

# Intangibles, Inequality and Stagnation

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## Abstract

We examine how aggregate output and income distribution interact with accumulation of intangible capital over time and across individuals. We consider an overlapping generations economy in which managerial skill (intangible capital) is essential for production, and it is acquired by young workers through on-the-job training by old managers. We show that, when young trainees are not committed to staying in the same firms and repaying their debt, a small difference in initial endowment and ability of young workers leads to a large inequality in accumulation of intangibles and lifetime income. A negative shock to endowment or the degree of commitment generates a persistent stagnation and a rise in inequality.

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# 1 Introduction

In the last few decades, especially after the global financial crisis of 2007-9, we observe two major concerns: slower growth of many countries and rising inequality across households within country. In Japan, there are heated debates on why Japan stopped growing and what caused the rising inequality after it entered into a prolonged financial crisis with collapse of asset prices in the early 1990s. Although proposed explanations differ across researchers, the key phenomena to explain appear to be declining growth rate of total factor productivity and worsening labor market condition for young workers.

In this paper, we explore a hypothesis that the slower productivity growth and the worsening youth labor market are entwined with accumulation of intangible capital. For this purpose, we consider an overlapping generations economy in which skill of managers (intangible capital) is essential for production along with labor, and managerial skill is acquired by young workers when they are trained by old managers on the job. Unlike physical capital, intangible capital - particularly managerial skill - cannot be directly transferred between generations. We formulate the technology of accumulating intangible capital in a fairly general way: The outcome is the managerial skill acquired by young trainees, and inputs include final goods (or resources), the skill of old managers and the initial skill of young trainees (innate learning ability or ability acquired by earlier education). Because training is costly and takes time, productivity profile of trainee-managers is upward-sloping before becoming downward-sloping with age over the life cycle.

Intangible capital also tends to be hard to pledge as collateral. In our economy, managers offer young workers two options, a simple labor contract, which pays competitive wage without training, and a career path, which offers apprentice wage and training to be future managers. The initial endowment and skill are heterogeneous across young workers and are publicly observable. So, the career path package can depend upon the initial skill and endowment of the trainee. If trainee could commit to stay in the same firm and repay his debt, he would choose the one with a higher permanent income between the two options. Then, the training would only depend upon the initial skill and there would be no inequality in permanent income controlling the initial skill. In our baseline economy, however, the trainee is not committed to staying in the same firm in future. He will lose only a fraction of his managerial skill by moving to another firm or starting a new firm, and faces constraints in borrowing from the firm or market. Then, current managers are willing to cover only a

fraction of the training cost, and pay to its career workers the compensation that is close to the marginal product of labor net of training cost. Also future managers cannot smooth consumption well through financial markets over the life cycle.

In such an economy with limited commitment, we show the aggregate intangible investment is lower than the unconstrained economy for any given interest rate. Moreover, inequality in initial endowment of young leads to diverse career paths and unequal distribution of income even among those with the same initial skill. At the extensive margin, rich young workers with large initial endowment accept the lower apprentice wage and opt for the career path to become future managers, while poor young workers receive no training and work as routine workers for life. At the intensive margin, richer young workers receive more intensive training to acquire better managerial skill, which leads to a large inequality even among workers who receive training. Over time, a temporary adverse shock to initial endowment or the degree of commitment generates a persistent fall in intangible capital investment, aggregate production and rise in inequality.

The limited commitment is more severe when intangible capital becomes less firm-specific and moving across firms becomes easier for managers. This points to perhaps unintended consequences of liberalization of the labor market for skilled workers. Since European Union came into full force around 2000, skilled workers became more mobile across countries, especially from countries like Italy and Spain to countries like Germany and Britain. Before the 1990s financial crisis in Japan, skilled Japanese workers typically worked for a single firm for life. This practice changed after the crisis. Skilled workers switch jobs more across firms. While liberalization of the labor market of skilled workers improves matching between workers and employers, the induced limited commitment may reduce intangible capital accumulation. In Japan, the fraction of young workers who got career-type permanent jobs declined relative to temporary jobs and career-type workers appear to receive less intensive on-the-job training after the crisis.<sup>1</sup>

Taking as given the limited commitment, our theory also provides some other guidance for public policy. The competitive economy under limited commitment exhibits misallocation in matching between old managers and young workers with heterogeneous initial endowment and skill. Rich young workers receive more training regardless of their talent while poor but

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<sup>1</sup>Up to the early 1990s, Japanese large firms often sent their most promising career employees to the oversea graduate programs at the firms' expense. This practices became less common since the late 1990s.

talented young workers receive less training under financing constraint. If the government is better than private lenders in enforcing debt repayment so that it can relax the financing constraint, then the government can provide loan for workers to receive training, which improves the resource allocation. If government is no better than private lenders in enforcing debtors (old managers) to pay, the policy option becomes more delicate. Government can provide subsidy for training poor young. But because government has difficulty in enforcing old managers to pay their liabilities (including tax liability), the subsidy must be financed by taxing workers (like payroll tax). Then the training subsidy may lead to too much training compared to the efficient allocation, which must be offset by the rationing of training based on the initial skill of young workers.<sup>2</sup>

Our paper is related to a few lines of literature. First, our model is based on Boyd and Prescott (1987)[24] about firms as dynamic coalitions for intangible capital accumulation. Chari and Hopenhayn (1991)[10] apply Boyd and Prescott (1987) for endogenous technology adoption, while Kim (2006)[18] introduces financing constraint to Chari and Hopenhayn (1991) to show how difference in financing constraint leads to a large gap in TFP across countries. We introduce limited commitment and heterogeneous initial endowment and skill of young workers to Boyd and Prescott (1987). With these additional ingredients, we can study how small difference in initial conditions leads to a large inequality across workers and how a small shock to endowment or the degree of commitment leads to a persistent decrease in intangible capital accumulation and aggregate production.

Secondly related is a vast literature on wealth distribution, human capital accumulation and occupational choices in the presence of financial frictions. If we restrict attention to a most closely related literature, Galor and Zeira (1993)[13] examine how indivisible human capital accumulation and financial friction lead to endogenous wealth distribution when parents care about their children and leave bequest. Banerjee and Newman (1993)[3] show rich dynamics of wealth distribution and growth as a result of occupational choices. Although we have similar extensive margin of human capital accumulation through occupational choices, we introduce a richer technology for accumulating intangible capital which uses resources as

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<sup>2</sup>If people can change the initial skill level at the start of working life through education, then people would start investing earlier to acquire better initial skill. Young people with larger initial endowment would have an advantage of acquiring initial skill through better education. Government can improve basic education to improve the initial skill, to create equal opportunity instead of equal outcome across all workers. This is related to Benabou(2002)[4].

well as skills of managers and trainees as inputs for accumulating intangible capital. This leads to a richer distribution dynamics through the matching between skilled managers and heterogeneous young workers.<sup>3</sup>

The third related literature is the macro literature on financial friction and capital misallocation. Kiyotaki (1998)[19], Buera (2009)[5], Buera, Kaboski and Shin (2011)[6] and Buera and Shin (2013)[7] and Moll (2014)[23] for example study how financial frictions affect misallocation of capital and economic growth. Our research is complementary to theirs because they focus on the allocation and accumulation of tangible capital and we focus on intangible capital. This addition is relevant because financial frictions may be more severe for intangible capital which is a large component of skilled workers' asset.<sup>4</sup>

Our theory is consistent with empirical findings on the level and the slope of workers' income profile in recent papers. Kambourov and Manovski (2009)[17] find that an increase in occupational mobility explains substantially why life-cycle earning profile becomes flatter, the experience premium becomes smaller and the inequality rises within group for more recent cohorts. While they emphasize the role of increasing occupation specific risks, we attribute the flattening life-cycle earning profile to the slowdown in investment in intangibles.<sup>5</sup> Gouvenen, Karahan, Ozkan and Song (2016)[15] find that there is a strong positive association between the level of lifetime earning and how much earning grow over the life cycle.<sup>6,7</sup>

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<sup>3</sup>On the other hand, we abstract from the endogenous bequest until the last section. See Banerjee and Duflo (2005)[2] and Matsuyama (2007)[22] for survey of more literature. See also Lucas (1992)[21] and Ljungqvist and Sargent (2012)[20] for a literature of endogenous financing constraints due to private information and hidden action, which we abstract in our model.

<sup>4</sup>Caggese and Perez (2017)[8] study the implication of difference in collateralizability between intangible and tangible capital for the misallocation across firms. Caselli and Gennaioli (2013)[9] explore a similar mechanism by focusing on the allocation of the control right of dynastic firms. See also Eisfeldt and Papanikolaou (2013)[12] for the asset price implications of organization capital - a specific form of intangible capital.

<sup>5</sup>Consistent with the theory, Heckman, Lochner and Taber (1998)[16] show that to account for skill premium, it is important to differentiate the potential income and the actual income during on-the-job training.

<sup>6</sup>Guiso, Pistaferri and Schivardi (2013)[14] find that firms operating in less financially developed markets offer lower entry wages but faster wage growth than firms in more financially developed markets, which is consistent with Michelacci and Quadrini (2009) in the earlier footnote. Guiso et. al. (2013) also find managers' income profile is steeper in financially underdeveloped market, which is consistent with our theory.

<sup>7</sup>Our framework is also motivated by literature on growth accounting, such as Corrado, Hulten and Sichel

## 2 Model

### 2.1 Environment

We consider an overlapping generations (OLG) model. To simplify presentation, we focus in this section on a OLG model, where a unit measure of agents is born every period and lives for two periods. As will be seen later, the framework can be extended to an environment where agents live for more than two periods.

A new born agent is endowed with  $e$  units of final goods and  $\kappa$  units of initial skill. The final goods endowment and initial skill are exogenous, heterogeneous across agents, and publicly observable. They follow a joint distribution  $F_t(\kappa, e)$  for agents born in period  $t$ . The utility function of an agent born at date  $t$  is given by

$$U_t = U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o,$$

where  $c_t^y$  and  $c_{t+1}^o$  are consumption when young at date  $t$  and when old at date  $t + 1$ , and  $\beta \in (0, 1)$  is a utility discount factor. Everyone works for one unit of time without disutility, either as a worker or a manager.

There is a continuum of firms in the economy. Each firm is a coalition of current and future managers. A firm has two technologies; the technology to produce final goods and the technology to train young workers to become future managers. When a firm has a group of current managers with sum of their managerial skill (intangible capital) of  $k$  and hires  $l$  workers, it can produce

$$y = A_t k^\alpha l^{1-\alpha}, \tag{1}$$

units of final goods, where  $\alpha \in (0, 1)$ . The evolution of aggregate productivity  $A_t$  is deterministic.

When a firm with  $k$  units of intangible capital trains  $n$  number of young workers with identical initial skill  $\kappa$ , making  $i$  units of investment of final goods, it can train them to become future managers with intangible capital  $k'$  as

$$nk' = (i/b)^{\frac{1}{1+\psi}} [k^\eta (n\kappa)^{1-\eta}]^{\frac{\psi}{1+\psi}}, \tag{2}$$

where  $b > 0$  and  $\psi, \eta \in (0, 1)$  are constant parameters. There is no uncertainty about the outcome of training. The left-hand side (LHS) is output of training - total intangible capital  

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(2009)[11], which shows that intangible capital accumulation has become a dominant source of growth in labor productivity.

of the firm, and the right-hand side (RHS) are inputs of training - final goods  $i$ , intangible capital  $k$  and the aggregate initial skill of trainees  $n\kappa$ . Following Rothschild and White (1995)[25] on education, we consider trainees with initial skill  $\kappa$  as an input and trainees with intangible capital  $k'$  as output (while the other inputs are intangible capital of current managers (teachers) and final goods (resource)). In contrast with Rothschild and White (1995), we ignore peer group effect among the trainees and the training function is constant returns to scale. Thus we can think of training function of an individual trainee by dividing both sides of (2) by the number of trainees as

$$k' = \left(\tilde{i}/b\right)^{\frac{1}{1+\psi}} \left(\tilde{k}^\eta \kappa^{1-\eta}\right)^{\frac{\psi}{1+\psi}}, \quad (3)$$

where  $\tilde{i} = i/n$  and  $\tilde{k} = k/n$  are final goods and intangible capital used to train the individual trainee. More generally, our formulation allows the firm to split its intangible capital (training ability) to train multiple groups of trainees with different levels of initial skill  $\kappa$  to obtain different level of managerial skill  $k'$  - as long as the sum of intangible capital used to train trainees equals  $k$ . Solving (3) in terms of the required final goods, we get the investment cost function of an individual trainee as

$$\tilde{i} = b \left(\frac{k'}{\tilde{k}^\eta \kappa^{1-\eta}}\right)^\psi k' \equiv \Phi(k', \tilde{k}, \kappa). \quad (4)$$

The cost is increasing in intangible capital acquired and decreasing in intangible capital input from managers and the trainee's initial skill:  $\Phi_{k'} > 0$  and  $\Phi_{\tilde{k}}, \Phi_{\kappa} < 0$ .

A young agent supplies labor regardless of whether he receives training or not. The training of young period affects his occupation later in his life. If trained when young, the agent can become a manager when old. If not trained when young, the agent loses his initial skill, cannot be a manager when old, and continues to be a routine worker.

As noted before, each firm is a dynamic coalition of managers and trainees in current and future periods. There is no resource required for firms and employees to match, and there is no penalty for workers and managers to switch firms between periods. Intangible capital acquired through training, however, is partly specific to the firm: If a manager moves to another firm or start a new firm in the next period, his intangible capital will shrink from  $k'$  to  $(1 - \theta)k'$ . The parameter  $\theta \in (0, 1)$  is a measure of firm specificity. Conversely, if a firm recruits a manager from another firm in the next period, the firm needs to recruit a manager with intangible capital  $\frac{k'}{1-\theta}$  in order to replace a home-trained manager with intangible capital  $k'$ .

Many firms (or coalitions) compete for routine workers by offering spot wage rate and for managers and trainees by offering long-term contracts which specify life-time profile of earnings and on-the-job training. Because intangible capital is firm specific, it is allocated within firms without being traded in the external market in equilibrium. The coalition determines jointly training and earnings profiles for all trainees and managers in the coalition. Training decisions include which young workers should be trained (the extensive margin of training), and how much investment of final goods and managers' intangible capital should be allocated to train each young worker (the intensive margin of training). Equilibrium allocation of earnings and training must be coalition-proof. The within-firm allocation of intangible capital and training decisions is an equilibrium only if any subgroup of agents within the firm cannot be better off by forming a sub-coalition.<sup>8</sup> The coalition-proof equilibrium is supported by an internal market in which current managers rent their intangible capital for training young workers. We leave more detailed discussion of the equivalence to Section C in the Appendix.

Denote the rental rate of the intangible capital of firm- $f$  to be  $r_t^f$ . The return of a unit of intangible capital of firm- $f$  managers is

$$x_t^f \equiv \max_l (r_t^f + A_t l^{1-\alpha} - w_t l), \quad (5)$$

where  $w_t$  is the wage rate of routine labor in the competitive market. The first term in the RHS is the return on providing training (teaching), while the gap between the second and the third is the profit from production for a unit of intangible capital.

Agents can smooth their consumption by trading riskless bonds. Each bond promises one final good in the following period, and the price is denoted  $q_t$ . The amount of bond each agent can issue is constrained by the limitations for lenders to enforce the issuer (borrower) to repay his debt in future. Because there is no limitation for lenders to observe and seize the entire wage income of routine worker, a young routine worker can issue up to  $w_{t+1}$  units of bond at date  $t$ .

The borrowing of trainees is more limited. Although the firm observes trainee's intangible capital  $k'$ , the firm cannot prevent the trainee from moving to another firm with  $(1 - \theta)k'$  intangible capital to earn income. When we denote  $x_{t+1}$  as the equilibrium return of intangible capital at period  $t + 1$ , the trainee would be able to earn  $(1 - \theta)x_{t+1}k'$  by defaulting on

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<sup>8</sup>We assume intangible capital does not shrink when a coalition is split into two sub-coalitions.



his debt and move to another firm. Thus the trainee will repay the debt  $d$  to firm- $f$  in the next period if and only if the earning after repaying debt is at least as high as the outside income as:

$$x_{t+1}^f k' - d \geq (1 - \theta)x_{t+1} k'. \quad (6)$$

When intangible capital is more firm specific with a smaller  $\theta$ , the outside income is lower, and the trainee can sell more bonds at period  $t$ .

## 2.2 Equilibrium

Denote the expected utility of a young future manager of type  $(\kappa, e)$  at firm  $f$  in period  $t$  to be  $V_{y,t}^{m,f}(\kappa, e)$ . It solves the following optimization problem:

$$V_{y,t}^{m,f}(\kappa, e) = \max_{c^y, c^o, d, \tilde{k}, k'} [\ln c^y + E_t \beta \ln c^o], \quad (7)$$

$$s.t., \quad c^y = w_t + q_t d - \Phi(k', \tilde{k}, \kappa) - r_t^f \tilde{k} + e, \quad (8)$$

$$c^o = x_{t+1}^f k' - d, \quad (9)$$

$$x_{t+1}^f k' - d \geq (1 - \theta)x_{t+1} k', \quad (10)$$

where  $\Phi(k', \tilde{k}, \kappa)$  denotes the cost function of intangible capital investment given by (4).

From trainee's budget constraint when young (8), investment in intangible capital decreases a trainee's consumption when young. From his budget constraint when old (9), the investment increases his consumption when old. Although borrowing against his future income allows him to smooth his consumption profile over his life cycle, it is constrained by the limitation of commitment. When the induced borrowing constraint (10) is binding, he faces trade-off between intangible capital accumulation and steepness in his consumption profile. The trade-off is more severe when the intangible capital is less specific.

Denote the equilibrium payoff of a young trainee of type  $(\kappa, e)$  to be  $V_{y,t}^m(\kappa, e)$ . Because trainees are free to choose which firm to be trained by,

$$V_{y,t}^m(\kappa, e) \geq \max_f V_{y,t}^{m,f}(\kappa, e).$$

So, for a firm  $f$  to hire young trainees of type  $(\kappa, e)$  in equilibrium, we need  $V_{y,t}^{m,f}(\kappa, e) = V_{y,t}^m(\kappa, e)$ . Then, we can show that the rental rate of intangible capital for training and the

the return on intangible capital of firm are equalized across firms in equilibrium as:

$$\begin{aligned} r_t^f &= r_t, \\ x_t^f &= x_t. \end{aligned}$$

(See Section C of the Appendix for details.)

Denote the expected utility of a young agent who chooses to be a routine worker to be  $V_{y,t}^w(\kappa, e)$ . It solves the optimization problem:

$$V_{y,t}^w(\kappa, e) = \max_{c^y, c^o, d} [\ln c^y + \beta E_t \ln c^o], \quad (11)$$

$$s.t., \quad c^y = w_t + q_t d + e, \quad (12)$$

$$c^o = w_{t+1} - d, \quad (13)$$

$$d \leq w_{t+1}. \quad (14)$$

Because a routine worker can borrow against all of his future income as in (14) and does not need to accumulate intangible capital, he does not face the trade-off between intangible capital accumulation and consumption smoothing.

The occupational choice of a young agent of type  $(\kappa, e)$  solves the payoff of the young agent,  $V_{y,t}(\kappa, e)$ ,

$$V_{y,t}(\kappa, e) = \max \{V_{y,t}^m(\kappa, e), V_{y,t}^w(\kappa, e)\}. \quad (15)$$

Denote the set of young agents who choose to be trained as  $\Theta_t$ ,<sup>9</sup>

$$\Theta_t \equiv \{(\kappa, e) : V_{y,t}^m(\kappa, e) > V_{y,t}^w(\kappa, e)\}. \quad (16)$$

To summarize, decisions of agents at period  $t$  include young agents' consumption,  $c_t^y(\kappa, e)$ , young agents' occupational choice,  $\mathbb{I}\{(\kappa, e) \in \Theta_t\}$ , intangible capital accumulation decision,  $k' = k_t^+(\kappa, e)$ , the amount of intangible capital hired for training,  $\tilde{k} = \tilde{k}_t(\kappa, e)$ , the bond issue decision,  $d_t(\kappa, e)$ , and old agents' consumption,  $c_t^o(\kappa, e)$ .

The endogenous aggregate state variables are summarized by the aggregate supply of intangible capital  $K_t$ , and labor  $L_t$ . Given the agents' policy functions, the final goods market clearing condition is given by

$$\begin{aligned} A_t K_t^\alpha L_t^{1-\alpha} &= \int c_t^y(\kappa, e) dF_t(\kappa, e) + \int c_t^o(\kappa, e) dF_{t-1}(\kappa, e) \\ &+ \int_{(\kappa, e) \in \Theta_t} \Phi(k_t^+(\kappa, e), \tilde{k}_t(\kappa, e), \kappa) dF_t(\kappa, e). \end{aligned} \quad (17)$$

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<sup>9</sup>Here we assume young agents who are indifferent do not choose to be trained.

The first term in the RHS is consumption of young agents, the second is consumption of current old agents and the last term is investment of final goods. The wage rate for routine labor equals the marginal product of labor as

$$w_t = (1 - \alpha)A_t(K_t/L_t)^\alpha. \quad (18)$$

The equilibrium condition of the rental market of intangible capital for training is

$$K_t = \int_{(\kappa, e) \in \Theta_t} \tilde{k}_t(\kappa, e) dF_t(\kappa, e). \quad (19)$$

The laws of motion for aggregate capital and labor supply are

$$K_{t+1} = \int_{(\kappa, e) \in \Theta_t} k_t^+(\kappa, e) dF_t(\kappa, e) \quad (20)$$

$$L_{t+1} = 2 - \int_{(\kappa, e) \in \Theta_t} dF_t(\kappa, e). \quad (21)$$

**Definition 1.** Given the initial labor and capital supply  $K_0$  and  $L_0$ , a perfect foresight dynamic equilibrium is  $\{c_t^y(\kappa, e), c_t^o(\kappa, e), d_t(\kappa, e), k_t^+(\kappa, e), k_t(\kappa, e), V_{y,t}^m(\kappa, e), V_{y,t}^w(\kappa, e)\}_{\forall \kappa, e \text{ and } t \geq 0}$ ,  $\{r_t, x_t, q_t, w_t, \Theta_t\}_{t \geq 0}$ , and  $\{K_t, L_t\}_{t \geq 1}$ , such that

1. given prices  $r_t, x_t$  and  $q_t$ ,  $\{c_t^y(\kappa, e), c_t^o(\kappa, e), d_t(\kappa, e), k_t^+(\kappa, e), k_t(\kappa, e)\}_{\forall \kappa, e}$  solves the problem of type  $(\kappa, e)$  young agents at period  $t$ , with corresponding value functions  $V_{y,t}^m(\kappa, e)$  and  $V_{y,t}^w(\kappa, e)$ ;
2.  $(\kappa, e) \in \Theta_t$  if and only if  $V_{y,t}^m(\kappa, e) > V_{y,t}^w(\kappa, e)$ ;
3. all markets clear;
4. given  $k_t^+(\kappa, e)$  and  $\Theta_t$ ,  $K_{t+1}$  and  $L_{t+1}$  follow laws of motion, (20) and (21);
5. there does not exist  $r_t^f, x_t^f$  such that  $r_t^f \geq r_t, x_t^f \geq x_t, V_{y,t}^{mf}(\kappa, e) \geq V_{y,t}^m(\kappa, e)$  with some of the inequalities holding strictly.

### 3 Accumulation of Intangible Capital

In this section, we study intangible capital accumulation, first under full commitment as a benchmark and secondly under limited commitment as the main case. In both cases, the

total cost of training a young worker to accumulate intangible capital - sum of the costs of final goods and renting current manager's intangible capital for training - must be minimized as

$$\varphi_t(k_{t+1}; \kappa) = \min_{\tilde{k}_t} \left[ i_t + r_t \tilde{k}_t \right] = \min_{\tilde{k}_t} \left[ b \left( \frac{k_{t+1}}{\tilde{k}_t^\eta \kappa^{1-\eta}} \right)^\psi k_{t+1} + r_t \tilde{k}_t \right].$$

Using the first order condition with respect to  $\tilde{k}_t$ ,

$$r_t = \eta\psi \frac{i_t}{\tilde{k}_t} = \eta\psi b \left( \frac{k_{t+1}}{\tilde{k}_t^\eta \kappa^{1-\eta}} \right)^\psi \frac{k_{t+1}}{\tilde{k}_t},$$

we get the current manager's intangible capital used to train a young worker of type  $(\kappa, e)$  to acquire intangible capital  $k_{t+1}$  as,

$$\tilde{k}_t(\kappa, e) = \left[ b \frac{\eta\psi}{r_t} \left( \frac{k_{t+1}}{\kappa} \right)^{(1-\eta)\psi} \right]^{\frac{1}{1+\eta\psi}} k_{t+1},$$

and final goods used as

$$i_t(\kappa, e) = \left[ b \left( \frac{r_t}{\eta\psi} \right)^{\eta\psi} \left( \frac{k_{t+1}}{\kappa} \right)^{(1-\eta)\psi} \right]^{\frac{1}{1+\eta\psi}} k_{t+1}.$$

The total minimized cost becomes

$$\varphi_t(k_{t+1}; \kappa) = (1 + \eta\psi) \left[ b \left( \frac{r_t}{\eta\psi} \right)^{\eta\psi} \left( \frac{k_{t+1}}{\kappa} \right)^{(1-\eta)\psi} \right]^{\frac{1}{1+\eta\psi}} k_{t+1}.$$

### 3.1 Intangible Capital Accumulation under Full Commitment

When the intangible capital is entirely firm specific, i.e.,  $\theta = 1$ , managers can commit to repay debt from entire future profit. Under full commitment, trainees make intangible capital accumulation decisions to maximize permanent income. A trainee with talent  $\kappa$  chooses intangible capital accumulation to maximize the present value of the net returns:

$$X_t k_{t+1} - \varphi_t(k_{t+1}; \kappa),$$

where  $X_t$  is the discounted expected rate of return on intangible capital. In the two-period OLG model,  $X_t = q_t x_{t+1}$ .

Using the first order condition

$$X_t = \varphi'_t(k_{t+1}; \kappa) = (1 + \psi) \left[ b \left( \frac{r_t}{\eta\psi} \right)^{\eta\psi} \left( \frac{k_{t+1}}{\kappa} \right)^{(1-\eta)\psi} \right]^{\frac{1}{1+\eta\psi}},$$

the trainees' intangible capital when they are old is proportional to their initial skill on the intensive margin as,

$$\begin{aligned} k_{t+1} &= k_t^+(\kappa, e) = a_t \kappa, \text{ where} \\ a_t &= \left[ \frac{1}{b} \left( \frac{X_t}{1 + \psi} \right)^{1+\eta\psi} \left( \frac{\eta\psi}{r_t} \right)^{\eta\psi} \right]^{\frac{1}{(1-\eta)\psi}}. \end{aligned}$$

The present value of the net returns is

$$X_t k_{t+1} - \varphi_t(k_{t+1}; \kappa) = \frac{(1-\eta)\psi}{1+\psi} X_t k_{t+1} = \frac{(1-\eta)\psi}{1+\psi} X_t \cdot a_t \kappa,$$

where  $\frac{(1-\eta)\psi}{1+\psi}$  is the share of contribution of the agent's learning ability  $\kappa$  in accumulating intangible capital from (3).

Comparing the permanent income from being trained and being a routine worker for life, a young agent chooses to be trained if and only if he is talented enough,

$$\frac{(1-\eta)\psi}{1+\psi} X_t \cdot a_t \kappa > q_t w_{t+1}.$$

That is,  $(\kappa, e) \in \Theta_t$  if and only if

$$\kappa > \kappa_t^* \equiv \frac{q_t w_{t+1}}{\frac{(1-\eta)\psi}{1+\psi} X_t \cdot a_t}. \quad (22)$$

The training decision does not depend on his wealth endowment. Given the intensive margin choice,  $k_t^+(\kappa, e)$ , investment of final goods and the amount of managers' intangible capital used in training are proportional to the trainee's initial skill as:

$$\begin{aligned} i_t(\kappa, e) &= \left[ b \left( \frac{r_t}{\eta\psi} \right)^{\eta\psi} a_t^{(1-\eta)\psi} \right]^{\frac{1}{1+\eta\psi}} \kappa, \\ \tilde{k}_t(\kappa, e) &= \left[ b \frac{\eta\psi}{r_t} a_t^{(1-\eta)\psi} \right]^{\frac{1}{1+\eta\psi}} \kappa. \end{aligned}$$

Thus, the allocation of managers' intangible capital implies perfect assortative matching between trainee's initial skill and manager's productivity. The assortative matching result

is similar to that in Andersen and Smith (2010)[1]. We relax an assumption in Andersen and Smith (2010) that matching is one-to-one. Instead, a trainee can rent intangible capital from multiple managers and a manager can train multiple trainees. This makes the model more tractable. The distribution of intangible capital across managers is not an aggregate state variable, while the aggregate amount of intangible capital and labor supply are the only endogenous state variables.

### 3.2 Intangible Capital Accumulation under Binding Limited Commitment

When the borrowing constraint is binding for trainees because of low firm specificity of intangible capital, distortions show up on both extensive and intensive margin of training decisions.

Suppose the borrowing constraint for trainees is binding, then

$$c_t^y = e + w_t + \theta X_t k_{t+1} - \varphi_t(k_{t+1}; \kappa) \quad (23)$$

$$c_{t+1}^o = (1 - \theta) \frac{X_t}{q_t} k_{t+1} \quad (24)$$

The first order condition over  $k_{t+1}$  for trainees' problem, (7), is

$$\frac{\varphi'_t(k_{t+1}; \kappa) - \theta X_t}{c_t^y} = \beta \frac{(1 - \theta) \frac{X_t}{q_t}}{c_{t+1}^o} = \frac{\beta}{k_{t+1}}.$$

The LHS is the marginal cost and the RHS is the marginal benefit of acquiring intangible capital in terms of discounted utility. Using  $\varphi_t(k_{t+1}; \kappa) = \frac{1+\eta\psi}{1+\psi} \varphi'_t(k_{t+1}; \kappa) k_{t+1}$  and (23), we rewrite the first order condition as

$$\beta(e + w_t) = \Gamma_t \left( \frac{k_{t+1}}{\kappa} \right) k_{t+1}, \quad (25)$$

where

$$\Gamma_t \left( \frac{k_{t+1}}{\kappa} \right) \equiv \left( 1 + \beta \frac{1 + \eta\psi}{1 + \psi} \right) \varphi'_t(k_{t+1}; \kappa) - (1 + \beta)\theta X_t.$$

Solving (25) with respect to  $k_{t+1}$ , we find that on the intensive margin, trainees' intangible capital accumulation depends on both wealth endowment and initial skill,

$$k_{t+1} = k_t^+(e, \kappa) = \tilde{a}_t(\kappa, e)\kappa, \text{ where}$$

$$\tilde{a}_t(\kappa, e) \equiv \frac{\beta(e + w_t)}{\kappa \Gamma_t(k_t^+(e, \kappa)/\kappa)}.$$

Since  $\Gamma_t\left(\frac{k_{t+1}}{\kappa}\right)$  is an increasing function of  $\frac{k_{t+1}}{\kappa}$ , we can show that

$$\frac{d}{d\kappa}\tilde{a}_t(\kappa, e) < 0, \quad \frac{d}{de}\tilde{a}_t(\kappa, e) > 0$$

Because  $\tilde{a}(\kappa, e)$  is decreasing  $\kappa$ ,  $k_t^+(e, \kappa)$  is an increasing function of  $\kappa$  but not proportional to  $\kappa$ ,

$$\frac{\partial}{\partial\kappa}k_t^+(\kappa, e) > 0, \quad \frac{\partial^2}{\partial\kappa^2}k_t^+(\kappa, e) < 0.$$

This implies the intensive-margin distortion due to the borrowing constraint is more severe for trainees of higher skill. In addition,  $k_{t+1}$  is increasing in wealth endowment

$$\frac{\partial}{\partial e}k_t^+(\kappa, e) > 0,$$

which implies wealth endowment relaxes the intensive-margin distortion. Given  $k_t^+(\kappa, e)$ , we can solve for the value function of future manager  $V_{y,t}^m(\kappa, e)$  and the occupational choice by comparing it with value of a routine worker  $V_{y,t}^w(\kappa, e)$ . Because wealth endowment relaxes borrowing constraint and the intensive-margin distortion, it also affects the extensive margin. An young agent chooses to be trained if and only if

$$\kappa > \kappa_t^*(e), \quad \text{with } \kappa_t^{*'}(e) \leq 0.$$

Agents with higher wealth endowment are more likely to be trained. Figure 1 illustrates the occupational choice.

Because both investment in training and allocation of managers' intangible capital is distorted by the financial constraint,

$$i_t(\kappa, e) = \left[ \left( \frac{r_t}{\eta\psi} \right)^{\eta\psi} b\tilde{a}_t(\kappa, e)^{(1-\eta)\psi} \right]^{\frac{1}{1+\eta\psi}} \kappa,$$

$$\tilde{k}_t(\kappa, e) = \left[ \frac{\eta\psi}{r_t} b\tilde{a}_t(\kappa, e)^{(1-\eta)\psi} \right]^{\frac{1}{1+\eta\psi}} \kappa.$$

The perfect assortative matching between trainee's initial skill and manager's productivity in the case of efficient accumulation is now replaced by matching on two dimensions. Controlling for the initial skill, a young agent is more likely to receive training and be matched with more productive managers if his wealth endowment is high.

The efficiency loss due to the limitation for future managers to stay in the same firms is closely related to the link between profiles of marginal productivity, earning and consumption of a trainee over his life cycle, which we are going to examine next.

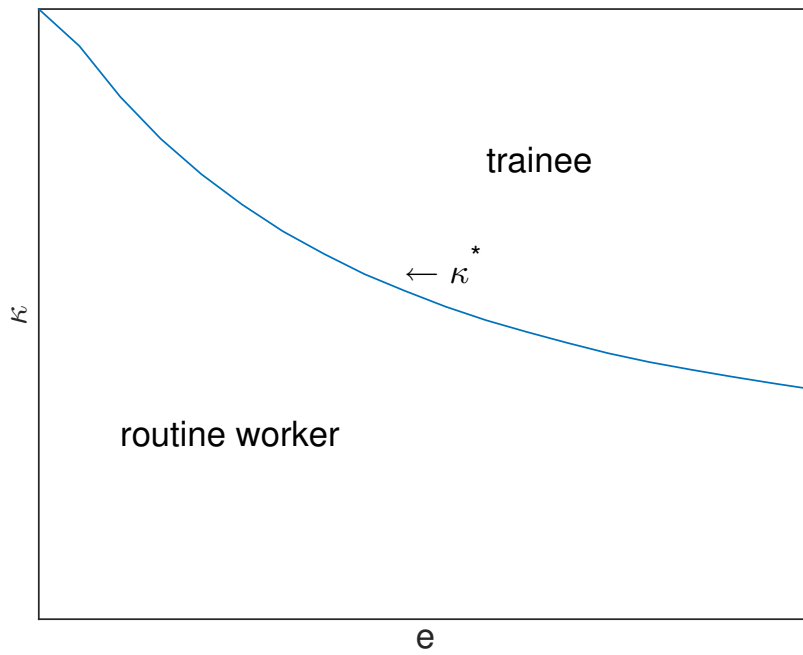


Figure 1: Occupational choice of young agents under limited commitment



## 4 Life-Cycle Patterns of Earnings, Productivity and Consumption

To examine the life cycle pattern of intangible capital accumulation, earnings and consumption, we now extend analysis to an OLG model in which everyone lives for three periods: young, middle and old periods. (The population size of each generation is unity as before.) Intangible capital accumulated when young depreciates between middle age and old age by factor  $\lambda \in (0, 1)$ . We emphasize the difference from the previous section, leaving the details in Section B of the Appendix.

Because the firm observes each manager's intangible capital that is firm specific, the firm can smooth of the consumption of the manager through the internal financial market. Let  $V_{y,t}^m(\kappa, e)$  be value function of a trainee of type  $(k, e)$  and  $V_{m,t}^m(k, d^f)$  be the value of middle-aged manager with intangible capital  $k$  and debt to the firm  $d^f$ . The trainee's problem is given by

$$V_{y,t}^{mf}(\kappa, e) = \max_{c^y, d^f, k'} \ln c^y + \beta V_{m,t+1}^m(k', d^f), \quad (26)$$

$$s.t., c^y = w_t - \varphi_t(k'; \kappa) + e + q_t d^f, \quad (27)$$

$$V_{m,t+1}^m(k', d^f) \geq V_{m,t+1}^m((1 - \theta)k', 0). \quad (28)$$

The second inequality (28) is the incentive constraint for the manager not to default on the debt to the firm by moving to another firm with reduced intangible capital, and is similar to (10) in the two-period OLG model.

Because the middle-aged manager faces a downward profit from intangible capital, he wants to smooth consumption by saving inside as well outside the firm. The saving inside the firm, however, is limited by the incentive constraint for the firm not to default and replace the old managers by hiring an outside manager. The rest of the saving is done through the external bond market,  $d_m^e \leq 0$ . Thus, the value function of the mid-age managers,  $V_{m,t}^m$ ,

solves the following problem,

$$V_{m,t}^m(k, d^f) = \max_{c^m, c^o, d_m^f, d_m^e} \ln c^m + \beta \ln c^o, \quad (29)$$

$$s.t., c^m = x_t k - d^f + q_t (d_m^f + d_m^e), \quad (30)$$

$$c^o = x_{t+1} \lambda k - d_m^f - d_m^e, \quad (31)$$

$$\lambda x_{t+1} k - d_m^f \leq \frac{1}{1-\theta} \lambda x_{t+1} k, \quad (32)$$

$$d_m^e \leq 0. \quad (33)$$

The second last inequality (31) is the incentive constraint for the firm not to replace the old managers with  $\lambda k$  units of intangible capital by hiring an outside manager with intangible capital  $\frac{\lambda k}{1-\theta}$ .

We can consider the life-cycle profile of the marginal product net of training cost of a trainee-manager as

$$(m_{y,t}^m(\kappa, e), m_{m,t+1}^m(\kappa, e), m_{o,t+2}^m(\kappa, e)) = (w_t - \varphi_t(k_t^+(\kappa, e); \kappa), x_{t+1} k_t^+(\kappa, e), x_{t+2} \lambda k_t^+(\kappa, e)).$$

Generally productivity (marginal product net of training cost) of a manager is humped shape. It is low when young because of a large training cost, is high in middle age with large intangible capital, before declining with depreciation of intangible capital in old age. The earning profile from the firm differs from the productivity by borrowing from the firm:

$$(y_{y,t}^m(\kappa, e), y_{m,t+1}^m(\kappa, e), y_{o,t+2}^m(\kappa, e)) = (m_{y,t}^m(\kappa, e) + q_t d_t^f(\kappa, e), m_{m,t+1}^m(\kappa, e) - d_t^f(\kappa, e) + q_{t+1} d_{m,t+1}^f(\kappa, e), m_{o,t+2}^m(\kappa, e) - d_{m,t+1}^f(\kappa, e)).$$

The earning profile is smoother (or less humped) than productivity over the life cycle, because the manager borrows from the firm when young, repays and saves in the firm when middle-aged, and is paid more than its marginal product by the return from saving in the firm,  $-d_{m,t+1}^f(\kappa, e) > 0$ .

The consumption over the life cycle is different from the earning by initial endowment as well as saving in the external bond market  $d_{m,t+1}^e < 0$  in middle age:

$$(c_{y,t}^m(\kappa, e), c_{m,t+1}^m(\kappa, e), c_{o,t+2}^m(\kappa, e)) = (y_{y,t}^m(\kappa, e) + e, y_{m,t+1}^m(\kappa, e) + q_{t+1} d_{m,t+1}^e(\kappa, e), y_{o,t+2}^m(\kappa, e) - d_{m,t+1}^e(\kappa, e)).$$

Thus consumption is even more smooth than the earning from the firm. Therefore, the productivity is most humped, the earning from the firm is less humped, and the consumption is most smooth over the life cycle. This general pattern of life cycle profiles can be verified empirically.

In the previous section, we show the intensity in intangible capital accumulation depends on both initial skill,  $\kappa$ , and wealth endowment,  $e$ . The exactly the same analysis holds for the OLG model with 3-period lived agents, if we denote the expected discounted return on intangible capital at young period as

$$X_t = q_t (x_{t+1} + \lambda q_{t+1} x_{t+2}).$$

The first term in the RHS is the discounted return in middle age and the last term is the discounted return in old age.

We use a numerical example to illustrate the life-cycle patterns of productivity, earning and consumption. The parameter values in the numerical example is reported in Table 1. We assume that initial skill and wealth endowment are independent from each other,  $F_t(\kappa, e) = G_t(e)H(\kappa)$ . The initial skill distribution is uniform,  $H(\kappa) \sim U([\kappa_L, \kappa_H])$ . There is a mass,  $1 - \omega$ , of agents with not initial wealth endowment. Conditional on receiving positive wealth endowment, the endowment follows a uniform distribution  $G_t(e) = (1 - \omega)\mathbb{I}(e \geq e_L) + \omega \frac{e - e_L}{e_H - e_L} \mathbb{I}(e_H \geq e \geq e_L)$ . Most other parameters are standard. We think of a period as 12 years. So the annualized discount factor is 0.976. The income share of the intangible capital is set to  $\alpha = 0.3$ . We assume that specificity parameter of the intangible capital is  $\theta = 0.1$ . Other findings in later sections are also computed using these parameter values as a benchmark.

Figure 2 illustrates how different initial wealth endowment affects the life-cycle profile of productivity, earning and consumption of among equally talented trainees. It shows that as a trainee has a larger wealth endowment, his life-cycle profiles are more hump-shaped, a result of more intensive intangible capital accumulation. Figure 3 illustrates how different initial skill affects the life-cycle profile among trainees with equally large initial wealth endowment. It shows that trainees with higher initial skill accumulates more intangible capital, and their life cycle profiles are more hump-shaped, controlling the initial wealth endowment.

While intangible capital accumulation is an increasing function of both wealth endowment and initial skill of trainees, the earning of young trainees is different. The earning of young trainees is an increasing function of the initial talent, while decreasing function

fraction of positive endowment $\omega$	0.8
support of endowment, $[e_L, e_H]$	$[0, 1]$
support of initial skill $\kappa$ , $[\kappa_L, \kappa_H]$	$[0, 1]$
share of intangibles $\alpha$	0.3
depreciation rate of intangible capital $\lambda$	0.5
training cost parameter $b$	0.01
share parameter of skill composite $\psi$	2
share parameter of manager's skill $\eta$	0.5
utility discount factor $\beta$	0.75
specificity of intangible capital $\theta$	0.1

Table 1: Parameter values used in model simulation

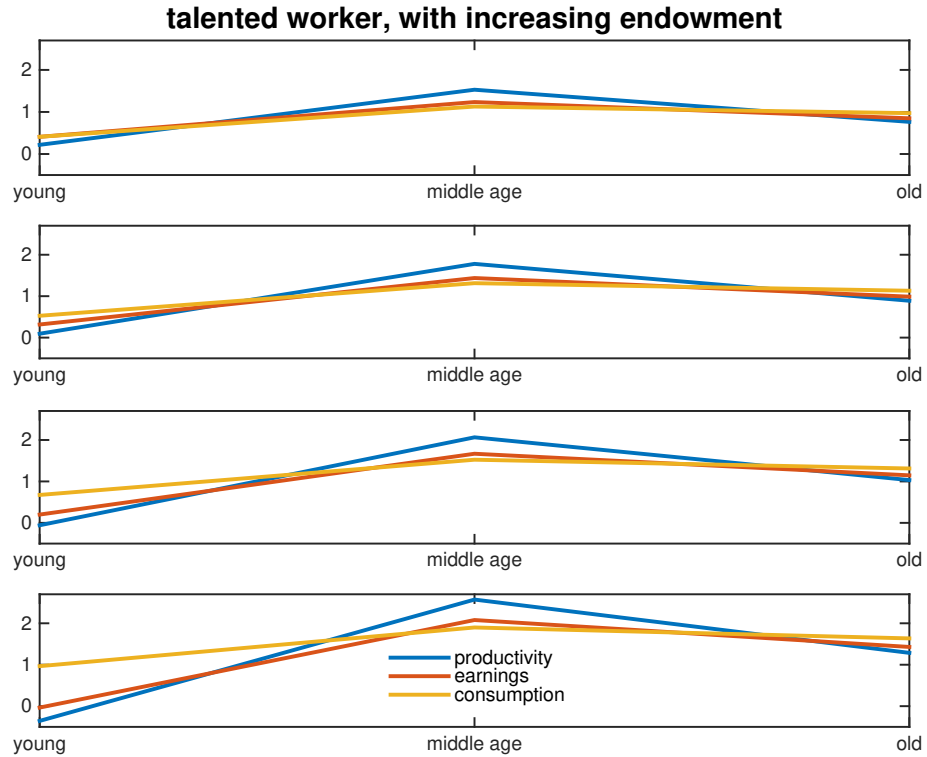


Figure 2: Life-cycle profiles of trainees of high initial skill.

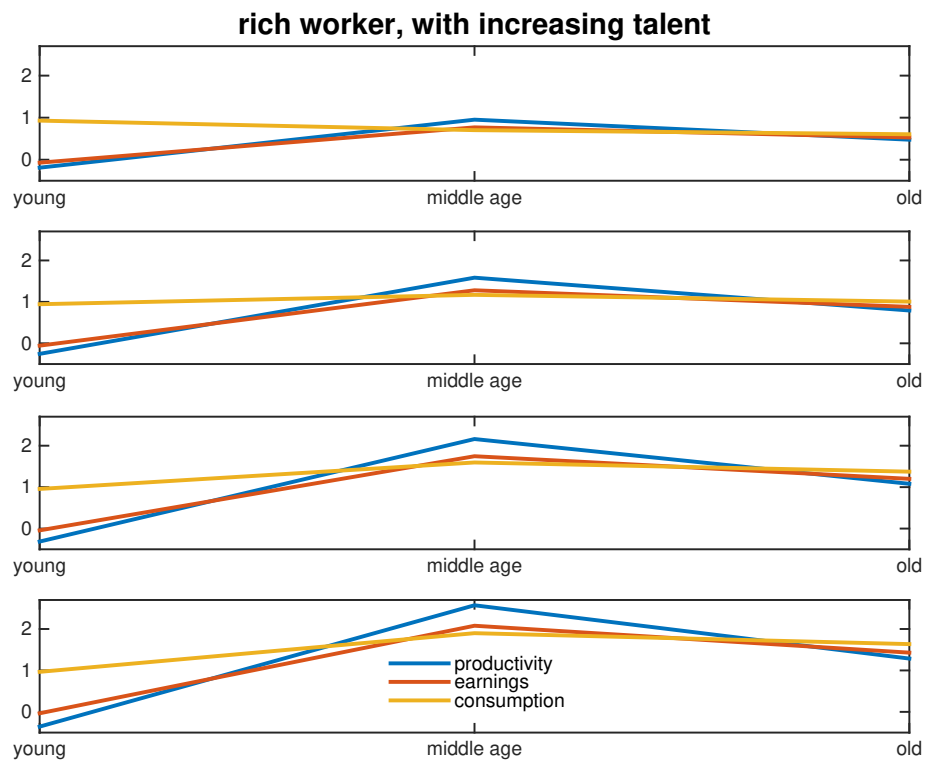


Figure 3: Life-cycle profiles of trainees of high wealth endowment.

of the wealth endowment. The difference arises from the cost of training. When wealthier trainees invest more in training, the cost of training increases more relative to the increase in their future productivity. They are trading current earning for future earnings. When more skilled trainees invest more in training, the cost of training decreases for the same amount of intangible capital accumulation. Both the current and future income of a more skilled trainee could be higher than a less skilled trainee.

## 5 Income Inequality and Intangible Capital Accumulation

In this section, we study the effect of intangible capital accumulation on inequality in the present value of lifetime income. Denote the present value of life-time income to be  $\mathcal{Y}(\kappa, e)$ .

$$\mathcal{Y}(\kappa, e) = \begin{cases} w(1 + q + q^2), & \text{if } \kappa \leq \kappa^*(e), \\ y_y^m(\kappa, e) + qy_m^m(\kappa, e) + q^2y_o^m(\kappa, e), & \text{if } \kappa > \kappa^*(e). \end{cases}$$

As is shown in Section 3.1, efficient accumulation can be implemented when intangible capital is entirely firm specific. In this case,  $\kappa^*(e)$  is independent of  $e$ . Young agents' occupational choice depends only on their present value of income. An agent chooses to be trained if and only if the present value of income from being a trainee is higher than that from being a routine worker for life. Among trainees, their present value of income is linear in their initial skill. These features are illustrated in Figure 4. The present value of lifetime income of the most talented agent is about 14% higher than that of a routine worker in our numerical example. Intangible capital accumulation does not induce too much inequality in the present value of income.

When intangible capital is only partially firm specific so that the borrowing constraint is binding for trainees, the wealth endowment and initial skill have a much bigger effect on their present value of income through their effect on intangible capital accumulation. In this case, young agents who receive training have upward-sloping consumption profiles from young to middle age, and the slope becomes steeper with more intensive training. To compensate for the rising slope, the “premium” in the present value of income of a trainee needs to be increasing in intangible capital accumulation. These features are illustrated in Figure 5.

Young workers with initial skill and wealth endowment of  $(\kappa^*(e), e)$  are indifferent between

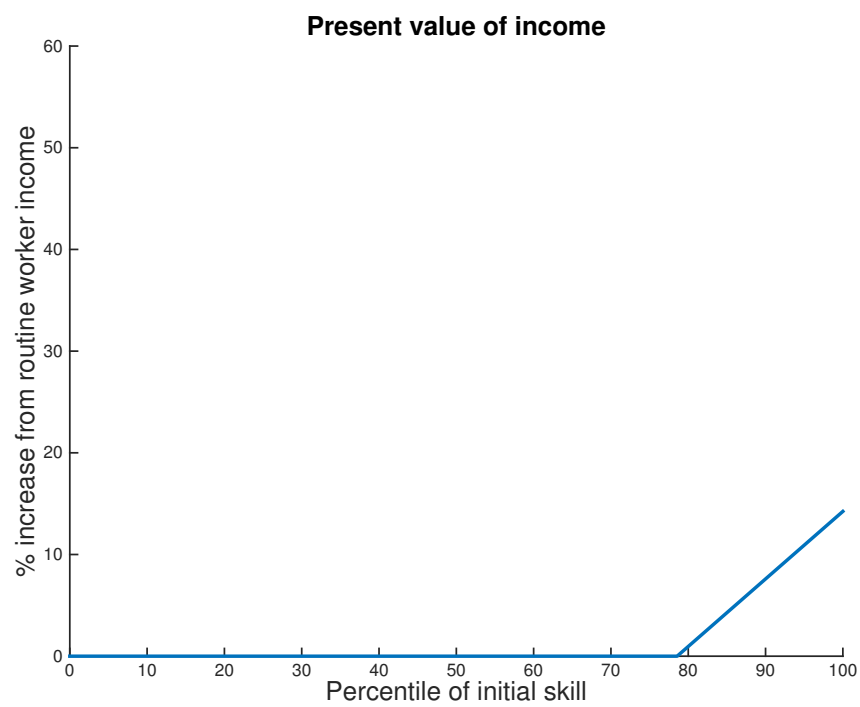


Figure 4: Income distribution under efficient intangible capital accumulation.

being a routine worker and a trainee. In order to make them indifferent, they need to receive premia in permanent income from receiving training. Moreover, the premia in the permanent income at the threshold is larger for those who have smaller wealth endowment. For example, for those with high wealth endowment of  $e = 1$ , the skill threshold equals  $\kappa^*(1) = 0.38$  and the premium is about 4% of the present value of routine workers. For those with low wealth of  $e = 0.05$ , the skill threshold is  $\kappa^*(0.05) = 0.93$  and the premium is about 15%. The borrowing constraint for young trainees is tighter when they have smaller wealth and receive more intensive training with higher initial skill at  $(\kappa^*(e), e) = (0.93, 0.05)$  than when their type is low skill and high wealth at  $(0.38, 1)$ .

Controlling for the initial skill, the income premium for trainees is an increasing function of wealth endowment. For trainees with the highest initial skill ( $\kappa = 1$ ), the income premium equals 15% with  $e = 0.05$  and equals to 55% with  $e = 1$ . Controlling for the initial skill, a trainee with higher wealth endowment accumulates more intangible capital and receives a larger premium to compensate for steeper consumption profile. When extended to an economy with endogenous bequest, the complementarity between wealth endowment and intangible capital accumulation would have profound implications on social mobility and misallocation of intangible capital across generations.

The binding constraint of limited commitment is critical to explain a large gap in the permanent income. The trainees with the highest skill and highest wealth endowment receive an income premium of 55% over the permanent income of routine worker when the constraint is binding, whereas the highest income premium is around 14% when the constraint is not binding.

## 6 Slow Recovery and Intangible Capital Accumulation

In this section, we study the time series implications of our framework for misallocation of intangible capital and economic fluctuations, by conducting two numerical experiments: first, an unexpected negative shock to agents' wealth endowment; second, an unexpected negative shock to specificity of intangible capital. Experiment 1 is meant to capture the effect of collapse of asset values and wealth endowment perhaps due to financial crisis. Experiment 2 tries to examine the effect of changes in the labor market. During "the lost two decades" of mid-1990s and mid-2010s in Japan, their labor market underwent a structural change:



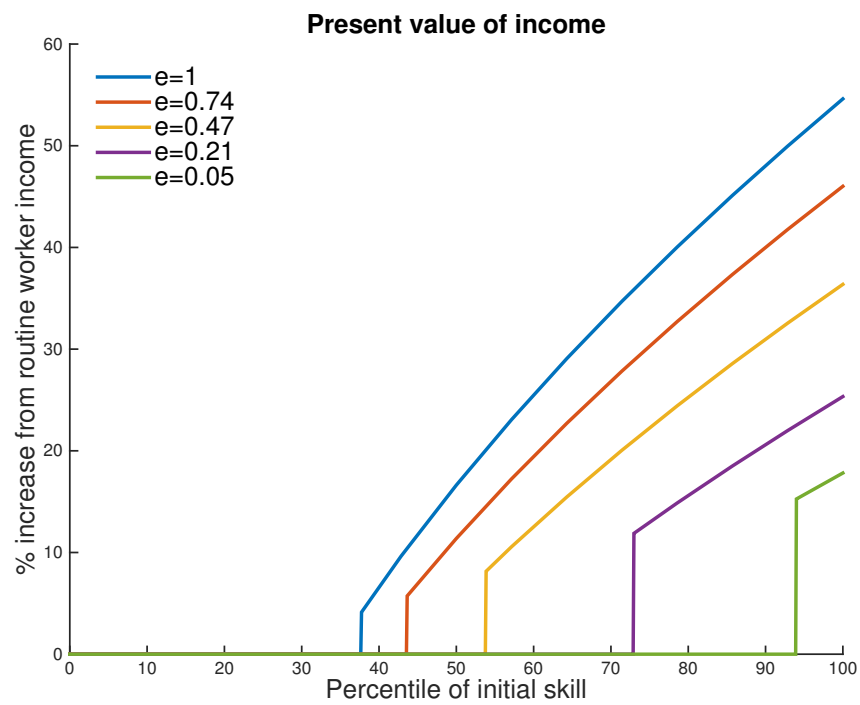


Figure 5: Income distribution under binding limited commitment.

the relationship between workers and firms becomes less likely to last for life-time, and permanent workers are more mobile with the development of labor market for mid-career workers - a sign of declining specificity of intangible capital.

## 6.1 Experiment 1: Negative Shock to Endowment

The negative shock to endowment is modeled as a shock to the total measure of young agents with positive endowment,  $\omega_t$ , keeping fixed the conditional distribution of young agents with positive endowment. Initially  $\omega_t$  drops by 10% from 0.8 to 0.72. After the initial shock,  $\omega_t$  converges gradually to the original level, with a half life of about 2 periods.

Figure 6 illustrates the dynamic responses of intangible capital,  $K_t$ , output,  $Y_t$ , occupational choice,  $\kappa_t^*(e)$ , and trainees' intangible capital accumulation,  $k_t^+(\kappa, e)$ . The dotted lines are aggregate responses in an unconstrained economy where there is no constraint on commitment and borrowing. Compared to the unconstrained benchmark, the economy with binding limited commitment recovers more sluggishly. The half-life of intangible capital decline is about 6 to 7 periods in the economy with binding limited commitment, while it is about 4 periods in the unconstrained benchmark. As a result, the recession measured in aggregate output is deeper and lasts longer. The initial drop in aggregate output is about 0.5% in the economy with binding limited commitment while it is about 0.3% in the unconstrained benchmark.<sup>10</sup>

The slow recovery is related to misallocation of intangible capital both on the extensive margin and intensive margin on the transition. On the extensive margin, training is allocated to agents with high initial wealth but low ability.  $\kappa_t^*(e)$  dropped by 4% to 5% for agents with high wealth endowment but decreases by 2.5% for agents with low wealth endowment. On the intensive margin, agents with high wealth endowment receives relatively even more training on the transition path than in the steady state. While training for high-skill trainees with low wealth endowment drops by 0.5% at the trough, it drops by 0.2% for high-skill trainees with high wealth endowment.

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<sup>10</sup>In the unconstrained economy, initially intangible capital initially decreases sharper than constrained economy and the recovery of output is non-monotonic. In the unconstrained economy, intangible capital serves as a buffer to smooth consumption against endowment shock more, reducing their investment at both intensive and extensive margin initially, resulting more workers in the following period.

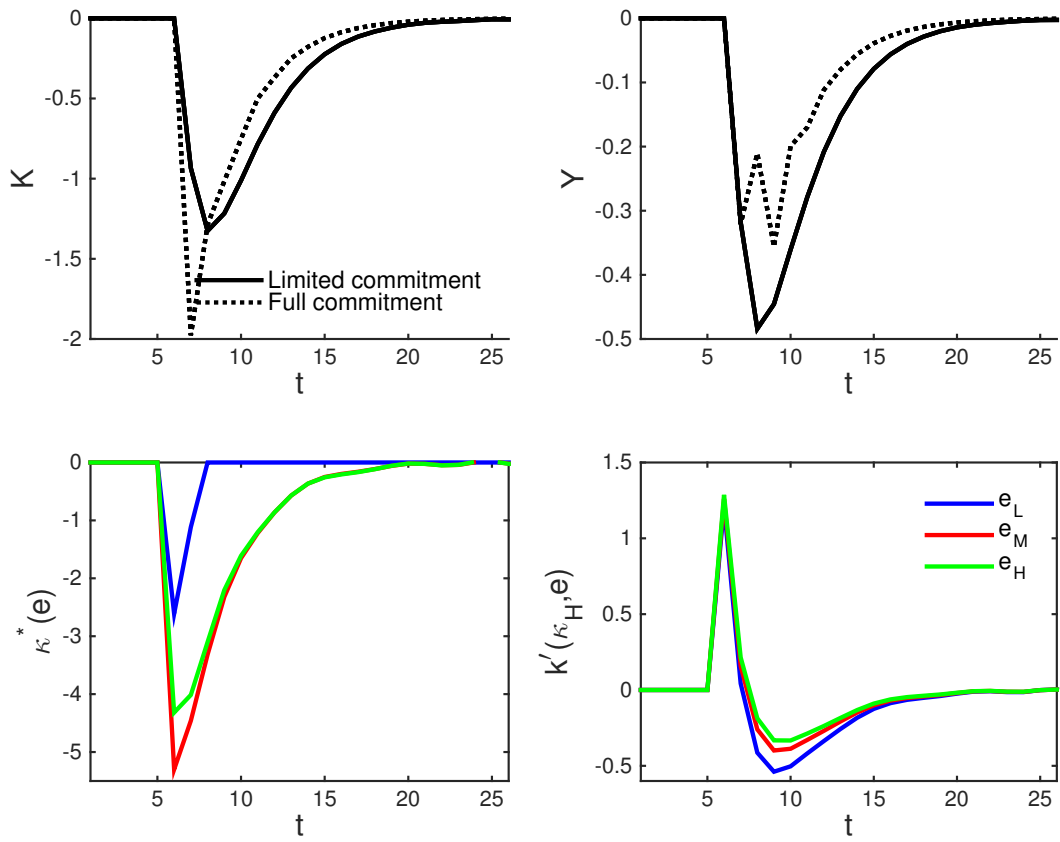


Figure 6: Dynamic response to negative endowment shock.

## 6.2 Experiment 2: Negative Shock to Specificity of the Intangible Capital

The negative shock to the specificity of intangible capital is modeled as a shock to  $\theta_t$ , with a 10% initial drop and a half life of about 2 periods.

When the specificity decreases, aggregate intangible capital stock and output decrease significantly and persistently with binding constraint of limited commitment. In contrast, in an unconstrained benchmark, all equilibrium variables remains constant as long as the constraint of limited commitment is not binding.

The misallocation of intangible capital on extensive and intensive margin along the transition path is clearer than in Experiment 1.  $\kappa_t^*(e)$  drops by more than 2% for agents with high wealth endowment while there is no response in  $\kappa_t^*(e)$  for agents with low wealth endowment. Among agents with high initial skill, the decline in intangible capital accumulation on the intensive margin is more severe for agents with low wealth endowment. At the trough, the intangible capital accumulation drops by 2.4% for those with low wealth endowment while it drops by 1.7% for those with high wealth endowment.

## 7 Conclusion

Our paper offers a tractable framework to study how intangible capital accumulation within firms interacts with income and consumption of managers at the micro level and aggregate productivity growth at the macro level. We show that when there is a negative shock to endowment or degree of firm specificity of intangible capital, labor productivity falls and income becomes more unequal persistently as we observe in developed countries in recent decades.

Two particular features of intangible capital (managerial skill) contribute to the interaction. First, intangible capital is not directly transferrable and needs to be accumulated through costly training on the job. Then, accumulation of intangible capital leads to a humped-shape profile of trainee-managers' productivity over the life cycle. Second, intangible capital is hard to pledge as collateral because future managers cannot be forced to stay and work in the same firm. This makes it harder for future managers to smooth consumption over lifetime, and in turn reduces intangible capital accumulation and increases income inequality because intangible capital accumulation must be compensated for the induced

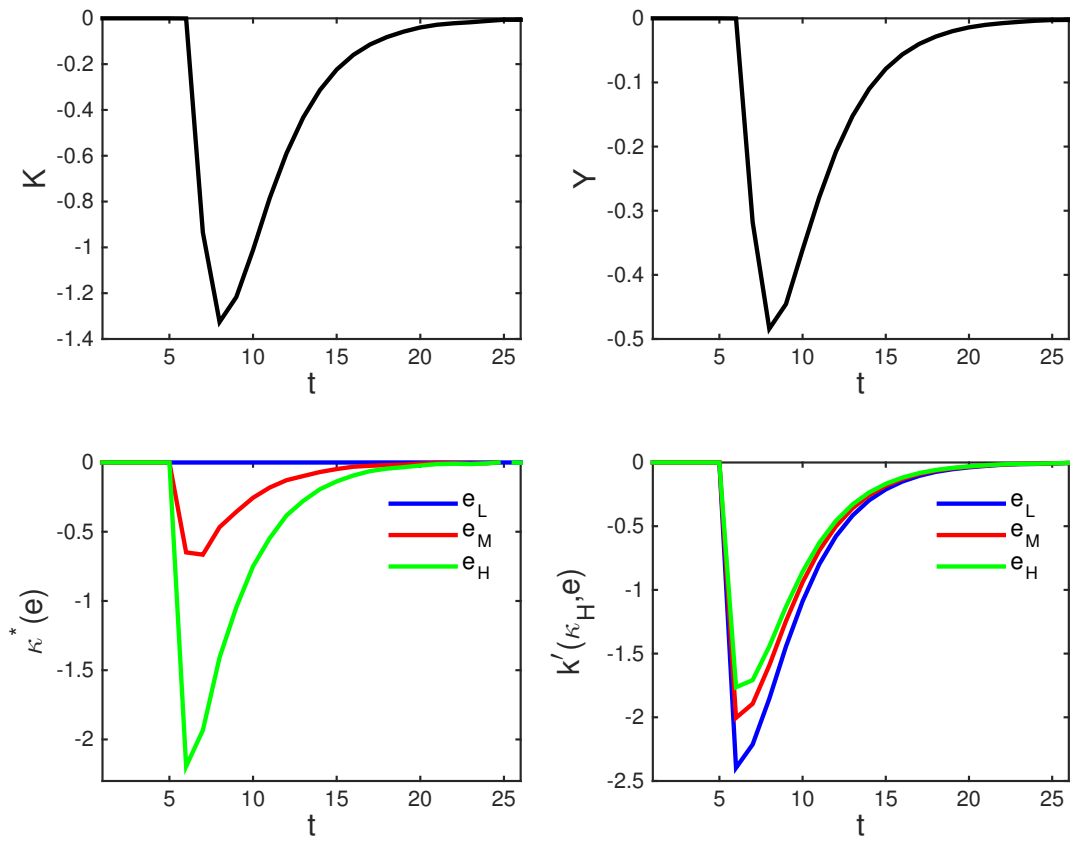


Figure 7: Dynamic response to negative shock to asset specificity.

non-smooth consumption profile.

The limited commitment becomes severer when intangible capital is less firm-specific and managers are consequently more mobile. Exploring the policy implications of the lower firm-specificity of human capital and the higher mobility of skilled workers is a topic for the future research.

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## A Two-Period OLG model

### A.1 Equilibrium Analysis with $\theta = 1$ .

First we complement the description of the equilibrium under full commitment in Section 3.1. The endogenous state variables at the beginning of period  $t$  are aggregate supply of labor and intangible capital  $(L_t, K_t)$ . Equilibrium wage rate is,

$$w_t = A(1 - \alpha) \left( \frac{K_t}{L_t} \right)^\alpha. \quad (34)$$

The rate of return on intangible capital  $x_t$  is

$$x_t = r_t + \alpha A_t \left( \frac{L_t}{K_t} \right)^{1-\alpha}. \quad (35)$$

Let  $F_t(\kappa, e) \equiv G_t(e)H_t(\kappa)$  and  $K_t^y$  be the supply of intangible capital acquired by current-period young trainees

$$\begin{aligned} K_t^y &= \int \int_{\kappa_t^*}^{\kappa_H} k_t^+(\kappa, e) dF_t(\kappa, e) \\ &= a_t \int_{\kappa_t^*}^{\kappa_H} \kappa dH_t(\kappa), \end{aligned} \quad (36)$$

where

$$a_t = a(X_t, r_t) = \left[ \frac{1}{b} \left( \frac{X_t}{1 + \psi} \right)^{1+\eta\psi} \left( \frac{\eta\psi}{r_t} \right)^{\eta\psi} \right]^{\frac{1}{(1-\eta)\psi}}, \quad (37)$$

$$(\kappa, e) \in \Theta_t \text{ iff } \kappa > \kappa_t^* = \frac{q_t w_{t+1}}{\frac{(1-\eta)\psi}{1+\psi} X_t \cdot a_t} \quad (38)$$



as in the text. The aggregate labor and intangible capital of the next period are

$$L_{t+1} = 1 + H_t(\kappa_t^*) \quad (39)$$

$$K_{t+1} = K_t^y. \quad (40)$$

The market equilibrium for intangible capital for training is

$$K_t = \int_{\kappa_t}^{\kappa_H} \tilde{k}_t(\kappa, e) dH_t(\kappa) = \frac{\eta\psi}{r_t} \frac{X_t}{1+\psi} K_t^y.$$

So,

$$r_t = \frac{\eta\psi}{1+\psi} \frac{X_t K_t^y}{K_t}. \quad (41)$$

In order to consider the market equilibrium in the dynamic setting (including the effect of unanticipated shocks), let's denote the short-hand notation

$$E_t(y_s) = y_{t,s}^e, \text{ for } s > t.$$

Then

$$X_t = q_t E_t(x_{t+1}) = q_t x_{t,t+1}^e. \quad (42)$$

The consumption of young agent is given by

$$c_t^y(\kappa, e) = \begin{cases} \frac{1}{1+\beta} \left[ e + w_t + X_t \frac{(1-\eta)\psi}{1+\psi} a_t \kappa \right], & \text{if } \kappa > \kappa_t^*, \\ \frac{1}{1+\beta} (e + w_t + q_t w_{t,t+1}^e), & \text{if } \kappa \leq \kappa_t^*. \end{cases}$$

Let  $S_t^y$  be the aggregate net worth of young generation at the end of period t. Because the net worth of the old generation equals zero at the end of period t, the market clearing implies

$$S_t^y = 0.$$

Let  $e_t^a$  be the aggregate (or average) endowment of young agents.

$$e_t^a \equiv \int e dG_t(e).$$

Then the market clearing condition for aggregate net worth of young generation is

$$\begin{aligned} 0 &= S_t^y = e_t^a + w_t - \int_{\kappa_t^*}^{\kappa_H} \varphi_t(a_t \kappa; \kappa) dH_t(\kappa) - \int c_t^y(\kappa) dF_t(\kappa, e) \\ &= e_t^a + w_t - \frac{1+\eta\psi}{1+\psi} X_t K_t^y - \frac{1}{1+\beta} \left[ e_t^a + w_t + \frac{(1-\eta)\psi}{1+\psi} X_t K_t^y + H_t(\kappa_t^*) q_t w_{t,t+1}^e \right] \\ &= \frac{\beta}{1+\beta} (e_t^a + w_t) - H_t(\kappa_t^*) \frac{q_t w_{t,t+1}^e}{1+\beta} - \left[ \frac{1+\eta\psi}{1+\psi} + \frac{(1-\eta)\psi}{1+\beta} \right] X_t K_t^y. \end{aligned} \quad (43)$$

The dynamic equilibrium of the aggregate economy under full commitment is given by ten endogenous variables  $(w_t, r_t, x_t, q_t, X_t, a_t, \kappa_t^*, K_t^y, K_{t+1}, L_{t+1})$  as a function of the state variable  $(K_t, L_t, A_t, e_t^a)$  which satisfies ten equations (34)–(43). Then all the individual choice  $\{\Theta_t, c_t^y(\kappa, e), k_t^+(\kappa, e)\}$  are determined as a function of aggregate state  $(K_t, L_t, A_t, e_t^a)$  and the individual characteristics  $(\kappa, e)$ .

## A.2 Equilibrium analysis with small $\theta$

Now we complement the description of equilibrium analysis under binding limited commitment in Section 3.2.

### Occupational choice.

From equations (25), (23) and (24), we have

$$\begin{aligned} k_{t+1} &= \frac{\beta}{\Gamma_t\left(\frac{k_{t+1}}{\kappa}\right)}(e + w_t) = k^+(\kappa, e; w_t, r_t, X_t) = k_t^+(\kappa, e) \\ c_t^y &= \frac{\varphi_t'(k_{t+1}; \kappa) - \theta X_t}{\Gamma_t\left(\frac{k_{t+1}}{\kappa}\right)}(e + w_t) = c^y(\kappa, e; w_t, r_t, X_t) = c_t^y(\kappa, e) \\ c_{t+1}^o &= (1 - \theta)\frac{X_t}{q_t}k_{t+1}, \end{aligned} \tag{44}$$

where

$$\begin{aligned} \Gamma_t\left(\frac{k_{t+1}}{\kappa}\right) &= \left(1 + \beta\frac{1 + \eta\psi}{1 + \psi}\right)\varphi_t'(k_{t+1}; \kappa) - (1 + \beta)\theta X_t \\ \varphi_t'(k_{t+1}; \kappa) &= (1 + \psi)\left[b\left(\frac{r_t}{\eta\psi}\right)^{\eta\psi}\left(\frac{k_{t+1}}{\kappa}\right)^{(1-\eta)\psi}\right]^{\frac{1}{1+\eta\psi}}, \end{aligned}$$

as in the text.  $k_{t+1} = k_t^+(\kappa, e)$  is given by  $k_{t+1}$  which solves (44). The discounted utility when the future manager is young is given by

$$\begin{aligned} V_{y,t}^m &= \ln c_t^y + \beta \ln c_{t+1}^o \\ &= (1 + \beta)\left[\ln(e + w_t) - \ln \Gamma_t\left(\frac{k_{t+1}}{\kappa}\right)\right] + \ln[\varphi_t'(k_{t+1}; \kappa) - \theta X_t] \\ &\quad + \beta[\ln X_t - \ln q_t + \ln(1 - \theta)] + \beta \ln \beta \end{aligned}$$

Comparing with (52), the agent chooses to become a manager if and only if  $V_{y,t}^m > V_{y,t}^w$ , or

$$\begin{aligned} & (1 + \beta) \left[ \ln(e + w_t) - \ln \Gamma_t \left( \frac{k_{t+1}}{\kappa} \right) \right] + \ln [\varphi'_t(k_{t+1}; \kappa) - \theta X_t] + \beta [\ln X_t + \ln(1 - \theta)] \\ & - (1 + \beta) [\ln(e + w_t + q_t w_{t,t+1}^e) - \ln(1 + \beta)] \\ & \equiv LHS \left( e, \frac{k_{t+1}}{\kappa} \right) > 0 \end{aligned} \quad (45)$$

We can show

$$\frac{\partial}{\partial e} LHS \left( e, \frac{k_{t+1}}{\kappa} \right) > 0, \text{ and } \frac{\partial}{\partial \frac{k_{t+1}}{\kappa}} LHS \left( e, \frac{k_{t+1}}{\kappa} \right) < 0.$$

From (44),  $\frac{k_{t+1}}{\kappa}$  is a decreasing function of  $\kappa$ . Therefore young agent chooses to become a manager if and only if

$$(\kappa, e) \in \Theta_t \equiv \{(\kappa, e) : \kappa > \kappa_t^*(e)\}, \quad (46)$$

where  $\kappa_t^*(e)$  solves

$$LHS \left( e, \frac{k_t^+(\kappa_t^*(e), e)}{\kappa_t^*(e)} \right) = 0, \quad (47)$$

and

$$\kappa_t^{*'}(e) \leq 0.$$

### Market clearing condition

As before, the endogenous state variables for the aggregate economy are aggregate labor and intangible capital stock  $(L_t, K_t)$ . Aggregate labor and intangible capital stock of the next period are:

$$L_{t+1} = 1 + \int_{(\kappa, e) \notin \Theta_t^m(\kappa, e)} dF_t(\kappa, e). \quad (48)$$

$$K_{t+1} = \int_{(\kappa, e) \in \Theta_t^m(\kappa, e)} k_t^+(\kappa, e) dF_t(\kappa, e). \quad (49)$$

The market clearing condition for training service is

$$K_t = \int_{(\kappa, e) \in \Theta_t^m} \tilde{k}(\kappa, e) dF_t(\kappa, e) = \int_{(\kappa, e) \in \Theta_t^m} \left[ \frac{\eta \psi}{r_t} b \frac{k_t^+(\kappa, e)^{1+\psi}}{\kappa^{(1-\eta)\psi}} \right]^{\frac{1}{1+\eta\psi}} dF_t(\kappa, e). \quad (50)$$

The consumption of young agents is

$$c_t^y(\kappa, e) = \begin{cases} \frac{\varphi'_t(k_t^+(\kappa, e); \kappa) - \theta X_t}{\beta} k_t^+(\kappa, e), & \text{if } (\kappa, e) \in \Theta_t^m(\kappa, e), \\ \frac{1}{1+\beta} (e + w_t + q_t w_{t,t+1}^e), & \text{otherwise.} \end{cases}$$

The market clearing condition of funds is that the net worth of young agents at the end of date  $t$  equals zero, or

$$\begin{aligned}
0 &= S_t^y \\
&= e_t^a + w_t - \int_{\Theta_t} \varphi_t(k_t^+(\kappa, e); \kappa) dF_t(\kappa, e) - \int c_t^y(\kappa) dF_t(\kappa, e) \\
&= \frac{\beta}{1+\beta} (e_t^{a,w} + w_t L_t^s) + e_t^{a,m} + w_t(1 - L_t^s) - \frac{w_{t,t+1}^e}{1+\beta} q_t L_t^s \\
&\quad - \int_{\Theta_t^m} \left[ \varphi_t(k_t^+(\kappa, e); \kappa) + \frac{\varphi_t'(k_t^+(\kappa, e); \kappa) - \theta X_t}{\beta} k_t^+(\kappa, e) \right] dF_t(\kappa, e), \tag{51}
\end{aligned}$$

where  $e_t^{a,w}$  and  $e_t^{a,m}$  are aggregate endowment of simple workers and trainees as

$$e_t^{a,m} \equiv \int_{(\kappa, e) \in \Theta_t^m} e dF_t(\kappa, e), \text{ and } e_t^{a,w} \equiv e_t^a - e_t^{a,m}.$$

The dynamic equilibrium of the aggregate economy under full commitment is given by seven endogenous variables  $(w_t, r_t, x_t, q_t, X_t, K_{t+1}, L_{t+1})$  and one function  $\kappa_t^*(e)$  as a function of the state variable  $(K_t, L_t, A_t, e_t^a)$  which satisfies ten equations (34, 35, 42), (47)–(51). Then all the individual choice  $\{\Theta_t, c_t^y(\kappa, e), k_t^+(\kappa, e)\}$  are determined as a function of aggregate state  $(K_t, L_t, A_t, e_t^a)$  and the individual characteristics  $(\kappa, e)$ .

## B Three-Period OLG Model

If a young agent chooses to be a routine worker, the value is

$$V_{yt}^w(\kappa, e) = \max_{c_t, c_{t,t+1}^e, c_{t,t+2}^e} [\ln c_t + \beta \ln c_{t,t+1}^e + \beta^2 \ln c_{t,t+2}^e],$$

subject to the budget constraint

$$c_t + q_t c_{t,t+1}^e + q_t q_{t,t+1}^e c_{t,t+2}^e = e + w_t + q_t w_{t,t+1}^e + q_t q_{t,t+1}^e w_{t,t+2}^e.$$

Then we get

$$\begin{aligned}
c_t &= \frac{1}{1+\beta+\beta^2} [e + w_t + q_t w_{t,t+1}^e + q_t q_{t,t+1}^e w_{t,t+2}^e], \\
c_{t,t+1}^e &= \frac{\beta/q_t}{1+\beta+\beta^2} [e + w_t + q_t w_{t,t+1}^e + q_t q_{t,t+1}^e w_{t,t+2}^e], \\
c_{t,t+2}^e &= \frac{\beta^2/(q_t q_{t,t+1}^e)}{1+\beta+\beta^2} [e + w_t + q_t w_{t,t+1}^e + q_t q_{t,t+1}^e w_{t,t+2}^e],
\end{aligned}$$

and

$$\begin{aligned}
V_{y,t}^w(\kappa, e) &= (1+\beta+\beta^2) [\ln(e + w_t + q_t w_{t,t+1}^e + q_t q_{t,t+1}^e w_{t,t+2}^e) - \ln(1+\beta+\beta^2)] \\
&\quad - (\beta+\beta^2) \ln q_t - \beta^2 \ln q_{t,t+1}^e + (\beta+2\beta^2) \ln \beta. \tag{52}
\end{aligned}$$

## B.1 Equilibrium under full commitment

Let  $X_t$  be the discounted expected rate of return on intangible capital in the middle and old period as

$$X_t = q_t x_{t,t+1}^e + \lambda q_t q_{t,t+1}^e x_{t,t+2}^e. \quad (53)$$

Choosing intangible capital to maximize the return  $X_t k_{t+1} - \varphi_t(k_{t+1}; \kappa)$ , the first order condition for the future manager is

$$X_t = \varphi'_t(k_{t+1}; \kappa)$$

Thus, similar to the two-period OLG model, we get

$$k_t^+(\kappa, e) = a_t \kappa, \text{ where}$$

$$a_t = a(X_t, r_t) = \left[ \frac{1}{b} \left( \frac{X_t}{1+\psi} \right)^{1+\eta\psi} \left( \frac{\eta\psi}{r_t} \right)^{\eta\psi} \right]^{\frac{1}{(1-\eta)\psi}}, \text{ and}$$

$$X_t k_t^+(\kappa, e) - \varphi_t(k_t^+(\kappa, e); \kappa) = \frac{(1-\eta)\psi}{1+\psi} X_t \cdot a_t \kappa.$$

Thus young agent chooses to become a manager if and only if

$$\frac{(1-\eta)\psi}{1+\psi} X_t \cdot a_t \kappa \geq q_t w_{t,t+1}^e + q_t q_{t,t+1}^e w_{t,t+2}^e, \text{ or}$$

$$\kappa \geq \kappa_t^* \equiv \frac{q_t w_{t,t+1}^e + q_t q_{t,t+1}^e w_{t,t+2}^e}{\frac{(1-\eta)\psi}{1+\psi} X_t \cdot a(X_t, r_t)}. \quad (54)$$

The future manager's consumption becomes

$$c_t^y = \frac{1}{1+\beta+\beta^2} \left[ w_t + \frac{(1-\eta)\psi}{1+\psi} X_t \cdot a_t \kappa \right]$$

$$c_{t,t+1}^{m,e} = \frac{\beta/q_t}{1+\beta+\beta^2} \left[ w_t + \frac{(1-\eta)\psi}{1+\psi} X_t \cdot a_t \kappa \right]$$

$$c_{t,t+2}^{o,e} = \frac{\beta^2/(q_t q_{t,t+1}^e)}{1+\beta+\beta^2} \left[ w_t + \frac{(1-\eta)\psi}{1+\psi} X_t \cdot a_t \kappa \right].$$

### B.1.1 Market clearing conditions.

Market clearing conditions at period  $t$ . There are four markets, labor market, rental market of intangible capital for training, the internal loan market and the consumption goods market.

The population of young agents who choose to become routine workers is

$$L_t^s = H_t(\kappa_t^*). \quad (55)$$

The aggregate labor supply of the next period equals

$$L_{t+1} = 1 + L_{t-1}^s + L_t^s, \quad (56)$$

where the last two terms in the RHS are old and middle aged routine workers in the next period. As before, we have

$$w_t = A_t(1 - \alpha) \left( \frac{K_t}{L_t} \right)^\alpha \quad (57)$$

$$x_t = r_t + \alpha A_t \left( \frac{L_t}{K_t} \right)^{1-\alpha}. \quad (58)$$

Let  $K_t^y$  be the supply of capital for the next period by present young trainees

$$\begin{aligned} K_t^y &= \int \int_{\kappa_t^*}^{\kappa_H} k_t^+(\kappa, e) dF_t(\kappa, e) \\ &= a_t \int_{\kappa_t^*}^{\kappa_H} \kappa dH_t(\kappa). \end{aligned} \quad (59)$$

The aggregate intangible capital of the next period is

$$K_{t+1} = K_t^y + \lambda K_{t-1}^y. \quad (60)$$

The demand for intangible capital for receiving training is from the worker's optimal training decisions,

$$\int_{\kappa_t^*}^{\kappa_H} \tilde{k}_t(\kappa, e) dH_t(\kappa) = \frac{\eta\psi}{r_t} \frac{X_t}{1+\psi} K_t^y.$$

So,

$$r_t = \frac{\eta\psi}{1+\psi} \frac{X_t K_t^y}{K_t}. \quad (61)$$

The consumption of young agent is given by

$$c_t^y(\kappa, e) = \begin{cases} \frac{1}{1+\beta+\beta^2} \left[ e + w_t + X_t \frac{(1-\eta)\psi}{1+\psi} a_t \kappa \right] & \text{if } \kappa > \kappa_t^* \\ \frac{1}{1+\beta+\beta^2} (e + w_t + q_t w_{t,t+1}^e + q_t q_{t,t+1}^e w_{t,t+2}^e) & \text{if } \kappa \leq \kappa_t^* \end{cases}.$$

Let  $S_t^y$  and  $S_t^m$  be the aggregate net worth of young and middle generation at the end of period  $t$ . Because the net worth of the old generation equals zero at the end of period  $t$ , the market clearing implies

$$S_t^y + S_t^m = 0.$$

Let  $e_t^a$  be the aggregate (or average) endowment of young agents. Then aggregate net worth of young generation is

$$\begin{aligned}
S_t^y &= e_t^a + w_t - \int_{\kappa_t}^{\kappa_H} \varphi_t(a_t \kappa; \kappa) dH_t(\kappa) - \int c_t^y(\kappa) dF_t(\kappa, e) \\
&= e_t^a + w_t - \frac{1 + \eta\psi}{1 + \psi} X_t K_t^y \\
&\quad - \frac{1}{1 + \beta + \beta^2} \left[ e_t^a + w_t + \frac{(1 - \eta)\psi}{1 + \psi} X_t K_t^y + q_t (w_{t,t+1}^e + q_{t,t+1}^e w_{t,t+2}^e) L_t^s \right] \\
&= \frac{\beta + \beta^2}{1 + \beta + \beta^2} (e_t^a + w_t) - q_t \frac{w_{t,t+1}^e + q_{t,t+1}^e w_{t,t+2}^e}{1 + \beta + \beta^2} L_t^s - \left[ \frac{1 + \eta\psi}{1 + \psi} + \frac{(1 - \eta)\psi}{1 + \beta + \beta^2} \right] X_t K_t^y \quad (62)
\end{aligned}$$

Generally, aggregate consumption of middle age agents is

$$\int c_t^m(\kappa) dF_t(\kappa, e) = \frac{1}{1 + \beta} \left[ \frac{S_{t-1}^y}{q_{t-1}} + (w_t + q_t w_{t,t+1}^e) L_{t-1}^s + (x_t + \lambda q_t x_{t,t+1}^e) K_{t-1}^y \right].$$

Thus, the aggregate net worth of the middle age agents at the end of date t is

$$\begin{aligned}
S_t^m &= \frac{1}{q_{t-1}} S_{t-1}^y + w_t L_{t-1}^s + x_t K_{t-1}^y - \int c_t^m(\kappa) dF_t(\kappa, e) \\
&= \frac{\beta}{1 + \beta} \frac{S_{t-1}^y}{q_{t-1}} + \frac{1}{1 + \beta} [(\beta w_t - q_t w_{t,t+1}^e) L_{t-1}^s + (\beta x_t - \lambda q_t x_{t,t+1}^e) K_{t-1}^y]
\end{aligned}$$

Therefore the market clearing condition for fund is

$$\begin{aligned}
0 &= S_t^y + S_t^m \\
&= S_t^y + \frac{\beta}{1 + \beta} \frac{S_{t-1}^y}{q_{t-1}} + \frac{1}{1 + \beta} [(\beta w_t - q_t w_{t,t+1}^e) L_{t-1}^s + (\beta x_t - \lambda q_t x_{t,t+1}^e) K_{t-1}^y]. \quad (63)
\end{aligned}$$

We consider  $(L_t, K_t, L_{t-1}^s, K_{t-1}^y, \frac{S_{t-1}^y}{q_{t-1}})$  as endogenous state variables. Then eleven endogenous variables  $(L_{t+1}, K_{t+1}, L_t^s, K_t^y, \kappa_t^*, w_t, x_t, r_t, X_t, S_t^y, q_t)$  are determined as functions of the endogenous and exogenous state variables which satisfies eleven equations (53) – (63).

## B.2 Equilibrium with Financing Constraint

Because middle-aged agent does not face the borrowing constraint, his expected utility only depends upon the wealth as

$$V_t(W_t) = \max_{c_t, c_{t+1}} \ln c_t + \beta \ln c_{t+1},$$

subject to

$$c_t + q_t c_{t+1} = W_t.$$

Thus

$$\begin{aligned} c_t &= \frac{1}{1+\beta} W_t \\ c_{t+1} &= \frac{\beta/q_t}{1+\beta} W_t \end{aligned}$$

$$V_t(W_t) = (1+\beta)[\ln W_t - \ln(1+\beta)] + \beta(\ln \beta - \ln q_t).$$

Thus the young trainee chooses

$$\max_{c_t, k_{t+1}, W_{t+1}} \ln c_t + \beta(1+\beta) \ln W_{t+1},$$

subject to

$$\begin{aligned} c_t + \varphi_t(k_{t+1}; \kappa) &= e + w_t + q_t d_t, \\ W_{t+1} &= \frac{X_t}{q_t} k_{t+1} - d_t, \\ d_t &\leq \theta \frac{X_t}{q_t} k_{t+1}. \end{aligned}$$

Assume that the borrowing constraint is binding. Then

$$\begin{aligned} c_t &= e + w_t + \theta X_t k_{t+1} - \varphi_t(k_{t+1}; \kappa) \\ W_{t+1} &= (1-\theta) \frac{X_t}{q_t} k_{t+1} \end{aligned}$$

The first order condition for  $k_{t+1}$  is

$$\frac{\beta(1+\beta)}{k_{t+1}} = \frac{\varphi'_t(k_{t+1}; \kappa) - \theta X_t}{c_t}$$

Using

$$\begin{aligned} \varphi_t(k_{t+1}; \kappa) &= \frac{1+\eta\psi}{1+\psi} \varphi'_t(k_{t+1}; \kappa) k_{t+1}, \text{ and} \\ c_t &= e + w_t + \theta X_t k_{t+1} - \frac{1+\eta\psi}{1+\psi} \varphi'_t(k_{t+1}; \kappa) k_{t+1}, \end{aligned}$$

we can write this first order condition as

$$\beta(1+\beta)(e+w_t) = \Gamma_t \left( \frac{k_{t+1}}{\kappa} \right) k_{t+1}, \text{ where} \tag{64}$$

$$\Gamma_t \left( \frac{k_{t+1}}{\kappa} \right) = \left[ 1 + (\beta + \beta^2) \frac{1+\eta\psi}{1+\psi} \right] \varphi'_t(k_{t+1}; \kappa) - (1+\beta+\beta^2)\theta X_t = \tilde{\Gamma} \left( \frac{k_{t+1}}{\kappa}, X_t, r_t \right)$$



Solving (64) with respect to  $k_{t+1}$ , we get

$$\begin{aligned} k_{t+1} &= k_t^+(\kappa, e), \text{ where} \\ \frac{\partial}{\partial \kappa} k_t^+(\kappa, e) &> 0 \text{ and } \frac{\partial}{\partial e} k_t^+(\kappa, e) > 0. \end{aligned} \quad (65)$$

Since  $\Gamma_t\left(\frac{k_{t+1}}{\kappa}\right)$  is an increasing function of  $\frac{k_{t+1}}{\kappa}$ ,  $k_{t+1}$  is an increasing function of  $\kappa$  but not proportional with  $\kappa$ , i.e.,  $\frac{\partial^2}{\partial \kappa^2} k^+(\kappa, e) < 0$ .

Alternatively, we can derive the implicit relationship as

$$\begin{aligned} k_{t+1} &= \frac{\beta(1+\beta)}{\Gamma_t\left(\frac{k_{t+1}}{\kappa}\right)} (e + w_t), \\ c_t &= \frac{\varphi'_t(k_{t+1}; \kappa) - \theta X_t}{\Gamma_t\left(\frac{k_{t+1}}{\kappa}\right)} (e + w_t), \\ W_{t+1} &= (1 - \theta) \frac{X_t}{q_t} k_{t+1}. \end{aligned}$$

Then, the discounted utility when the future manager is young is given by

$$\begin{aligned} V_{y,t}^m &= \ln c_t + \beta(1+\beta) \ln \left[ (1 - \theta) \frac{X_t}{q_t} k_{t+1} \right] + \beta [\beta \ln \beta - \beta \ln q_{t,t+1}^e - (1 + \beta) \ln(1 + \beta)] \\ &= (1 + \beta + \beta^2) \left[ \ln(e + w_t) - \ln \Gamma_t\left(\frac{k_{t+1}}{\kappa}\right) \right] + \ln [\varphi'_t(k_{t+1}; \kappa) - \theta X_t] \\ &\quad + (\beta + \beta^2) [\ln X_t - \ln q_t + \ln(1 - \theta)] - \beta^2 \ln q_{t,t+1}^e + (\beta + 2\beta^2) \ln \beta \end{aligned}$$

Comparing with (52), the agent chooses to become a manager if and only if  $V_t^{y,m} \geq V_t^{y,w}$ , or

$$\begin{aligned} 0 &\leq RHS\left(e, \frac{k_{t+1}}{\kappa}\right) \equiv \\ &(1 + \beta + \beta^2) \left[ \ln(e + w_t) - \ln \Gamma_t\left(\frac{k_{t+1}}{\kappa}\right) \right] + \ln [\varphi'_t(k_{t+1}; \kappa) - \theta X_t] + (\beta + \beta^2) [\ln X_t + \ln(1 - \theta)] \\ &\quad - (1 + \beta + \beta^2) [\ln(e + w_t + q_t w_{t,t+1}^e + q_t q_{t,t+1}^e w_{t,t+2}^e) - \ln(1 + \beta + \beta^2)]. \end{aligned} \quad (66)$$

where  $k_{t+1} = k_t^+(\kappa, e)$  solves the equation (64). Together, we have young agent chooses to become a manager if and only if

$$(\kappa, e) \in \Theta_t^m(\kappa, e). \quad (67)$$

We can show

$$\begin{aligned} \frac{\partial}{\partial e} RHS\left(e, \frac{k_{t+1}}{\kappa}\right) &> 0, \\ \frac{\partial}{\partial \frac{k_{t+1}}{\kappa}} RHS\left(e, \frac{k_{t+1}}{\kappa}\right) &< 0. \end{aligned}$$

Because we know  $\frac{k_{t+1}}{\kappa}$  is a decreasing function of  $\kappa$  from (64), we learn (67) is equivalent to

$$\kappa > \kappa_t^*(e), \text{ where } \kappa_t^{*'}(e) < 0.$$

### B.2.1 Market clearing condition

Let  $L_t^s$  be population of young agents who choose to become routine workers:

$$L_t^s = \int_{(\kappa, e) \notin \Theta_t^m(\kappa, e)} dF_t(\kappa, e). \quad (68)$$

The labor supply of the next period is

$$L_{t+1} = 1 + L_t^s + L_{t-1}^s. \quad (69)$$

As before, let  $K_t^y$  be the supply of capital for the next period by present young trainees.

$$K_t^y = \int_{(\kappa, e) \in \Theta_t^m(\kappa, e)} k_t^+(\kappa, e) dF_t(\kappa, e). \quad (70)$$

where

$$k_{t+1} = k^+(\kappa, e; w_t, X_t, r_t) = k_t^+(\kappa, e).$$

The supply of intangible capital of the next period is

$$K_{t+1} = K_t^y + \lambda K_{t-1}^y. \quad (71)$$

The demand is from the worker's optimal training decisions,

$$\int_{(\kappa, e) \in \Theta_t^m(\kappa, e)} \tilde{k}_t(\kappa, e) dF_t(\kappa, e) = \int_{(\kappa, e) \in \Theta_t^m(\kappa, e)} \left[ \frac{\eta\psi}{r_t} b \frac{k_t^+(\kappa, e)^{1+\psi}}{\kappa^{(1-\eta)\psi}} \right]^{\frac{1}{1+\eta\psi}} dF_t(\kappa, e).$$

Thus the market clearing condition for training service is

$$K_t = \int_{(\kappa, e) \in \Theta_t^m(\kappa, e)} \left[ \frac{\eta\psi}{r_t} b \frac{k_t^+(\kappa, e)^{1+\psi}}{\kappa^{(1-\eta)\psi}} \right]^{\frac{1}{1+\eta\psi}} dF_t(\kappa, e). \quad (72)$$

The consumption of young agents is

$$c_t^y(\kappa) = \begin{cases} \frac{\varphi'_t(k_t^+(\kappa, e); \kappa) - \theta X_t}{\beta + \beta^2} k_t^+(\kappa, e), & \text{if } (\kappa, e) \in \Theta_t^m(\kappa, e), \\ \frac{1}{1 + \beta + \beta^2} (e + w_t + q_t w_{t,t+1}^e + q_t q_{t,t+1}^e w_{t,t+2}^e), & \text{otherwise.} \end{cases}$$

The net worth of young agents at the end of date t is

$$\begin{aligned} S_t^y &= e_t^a + w_t - \int_{\Theta_t^m(\kappa, e)} \varphi_t(k_t^+(\kappa, e); \kappa) dF_t(\kappa, e) - \int c_t^y(\kappa) dF_t(\kappa, e) \\ &= \frac{\beta + \beta^2}{1 + \beta + \beta^2} (e_t^{a,w} + w_t L_t^s) + e_t^{a,m} + w_t (1 - L_t^s) - \frac{w_{t,t+1}^e + q_{t,t+1}^e w_{t,t+2}^e}{1 + \beta + \beta^2} q_t L_t^s \\ &\quad - \int_{\Theta_t^m} \left[ \varphi_t(k_t^+(\kappa, e); \kappa) + \frac{\varphi'_t(k_t^+(\kappa, e); \kappa) - \theta X_t}{\beta + \beta^2} k_t^+(\kappa, e) \right] dF_t(\kappa, e), \end{aligned} \quad (73)$$

where  $e_t^{a,w}$  and  $e_t^{a,m}$  are aggregate endowment of routine workers and trainees as

$$\begin{aligned} e_t^{a,w} &= \int_{(\kappa,e) \notin \Theta_t^m(\kappa,e)} e \cdot dF_t(\kappa, e), \\ e_t^{a,m} &= \int_{(\kappa,e) \in \Theta_t^m(\kappa,e)} e \cdot dF_t(\kappa, e). \end{aligned}$$

Because aggregate consumption of middle age agents is

$$\int c_t^m(\kappa) dF_t(\kappa, e) = \frac{1}{1+\beta} \left[ \frac{S_{t-1}^y}{q_{t-1}} + (w_t + q_t w_{t,t+1}^e) L_{t-1}^s + (x_t + \lambda q_t x_{t,t+1}^e) K_{t-1}^y \right],$$

the net worth of middle age agents at the end of date t is

$$\begin{aligned} S_t^m &= \frac{1}{q_{t-1}} S_{t-1}^y + w_t L_{t-1}^s + x_t K_{t-1}^y - \int c_t^m(\kappa) dF_t(\kappa, e) \\ &= \frac{\beta}{1+\beta} \frac{S_{t-1}^y}{q_{t-1}} + \frac{1}{1+\beta} [(\beta w_t - q_t w_{t,t+1}^e) L_{t-1}^s + (\beta x_t - \lambda q_t x_{t,t+1}^e) K_{t-1}^y]. \end{aligned}$$

The market clearing condition of funds is

$$S_t^y + S_t^m = 0, \text{ or,}$$

$$0 = S_t^y + \frac{\beta}{1+\beta} \frac{S_{t-1}^y}{q_{t-1}} + \frac{1}{1+\beta} [(\beta w_t - q_t w_{t,t+1}^e) L_{t-1}^s + (\beta x_t - \lambda q_t x_{t,t+1}^e) K_{t-1}^y]. \quad (74)$$

As before, we consider  $(L_t, K_t, L_{t-1}^s, K_{t-1}^y, \frac{S_{t-1}^y}{q_{t-1}})$  as endogenous state variables. Then ten endogenous variables  $(L_{t+1}, K_{t+1}, L_t^s, K_t^y, w_t, x_t, r_t, X_t, S_t^y, q_t)$  and a set  $\Theta_t^m(\kappa, e)$  are determined as functions of the endogenous and exogenous state variables which satisfies eleven equations (53), (57), (58) and (67) – (74).

## C Equivalence to coalition-proof equilibrium

An equilibrium is coalition-proof if no agent within a firm can form a Pareto-improving sub-coalition within the firm. In other words, the highest possible private payoff an agent obtains from forming a sub-coalition is her equilibrium payoff. In this section, we will confirm this equilibrium property in the OLG model with two-period lived agents. It is straightforward to extend the analysis to the OLG model with three-period lived agents.

We assume that agents within a deviating coalition can reallocate income through the internal financial market within the subcoalition without relying on the financial market of the firm. Because the agent's incentive to repay the debt to the internal financial market depends on his incentive

to leave the firm, the incentive to repay the debt does not depend upon whether the agent forms subcoalition within the firm or not.

For the rest of the section, we will conjecture and verify that there exists a coalition proof equilibrium where  $r_t^f = r_t$  and  $x_t^f = x_t$  and the equilibrium values and allocations are the same as in the competitive equilibrium defined by Definition 1.

Anticipating that the equilibrium is coalition proof, we abuse notation by denoting the value function of a young trainee of type  $(\kappa, e)$  from deviation at period  $t$  to be  $V_{y,t}^m(\kappa, e)$  and its equilibrium payoff to be  $V_{y,t}^{m*}(\kappa, e)$ . Similarly for an old agents who holds intangible capital and debt  $(k, d)$ , we denote  $V_{o,t}^m(k, d)$  for the deviator and  $V_{o,t}^{m*}(k, d)$  for the equilibrium value. Denote also the cost of investment for a trainee with initial skill  $\kappa$  to acquire intangible capital  $k'$  using  $\tilde{k}$  units of current managers' skill for training to be

$$\tilde{i} = b \left( \frac{k'}{\kappa^\eta \tilde{k}^{1-\eta}} \right) k' \equiv \Phi(k', \tilde{k}, \kappa).$$

For a young trainee of type  $(\kappa, e)$ , its payoff from forming a subcoalition with  $n^o$  number of old managers with intangible capital of  $k^o$  solves the following problem:

$$\begin{aligned} V_{y,t}^m(\kappa, e) = & \max_{n^o, y^o, l, \tilde{k}, k', d'} \{ \ln[A\tilde{k}^\alpha (l+1)^{1-\alpha} - w_t l - n^o y^o \\ & - \Phi(k', \tilde{k}, \kappa) + q_t d' + e] + \beta V_{o,t+1}^{m*}(k', d') \}, \end{aligned} \quad (75)$$

subject to

$$n^o k^o = \tilde{k},$$

$$V_{o,t+1}^{m*}(k', d') \geq V_{o,t+1}^{m*}((1-\theta)k', 0), \quad (76)$$

$$\ln(y^o - d^o) \geq V_{o,t}^{m*}(k^o, d^o). \quad (77)$$

Constraint (76) is the incentive constraint for the manger not to default and move to another firm. (77) is the participation constraint for old managers to form a subcoalition for the compensation of  $y^o$ .

Conjecture and verify later that the equilibrium compensation for the manager is proportional to the intangible capital at the rate  $x_t$ . Then (77) becomes  $\ln(y^o - d^o) \geq \ln(x_t k^o - d^o)$ , or

$$y^o \geq x_t k^o. \quad (78)$$

Similarly (76) becomes  $\ln(x_{t+1} k' - d') \geq \ln[x_{t+1}(1-\theta)k']$  or

$$d' \leq \theta x_{t+1} k'. \quad (79)$$

Because reducing the payoff to the old managers would increase the young trainee's payoff from forming a subcoalition, (78) holds with equality. Conjecture and verify later that (79) is binding. Then the problem (75) becomes

$$V_{y,t}^m(\kappa, e) = \max_{l, \tilde{k}, k'} \{ \ln[A\tilde{k}^\alpha(l+1)^{1-\alpha} - w_t l - x_t \tilde{k} - \Phi(k', \tilde{k}, \kappa) + q_t \theta x_{t+1} k' + e] + \beta \ln[x_{t+1}(1-\theta)k'] \}. \quad (80)$$

The first order conditions of (80) with respect to  $l$  is

$$w_t = (1-\alpha)A \left( \frac{\tilde{k}}{\tilde{l}} \right)^\alpha, \text{ or } \frac{\tilde{l}}{\tilde{k}} = \left[ \frac{(1-\alpha)A}{w_t} \right]^{\frac{1}{\alpha}},$$

where  $\tilde{l} = l + 1$ . The first order conditions of (80) with respect  $\tilde{k}$  is

$$\begin{aligned} -\frac{\partial}{\partial \tilde{k}} \Phi(k', \tilde{k}, \kappa) &= \eta \psi b \frac{k'^{1+\psi}}{\tilde{k}^{1+\eta\psi} \kappa^{(1-\eta)\psi}} \\ &= x_t - \alpha A \left( \frac{\tilde{l}}{\tilde{k}} \right)^{1-\alpha} = x_t - \alpha \left[ \left( \frac{1-\alpha}{w_t} \right)^{1-\alpha} A \right]^{\frac{1}{\alpha}} \equiv r_t, \text{ or,} \\ \tilde{k} &= \left[ b \frac{\eta \psi}{r_t} \left( \frac{k'}{\kappa} \right)^{(1-\eta)\psi} \right]^{\frac{1}{1+\eta\psi}} k'. \end{aligned} \quad (81)$$

Then (80) becomes

$$V_{y,t}^m(\kappa, e) = \max_{k'} \{ \ln[w_t + e - \varphi_t(k'; \kappa) + q_t \theta x_{t+1} k'] + \beta \ln[x_{t+1}(1-\theta)k'] \}, \text{ where}$$

$$\varphi_t(k'; \kappa) = \Phi(k', \tilde{k}, \kappa) + r_t \tilde{k} = (1 + \eta \psi) \left[ b \left( \frac{r_t}{\eta \psi} \right)^{\eta \psi} \left( \frac{k'}{\kappa} \right)^{(1-\eta)\psi} \right]^{\frac{1}{1+\eta\psi}} k'.$$

Thus the first order condition for  $k'$  implies

$$\frac{\varphi'_t(k'; \kappa) - q_t \theta x_{t+1}}{w_t + e - \varphi_t(k'; \kappa) + q_t \theta x_{t+1} k'} = \frac{\beta}{k'}.$$

These are the same conditions as those for the competitive equilibrium. Therefore, we learn

$$V_{y,t}^m(\kappa, e) = V_{y,t}^{m*}(\kappa, e).$$

For an old manager with capital and debt  $(k, d)$ , his payoff from forming a subcoalition with  $n^y$  number of trainee of type  $(\kappa, e)$  solves the following problem:

$$\begin{aligned} V_{o,t}^m(k, d) &= \max_{l, n^y, y^y, k'} \ln \left[ Ak^\alpha(l + n^y)^{1-\alpha} - wl - n^y y^y - n^y \Phi \left( k', \frac{k}{n^y}, \kappa \right) - d \right] \\ \text{s.t. } &W_{y,t}(y^y, k', e) \geq V_{y,t}^{m*}(\kappa, e), \end{aligned} \quad (82)$$

where the indirect utility for a young trainee solves the following problem:

$$W_{y,t}(y^y, k', e) = \max_{d'} [\ln(y^y + e + q_t d') + \beta \ln(x_{t+1} k' - d')] \\ \text{s.t. } d' \leq \theta x_{t+1} k'.$$

Conjecture and verify later that the trainees are borrowing constrained, we get

$$W_{y,t}(y^y, k', e) = \ln(y^y + e + \theta q_t x_{t+1} k') + \beta \ln[(1 - \theta)x_{t+1} k'].$$

Define  $\tilde{l} = l + n^y$ . Using the Lagrangian,

$$\mathcal{L} = Ak^\alpha \tilde{l}^{1-\alpha} - w\tilde{l} + (w - y^y)n^y - n^y \Phi\left(k', \frac{k}{n^y}, \kappa\right) \\ + \lambda \left\{ \ln[y^y + e + \theta q_t x_{t+1} k'] + \beta \ln[(1 - \theta)x_{t+1} k'] - V_{y,t}^{m*}(\kappa, e) \right\},$$

we derive the first order conditions

$$w_t = (1 - \alpha)A \left(\frac{k}{\tilde{l}}\right)^\alpha, \text{ or } \frac{\tilde{l}}{k} = \left[\frac{(1 - \alpha)A}{w_t}\right]^{\frac{1}{\alpha}}, \quad (83)$$

$$w_t - y^y = \Phi(k', \tilde{k}, \kappa) + \tilde{k} \left[-\frac{\partial}{\partial \tilde{k}} \Phi(k', \tilde{k}, \kappa)\right], \quad (84)$$

where  $\tilde{k} = k/n^y$ . Denote

$$c^y = y^y + e + \theta q_t x_{t+1} k'.$$

The first order conditions with respect to  $y^y$  and  $k'$  imply

$$\frac{\frac{1}{c^y}}{n^y} = \frac{\frac{\theta q_t x_{t+1}}{c^y} + \frac{\beta}{k'}}{n^y \frac{\partial}{\partial k'} \Phi(k', \tilde{k}, \kappa)}, \text{ or} \quad (85)$$

$$\frac{\partial}{\partial k'} \Phi(k', \tilde{k}, \kappa) - \theta q_t x_{t+1} = \beta \frac{e + w_t + \theta q_t x_{t+1} k' - \left\{ \Phi(k', \tilde{k}, \kappa) + \tilde{k} \left[-\frac{\partial}{\partial \tilde{k}} \Phi(k', \tilde{k}, \kappa)\right] \right\}}{k'}. \quad (86)$$

Denote

$$r_t = -\frac{\partial}{\partial \tilde{k}} \Phi(k', \tilde{k}, \kappa).$$

Then,  $\tilde{k}$  solves

$$\varphi_t(k'; \kappa) = \min_{\tilde{k}} \left[ \Phi(k', \tilde{k}, \kappa) + r_t \tilde{k} \right], \text{ and} \\ \tilde{k} = \left[ b \frac{\eta \psi}{r_t} \left(\frac{k'}{\kappa}\right)^{(1-\eta)\psi} \right]^{\frac{1}{1+\eta\psi}} k'.$$

Then, condition (86) becomes

$$\begin{aligned}\varphi'_t(k'; \kappa) - \theta q_t x_{t+1} &= \frac{\beta}{k'} [w_t + e - \varphi_t(k'; \kappa)], \text{ and} \\ \beta(w_t + e) &= \Gamma_t \left( \frac{k'}{\kappa} \right) k', \text{ where} \\ \Gamma_t \left( \frac{k'}{\kappa} \right) &= \left( 1 + \beta \frac{1 + \eta\psi}{1 + \psi} \right) \varphi'_t(k'; \kappa) - (1 + \beta)\theta q_t x_{t+1}.\end{aligned}$$

Therefore,

$$W_{y,t}(y^y, k', e) = V_{y,t}^{m*}(\kappa, e), \text{ for } (\kappa, e) \in \Theta_t,$$

if  $(w_t, r_t)$  are the same as in the competitive equilibrium. Also, we learn

$$\begin{aligned}V_{o,t}^m(k, d) &= \ln \left[ Ak^\alpha (l + n^y)^{1-\alpha} - wl - n^y y^y - n^y \Phi \left( k', \frac{k}{n^y}, \kappa \right) - d \right] \\ &= \alpha A \left( \frac{\tilde{l}}{k} \right)^{1-\alpha} k - \frac{\partial}{\partial \tilde{k}} \Phi \left( k', \tilde{k}, \kappa \right) k - d \\ &= \left\{ \alpha \left[ \left( \frac{1-\alpha}{w_t} \right)^{1-\alpha} A \right]^{\frac{1}{\alpha}} + r_t \right\} k - d \\ &= x_t k - d = V_{o,t}^{m*}(k, d).\end{aligned}$$

Therefore the competitive equilibrium is coalition-proof. Since the value functions and allocations in the coalition proof equilibrium are the same as in the competitive equilibrium with  $r_t^f = r_t$  and  $x_t^f = x_t$ , these prices can be supported in equilibrium.