# Credit Horizons

Nobuhiro Kiyotaki, John Moore and Shengxing Zhang
Princeton, Edinburgh and LSE

## Questions

To finance investment, entrepreneurs raise external funds against their future revenues – largely against *near-term* revenues

Why are credit horizons short, even when projects are long?

How do credit horizons interact with firm dynamics?

Could a drop in long-term real interest rates lead to secular stagnation?

## Approach

Human capital of entrepreneurs/engineers is essential for constructing and then maintaining production facilities/plant

Their human capital is inalienable. To finance investment, engineer sells plant ownership to saver. Engineers cannot commit to work for less than their share of contribution

For more distant future, the fraction of output attributable to engineers' cumulative maintenance is larger

The contribution and share of the initial plant owner is smaller

$\longrightarrow$ Price of new plant ($=$ fund-raising capacity of entrepreneurs) is governed largely by near-term revenues

## Model

Small open economy with an exogenous real interest rate $R$

Homogeneous perishable consumption/investment good at each date $t = 0, 1, 2, ...$(numeraire)

Continuum of agents, with common discount factor $\beta < 1/R$

Each agent sometimes has an investment opportunity (entrepreneur/engineer) and sometimes not (saver)

At each date, an engineer E can jointly produce plant and tools from goods and building: within the period, per unit of plant,

$$\left.\begin{array}{c} x \text{ goods} \\ 1 \text{ building} \end{array}\right\} \rightarrow \left\{\begin{array}{c} \text{plant of productivity } 1 \\ 1 \text{ E-tool} \end{array}\right.$$

Engineer raises funds by selling the plant to savers. Match between plant and engineer is not specific $\longrightarrow$ Plant owner hires engineers for maintenance in a competitive market at "wage" $w$. Engineer cannot precommit to work for less

At each date, the owner of plant of productivity $z$ can hire any number $h$ of tools (hiring each tool along with the engineer who knows how to use it) to produce goods and maintain plant productivity: within the period, per unit of plant,

$$\left.\begin{array}{c}\text{productivity } z \text{ plant} \\ h \text{ tools}\end{array}\right\} \longrightarrow \left\{\begin{array}{c} y = az \quad \text{goods} \\ \lambda \text{ productivity } z' = z^{\theta} h^{\eta} \text{ plant} \\ \lambda h \text{ tools}\end{array}\right.$$

New buildings are supplied by foreigners

Alternative use of building by foreigners:

$$1 \text{ building} \longrightarrow \begin{cases} \;\; f \;\; \text{goods} \\ \lambda \text{ building} \end{cases}$$

$\longrightarrow$ Price of buildings

$$q = \frac{f}{R - \lambda}$$

The plant owner always has the option to stop and liquidate his plant into generic building. So his value of a unit of plant of productivity $z$ at the end of the period is given by

$$V(z) = Max\left\{q,\ \frac{1}{R}\max_h\left[az - wh + \lambda V\left(z^\theta h^\eta\right)\right]\right\}$$

The plant owner must devise a long-term plan:

stop after a finite number of periods $T$, or

continue forever $(T = \infty)$?

An engineer raises fund by selling a new plant at price $b = V(1)$

The budget constraint of an agent at date $t$ who has $h_t$ tools and $d_t$ financial assets (maturing one-period discount bonds plus returns to plant ownership) is

$$c_t + (x + q - b)i_t + \frac{d_{t+1}}{R} = wh_t + d_t,$$

where $h_t$ is positive iff the agent was engineer yesterday. Iff the agent is an engineer today, investment $i_t$ is positive, and her tools tomorrow will be

$$h_{t+1} = \lambda h_t + i_t$$

The budget constraint can be written as

$$c_t + (x + q - b)h_{t+1} + \frac{d_{t+1}}{R} = [w + \lambda(x + q - b)]h_t + d_t \equiv n_t,$$

where $n_t$ is net worth

When the rate of return on investment with maximal leverage, $R^E$, exceeds the interest rate

$$R^E = \frac{w + \lambda(x + q - b)}{x + q - b} > R,$$

the engineer's consumption and investment are

$$\begin{aligned} c_t &= (1 - \beta)n_t \\ (x + q - b)h_{t+1} &= \beta n_t \end{aligned}$$

A steady state equilibrium of our small open economy is characterized by the wage rate $w$ and new-plant price $b$, together with the quantity choices of savers/plant owners $(c, d, h, z, y)$, engineers $(c, h, i)$, and foreigners (who have net asset holdings $D^*$), such that the markets for goods, tools, plant, and bonds all clear

**Proposition 1. Pure Equilibrium with No Stopping: Low opportunity cost** $f < f^{\text{critical}}$

(a) No plant owner stops

(b) Aggregate ratio of tools-to-plant stays one-to-one (because equal initial supply, equal depreciation, no stopping): $h_t = 1$

(c) All plant is maintained at initial productivity: $z_t = 1$

(d) All plan has output: $y_t = a$

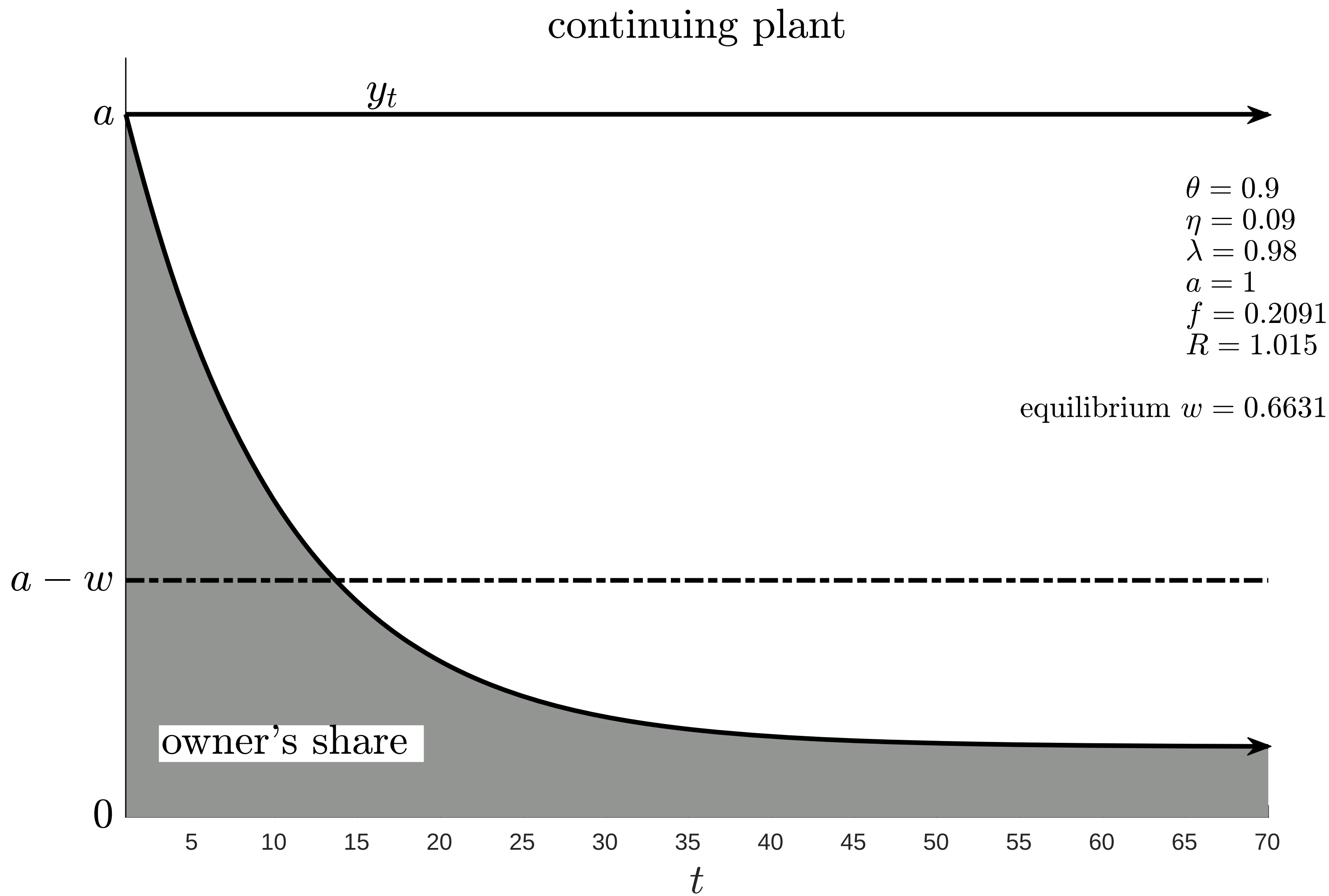
Optimal maintenance choice ($z_{t+1} = z_t^{\theta} h_t^{\eta}$ and $h_t = z_t = 1$)

$$w = 0 + \frac{\lambda}{R}\eta a + \frac{\lambda^2}{R^2}\eta\theta a + \frac{\lambda^3}{R^3}\eta\theta^2 a + ...$$

$$\begin{aligned} b \; = \; & \frac{1}{R}a \; + \; \frac{\lambda}{R^2}a(1-\eta) + \frac{\lambda^2}{R^3}a(1-\eta-\eta\theta) \\ & + \frac{\lambda^3}{R^4}a(1-\eta-\eta\theta-\eta\theta^2) + ... \end{aligned}$$

Engineers' share of gross output rises with horizon as 0, $\eta$, $\eta + \eta\theta$, $\eta + \eta\theta + \eta\theta^2, ...$

Plant owner's share from present plant declines with horizon as 1, $1-\eta$, $1-\eta-\eta\theta$, $1-\eta-\eta\theta-\eta\theta^2, ...$


continuing plant
$y_t$
$a$
$a - w$
$0$
$\theta = 0.9$
$\eta = 0.09$
$\lambda = 0.98$
$a = 1$
$f = 0.2091$
$R = 1.015$
equilibrium $w = 0.6631$
owner's share
5
10
15
20
25
30
35
40
45
50
55
60
65
70
$t$

**Proposition 2. Mixed Equilibrium: High opportunity cost** $f > f^{critical}$

(a) Plant owners are initially indifferent between stopping in some finite time and continuing forever

(b) Aggregate ratio of tools-to-plant is larger than one-to-one for continuing plant: $h_t > 1$

(c) The productivity of continuing plant increases over time

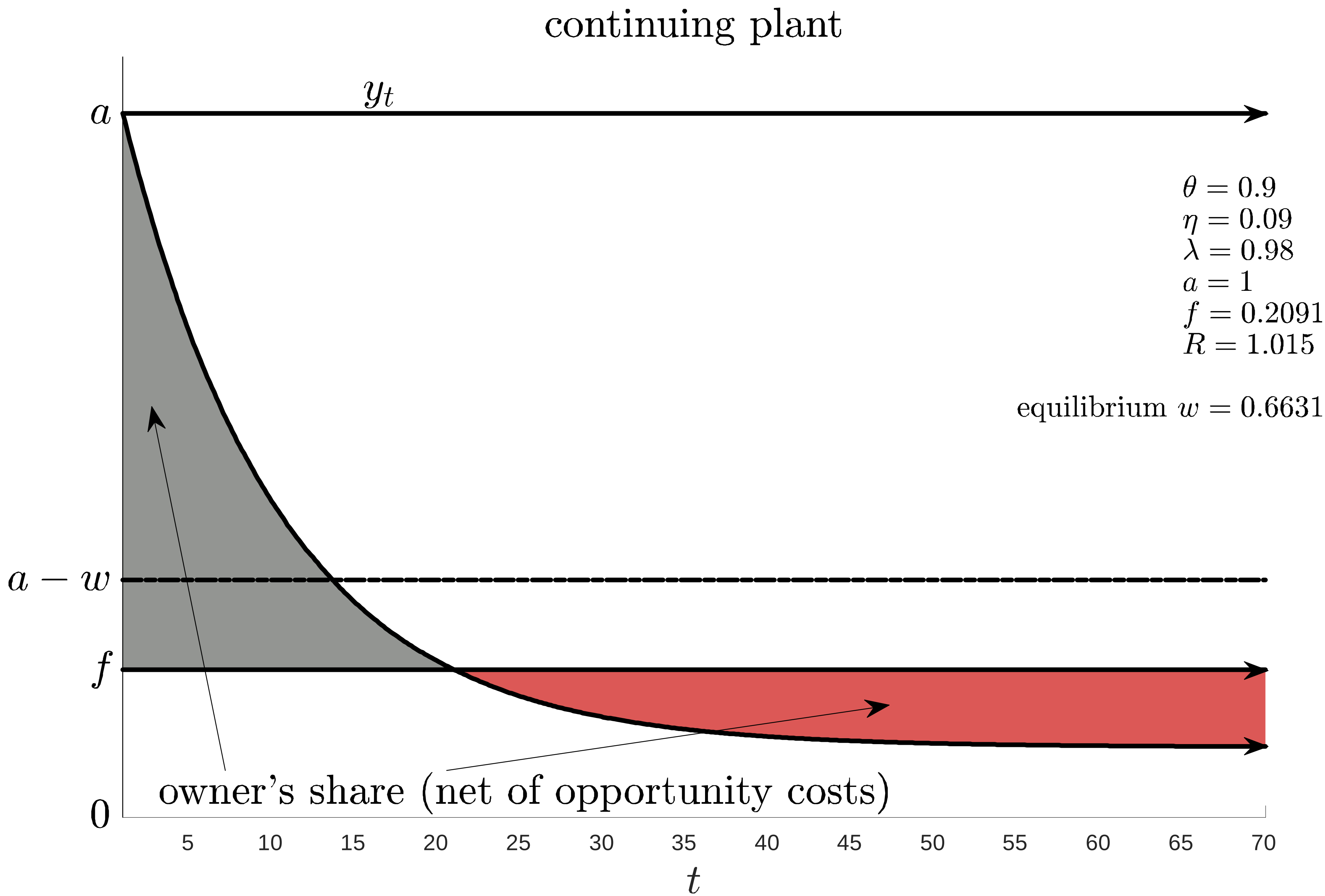
(d) The productivity of stopping plant decreases over time

**Lemma:** There is no equilibrium in which all plant shut down in finite time

## Proposition 3 (Pure Equilibrium with No Stopping):

(a) For an open set of parameters (particularly $\lambda$ and $R$ not too far from 1), there is a pure equilibrium with no stopping such that an unexpected permanent drop in the interest rate $R$ leads to a lower steady state growth rate $G$

(b) Immediately following the drop in $R$, the economy can experience a temporary boom, but all agents (engineers and savers) can be strictly worse off


continuing plant
$y_t$
$a$
$a-w$
$f$
0
$\theta = 0.9$
$\eta = 0.09$
$\lambda = 0.98$
$a = 1$
$f = 0.2091$
$R = 1.015$
equilibrium $w = 0.6631$
owner's share (net of opportunity costs)
5
10
15
20
25
30
35
40
45
50
55
60
65
70
$t$

In particular, with constant returns to scale, $\eta + \theta = 1$,

$$\underbrace{b}_{\text{engineer's fund-raising capacity}} = \underbrace{\frac{a}{R - \lambda\theta}}_{\text{PV of plant owner's share}}$$

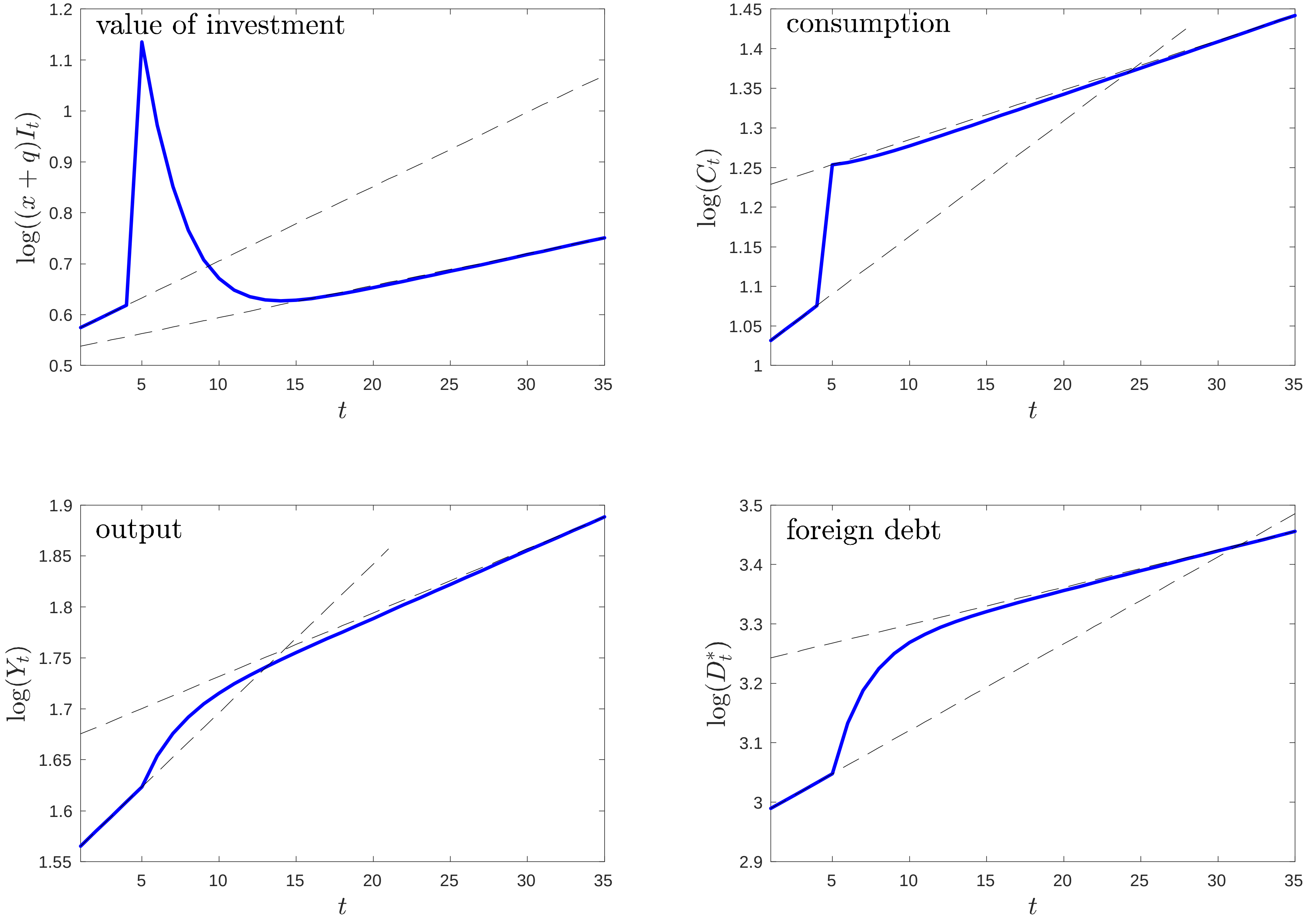
Because $\theta < 1$, the fall in $R$ may not increase the engineer's fund-raising capacity as much as building price

$$q = \frac{f}{R - \lambda}$$

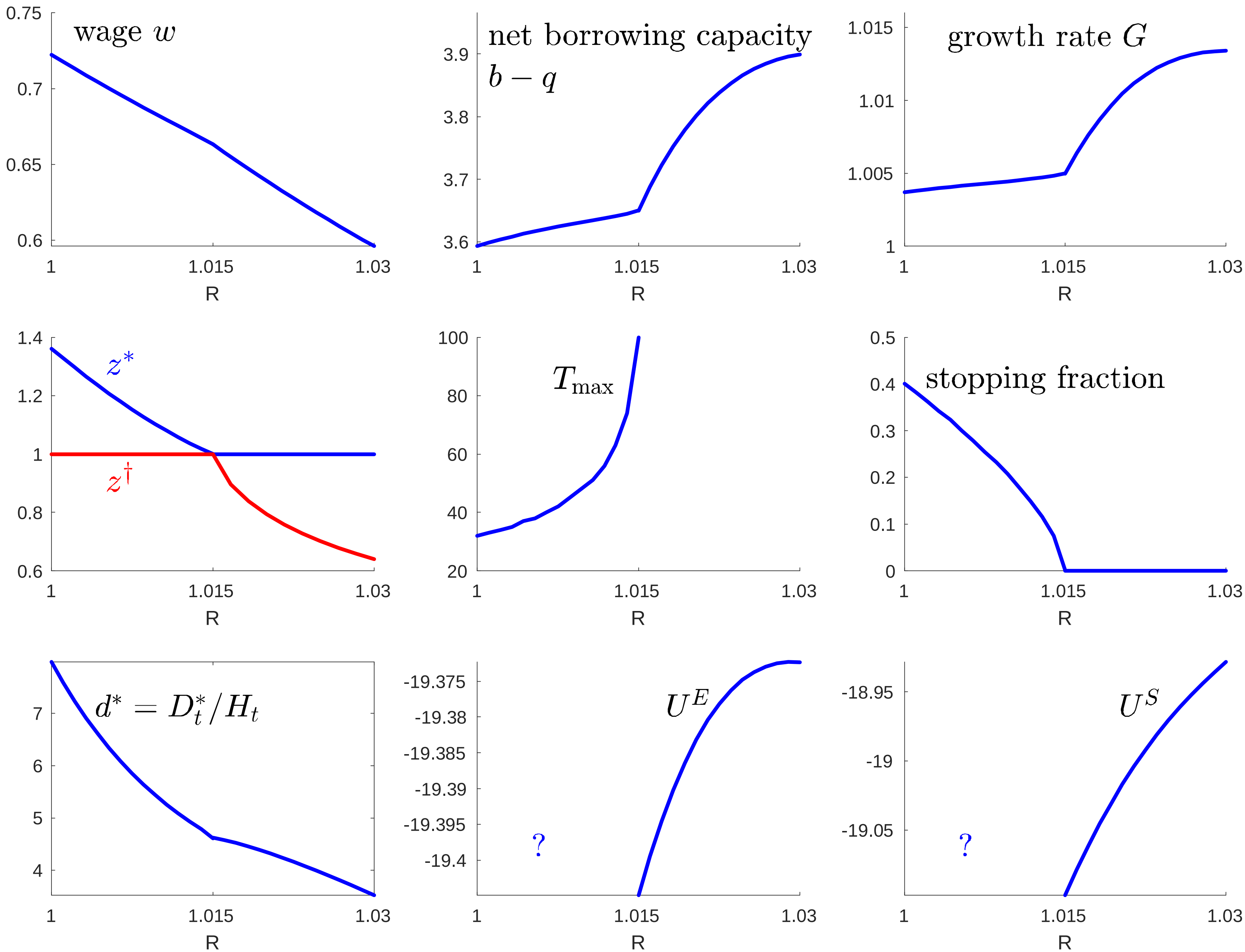
This effect can be strong enough – overcoming rise in net worth – to stifle investment and growth:

$$\text{gross investment} \downarrow \; =$$

$$\beta \times \frac{\text{net worth of engineers} \uparrow}{\text{investment cost } \; (x + q) \uparrow\uparrow - \; \text{fund-raising capacity } \; (b) \uparrow}$$



value of investment
$\log((x+q)I_t)$
$t$
consumption
$\log(C_t)$
$t$
output
$\log(Y_t)$
$t$
foreign debt
$\log(D_t^*)$
$t$



wage $w$
net borrowing capacity $b - q$
growth rate $G$
$z^*$
$z^\dagger$
$T_{\max}$
stopping fraction
$d^* = D_t^* / H_t$
$U^E$
$U^S$
?
?
R

## Extension: Idiosyncratic Shocks:

$$\left.\begin{array}{c}\text{productivity } z \text{ plant}\\ h \text{ tools}\end{array}\right\} \rightarrow \left\{\begin{array}{c} y = az \quad \text{goods} \\ \lambda \text{ productivity } z' = \epsilon z^{\theta} h^{\eta} \text{ plant} \\ \lambda h \text{ tools}\end{array}\right.$$

$$\ln \epsilon \sim N\left(-\frac{\sigma^2}{2}, \sigma^2\right), \text{ iid. across plant and time}$$

For a small variance $\sigma^2$, there is a cutoff plant productivity $z^{\dagger}$ at which the engineers' maintenance increases discontinuously with plant productivity

For a large variance $\sigma^2$, the engineers' maintenance is a smooth increasing function of plant productivity

## Policy

Non-exclusivity is the sole departure from Arrow-Debreu: impossible to keep track of each engineer's trading history

If plant is easy to locate, then perhaps government could tax the plant owner's payroll at rate $\tau$

Use the revenue to subsidize investment at rate $s$, where

$$sI = \tau w H$$

In Pure Equilibrium with No Stopping, $\frac{\partial G}{\partial \tau} > 0$

Defining welfare as the population-weighted average of the expected discounted utilities of engineers and savers, we can show that (for small $\tau$) welfare rises with $\tau$