnerability of the financial system.[35]

In particular, at time t^N bankers receive news that there may be a high return on capital in the form of a large capital quality shock. But they do not know for sure (1) whether the shock will occur and (2) conditional on being realized, when it will occur. If the shock is realized at some time $\tau > t^N$, it takes the form of a one-time impulse to the capital quality shock process of size B. Formally, the news bankers receive is that the capital quality will follow the process

$$\xi_\tau = 1-\rho_\xi+\rho_\xi \xi_{\tau-1}+\epsilon_\tau+\widetilde{B}_\tau \text{ for } \tau > t^N,$$

where $\widetilde{B}_\tau = B$ if the large shock realizes at τ and $\widetilde{B}_\tau = 0$, otherwise. Given the capital quality shock is serially correlated, there will be a persistent effect of B. However, given it is a one-time shock, if it occurs, there will be no subsequent realizations of this impulse.

When they receive the news at t^N, bankers' prior probability that a shock will eventually occur is given by $\bar{P}$. Conditional on the shock happening, the future date when it will happen, $\tau \in \{t^N+1, t^N+2, ..., t^N+T\}$, is distributed according to a probability mass function ζ_τ which we assume to be a discrete approximation of a normal with mean μ^B and standard deviation σ^B. Thus at date t^N, the probability that the shock happens at τ is given by

$$prob_{t^N}(\widetilde{B}_\tau = B) = \begin{cases} \bar{P}\cdot\zeta_\tau, \text{ for } \tau = t^N+1, t^N+2, ..., t^N+T \\ 0, \text{ for } \tau > t^N+T \end{cases}.$$

As long as no shock is observed, bankers update their beliefs using Bayes rule:

$$\begin{aligned} prob_t(\widetilde{B}_\tau = B) &= \frac{\bar{P}\cdot\zeta_\tau}{1-\sum_{j=t^N+1}^{t}\bar{P}\cdot\zeta_j} \\ &= \frac{\sum_{j=t+1}^{t_N+T}\bar{P}\cdot\zeta_j}{1-\sum_{j=t^N+1}^{t}\bar{P}\cdot\zeta_j}\cdot\frac{\zeta_\tau}{\sum_{j=t+1}^{t_N+T}\zeta_j} \end{aligned}$$

for $\tau = t_N+1, ..., t_N+T$, and $prob_t(\widetilde{B}_\tau = B) = 0$ for $\tau > t_N+T$. The first term in the RHS is the posterior probability of the shock ever happening, which we denote by $\overline{P}_t$ and which is decreasing

34. See Christiano Motto and Rostagno (2014) for a medium scale estimated DSGE model in which news shocks play an important role in explaining fluctuations in financial variables.

35. Technically, we assume that households are aware that bankers became optimistic but do not change their beliefs about the quality of capital, *i.e.* they do not believe the news. This allows us to have diverse beliefs without having households extract information from prices. A similar assumption is made for the same reason, for instance, in Cogley and Sargent (2009). Because households know bankers are more optimistic, they understand that there is less danger for bankers to divert their assets and loose their franchise. This allows bankers to raise their leverage multiple.

with t. The second term is the probability that the shock realizes at τ conditional on the shock eventually happening. The latter is increasing with t until $t = t_N + T$, before becoming zero.

Observe that the process will generate a burst of optimism that will eventually fade if the good news is not realized. Early on, bankers will steadily raise their forecasts of the near term return to capital as they approach the date where, a priori, the shock is most likely to occur. As time passes without the realization of the shock, bankers' become less certain it will ever occur, especially as they move past the peak time of when it was most likely to occur: the optimism proceeds to vanish.

We now illustrate how with the belief mechanism just described, the model generates a boom/bust scenario. We assume that the time the bankers receive the optimistic news, t^N, is the first quarter of 2005, in line with the description of events in Bordalo *et al.* We pick the mean of the conditional distribution for τ, μ^B, so that the prior on when the shock is most likely to occur is 2007Q2, which coincides roughly with the peak of house prices. We pick the standard deviation σ^B to ensure that by the end of 2008, if the shock has not occurred, bankers' will completely give up hope that it will ever occur.[36] Next, we set the size of the impulse B to equal a two standard deviation shock, that is, a shock which is unusually large but not beyond the realm of possibility. Finally, we pick the prior probability that the shock will even occur $\bar{P}$, to ensure that economy reaches the crisis zone in 2008Q4 without any fundamental shocks. As external validation, we check whether the forecast errors generated by our belief process (in conjunction with the model) generates forecast errors consistent with the survey evidence in Bordalo *et al.*

Figure 6 characterizes the dynamics of beliefs and the credit boom that can emerge absent any fundamental shocks. The top left panel gives the prior distribution for the time the shock will happen, conditional on it happening for sure, *i.e.* $\left\{\zeta_{t^N+i}\right\}_{i=1}^{T}$. The middle panel then illustrates the ingredients bankers use to forecast the shock. The blue solid line in the top middle panel gives the probability the shock will eventually happen, $\overline{P}_t$. When the news is received in 2005Q1, the probability jumps to its prior value near unity. Time passing without the shock occurring leads bankers' to reduce this probability. The optimism fades rapidly as time passes 2007Q2, the most likely time the shock was expected to occur. The dashed red line then gives the probability the shock will occur in the subsequent period, conditional on it eventually happening. The estimate that the shock will occur in the subsequent period is then the product of the blue solid and red dashed lines.

To illustrate the boom/bust nature of beliefs, the right panel portrays the year ahead forecast of the capital quality shock (the dashed red line). After receiving the news in 2005Q1, optimism steadily builds, peaking just before 2007Q2. However, as time continues to pass beyond the most likely time, the optimism fades quickly, effectively vanishing by 2008Q4. Note that throughout the boom and bust in beliefs, the true fundamental shock (the blue solid line), is unchanged. Thus, there is serial correlation in the forecast errors of the capital quality. Shortly, we show that this generates a behaviour of forecast errors of excess returns that is consistent with the evidence.

The bottom left panel shows the response of output to the news. The optimism leads to an increase in asset values, which boosts investment, in turn leading to a modest increase in output of about 0.3%.[37] There is however a non-trivial debt buildup of 10%. Because bankers become optimistic relative to households, they increase their relative holdings of capital and they fund this increase by issuing debt. In turn, the increase in leverage significantly raises the likelihood the

36. Given our discrete approximation of the normal distribution, a choice of σ^B translates into a maximum numbers of periods within which the shock can occur.

37. Note that asset values depend on the expected discount forecast of the shock over the entire future. While the near term estimates of the shock increase slowly, longer horizon estimates will jump more rapidly.

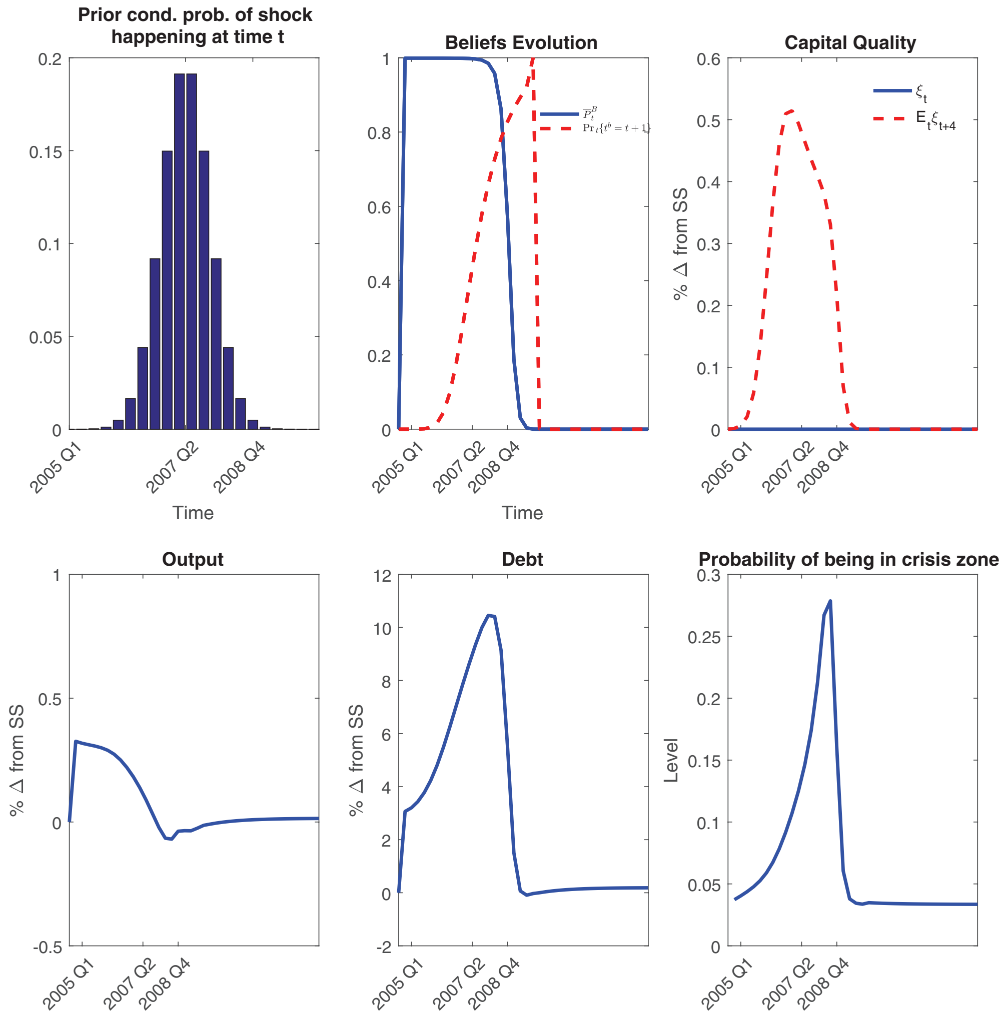
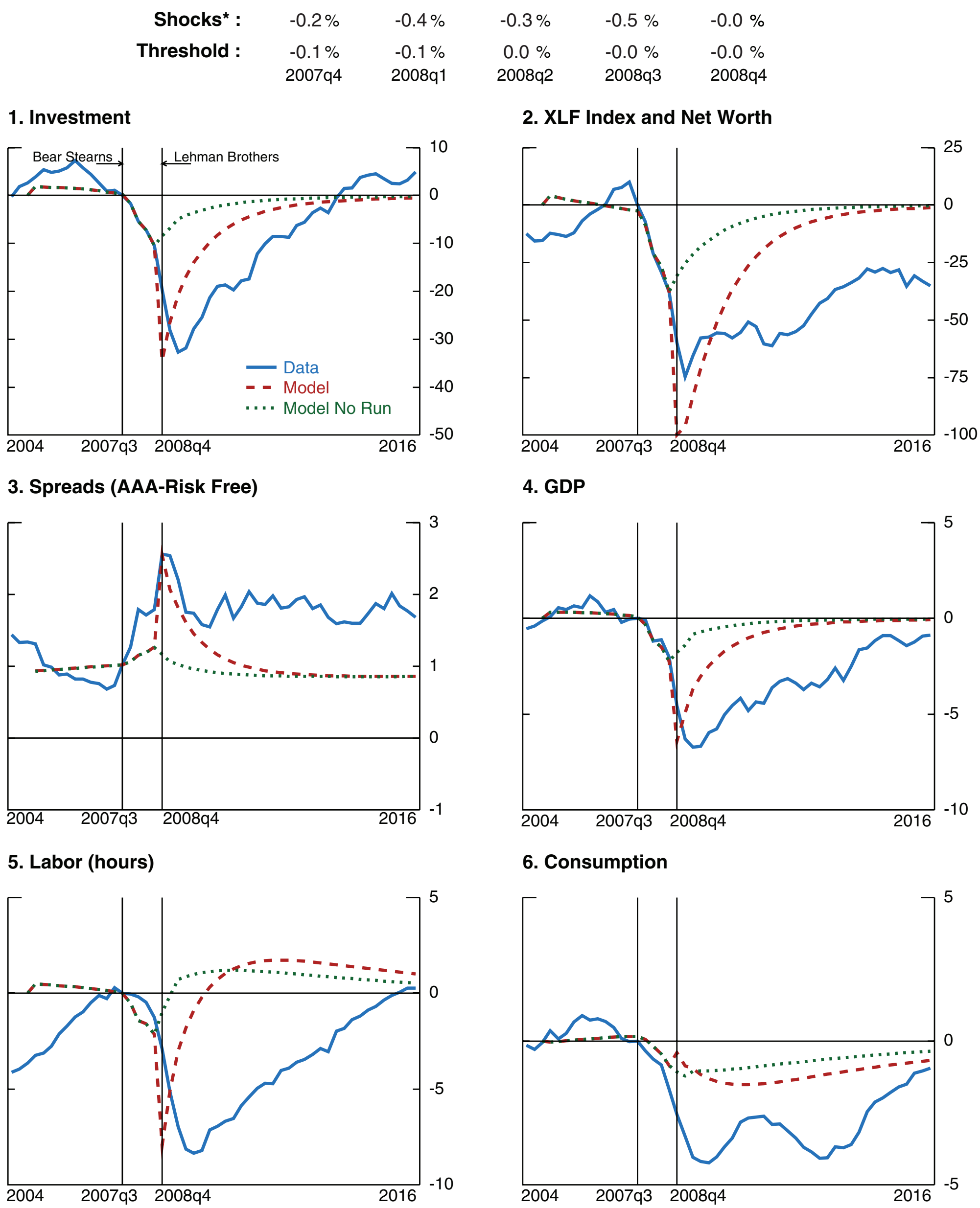


FIGURE 6
Belief dynamics and credit boom

economy moves into a crisis zone in the subsequent period, peaking at close to 30% by 2008Q4.[38] In this respect, the boom lays the seeds of the bust.

We now repeat the crisis experiment described in Section 4.2.4, this time allowing for the wave of optimism beginning in 2005Q1 that we have just characterized. As before, we allow for a series of modest capital quality shocks from 2007Q4 to 2008Q3 so that the model tracks the behaviour of investment over this period. But unlike before, we do not introduce a fundamental shock in 2008Q4. Due to the debt buildup in this case, the economy reaches the crisis zone in 2008Q4 absent an additional shock.

38. As discussed by Gertler Kiyotaki and Prestipino (2016), there were additional factors contributing to the leverage buildup, including financial innovation. For simplicity we abstract from these factors and note only that including them would increase the debt buildup further and the resulting degree of fragility.



NOTE: The data for GDP, Investment, and Consumption are computed as logged deviations from trend where the trend is the CBO potential GDP. Labor data is computed as logged deviations from trend where the trend is the CBO potential hours worked. The XLF Index data is computed as the percent deviation from its 2007q3 level.

FIGURE 7

Financial crisis after credit boom: model versus data

Figure 7 illustrates the experiment. The wave of optimism induces a rise in investment prior to the recession along with the debt buildup portrayed in Figure 6. The latter helps move the economy into the crisis zone as the recession hits. This can be seen from the top two lines that list the sequence of shocks that we feed along with the threshold values of the shocks needed to push the economy into the crisis zone. In contrast to the previous case, the economy is in the

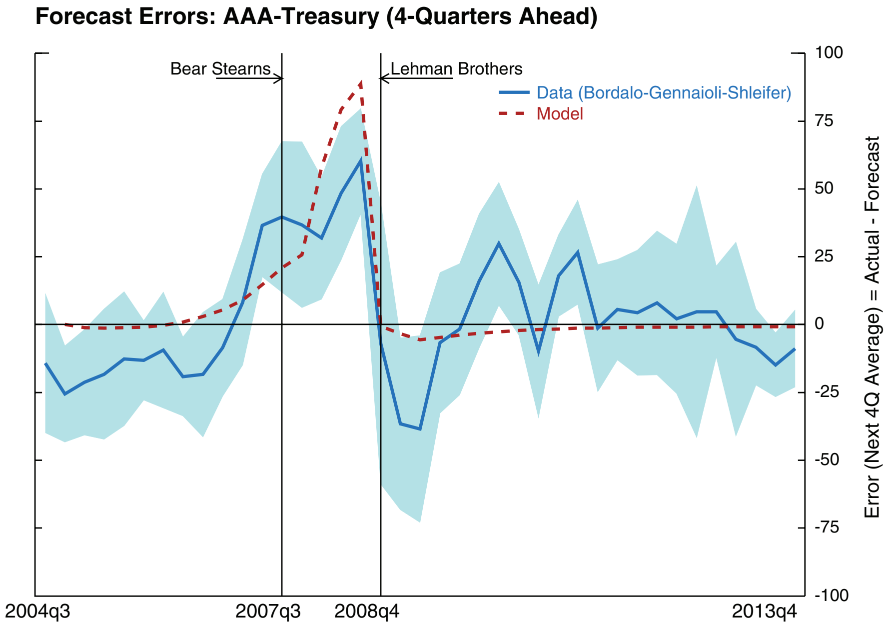


FIGURE 8

Forecast errors in excess returns: model versus data

crisis zone throughout the recession. In 2007Q4 and 2008Q1, modest negative shocks push the economy into the crisis zone. From 2008Q2 to 2008Q4, the economy is in the crisis zone absent any new shocks.

Given the economy is in the region where a panic is feasible, we again suppose the rollover crisis takes place in 2008Q4. Again, this time we do not require a fundamental shock to move the economy to a crisis zone, so we do without it. Overall, the effect of the banking crisis is very similar to the case without the debt boom. The contractions in output, investment, employment and consumption in terms of both amplitude and persistence are all close to what happens in the previous case. There is also a similar increase in the credit spread and contraction in net worth. In the end, what the credit boom does is to raise the likelihood of the subsequent crisis.

Finally, as a check on our belief mechanism, we compare the forecast errors for excess returns generated by the model with evidence from the Survey of Professional Forecasters. Following Bordalo Gennaioli and Schleifer (2017), the dark line in Figure 8 reports errors in the mean forecast over the year ahead of the spread between the AAA bond rate and the 10-year government bond rate. Specifically, a point at time t is the date-t consensus forecast of AAA-Treasury yield spreads averaged across forecast horizons going from $t+1$ to $t+4$.[39] The shaded area is a two standard deviation error band, based on the dispersion of the individual forecasters expectations. As the survey data shows, investors were overly optimistic about credit spreads in the near future from mid 2007 to the Lehman collapse: Over this period, they persistently underestimated the credit spread over the following one year horizon. Following the Lehman collapse, there is a modest swing towards excess pessimism, though not as significant or persistent as the optimism wave.

The dashed line represents the analogous forecast error in the excess return that the model generates. As bankers become optimistic and expect a boom that fails to materialize, their forecast

39. Consensus forecasts are cross-sectional averages of individual respondents forecasts.

errors about the yield spreads over 1 year horizon follow a pattern that is similar in duration and magnitude to the data in the pre-Lehman period. The model forecast stays within the standard error bands for the most part, though it creeps slightly above a few quarters before the collapse. There is also very modest period of pessimism following the collapse. This is due to the fact that following the run in the model, the probability of a second run increases (and this second run does not materialize). The standard error bands are sufficiently wide following the collapse, so that the difference between the data and model simulations are not significant. Overall, the belief mechanism we used is roughly in line with the evidence.

5. CONCLUSION

We have developed a macroeconomic model with a banking sector where costly financial panics can arise. A panic or run in our model is a self-fulfilling failure of creditors to roll over their short-term credit to banks. When the economy is close to the steady state a self-fulfilling rollover crises cannot happen because banks have sufficiently strong balance sheets. In this situation, "normal size" business cycle shocks do not lead to financial crises. However, in a recession, banks may have sufficiently weak balance sheet so as to open up the possibility of a run. Depending on the circumstances either a small shock or no further shock can generate a run that has devastating consequences for the real economy. We show that our model generates the highly nonlinear contraction in economic activity associated with financial crises. It also captures how crises may occur even in the absence of large exogenous shocks to the economy. We then illustrate that the model is broadly consistent with the recent financial crisis. Finally, we extend the framework to model the credit boom that leads to the bust. We do so by introducing a wave of optimism that leads to a leverage buildup that increases the fragility of the economy, measured explicitly by the exposure to a banking panic.

One issue we save for further work is the role of macroprudential policy. As with other models of macroprudential policy, externalities are present that lead banks to take more risk than is socially efficient. Much of the literature is based on the pecuniary analysed by Lorenzoni (2008), where individual banks do not properly internalize the exposure of the system to asset price fluctuations that generate inefficient volatility, but not runs. A distinctive feature of our model is that the key externality works through the effect of leverage on the bank run probability: Because the run probability depends on the leverage of the banking system as a whole, individual banks do not fully take into account the impact of their own leverage decisions on the exposure of the entire system. In this environment, the key concern of the macroprudential policy becomes reducing the possibility of a financial collapse in the most efficient way. Our model will permit us to explore the optimal design of policies qualitatively and quantitatively.

A. APPENDIX

This Appendix describes the details of the equilibrium.

The aggregate state of the economy is summarized by the vector of state variables $\tilde{\mathcal{M}}_t = (S_{t-1}, S^b_{t-1}, \overline{R}_t D_{t-1}, \xi_t)$, with sunspot realization ι_t at time t, where $S_{t-1} =$ capital stock at the end of $t-1$; S^b_{t-1} = bank capital holdings in $t-1$; $\overline{R}_t D_{t-1}$ = bank deposit obligation at the beginning of t; and $\xi_t =$ capital quality shock realized in t.

A.1. *Producers*

As described in the text, the capital stock for production in t is given by

$$K_t = \xi_t S_{t-1}, \tag{A.60}$$

The capital quality shock is serially correlated as follows

$$\xi_{t+1} \sim F\left(\xi_{t+1} \,|\, \xi_t\right) = F_t\left(\xi_{t+1}\right)$$

with a continuous density:

$$F_t'\left(\xi_{t+1}\right) = f_t\left(\xi_{t+1}\right), \text{ for } \xi_{t+1} \in (0, \infty).$$

Capital at the end of period is

$$S_t = \Gamma(I_t)K_t + (1-\delta)K_t. \tag{A.61}$$

As we described in the text, capital goods producer's first order condition for investment is

$$Q_t\Gamma'(I_t) = 1. \tag{A.62}$$

A final goods firms chooses intermediate goods $\{Y_t(f)\}$ to minimize the cost

$$\int_0^1 P_t(f)Y_t(f)df$$

subject to the production function:

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{A.63}$$

The cost minimization then yields a demand function for each intermediate good f :

$$Y_t(f) = \left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon} Y_t, \tag{A.64}$$

where P_t is the price index, given by

$$P_t = \left[\int_0^1 P_t(f)^{1-\varepsilon} df\right]^{\frac{1}{1-\varepsilon}}.$$

Conversely, an intermediate goods producer f chooses input to minimize the production cost

$$w_t L_t(f) + Z_t K_t(f)$$

subject to

$$A_t[K_t(f)]^{\alpha}[L_t(f)]^{1-\alpha} = Y_t(f).$$

The first-order conditions yield

$$\frac{K_t(f)}{L_t(f)} = \frac{\alpha}{1-\alpha}\frac{w_t}{Z_t} = \frac{K_t}{L_t}, \tag{A.65}$$

and the following relation for marginal cost:

$$MC_t = \frac{1}{A_t}\left(\frac{Z_t}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}. \tag{A.66}$$

Each period, the intermediate goods producer chooses $P_t(f)$ and $Y_t(f)$ to maximize the expected discounted value of profits:

$$E_t\left\{\sum_{\tau=t}^{\infty}\widetilde{\Lambda}_{t,\tau}\left[\left(\frac{P_\tau(f)}{P_\tau}-MC_\tau\right)Y_\tau(f)-\frac{\rho^r}{2}Y_\tau\left(\frac{P_\tau(f)}{P_{\tau-1}(f)}-1\right)^2\right]\right\},$$

subject to the demand curve (A.64), where $\widetilde{\Lambda}_{t,\tau}=\beta^{\tau-t}(C_\tau/C_t)^{-\gamma_h}$ is the discount factor of the representative household. Taking the firm's first order condition for price adjustment and imposing symmetry implies the following forward looking Phillip's curve:

$$(\pi_t-1)\pi_t=\frac{\varepsilon}{\rho^r}\left(MC_t-\frac{\varepsilon-1}{\varepsilon}\right)+E_t\left[\widetilde{\Lambda}_{t,t+1}\frac{Y_{t+1}}{Y_t}(\pi_{t+1}-1)\pi_{t+1}\right], \tag{A.67}$$

where $\pi_t=\frac{P_t}{P_{t-1}}$ is the realized gross inflation rate at date t. The cost minimization conditions with symmetry also imply that aggregate production is simply

$$Y_t=A_tK_t{}^{\alpha}L_t{}^{1-\alpha}. \tag{A.68}$$

A.2. *Households*

We modify the household's maximization problem in the text by allowing for a riskless nominal bond which will be in zero supply. We do so to be able the pin down the riskless nominal rate R_t^n. Let B_t be real value of this riskless bond. The household then chooses C_t, L_t, B_t, D_t, and S_t^h to maximize expected discounted utility U_t:

$$U_t=E_t\left\{\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left[\frac{(C_\tau)^{1-\gamma_h}}{1-\gamma_h}-\frac{(L_\tau)^{1+\varphi}}{1+\varphi}-\varsigma(S_\tau^h,S_\tau)\right]\right\},$$

subject to the budget constraint

$$C_t+D_t+Q_tS_t^h+B_t=w_tL_t-T_t+\Pi_t+R_tD_{t-1}+\frac{R_{t-1}^n}{\pi_t}B_{t-1}+\xi_t[Z_t+(1-\delta)Q_t]S_{t-1}^h.$$

As explained in the text, the rate of return on deposits is given by

$$\begin{aligned}R_t&=\mathrm{Max}\left\{\overline{R}_t,\frac{\xi_t[Z_t+(1-\delta)Q_t]S_{t-1}^b}{D_{t-1}}\right\}\\&=\mathrm{Max}\left\{\overline{R}_t,\frac{\xi_t[Z_t+(1-\delta)Q_t]}{Q_{t-1}}\frac{Q_{t-1}S_{t-1}^b}{Q_{t-1}S_{t-1}^b-N_{t-1}}\right\}\\&=\mathrm{Max}\left(\overline{R}_t,R_t^b\frac{\phi_{t-1}}{\phi_{t-1}-1}\right),\end{aligned}$$

where $R_t^b=\frac{\xi_t[Z_t+(1-\delta)Q_t]}{Q_{t-1}}$ and where $\phi_t=Q_tS_t^b/N_t$ is the bank leverage multiple.

We obtain the first order conditions for labour, riskless bonds, deposits, and direct capital holding, as follows:

$$w_t=(C_t)^{\gamma_h}(L_t)^{\varphi} \tag{A.69}$$

$$E_t\left(\Lambda_{t+1}\frac{R_t^n}{\pi_{t+1}}\right)=1 \tag{A.70}$$

$$E_t\left[\Lambda_{t+1}\mathrm{Max}\left(\overline{R}_{t+1},R_{t+1}^b\frac{\phi_t}{\phi_t-1}\right)\right]=1 \tag{A.71}$$

$$E_t\left\{\Lambda_{t+1}\xi_{t+1}\frac{Z_{t+1}+(1-\delta)Q_{t+1}}{Q_t+\frac{\partial}{\partial S_t^h}\varsigma(S_t^h,S_t)\cdot C_t{}^{\gamma_h}}\right\}=1, \tag{A.72}$$

where

$$\Lambda_{t+1} = \widetilde{\Lambda}_{t,t+1}=\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma_h}, \text{ and}$$

$$\frac{\partial}{\partial S_t^h}\varsigma(S_t^h,S_t) = \mathrm{Max}\left[\chi\left(\frac{S_t^h}{S_t}-\gamma\right),\ 0\right].$$

A.3. *Bankers*

For ease of exposition, the description of the banker's problem in the text does not specify how the individual choice of bank's leverage affects its own probability of default. This was possible because, as argued in footnote 10, the indirect marginal effect of leverage on the objective of the firm, V_t, through the change in p_t is zero. Therefore the first order conditions for the bank's problem, equations (2.27) and (2.28), can be derived irrespectively of how the individual choice of bank's leverage affects its own probability of default.

We now formalize the argument in footnote 10 and describe how the default thresholds for individual banks vary with individual bank leverage. As will become clear in Section A.5 below, this analysis is key in order to study global optimality of the leverage choice selected by using the first order conditions in the text, equations (2.27) and (2.28).

As in the text, ι is a sunspot which takes on values of either unity or zero. We can then express the rate of return on bank capital R_{t+1}^b

$$R_{t+1}^b=\xi_{t+1}\frac{Z_{t+1}+(1-\delta)Q_{t+1}}{Q_t}=R_{t+1}^b(\xi_{t+1},\iota_{t+1}),$$

The individual bank defaults at date $t+1$ if and only if

$$1>\frac{\xi_{t+1}[Z_{t+1}+(1-\delta)Q_{t+1}]s_t^b}{\overline{R}_{t+1}d_t}=\frac{R_{t+1}^b(\xi_{t+1},\iota_{t+1})}{\overline{R}_{t+1}}\frac{Q_ts_t^b}{Q_ts_t^b-n_t},$$

or

$$R_{t+1}^b(\xi_{t+1},\iota_{t+1})<\overline{R}_{t+1}\frac{\phi_t}{\phi_t-1}.$$

Let $\Xi_{t+1}^D(\phi)$ be the set of capital quality shocks and sunspot realizations which make the individual bank with a leverage multiple of ϕ default and conversely let $\Xi_{t+1}^N(\phi)$ be the set that leads to non-default at date $t+1$:

$$\Xi_{t+1}^D(\phi)=\left\{(\xi_{t+1},\iota_{t+1})\,|\,R_{t+1}^b(\xi_{t+1},,\iota_{t+1})<\frac{\phi-1}{\phi}\overline{R}_{t+1}(\phi)\right\},$$

$$\Xi^N_{t+1}(\phi)=\left\{(\xi_{t+1},\iota_{t+1})\,|\,R^b_{t+1}(\xi_{t+1},\iota_{t+1})\geq\frac{\phi-1}{\phi}\overline{R}_{t+1}(\phi)\right\},$$

where $\overline{R}_{t+1}(\phi)$ is the promised deposit interest rate when the individual bank chooses ϕ which satisfies the condition for the household to hold deposits:

$$1=\overline{R}_{t+1}(\phi)\int_{\Xi^N_{t+1}(\phi)}\Lambda_{t+1}d\widetilde{F}_t+\frac{\phi}{\phi-1}\int_{\Xi^D_{t+1}(\phi)}\Lambda_{t+1}R^b_{t+1}(\xi_{t+1},\iota_{t+1})d\widetilde{F}_t. \tag{A.73}$$

Here $\widetilde{F}_t(\xi_{t+1},,\iota_{t+1})$ denotes the distribution function of (ξ_{t+1},ι_{t+1}) conditional on date t information.

Assume that the aggregate leverage multiple is given by $\overline{\phi}_t$. When the individual banker chooses the leverage multiple ϕ, which can be different from $\overline{\phi}_t$, the individual bank defaults at date $t+1$ if and only if

$$\begin{gathered}\xi_{t+1}<\xi^I_{t+1}(\phi)\text{ and }\iota_{t+1}=0\\ \text{where}\\ \xi^I_{t+1}(\phi)R^b_{t+1}(\xi^I_{t+1}(\phi),0)=\tfrac{\phi-1}{\phi}\overline{R}_{t+1}(\phi)\end{gathered} \tag{A.74}$$

or

$$\begin{gathered}\xi_{t+1}<\xi^R_{t+1}(\phi)\text{ and }\iota_{t+1}=1\\ \text{where}\\ \xi^R_{t+1}(\phi)=\sup\left\{\xi_{t+1}\text{ s.t. }\xi_{t+1}R^b_{t+1}\left(\xi_{t+1},1\right)<\tfrac{\phi-1}{\phi}\overline{R}_{t+1}(\phi)\right\}.\end{gathered} \tag{A.75}$$

Thus, the set of capital quality shocks and sunspots which make the individual bank default $\Xi^D_{t+1}(\phi)$ is

$$\Xi^D_{t+1}(\phi)=\left\{(\xi_{t+1},\iota_{t+1})\left|\begin{array}{c}\xi_{t+1}<\xi^I_{t+1}(\phi)\text{ and }\iota_{t+1}=0\\ \text{or}\\ \xi_{t+1}<\xi^R_{t+1}(\phi)\text{ and }\iota_{t+1}=1\end{array}\right.\right\}. \tag{A.76}$$

$$\Xi^N_{t+1}(\phi)=\left\{(\xi_{t+1},\iota_{t+1})\left|\begin{array}{c}\xi_{t+1}\geq\xi^I_{t+1}(\phi)\text{ and }\iota_{t+1}=0\\ \text{or}\\ \xi_{t+1}\geq\xi^R_{t+1}(\phi)\text{ and }\iota_{t+1}=1\end{array}\right.\right\}. \tag{A.77}$$

The behaviour of $\xi^I_{t+1}(\phi)$ is straightforward and can be easily characterized from (A.74) under the natural assumption that R^b_{t+1} is increasing in the quality of capital at $t+1$. This gives:

$$\begin{gathered}\frac{d\xi^I_{t+1}(\phi)}{d\phi}>0,\text{ for }\phi\in(1,\infty)\\ \lim_{\phi\downarrow1}\xi^I_{t+1}(\phi)=0.\end{gathered} \tag{A.78}$$

The behaviour of $\xi^R_{t+1}(\phi)$ is more complicated because, when a sunspot is observed, the function $R^b_{t+1}\left(\xi_{t+1},1\right)$ that determines returns on bank's assets as a function of the capital quality is discontinuous around the aggregate run threshold $\xi^R_{t+1}=\xi^R_{t+1}(\bar{\phi}_t)$: at the threshold ξ^R_{t+1} asset prices jump from liquidation prices up to their normal value (see Figure 5):

$$\lim_{\xi_{t+1}\downarrow\xi^R_{t+1}}R^b_{t+1}\left(\xi_{t+1},1\right)=R^b_{t+1}\left(\xi^R_{t+1},0\right)>\lim_{\xi_{t+1}\uparrow\xi^R_{t+1}}R^b_{t+1}\left(\xi_{t+1},1\right). \tag{A.79}$$

This implies that, if the capital quality shock is at the aggregate run threshold ξ^R_{t+1}, an increase in leverage from the value that makes the recovery rate equal to unity at liquidation prices, does not induce default as long as it is not so large that the bank becomes insolvent even at normal prices.

By definition of the run threshold ξ^R_{t+1}, the value of leverage that makes the recovery rate at liquidation prices equal to unity is exactly the aggregate leverage $\bar{\phi}_t$, that is

$$\frac{\bar{\phi}_t-1}{\bar{\phi}_t}\overline{R}_{t+1}\left(\bar{\phi}_t\right)=\lim_{\xi_{t+1}\uparrow\xi^R_{t+1}} R^b_{t+1}\left(\xi_{t+1},1\right).$$

On the other hand, we let $\hat{\phi}_t$ denote the value above which the bank defaults at the aggregate run threshold ξ^R_{t+1} even at normal prices. This value satisfies

$$\frac{\hat{\phi}_t-1}{\hat{\phi}_t}\overline{R}_{t+1}\left(\hat{\phi}_t\right)=R^b_{t+1}\left(\xi^R_{t+1},0\right)$$

and (A.79) implies that $\hat{\phi}_t>\bar{\phi}_t$.

For any value of leverage above the aggregate level $\bar{\phi}_t$ but below $\hat{\phi}_t$, when a sunspot is observed, the bank defaults if and only if a system wide run happens. That is, $\xi^R_{t+1}(\phi)$ is insensitive to variation in individual bank's leverage in this region:

$$\xi^R_{t+1}(\phi)=\xi^R_{t+1}(\overline{\phi}_t)\; for\; \phi\in[\overline{\phi}_t,\widehat{\phi}_t].$$

For values of leverage above $\widehat{\phi}_t$ the bank is always insolvent even at non-liquidation prices whenever defaults, *i.e.* $\xi^R_{t+1}(\phi)=\xi^I_{t+1}(\phi)$ for $\phi>\hat{\phi}_t$. When ϕ is smaller than aggregate $\overline{\phi}_t$, the bank is less vulnerable to the run so that $\xi^R_{t+1}(\phi)<\xi^R_{t+1}$. In the extreme when the leverage multiple equals unity, the individual bank is not vulnerable to a run so that $\xi^R_{t+1}(1)=0$.

To summarize, the behaviour of $\xi^R_{t+1}(\phi)$ can be characterized as follows:

$$\begin{gathered}\lim_{\phi\downarrow 1}\xi^R_{t+1}(\phi)=0\\ \frac{d\xi^R_{t+1}(\phi)}{d\phi}>0,\text{ for }\phi\in\left(1,\overline{\phi}_t\right)\\ \xi^R_{t+1}(\phi)=\xi^R_{t+1},\text{ for }\phi\in[\overline{\phi}_t,\widehat{\phi}_t]\text{ where }\xi^I_{t+1}(\widehat{\phi}_t)=\xi^R_{t+1}\\ \xi^R_{t+1}(\phi)=\xi^I_{t+1}(\phi),\text{ for }\phi\in[\widehat{\phi}_t,\infty).\end{gathered}\tag{A.80}$$

See Figure A1.

We can now rewrite the problem of the bank as in the text, but incorporating explicitly the dependence of the default and non-default sets on the individual choice of leverage, as captured by $\Xi^D_{t+1}(\phi)$ and $\Xi^N_{t+1}(\phi)$:

$$\max_{\phi}\;(\mu_t\phi+\nu_t),\tag{A.81}$$

subject to the incentive constraint:

$$\theta\phi\le\mu_t\phi+\nu_t,\tag{A.82}$$

the deposit rate constraint obtained from (A.73) :

$$\overline{R}_{t+1}(\phi)=\frac{\left[1-\frac{\phi}{\phi-1}\int_{\Xi^D_{t+1}(\phi)}\Lambda_{t+1}R^b_{t+1}(\xi_{t+1},\iota_{t+1})d\widetilde{F}_t\right]}{\int_{\Xi^N_{t+1}(\phi)}\Lambda_{t+1}d\widetilde{F}_t}.\tag{A.83}$$

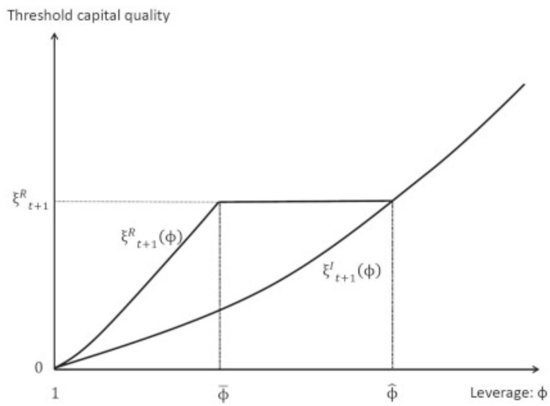


FIGURE A1

μ_t and ν_t given by

$$\mu_t = \int_{\Xi^N_{t+1}(\phi)} \Omega_{t+1}[R^b_{t+1} - \overline{R}_{t+1}(\phi)]d\widetilde{F}_t \tag{A.84}$$

$$\nu_t = \int_{\Xi^N_{t+1}(\phi)} \Omega_{t+1}\overline{R}_{t+1}(\phi)d\widetilde{F}_t \tag{A.85}$$

and where $\Xi^D_{t+1}(\phi)$ and $\Xi^N_{t+1}(\phi)$ are given by (A.76)$-$(A.77), $\xi^I_{t+1}(\phi)$ and $\xi^R_{t+1}(\phi)$ satisfy (A.74)–(A.75).

Using (A.84)–(A.85) in the objective, we can write the objective function as

$$\Psi_t(\phi) = \int_{\Xi^N_{t+1}(\phi)} \Omega_{t+1}\{[R^b_{t+1}(\xi_{t+1}, \iota_{t+1}) - \overline{R}_{t+1}(\phi)]\phi + \overline{R}_{t+1}(\phi)\}d\widetilde{F}_t. \tag{A.86}$$

Before proceeding with differentiation of the objective above, we introduce some notation that will be helpful in what follows. For any function $G(\phi, \xi_{t+1}, \iota_{t+1})$ and for any ϕ different from $\bar{\phi}_t$ or $\hat{\phi}_t$ we let

$$(G)^*_{\phi t} \equiv \frac{d}{d\phi}\left[\int_{\Xi^D_{t+1}(\phi)} G(\phi,\xi,\iota)d\widetilde{F}_t(\xi,\iota)\right] - \int_{\Xi^D_{t+1}(\phi)} \frac{\partial G}{\partial \phi}(\phi,\xi,\iota)d\widetilde{F}_t(\xi,\iota)$$

$$= (1-\varkappa)G\left(\phi, \xi^I_{t+1}(\phi), 0\right)f_t\left(\xi^I_{t+1}(\phi)\right)\frac{d\xi^I_{t+1}(\phi)}{d\phi} + \varkappa G\left(\phi, \xi^R_{t+1}(\phi), 1\right)f_t\left(\xi^R_{t+1}(\phi)\right)\frac{d\xi^R_{t+1}(\phi)}{d\phi} \tag{A.87}$$

denote the marginal effect of ϕ on G only through its effect on the default probability. Then, we know that as long as $G(\cdot)$ is continuous at $\xi^I_{t+1}(\phi)$ and $\xi^R_{t+1}(\phi)$ we have

$$\frac{d}{d\phi}\left[\int_{\Xi^N_{t+1}(\phi)} G(\phi,\xi,\iota)d\widetilde{F}_t(\xi,\iota)\right] - \int_{\Xi^N_{t+1}(\phi)} \frac{\partial G}{\partial \phi}(\phi,\xi,\iota)d\widetilde{F}_t(\xi,\iota) = -(G)^*_{\phi t}.$$

Notice that we have not defined $(G)^*_{\phi t}$ for $\phi_t = \bar{\phi}_t$ or $\phi_t = \hat{\phi}_t$ because $\frac{d\xi^R_{t+1}(\phi)}{d\phi}$ does not exist at that point.

Differentiation of (A.86) at any value different from $\bar{\phi}_t$ and $\hat{\phi}_t$ yields

$$\Psi_t'(\phi)=\mu_t-(\phi-1)\frac{\nu_t}{\overline{R}_{t+1}}\frac{d\overline{R}_{t+1}(\phi)}{d\phi}-\left(\Omega_{t+1}\left\{[R^b_{t+1}(\xi_{t+1},\iota_{t+1})-\overline{R}_{t+1}(\phi)]\phi+\overline{R}_{t+1}(\phi)\right\}\right)^*_{\phi t}.$$

Now notice that for $\phi\in[1,\bar{\phi}_t)$ and $\phi>\hat{\phi}_t$ we have that the bank networth is zero at both thresholds, that is

$$[R^b_{t+1}(\xi^I_{t+1}(\phi),0)-\overline{R}_{t+1}(\phi)]\phi+\overline{R}_{t+1}(\phi)=0$$
$$[R^b_{t+1}(\xi^R_{t+1}(\phi),1)-\overline{R}_{t+1}(\phi)]\phi+\overline{R}_{t+1}(\phi)=0$$

implying $\left(\Omega_{t+1}\left\{[R^b_{t+1}(\xi_{t+1},\iota_{t+1})-\overline{R}_{t+1}(\phi)]\phi+\overline{R}_{t+1}(\phi)\right\}\right)^*_{\phi t}=0.$

For $\phi\in\left(\bar{\phi}_t,\hat{\phi}_t\right)$, we have that at the insolvency threshold net worth is still zero

$$[R^b_{t+1}(\xi^I_{t+1}(\phi),0)-\overline{R}_{t+1}(\phi)]\phi+\overline{R}_{t+1}(\phi)=0$$

while the run threshold is fixed at the aggregate level

$$\frac{d\xi^R_{t+1}(\phi)}{d\phi}=0$$

so that again $\left(\Omega_{t+1}\left\{[R^b_{t+1}(\xi_{t+1},\iota_{t+1})-\overline{R}_{t+1}(\phi)]\phi+\overline{R}_{t+1}(\phi)\right\}\right)^*_{\phi t}=0.$

Therefore, we have that for all ϕ different from $\bar{\phi}_t$ and $\hat{\phi}_t$

$$\Psi_t'(\phi)=\mu_t-(\phi-1)\frac{\nu_t}{\overline{R}_{t+1}}\frac{d\overline{R}_{t+1}(\phi)}{d\phi}$$

and by continuity of $\Psi_t(\phi)$ and $\mu_t-(\phi-1)\frac{\nu_t}{\overline{R}_{t+1}}\frac{d\overline{R}_{t+1}(\phi)}{d\phi}$, it can be extended to $\bar{\phi}_t$ and $\hat{\phi}_t$ as well.

Then, as reported in the text, the first-order condition is

$$\begin{aligned}\phi_t=\tfrac{\nu_t}{\theta-\mu_t},\ \text{if}\ \mu^r_t>0,\ \text{and}\\ \mu^r_t=0,\ \text{if}\ \phi_t<\tfrac{\nu_t}{\theta-\mu_t},\end{aligned}\tag{A.88}$$

$$\mu^r_t=\mu_t-(\phi_t-1)\frac{\nu_t}{\overline{R}_{t+1}}\frac{d\overline{R}_{t+1}(\phi_t)}{d\phi_t}.\tag{A.89}$$

(Here, we assume $\mu_t<\theta$ which we will verify later).

As explained below in Section A.5, we make assumptions such that conditions (A.88)–(A.89) characterize the unique global optimum for the bank's choice of leverage. Since these conditions don't depend on the individual net worth of a banker, every banker chooses the same leverage multiple and has the same Tobin's Q

$$\psi_t=\mu_t\phi_t+\nu_t.\tag{A.90}$$

Thus from the discussion in the text, it follows that there is a system wide default if and only if

$$R^b_{t+1}(\xi_{t+1},0)=\xi_{t+1}\frac{Z_{t+1}+(1-\delta)Q_{t+1}}{Q_t}<\frac{\phi_t-1}{\phi_t}\overline{R}_{t+1}(\phi_t),\ \text{or}$$

$$R^b_{t+1}(\xi_{t+1},1)=\xi_{t+1}\frac{Z^*_{t+1}+(1-\delta)Q^*_{t+1}}{Q_t}<\frac{\phi_t-1}{\phi_t}\overline{R}_{t+1}(\phi_t),$$

where $\overline{R}_{t+1}(\phi_t)$ is the aggregate promised deposit interest rate.

A systemic default occurs if and only if

$$\xi_{t+1} < \xi^I_{t+1}, \text{ where } \xi^I_{t+1}\frac{Z_{t+1}+(1-\delta)Q_{t+1}}{Q_t} = \frac{\phi_t - 1}{\phi_t}\overline{R}_{t+1}(\phi_t), \tag{A.91}$$

or

$$\xi_{t+1} < \xi^R_{t+1} \text{ and } \iota_{t+1} = 1, \text{ where } \xi^R_{t+1}\frac{Z^*_{t+1}+(1-\delta)Q^*_{t+1}}{Q_t} = \frac{\phi_t - 1}{\phi_t}\overline{R}_{t+1}(\phi_t). \tag{A.92}$$

It follows that the probability of default at date $t+1$ conditional on date t information in the symmetric equilibrium is given by

$$p_t = F_t(\xi^I_{t+1}) + \varkappa\left[F_t(\xi^R_{t+1}) - F_t(\xi^I_{t+1})\right]. \tag{A.93}$$

The aggregate capital holding of the banking sector is proportional to the aggregate net worth as

$$Q_t S^b_t = \phi_t N_t. \tag{A.94}$$

The aggregate net worth of banks evolves as

$$N_t = \begin{cases} \sigma \max\left\{\xi_t (Z_t + (1-\delta)Q_t) S^b_{t-1} - \overline{R}_t D_{t-1}\ ,\ 0\right\} + \zeta S_{t-1} & \text{if no run at } t \\ 0 & \text{otherwise} \end{cases}. \tag{A.95}$$

Banks finance capital holdings by net worth and deposit, which implies

$$D_t = (\phi_t - 1)N_t. \tag{A.96}$$

A.4. *Market clearing*

The market for capital holding implies

$$S_t = S^b_t + S^h_t. \tag{A.97}$$

The final goods market clearing condition implies

$$Y_t = C_t + I_t + \frac{\rho^r}{2}{\pi_t}^2 Y_t + G. \tag{A.98}$$

As is explained in the text, the monetary policy rule is given by

$$R^n_t = \frac{1}{\beta}(\pi_t)^{\varphi_\pi}\left(\frac{MC_t}{\frac{\varepsilon-1}{\varepsilon}}\right)^{\varphi_y}. \tag{A.99}$$

The recursive equilibrium is given by a set of ten quantity variables $(K_t, S_t, I_t, L_t, Y_t, C_t, S^h_t, S^b_t, D_t, N_t)$, seven price variables $(w_t, Z_t, MC_t, \pi_t, \overline{R}_{t+1}, Q_t, R^n_t)$, and eight bank coefficients $\left(\psi_t, \mu_t, \nu_t, \mu^r_t, \phi_t, p_t, \xi^R_{t+1}, \xi^I_{t+1}\right)$ as a function of the four state variables $\widetilde{M}_t = (S_{t-1}, S^b_{t-1}, \overline{R}_t D_{t-1}, \xi_t)$ and a sunspot variable ι_t, which satisfies twenty five equations, given by: (A.60,A.61,A.62,A.65,A.66,A.67,A.68,A.69,A.70,A.71,A.72,A.74,A.75,A.84,A.85,A.88,A.89, A.90,A.93,A.94,A.95,A.96,A.97,A.98,A.99). Here, the capital quality shocks follow a Markov process $\xi_{t+1} \sim F\left(\xi_{t+1} \mid \xi_t\right)$ and the sunspot is iid. with $\iota_t = 1$ with probability $\varkappa$.

A.5. *On the global optimum for individual bank's choice*

To study global optimality of the individual leverage choice selected by the first order conditions in (A.88), we need to analyse the curvature of the objective function $\Psi_t(\phi)$ in (A.86).

To do so, we use (A.83) to derive an expression for $\frac{d\bar{R}}{d\phi}$ and substitute it into (A.89) to obtain

$$\Psi_t'(\phi)=\int_{\Xi^N_{t+1}(\phi)}\Omega_{t+1}R^b_{t+1}d\widetilde{F}_t-\left[1-\int_{\Xi^D_{t+1}(\phi)}\Lambda_{t+1}R^b_{t+1}d\widetilde{F}_t\right]\frac{\int_{\Xi^N_{t+1}(\phi)}\Omega_{t+1}d\widetilde{F}_t}{\int_{\Xi^N_{t+1}(\phi)}\Lambda_{t+1}d\widetilde{F}_t},\tag{A.100}$$

Proceeding as in Section A.3 to differentiate (A.100) for any value of ϕ different from $\bar{\phi}_t$ and $\hat{\phi}_t$, we get

$$\Psi_t''(\phi)=\begin{array}{c}-\left(\Omega_{t+1}R^b_{t+1}\right)^*_{\phi t}+\left(\Lambda_{t+1}R^b_{t+1}\right)^*_{\phi t}\cdot\frac{\int_{\Xi^N_{t+1}(\phi)}\Omega_{t+1}d\widetilde{F}_t}{\int_{\Xi^N_{t+1}(\phi)}\Lambda_{t+1}d\widetilde{F}_t}\\+\\\left[1-\int_{\Xi^D_{t+1}(\phi)}\Lambda_{t+1}R^b_{t+1}d\widetilde{F}_t\right]\frac{\int_{\Xi^N_{t+1}(\phi)}\Omega_{t+1}d\widetilde{F}_t}{\int_{\Xi^N_{t+1}(\phi)}\Lambda_{t+1}d\widetilde{F}_t}\left[\frac{(\Omega_{t+1})^*_{\phi t}}{\int_{\Xi^N_{t+1}(\phi)}\Omega_{t+1}d\widetilde{F}_t}-\frac{(\Lambda_{t+1})^*_{\phi t}}{\int_{\Xi^N_{t+1}(\phi)}\Lambda_{t+1}d\widetilde{F}_t}\right].\end{array}\tag{A.101}$$

Note that for $\phi\in[1,\bar{\phi}_t)$

$$R^b_{t+1}\left(\xi^I_{t+1}(\phi),\iota\right)=R^b_{t+1}\left(\xi^R_{t+1}(\phi),1\right)=\frac{\phi-1}{\phi}\overline{R}_{t+1}(\phi).$$

For $\phi\in\left(\bar{\phi}_t,\hat{\phi}_t\right)$, we have $\frac{d\xi^R_{t+1}(\phi)}{d\phi}=0$ which implies that for any function $G(\xi_{t+1},\iota_{t+1})$

$$(G)^*_{\phi t}=(1-\varkappa)G(\phi,\xi^I_{t+1}(\phi),0)f_t\left(\phi,\xi^I_{t+1}(\phi)\right)\frac{d\xi^I_{t+1}(\phi)}{d\phi}\text{ for }\phi\in\left(\bar{\phi}_t,\hat{\phi}_t\right)\tag{A.102}$$

and also

$$R^b_{t+1}\left(\xi^I_{t+1}(\phi),0\right)=\frac{\phi-1}{\phi}\overline{R}_{t+1}(\phi)$$

Then, we learn

$$\left(\Omega_{t+1}R^b_{t+1}\right)^*_{\phi t}=\left(\Omega_{t+1}\right)^*_{\phi t}\cdot\frac{\phi-1}{\phi}\overline{R}_{t+1}(\phi)$$

$$\left(\Lambda_{t+1}R^b_{t+1}\right)^*_{\phi t}=\left(\Lambda_{t+1}\right)^*_{\phi t}\cdot\frac{\phi-1}{\phi}\overline{R}_{t+1}(\phi).$$

Substituting this back into (A.101) and using (A.83) to substitute for $\overline{R}_{t+1}(\phi)$, we get

$$\Psi_t"(\phi) = \frac{1}{\phi} \frac{\int_{\Xi^N_{t+1}(\phi)} \Omega_{t+1} d\widetilde{F}_t}{\int_{\Xi^N_{t+1}(\phi)} \Lambda_{t+1} d\widetilde{F}_t} \left[\frac{\left(\Omega_{t+1}\right)^*_{\phi t}}{\int_{\Xi^N_{t+1}(\phi)} \Omega_{t+1} d\widetilde{F}_t} - \frac{\left(\Lambda_{t+1}\right)^*_{\phi t}}{\int_{\Xi^N_{t+1}(\phi)} \Lambda_{t+1} d\widetilde{F}_t} \right] \tag{A.103}$$

for any ϕ different from $\bar{\phi}_t$ and $\hat{\phi}_t$.[40]

We assume that a bank that individually survives a systemic bank run by choosing its own leverage below the aggregate level $\overline{\phi}_t$ behaves just like new entrants during the panic: it stores its net worth and starts operating the period right after the crisis. Given that both leverage and spreads increase dramatically after a crisis, new banker's Tobin's Q is very high during a crisis so that

$$\frac{\Omega_{t+1}\left(\xi_{t+1}, 1\right)}{\int_{\Xi^N_{t+1}(\phi)} \Omega_{t+1} d\widetilde{F}_t} > \frac{\Lambda_{t+1}\left(\xi_{t+1}, 1\right)}{\int_{\Xi^N_{t+1}(\phi)} \Lambda_{t+1} d\widetilde{F}_t} \text{ for } \xi_{t+1} = \xi^R_{t+1}(\phi) < \xi^R_{t+1}.$$

By the same argument, we also have that

$$\frac{\Omega_{t+1}\left(\xi_{t+1}, 0\right)}{\int_{\Xi^N_{t+1}(\phi)} \Omega_{t+1} d\widetilde{F}_t} > \frac{\Lambda_{t+1}\left(\xi_{t+1}, 0\right)}{\int_{\Xi^N_{t+1}(\phi)} \Lambda_{t+1} d\widetilde{F}_t} \text{ for } \xi_{t+1} = \xi^I_{t+1}(\phi) < \xi^I_{t+1}.$$

Given this, equation (A.103) and (A.87) imply that the objective function of the banker is strictly convex in the region where leverage is below the aggregate level $\bar{\phi}_t$, that is $\Psi_t"(\phi) > 0$ for $\phi \in [1, \bar{\phi}_t)$, as long as the probability of a run is still positive, *i.e.* $f_t\left(\xi^R_{t+1}(\phi)\right) > 0$. If, on the other hand, leverage is so low that default is not possible, *i.e.* $f_t\left(\xi^R_{t+1}(\phi)\right) = f_t\left(\xi^I_{t+1}(\phi)\right) = 0$, the second derivative is zero.

For $\phi \in \left(\bar{\phi}_t, \hat{\phi}_t\right)$, equations (A.102) and (A.103) imply that $\Psi_t"(\phi)$ depends on the relative increase in the marginal value of wealth of the banker and of the households *only at the insolvency threshold,* see Figure A1. Therefore, in this case we have that the objective is convex, $\Psi_t"(\phi) > 0$, as long as $f_t\left(\xi^I_{t+1}(\phi)\right) > 0$.

Summing up, we have:

$$\Psi_t"(\phi) \begin{cases} = 0 & \text{if } \phi \in [1, \bar{\phi}_t) \text{ and } f_t\left(\xi^R_{t+1}(\phi)\right) = 0 = f_t\left(\xi^I_{t+1}(\phi)\right) \\ > 0 & \text{if } \phi \in [1, \bar{\phi}_t) \text{ and } f_t\left(\xi^R_{t+1}(\phi)\right) > 0 \\ = 0 & \text{if } \phi \in \left(\bar{\phi}_t, \hat{\phi}_t\right) \text{ and } f_t\left(\xi^I_{t+1}(\phi)\right) = 0 \\ > 0 & \text{if } \phi \in \left(\bar{\phi}_t, \hat{\phi}_t\right) \text{ and } f_t\left(\xi^I_{t+1}(\phi)\right) > 0 \end{cases}. \tag{A.104}$$

40. Notice that $(\Omega_{t+1})^*_{\phi t}$ and $(\Lambda_{t+1})^*_{\phi t}$ are not continuous at $\bar{\phi}_t$ since, for instance

$$\lim_{\phi \uparrow \bar{\phi}_t} (\Omega_{t+1})^*_{\phi t} = (1-\varkappa)\Omega_{t+1}(\xi^I_{t+1}, 0) f_t\left(\xi^I_{t+1}\right) \frac{d\xi^I_{t+1}(\phi)}{d\phi} + \varkappa \Omega_{t+1}(\xi^R_{t+1}, 1) f_t\left(\xi^R_{t+1}\right) \left[\frac{d\xi^R_{t+1}(\phi)}{d\phi}\right]_-$$

$$> (1-\varkappa)\Omega_{t+1}(\xi^I_{t+1}, 0) f_t\left(\xi^I_{t+1}\right) \frac{d\xi^I_{t+1}(\phi)}{d\phi} = \lim_{\phi \downarrow \bar{\phi}_t} (\Omega_{t+1})^*_{\phi t},$$

where $\left[\frac{d\xi^R_{t+1}(\phi)}{d\phi}\right]_-$ is the left derivative of $\xi^R_{t+1}(\phi)$ at $\bar{\phi}_t$. This implies that $\Psi"(\phi)$ does not exist at $\bar{\phi}_t$.

Equation (A.104) implies that the objective of the bank is weakly convex. Thus, to study global optimality it is sufficient to compare the equilibrium choice of leverage, $\bar{\phi}_t$, to deviations to corner solutions.

When the incentive constraint is binding, *i.e.* $\Psi_t'\left(\overline{\phi}_t\right)=\mu_t^r>0$ at $\overline{\phi}_t=\frac{\nu_t}{\theta-\mu_t}$, a bank cannot increase its own leverage above $\bar{\phi}_t$ so that the only deviation that we need to check is $\phi=1$. Therefore, the condition for global optimality in this case is:

$$\Psi_t(1)<\Psi_t\left(\frac{\nu_t}{\theta-\mu_t}\right). \tag{A.105}$$

When the constraint is not binding, *i.e.* $\Psi_t'\left(\overline{\phi}_t\right)=\mu_t^r=0$ and $\overline{\phi}_t<\frac{\nu_t}{\theta-\mu_t}$, an individual bank could deviate to either $\phi=1$ or $\phi=\phi_t^{IC}$, where ϕ_t^{IC} is the maximum level of leverage compatible with incentive constraints, *i.e.* $\Psi_t(\phi^{IC})=\theta\phi^{IC}$. In this case, given weak convexity of the objective, the global optimality condition is satisfied if and only if

$$\Psi_t(1)=\Psi_t\left(\bar{\phi}_t\right)=\Psi_t\left(\phi_t^{IC}\right). \tag{A.106}$$

Notice that equation (A.104) implies that the above equality is satisfied if and only if the probability of default is zero for any feasible choice of leverage $\phi\in\left[1,\phi_t^{IC}\right]$ which would result in a flat objective function.

We verify numerically that condition (A.105) is satisfied in the neighbourhood of the risk-adjusted steady state, where the constraint is binding. Moreover, in our calibration, whenever the incentive constraint is not binding in equilibrium the probability of insolvency is zero for any feasible choice of leverage above the equilibrium level, *i.e.* $f_t\left(\xi_{t+1}^I(\phi)\right)=0$ for $\phi\in(\bar{\phi}_t,\phi_t^{IC}]$, so that a deviation by an individual bank to a higher level of leverage is never strictly preferred.

However, the economy does occasionally transit to extreme states in which the constraint is binding but the probability of the run is high enough that equation (A.105) is violated and to states in which the constraint is slack and the probability of the run is positive thus violating (A.106). In such states, a bank would gain by a deviation to $\phi=1$, see Figure A2. The only equilibrium in these cases would then be one in which a fraction of banks decrease their leverage in anticipation of a run while all of the others are against the constraint, *i.e.* there is no symmetric equilibrium. In order to focus on the symmetric equilibrium, we introduce a small cost to a bank to deviating to a position of taking no leverage $\phi=1$. This cost could reflect expenses involved in a major restructuring of the bank's portfolio. It could also reflect reputation costs associated with the bank's refusal to accept deposits in a given period in order to survive a run in the subsequent period. In particular, we posit that the objective of the bank is given by

$$V_t(n_t)=\Psi_t(\phi)n_t\left(1-\tau\bar{\phi}_t\right)\ \textit{for}\ \phi\in[1,\bar{\phi}_t).$$

That is, a deviation of a bank that reduces leverage below the aggregate value $\bar{\phi}_t$ entails a fixed cost $\tau\bar{\phi}_t$ per unit of net worth. We check computationally that the deviation is never profitable, *i.e.* $\Psi_t\left(\bar{\phi}_t\right)>\Psi_t(1)$, in all of our experiments for values of τ which are greater than or equal to 0.77%.[41] Examining asymmetric equilibrium without such reputation cost is a topic of future research.

41. The value of deviating can increase in very extreme cases but in a simulation of 100 thousands periods it is still below 1.7% for 99% of the times.

A.6. *Computation*

It is convenient for computations to let the aggregate state of the economy be given by

$$\mathcal{M}_t = (S_{t-1}, N_t, \xi_t, \iota_t).$$

Notice that bank net worth replaces the specific asset and liability position of banks in the natural state that we have used so far $\tilde{\mathcal{M}}_t = (S_{t-1}, S^b_{t-1}, D_{t-1}\bar{R}_t, \xi_t)$. To see that this state is sufficient to compute the equilibrium we rewrite the evolution of net worth, equation (A.95), forward. Using the definition of the leverage multiple and the budget constraint of the banker, we get that whenever there is no run at time t, so that $N_t > 0$, the evolution of net worth is given by

$$N_{t+1} = \begin{cases} \sigma N_t \left\{ \phi_t \left(\xi_{t+1} \frac{Z_{t+1} + (1-\delta) Q_{t+1}}{Q_t} - \bar{R}_{t+1} \right) + \bar{R}_t \right\} + \zeta S_t & \begin{array}{c}\text{if there is no default:} \\ \xi_{t+1} \in \Xi^N_{t+1}(\phi_t)\end{array} \\ 0 & \begin{array}{c}\textit{if} \text{ there is a run:} \\ \xi_{t+1} < \xi^R_{t+1} \text{ and } \iota_{t+1} = 1\end{array} \\ \zeta S_t & \begin{array}{c}\textit{if} \text{ banks are insolvent:} \\ \xi_{t+1} < \xi^I_{t+1} \text{ and } \iota_{t+1} = 0\end{array} \end{cases} . \tag{A.107}$$

Otherwise, if a run has happened at time t so that $N_t = 0$, the evolution of net worth is given by equation (2.34), which we report for convenience:

$$N_{t+1} = \zeta S_t \left(1 + \sigma \frac{S_{t-1}}{S_t} \right). \tag{A.108}$$

We can then look for a recursive equilibrium in which each equilibrium variable is a function of $\mathcal{M}_t$ and the evolution of net worth is given by a function $N_{t+1}(\mathcal{M}_t;\ \xi_{t+1}, \iota_{t+1})$ that depends on the realization of the exogenous shocks (ξ_{t+1}, ι_{t+1}) and satisfies equations (A.107) and (A.108) above.

We use time iteration in order to approximate the functions

$$\vartheta = \left\{ \mathbf{Q}(\mathcal{M}), \mathbf{C}(\mathcal{M}), \psi(\mathcal{M}), \xi^R_{t+1}(\mathcal{M}), \xi^I_{t+1}(\mathcal{M}), \mathbf{T}(\mathcal{M}; \epsilon', \iota') \right\},$$

where $\mathbf{T}(\mathcal{M}_t; \epsilon', \iota')$ is the transition law determining the stochastic evolution of the state.

The computational algorithm proceeds as follows:

1. Determine a functional space to use for approximating equilibrium functions. (We use piece-wise linear.)
2. Fix a grid of values for the state $G \subset [S^m, S^M] \times [0, N^M] \times [1 - 4\sigma^\xi, 1 + 4\sigma^\xi] \times \{0, 1\}$
3. Set $j = 0$ and guess initial values for the equilibrium objects of interest on the grid

$$\vartheta_j = \left\{ Q_j(\mathcal{M}), C_j(\mathcal{M}), \psi_j(\mathcal{M}), \xi^R_{t+1,j}(\mathcal{M}), \xi^I_{t+1,j}(\mathcal{M}), T_j(\mathcal{M}; \xi', \iota') \right\}_{\mathcal{M} \in G}$$

4. Assume that ϑ_i has been found for $i < M$ where M is set to 10,000. Use ϑ_i to find associated functions ϑ_i in the approximating space, *e.g.* $\mathbf{Q}_i$ is the price function that satisfies $\mathbf{Q}_i(\mathcal{M}) = Q_i(\mathcal{M})$ for each $\mathcal{M} \in G$.
5. Compute all time $t+1$ variables in the system of equilibrium equations by using the functions ϑ_i from the previous step, *e.g.* for each $\mathcal{M} \in G$ let $Q_{t+1} = \mathbf{Q}_i(T_i(\mathcal{M}; \epsilon', \iota'))$, and then solve the system to get the implied ϑ_{i+1}
6. Repeat 4 and 5 until convergence of ϑ_i

A.7. *Impulse response functions*

The risk-adjusted steady state is $\bar{\mathcal{M}} = (\bar{S}, \bar{N}, 1, 0)$ which satisfies:

$$\bar{\mathcal{M}} = \mathbf{T}\left(\bar{\mathcal{M}}; 0, 0\right)$$

We compute responses to a sequence of n shocks $\left\{\epsilon_t^{irfs}, \iota_t^{irfs}\right\}_{t=1}^n$ by starting the economy in the risk-adjusted steady state, $\mathcal{M}_0 = \bar{\mathcal{M}}$, and computing the evolution of the state given the assumed shocks from time 1 to n and setting all future shocks to 0, *i.e.* $\epsilon_t = \iota_t = 0$ for $t \geq n+1$:

$$\mathcal{M}_{t+1} = \begin{cases} \mathbf{T}\left(\mathcal{M}_t; \epsilon_t^{irfs}, \iota_t^{irfs}\right) & \text{if } t \leq n \\ \mathbf{T}(\mathcal{M}_t; 0, 0) & \text{if } t > n \end{cases}$$

We then plot for each variable, the values of the associated policy function computed along this path for the state, *e.g.* $Q_t = \mathbf{Q}(\mathcal{M}_t)$. Notice that, given our nonlinear policy functions, these values are different from conditional expectations given the sequence of shocks $\left\{\epsilon_t^{irfs}, \iota_t^{irfs}\right\}_{t=1}^n$.

A.8. *Long rates and forecast errors of credit spreads*

We compute long rates as the average of future short rates. In particular, we let r_t^{AAA} denote the 10 years expected rate of return on bank assets

$$r_t^{AAA} = \frac{1}{40} E_t \sum_{i=1}^{40} \left(R_{t+i}^b - 1\right)$$

and r_t^T the 10 years risk free rate

$$r_t^T = \frac{1}{40} E_t \sum_{i=1}^{40} \left(R_{t+i}^f - 1\right).$$

We approximate expectations by simulating the model and then measure the spread $r_t^{AAA} - r_t^T$ by the difference between the AAA-rated corporate bonds and the 10-year Treasuries.

We follow Bordalo *et al.* (2017) to compute averages of the forecast errors over the year ahead. That is, letting $SP_t = r_t^{AAA} - r_t^T$, the forecast errors that we report at time t are given by

$$FE_t = \frac{1}{4} \sum_{i=1}^{4} \left(SP_{t+i} - E_t SP_{t+i}\right).$$

Notice that SP_t will be an average of the quarterly excess returns, $E_t\left(R_{t+1}^b - R_{t+1}^f\right)$ which, as shown *e.g.* in Figure 1, are countercyclical. This implies that periods of optimistic beliefs about the capital quality will translate into periods in which forecasts of the spread over the following 1 year (period from $t+1$ to $t+4$) will be low. As long as no boom is realized, the realization of the average spreads between periods $t+1$ and $t+4$ will then tend to be higher than the date-t forecast of the average spreads of the same periods so that positive date-t forecast errors are associated with periods of optimistic expectations.

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