Burden-sharing in Nonbinding Alliances *

Songying Fang
Department of Political Science
University of Minnesota
sfang@umn.edu

Kristopher W. Ramsay
Department of Politics
Princeton University
kramsay@Princeton.edu.

*We thank Kevin Clarke, Nevzat I. Eren, Mark Fey, Hein Goemans, David Primo,Thomas Ramsay, Curtis Signorino, Branislav Slantchev, Alexander Vostroknutov, three anonymous reviewers and the editors, Keith Krehbiel and Nolan McCarty for their comments and advice. Kris Ramsay acknowledges financial support from NSF grant SES-0413381. Any remaining errors are the sole responsibility of the authors.

Abstract

We develop a model of alliances with outside options to study burden-sharing in nonbinding alliance agreements. The analysis provides an explanation for the variance in ally contributions to NATO over time and why the post-Cold War period has seen an increase in the use of coalitions of the willing. Additionally, our analysis reveals something of an initiator's disadvantage in burden-sharing—the initiator of an alliance action pays a disproportionate cost of the military burden. Our argument provides an alternative explanation for why the United States has been consistently the largest contributor to NATO.

In 1981 the U.S. Congress included in its annual Defense Authorization Act a call for U.S. allies to increase their contribution to the common defense. Since then the Secretary of Defense has provided an annual report that describes disparities between the United States and allied contributions to various joint security endeavors and proposes policies aimed at eliminating inequalities. Such concerns about burden-sharing have never gone away from American foreign policy debate and are reflected in voluminous scholarly work on alliances.

The starting point for much of the literature on the economics of alliances is Olson and Zeckhauser's (1966) "An Economic Theory of Alliances." Olson and Zeckhauser argue that to understand alliance behavior, we must begin with a recognition of its purpose. Alliances coordinate offensive policies, such as wars or peacekeeping, or encourage collaboration to protect the members from aggression by a common enemy. Olson and Zeckhauser suggest that the common interests, which defines the alliance relationship, imbues such an agreement with the characteristics of a collective good.[1] Applying the logic of collective good to alliances, Olson and Zeckhauser find that the level of contribution of any member depends on its relative size. That is, larger and richer members tend to contribute more to joint pursuits, while smaller and poorer members contribute less, thus creating disproportionate defense burdens.

The economic theory of alliances, however, leaves unexplained three empirical puzzles. First, while the theory explains some alliance behavior, such as NATO burden-sharing in the 1950s and 1960s, it does not explain well the distribution of military burdens in pre-Cold War alliances like the Triple Alliance and Triple Entente just prior to World War I. In this case the

[1]Note that Olson and Zeckhauser's collective good is similar to but not the same as a public good. Collective goods are closely related to club goods. That is, goods that are non-rival, but excludable for non-club or non-alliance members. For consistency, however, we use the collective good rather than club good terminology.

more wealthy Great Britain and Germany, contributed less than the poorer Austria-Hungary and Russia. The Anglo-French Treaty is a similar counter-example from the interwar period. This alliance was based on a mutual recognition by Great Britain and France that their fates were closely connected to defending the Rhine, but throughout most of the interwar years the French bore most of the burden defending it. Second, the economic theory of alliance struggles to explain the change in non-U.S. NATO spending in the 1970s and 1980s. In the 1970s U.S. NATO spending went down with détente, but non-U.S. NATO spending increased. By 1980, the old pattern of higher U.S. burdens were reestablished. Finally, in the post-Cold War era, there has been another disproportionate change in burden-sharing within NATO, with European countries decreasing defense spending and contributions to NATO activities in larger proportions than the United States.

These puzzles have led to a significant literature that generalizes and extends Olson and Zeckhauser's basic economic theory of alliances. Most notably, Sandler and his co-authors have developed a series of joint production models that generalize the economic theory of alliances to circumstances where the alliance actions produce less than pure collective goods (Conybeare and Sandler 1990, Murdoch and Sandler 1982, Murdoch and Sandler 1984, Sandler 1977, Sandler and Hartley 1999, Sandler and Hartley 1995, Sandler and Hartley 2001). Conybeare (1994) extends the economic model in a different direction, treating alliances as producers of an impure collective good called defense, and asks what the portfolio benefits of alliances are in terms of military returns and the lowering of risk. Departing even further from the collective good model of alliance, Morrow (1994), Morrow (2000), and Smith (1995) have investigated and explained other dimensions of alliance behavior, such as the signaling value of alliance formation and the varying degrees of reliability of defensive alliance agreements. Our study builds on this line of research by relaxing an important assumption in the economic theory of alliances.

Like Smith's (1996) work on the credibility of defensive agreements in times of war, we consider how commitment problems affect burden-sharing within alliances. Specifically, while the existing burden-sharing literature implicitly assumes that allies do not have outside options, and must pursue foreign policy objectives within existing alliance structures, recent history suggests that there exists viable alternatives to established alliances for countries to pursue joint foreign policy goals. An alliance agreement, like any agreement between sovereigns, is binding only to the extent those involved choose to work within it.

As a first step toward relaxing this assumption, in our model the policy initiator has the option of undertaking a costly search for an *ad hoc* partner. With this modeling choice, we analyze a previously unstudied aspect of the bargaining process within alliances: the two-front bargaining problem faced by the initiator, where on one side there are existing allies, and on the other there are potential new partners.[2] The question then becomes: how does the option of searching for an *ad hoc* partner affect the existing ally's incentive to contribute to the joint foreign policy action? By considering the possibility of pursuing an outside option we gain insight into many of the empirical puzzles that can not be explained by the economic theory of alliances.

Introducing outside options into a model of alliances also allows us to consider how the structure of the international system might influence burden-sharing among allies. There is an intuitive connection between search cost and the configuration of alliance between countries. In what international relations theory calls a tight bipolar system, where there are two competing blocs of countries and an intense competition between them over strategic

[2]Although few have analyzed the effects of outside options in contexts other than war (an exception being Voeten (2001)), the economic literature on the subject is extensive (Binmore 1985, Lee 1994, Muthoo 1999, Muthoo 1995, Shaked 1994).

resources, we would expect search costs to be high. Under these circumstances there would be few unaligned countries and a potentially high cost of abandoning established allies. In what is called a lose bipolar system, with less intense competition between blocs and some small number of unaligned countries, we would expect the search cost to be lower. Finally, in a multi-polar system, characterized by a large number of unaligned or weakly aligned countries, we would expect search costs to be lower still.

Interpreting search costs in this way, the alliance bargaining model with outside options produces a unusual relationship between ally contributions and the flexibility of alliance configurations. Specifically, allies contribute more in loose bipolar conditions than in tight bipolar conditions, but do not contribute enough in the multi-polar setting to deter search. This pattern is an interesting implication of our analysis, and broadly consistent with the historical record. The result suggests that the breakdown of NATO relations in 2003 might be caused as much by strategic (structural) changes in the international system, as by inept management of the alliance by political leaders in the United States.

Furthermore, our analysis reveals something of an initiator's disadvantage in burden-sharing—the initiator of a foreign policy action in an alliance pays a disproportionate cost of the military burden. This departs from the conventional view that links country size and wealth to burden-sharing and propose an alternative explanation for why the United States has been consistently the largest contributor to NATO. If we consider only the Cold War actions of NATO, the country size proposition and the initiator disadvantage are observationally equivalent for the United States. If, however, we look at historical alliances, or alliances in the post-Cold War period, we see that the more consistent regularity is that initiators of multilateral security actions shoulder the largest burden.

A Model of Burden-Sharing with Search

We model the alliance bargaining process as one between an initiator and an ally over the amount of resource that the ally contributes to a joint foreign policy action that produces a collective good.[3] Call the country initiating the foreign policy act country 1 and the ally country 2. Consider the simple world where the countries in the alliance have the same policy preference and receive equal benefit from the proposed foreign policy act.[4] For some fee, ϕ, the allies can take an action that provides each member of the partnership a benefit b. Normalize the benefit such that if the collective good is produced, and a country is a member of the coalition, each country gets a payoff of 1, but gets 0 otherwise. Also assume $\phi \in (1, 3/2]$, such that no state wants to produce the good on its own, but if two countries contribute to its production both are better off.[5] This model focuses on the basic strategic trade-off faced by members of an alliance. On the one hand, both countries have a common interest in producing the policy outcome, modeled here as a unit payoff; on the other hand, both allies desire to free ride on the contribution of their alliance partner in order to maximize their individual gain from the alliance's action.

The alliance bargaining problem we consider consists of two periods. The game begins

[3]Lee (1994) develops a model that studies the role of searching for outside options in bargaining between a buyer and a seller over the price of an indivisible good. Our model differs in that it looks at situations in which a collective good is to be produced for a known cost, but where the good can be produced by different combinations of countries.

[4]We abstract away from differences in policy preferences to investigate the strategic effects of outside options on alliance behavior. Clearly, this is not the whole story, but it is interesting how many apparent anomalies in previous theories can be explained by our model.

[5]The logic that drives the equilibrium strategies is the same for all $\phi \in (1, 2)$, which is the feasible range for the cost of producing the collective good; however, $\phi \in (1, 3/2]$ is the harder case to solve than the case where $\phi \in (3/2,2)$.

when country 1 proposes a joint foreign policy action. In response, country 2 informs country 1 how much it is willing to contribute. Let $r_1 \in [0, 1]$ be country 2's proposed contribution, which country 1 could subtract from the fee for taking the proposed action. Given r_1, country 1 can decide whether or not to go forward with its proposal. If the policy action is taken with the ally, country 2 gets $1 - r_1$ and country 1 gets $1 - \phi + r_1$. Obviously, country 1 only produces the good if $1 - \phi + r_1 \geq 0$. As an alternative to accepting the proposed contribution from country 2, country 1 may either choose to search for contributions from potential partners outside the alliance or to advance the game to the second period and ask country 2 to make a second offer. If country 1 decides to search in the first period, it pays a cost $c \in (0, 1/2]$ and then calls for a contribution from a state outside the alliance.[6]

Now consider the search process. For simplicity, we assume that the contribution offered by the potential *ad hoc* ally, denoted as x_1, is unknown *ex ante* and drawn from a uniform distribution on $[0, 1]$.[7] Furthermore, we assume that the outside offer x_1 is a standing offer and can be accepted in period 1 or 2.[8] After a search, country 1 decides whether to accept r_1 or x_1. If it accepts either offer in the first period, then the game ends and payoffs are realized. Otherwise, country 1 moves the game to the second period by calling on country 2

[6]If $c > 1/2$, then the search will be an irrelevant option since it is too costly. For more details, see the proofs in the appendix.

[7]If the *ad hoc* partner is strategic and fully informed, then it does not offer any more (or less) of a contribution than the ally. Here we model a world where the *ad hoc* partner is uncertain about the cost of action. For example, the third party might have less or noisier information than the alliance members or does not know enough about intra-alliance negotiations to learn country 2's proposal. In such a world, an *ad hoc* partner makes an offer that *appears* to be a random variable.

[8]This assumption allows us to consider an important dynamic: if the outside offer can be taken in either period, the alliance partner has an opportunity to react to developments, i.e., it can condition its second period offer on the initial outside option.

to make a second proposal.

The second period is played in a similar fashion with country 2 making a new offer, $r_2 \in [0, 1]$. Given r_2, country 1 can choose not to proceed with the proposed policy, to proceed with the alliance partner at a fee $\phi - r_2$, or to proceed with the *ad hoc* partner at a fee $\phi - x_1$, all of which will end the game.[9] Alternatively, country 1 can search again and draw a second outside offer, x_2, at a cost c. After observing r_2, x_1, and x_2, country 1 must choose whether or not to produce the collective good, and with whom to produce it. The sequence of the game is also depicted in Figure 1.

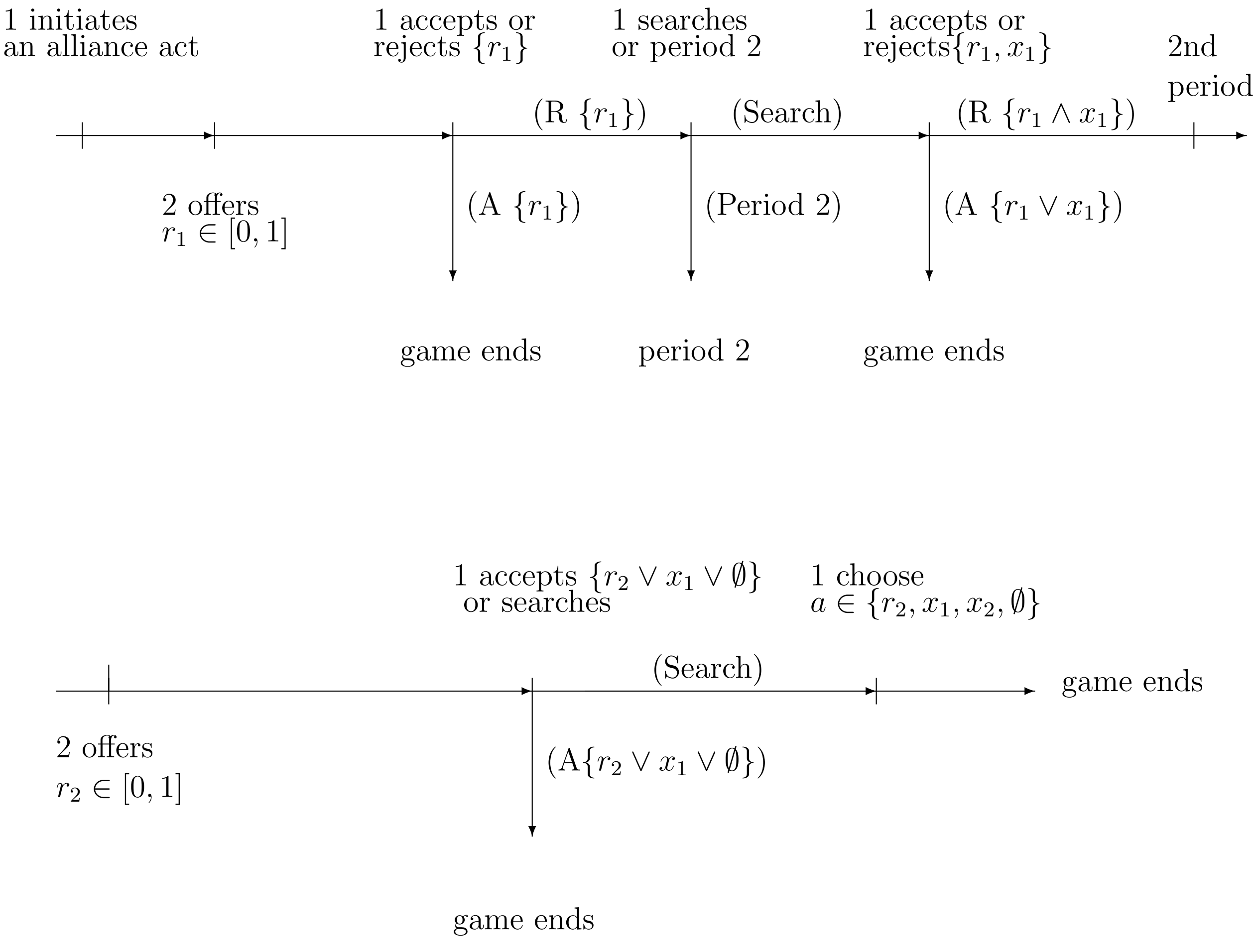
Countries in this dynamic game are assumed to discount future payoffs with a common discount factor $\delta \in (0, 1]$. That is, delay may be costly. When country 1 takes the joint policy action its utility is $\delta^{t-1}(1 - \phi + s_t)$, where s_t is either the ally's contribution in period t or an outside offer available. If there was search, then the search costs are subtracted from this payoff. Country 2's utility is $\delta^{t-1}(1 - r_t)$ if r_t is accepted in period t, and zero if country 1 does not accept country 2's proposal.

Before proceeding to the equilibrium analysis, some discussion of the search cost is in order. Note that any search for an *ad hoc* partner is costly, regardless of the given country's alliance membership. The question then becomes: what aspects of an alliance are captured by country 1's search cost? First, the cost reflects a decrease in political capital that countries have invested in an alliance relationship.[10] As was clearly the case in the run-up to the second Gulf War, statements by the Bush administration that it was willing to act outside NATO and the Security Council were damaging to relations between the U.S. and fellow

[9] If country 1 does not search in the first period, then without loss of generality, we can set $x_1 = 0$.

[10] This loss in political capital may come from domestic audience costs that arise from violating a previous commitment, or audience costs in the ally's country that discourage its leaders from working with the offending ally in the future.

Figure 1: Time Line of the Alliance Game



NATO allies, making future cooperation for joint gains more difficult. Second, failing to live up to alliance agreements may hurt a country's reputation for reliability: if a country abandons this ally, how can another ally be confident that it will not also be abandoned in the future? Finally, there exist incentives for other countries to counter-balance a new alliance. That is, after a new alliance is formed countries on the outside of the coalition may form alliances of their own.[11] If the creation of alliances leads non-members to respond in kind, then the creation of alliances induces rigidity into the international system. So, much as the formation of NATO led to the formation of the Warsaw Pact and the Triple Alliance energized the Triple Entente, building alliances can make it more difficult to find an *ad hoc* partner. This last factor hints to a larger point that search costs reflect the ease with which a new alliance partner can be found. We explore this idea below when we consider why countries might form alliances.

Equilibrium Analysis

We solve for the subgame perfect equilibrium of the game using backward induction.[12] We first examine the allies' second period equilibrium strategies conditional on what happened in the first period, and then analyze their first period strategies. The analysis results in two lemmas that characterize countries' equilibrium strategies in each of the two periods. The lemmas are the building blocks for the unique subgame perfect equilibrium for the entire game, and highlight some of the strategic incentives that arise when allies bargain in the shadow of outside options.

Our first result says that, given that country 1 *searched* in the first period and received

[11] For a discussion of this phenomenon, see Bruce (1990).

[12] All proofs are found in the appendix.

an outside offer, country 2 always matches the outside offer in the second period to keep herself in the running as a partner for the production of the collective good.

Lemma 1. *If in an equilibrium period two is reached and country 1 searched in the first period, then country 2 offers enough to be in the running for the production of the good in the second period. That is,* $r_2 \geq \max\{x_1, \phi - 1\}$.

The intuition behind the result is that, if country 1 searched in the first period, then it has an offer x_1 in hand, which may or may not be big enough for the production to take place. Country 1 needs to receive a contribution that is at least as big as $\phi - 1$ to produce the good. If $x_1 \geq \phi - 1$, then country 2 needs to match x_1 so that it has a chance to participate in the joint policy and receive the benefit. On the other hand, if $x_1 < \phi - 1$, it is in country 2's interest to offer at least $\phi - 1$, so that the good will be produced within the alliance. Therefore, country 2's offer in the second period has to be as big as both x_1 and $\phi - 1$.

We now fully characterize the countries' second period equilibrium strategies. By Lemma 1, the ally's second offer r_2 is the only relevant offer when country 1 considers whether or not to search in the second period. Let $r^* \in [0, 1]$ be the level of contribution by the ally that makes country 1 indifferent between searching and not searching in the second period. It is a function of the search cost c. If $r^* < \phi - 1$, then a high search cost makes searching worse than any scenario in which country 1 will produce the good with the existing ally. As a result, country 2 will offer exactly $\phi - 1$ so that the good will be produced, and country 2 takes the entire surplus. Now suppose $r^* \geq \phi - 1$. Then, in the second period country 1 searches if $r_2 < r^*$, and does not search if $r_2 \geq r^*$. So, if country 2 contributes at least r^*, then country 1 accepts the offer and country 2's payoff is $1 - r_2$; if country 2 contributes a smaller amount, then country 1 searches and the resulted payoff for country 2 is a lottery: it receives $1 - r_2$ with probability r_2 (the probability that the second outside offer x_2 is no larger

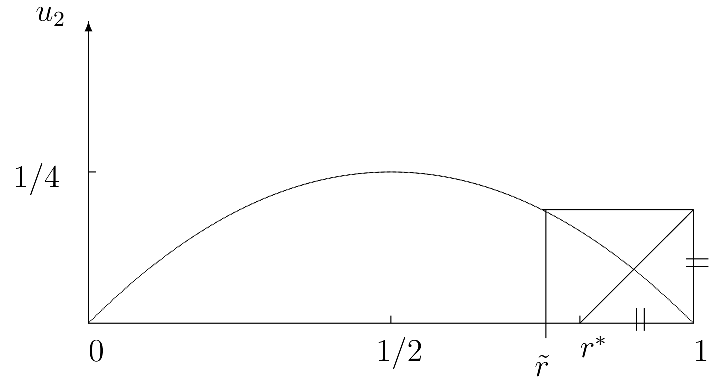


Figure 2: Country 2's utility in period 2 given search.

Note that the double-crossed lines are of equal length, so that the utility of offering r^* for country 2 is the same as the utility of offering $\tilde{r}$ and inducing country 1 to search. In other words, $1-r^* = (1-\tilde{r})\tilde{r}$.

than r_2), and receives 0 with probability $1 - r_2$. In this scenario, at most it gets 1/4, which is the maximum expected value of the lottery $(1 - r_2)r_2$ achieved at $r_2 = 1/2$. Country 2's best response, then, depends on the values of r^*, x_1, and 1/2. If $r^* < 3/4$, i.e., $1 - r^* > 1/4$, then country 2 offers $r_2 = max\{r^*, x_1\}$. This is a case where the most country 2 can get from inducing country 1 to search is less than what it receives from satisfying country 1 outright, but also by Lemma 1, country 1 has to offer at least x_1 to be in the running for the production of the good. If $r^* \geq 3/4$, i.e., $1 - r^* \leq 1/4$, then country 2 can be better off by inducing country 1 to search and the strategies are more involved.

Let $\tilde{r} > 1/2$ denote an offer by country 2 that leads country 1 to search but gives country 2 the same expected payoff as offering r^*. That is, $\tilde{r}$ is the solution to the following equality:

$$1 - r^* = (1 - \tilde{r})\tilde{r}.$$

The relationship between r^* and $\tilde{r}$ is shown in Figure 2. Because $u_2(r_2)$ is concave in r_2, $\tilde{r} \leq r^*$. The thresholds, $\tilde{r}$, r^*, and 1/2, mark the regions that the outside offer x_1 may fall and to which country 2 responds differently with its optimal offer. A complete characterization

Table 1: Equilibrium Strategies in Period 2 after a Search in Period 1.

	$r^* < 3/4$	(or $1 - r^* > 1/4$)		
Range of x_1	$[0, r^*]$		$(r^*, 1]$	
country 2's offer	r^*		x_1	
country 1's response	Accept		Accept	
	$r^* \geq 3/4$	(or $1 - r^* \leq 1/4$)		
Range of x_1	$[0, 1/2]$	$(1/2, \tilde{r}]$	$(\tilde{r}, r^*]$	$(r^*, 1]$
country 2's offer	$1/2$	x_1	r^*	x_1
country 1's response	Search	Search	Accept	Accept

of the countries' strategies at this stage is presented in Table 1. [13]

From country 2's strategy in the second period, we begin to see the dynamic effect of outside options on the alliance relationship. In particular, consider the more complicated case where $r^* \geq 3/4$. If country 1's first-period search went very badly, then country 2 can ignore the outside offer altogether and focus on choosing an offer that maximizes its expected payoff given that country 1 will search again. If the search result was somewhat improved, but country 1 still has a strong incentive to search again, then country 2 has to match the outside offer to keep itself in the running. If the first search result was quite good and country 1 only has a mild incentive to search again, then country 2 is better off proposing enough to deter country 1 from searching to avoid the risk of being replaced by a second outside offer. If the first search went exceedingly well, then country 1 does not search for a second time and country 2 has to match x_1 to receive any benefit. Overall, a successful search by country 1 decreases country 2's incentive to free-ride and increases country 2's

[13]The derivation of the equilibrium strategies in Table 1 can be found in the appendix.

willingness to contribute more.[14]

The preceding analysis assumes that country 1 searched in the first period. This is the more complicated case. If it did not search in the first period, then country 1's decision problem in the second period is similar to the above, but we need no longer consider the constraint that $r_2 \geq \max\{x_1, \phi - 1\}$. Therefore, if $r^* < 3/4$, then country 2 offers r^* and country 1 accepts; if $r^* \geq 3/4$, then country 2 offers 1/2 and country 1 searches.

The next lemma says that country 1, the initiator, never moves the game directly to the second period by asking its ally to make a second offer. Rather, country 1 always takes advantage of having the outside option in the first period if it is not initially satisfied with the ally's offer.

Lemma 2. *For δ sufficiently larger, country 1 accepts a contribution from country 2 in the first period if it is greater than r^*; otherwise, it rejects an offer and searches in the first period.*

Intuitively, if country 2's first offer r_1 is sufficiently large, then country 1 accepts it and produces the good; otherwise, country 1 can look for a better outside offer or asks country 2 to make a second offer. Lemma 2 states that when rejecting r_1, country 1 searches first rather than continues bargaining with the ally. Here, r^* again becomes the crucial offer that makes country 1 indifferent between accepting country 2's first offer and rejecting it in the first period. Comparing the payoffs from searching in the first period and moving directly to

[14] Note that country 1 has an incentive to reveal the *ad hoc* offer to its ally. Looking at Table 1, we know that if the offer were revealed, the worst that can happen to country 1 is that the outside offer is very low and country 2 offers 1/2 in response. If the outside offer is high, revealing the offer makes country 1 better off. If country 1 never reveals the offer, country 2's belief is defined by the expected value of the outside offer, which induces country 2 to offer 1/2. As a result, if country 1 reveals the outside offer, in no state of the world is it worse off, and in some states of the world it is better off.

Table 2: Country 1's Equilibrium Strategy in Period 1 after a Search.

		$r^* \leq 3/4$	(or $1 - r^* \geq 1/4$)		
Range of x_1	$[0, r_1^*]$	$(r_1^*, 1]$			
1's response in pd 1 after search	Accept r_1 if $r_1 \geq r_1^*$; else 2nd pd.	Accept $max\{x_1, r_1\}$			
		$r^* > 3/4$	(or $1 - r^* < 1/4$)		
Range of x_1	$[0, 1/2]$	$(1/2, \tilde{r}]$	$(\tilde{r}, r_1^*]$	$(r_1^*, r^*]$	$(r^*, 1]$
1's response in pd 1 after search	Accept r_1 if $r_1 \geq$ $\delta\theta(1/2) + \phi - 1$; else 2nd pd.	Accept r_1 if $r_1 \geq$ $\delta\theta(x_1) + \phi - 1$; else 2nd pd.	Accept r_1 if $r_1 \geq r_1^*$; else 2nd pd	Accept $max\{x_1, r_1\}$	Accept $max\{x_1, r_1\}$

Note: In the table, r_1^* *is country 2's first period offer that gives country 1 the same payoff as offering* r^* *in the second period. That is,* r_1^* *is the solution to the equation* $1 - \phi + r_1^* = \delta(1 - \phi + r^*)$*. The derivation of the equilibrium strategies in Table 2 can be found in the appendix.*

Table 3: Percentage of National Income Devoted to Arms, 1914

Triple Alliance		Triple Entente	
Germany	4.6	Great Britain	3.4
Austria-Hungary	6.1	France	4.8
Italy	3.5	Russia	6.3

the second period, it can be shown that, for any $r_1 < r^*$, country 1 is better of searching in the first period. The full characterization of country 1's equilibrium strategies at this stage is presented in Table 2.

With the above analysis, we can now discuss the unique equilibrium to this game. Proposition 1 characterizes the two cases in which the search cost for country 1 is sufficiently high so that the ally has an opportunity to offer an amount that will guarantee a joint production of the good.

Proposition 1. *For δ sufficiently large, if $1/2(2-\phi)^2 < c$, then country 2 offers $\phi - 1$ in the first period and country 1 accepts immediately; if $1/32 \leq c < 1/2(2-\phi)^2$, then country 2 offers $r_1 = r^*$ in the first period and country 1 accepts immediately.*

Recall that r^* is the level of contribution that makes country 1 indifferent between accepting the ally's offer and searching in the first period, and we find that $r^* = 1 - \sqrt{2c}$. When c is sufficiently high, greater than 1/32, the offer that makes country 1 indifferent between searching and not searching is relatively small. Consequently, it is in country 2's interest to satisfy country 1 outright and jointly produce the good. Additionally, since r^* is a decreasing function of c, as c decreases within the feasible range of the case the ally has to contribute more to satisfy country 1.

Proposition 1 offers a rather different explanation for disproportionate burden-sharing in alliances than the conventional wisdom. Specifically, in our model the initiator of an alliance action is in a weak position because its ally need only minimally satisfy the policy initiator to take part in the policy benefit. As we touched upon earlier, the prevailing view is that large, rich countries tend to shoulder a disproportionate share of the defense burden, while small countries free-ride. However, an examination of prominent alliances, such as the Triple Alliance and Triple Entente on the eve of World War I (Thies 1987, Conybeare and Sandler 1990) and NATO in the early Cold War period (Olson and Zeckhauser 1966), reveals a pattern that is consistent with our finding. Evidence in Table 3 is inconsistent with the largest power paying the largest share. Moreover, while it is not true for every possible value of the search costs, in our model the initiator of an alliance act often pays a large share–or more accurately it net share of benefit is smaller. In fact, the rigid alliance system in the early twentieth century, and the resulting military plans, would suggest search costs were high and countries like Russia and Austria-Hungary should be spending more on arms. [15] Similarly, we should expect the United States to consistently be the largest contributor to NATO, and to have spent the largest share of GDP on defense over the period from 1950–1989 (Olson and Zeckhauser 1966, Sandler and Hartley 1995). This is not simply because the United States is bigger and richer; rather, it is a result of United States being the initiator of most NATO actions.[16]

[15] Data from Taylor (1954) page xxix, also referenced in Thies (1987). For a discussion of the alliance system leading up to World War I, see Joll (1992).

[16] One might worry that the importance of the initiator is a product of our assumptions about the bargaining protocol. While in a formal sense this is true, in many ways the purpose of our model is to investigate how the availability of outside options changes the bargaining power of the non-free riders. As the concept of free riding is defined to be one where one player has all the bargaining power, we know formally, and believe conceptually, that the qualitative nature of this result will extend to other bargaining protocols where the supporting countries have more bargaining power than those initiating allied actions.

Furthermore, the inverse relationship between the search cost and the amount that the ally contributes in this equilibrium sheds new light on the dynamics of burden-sharing during the Cold War when the search cost was generally high. During the Cold War, burden-sharing between the United States and its NATO allies shifted over time. One existing explanation for this shift, presented by Sandler et al., is that as the United States moved from a policy of mutually assured destruction (i.e., offering deterrence) to flexible response (i.e., conventional and limited defense) and the nature of the good provided by NATO changed from a pure public good to a impure public good.

As O'neal and Elrod (1989) have pointed out, this argument is inconsistent with the empirical evidence. In particular, Sandler et al.'s argument predicts a change in burden-sharing as early as 1967, and could likely be extended back to the missile gap debate in the late 1950s. This disparity between the joint production model's prediction and the change in burden-sharing suggests something else was the cause. One weakness of the joint production model is that it is largely apolitical. That is, it focuses on how technological change shapes NATO's defense policy, leaving political change out of the equation. Second, although the shift from a strategy of mutually assured destruction to flexible response seems to be a reasonable explanation for changes in the public-ness of the common defense in 1967, the relatively minor change to a *deep strike* strategy in the 1980s does not seem significant enough to explain the reemergence of disproportionality in the 1980s.[17]

Our model, on the other hand, points to an obvious change that coincides with both developments—détente and its decline. In fact, we find results similar to the informal results

[17] There are numerous studies on the trends in military burdens over time. The general trend in military spending shows that from 1949–1970 there existed a strong correlation between country size and defense burden, which then decreased in the 1970s, but returned in the 1980s. For more discussion, see O'neal and Elrod (1989) and Sandler and Murdoch (2000).

of Snyder (1984). That is, alliance behavior is strongly correlated with the flexibility of the international system.

To see this, recall that in 1967 the Johnson administration approached the Soviet Union on both the issues of Vietnam and arms control. As both countries' economies were suffering from the increased cost of defense, agreement on a reduction in tensions became possible. In addition, as the split between the Soviet Union and the Peoples Republic of China (PRC) grew, there was finally room for the United States to pursue the *possibility* of coordinating policy with non-NATO actors, as seen in the Shanghai communique in 1972 and the February joint communique in 1973. So as the United States' search costs decreased during détente, the consequence was an increase in the share of the military burden taken on by NATO allies. Notice that this comparative static exists even without the use of the outside option, implying that changes in search costs can have distributional consequences within an alliance without ever leading to new coalitions.

By 1980, however, the deterioration of relations between the United States and the Soviet Union, particularly following the invasion of Afghanistan in December of 1979, again increased the costs of searching for the United States. As a result, the smaller NATO allies began to free ride and the defense burden, measured as the ratio of defense spending to GDP, was carried by the initiator of allied actions, which was the United States.

Moreover, as we move into the post-Cold War period, the joint production model has less to say about the configuration of burden sharing. In particular, the implicit assumption that alliance agreements are binding is problematic. So, if the end of the Cold War implies that countries have more options when it comes to producing security, which recent trends suggest that they do, we may wonder what happens as the search cost decreases further.

By the relationship between the search cost and the ally's equilibrium strategy, as the search cost decreases, the level of contribution required to deter country 1 from searching is higher. This in turn makes country 2 less willing to satisfy country 1 outright, and more willing to gamble on country 1 having a bad draw from searching. Proposition 2 characterizes the unique equilibrium that results when satisfying country 1 immediately is too costly for country 2.

Proposition 2. *Let $\hat{r}_1$ be the solution of the following constrained maximization problem:*

$$\begin{aligned} \max \omega(r_1) = & \int_0^{x(r_1)} (1-r_1)dx_1 + \int_{x(r_1)}^{\tilde{r}} \delta(x_1(1-x_1))dx_1 + \int_{\tilde{r}}^{r_1^*} \delta(1-r^*)dx_1 \\ s.t. \qquad & \delta\theta(1/2) - 1 + \phi \leq r_1 \leq \delta\theta(\tilde{r}) - 1 + \phi. \end{aligned}$$

where $x(r_1)$ solves $1-\phi+r_1 = \delta\theta(x_1)$. For δ sufficiently large and $c < 1/32$, country 2 offers $\hat{r}_1$ and country 1 searches in the first period.

The proposition states that when r^* is high, or when the search cost is low (i.e., $c \in (0, 1/32]$), country 2 may offer $\hat{r}_1$, which is less than enough to make country 1 satisfied outright. What is interesting about this equilibrium is that all sorts of outcomes are possible. Specifically, in the cases when country 1 searches, if the outside offer is sufficiently small , i.e., $x_1 \in [0, 1/2]$, then country 1 accepts $\hat{r}_1$ in the first period. If, however, the outside offer is somewhat better, then country 1 moves the game to the second period and forces country 2 to match the outside offer. If the first period search goes well, then country 1 can move the game to the second period and induce its ally to offer r^*, and country 1 gets a bonus from its ally's bid-jumping incentive. Finally, the first period search could go exceedingly well, in fact, so well that country 1 decides to break with its ally and takes the joint action with the outside partner. So, even in an alliance without private information, if anarchy implies that countries have outside options, then there is positive probability that alliances break down. On the other hand, the existence of outside options reduces country 2's incentive to free ride and, in equilibrium, it offers a resource contribution greater than the minimum needed to make its ally willing to take the joint action.

Given this result, what should we expect to happen with the end of the Cold War? Because the international system went from one of rigidity and intense competition between the Soviet Union and the United States to a unipolar system, where the United States has much more flexibility in choosing the means for pursuing foreign policy goals, it seems reasonable to conclude that search costs have decreased significantly. In such a situation, we should see that NATO allies become less eager to pursue collective foreign policy goals led by the United States, and that there is a greater risk that major powers take actions with *ad hoc* coalitions, rather than with existing allies. This, in fact, occurs, with Italy's *ad hoc* coalition Operation Alba in the Balkans, Australia's various operations in the South Pacific and Timor Sea, and the United States' operation in Iraq.[18]

One measurable manifestation of the decreased willingness of allies to contribute to the common defense is a disproportionate decrease in spending on defense among allies. In fact, this is exactly what has happened. Between 1990 and 1998, the defense spending of non-U.S. members of NATO, as a percentage of GDP, decreased by 31.4%, and allies in the Middle East similarly have decreased their spending by 55.4% (US Department of Defense 1999). The one region where there has been little change is in Asia. This may be the result of the existence of a major power with competing interests in the region, i.e. the PRC.

As stated in Propositions 1 and 2, there are two possible equilibrium stories, depending

[18] One may wonder how to empirically differentiate our model of costly search from a model that simply says burden-sharing will depend on the level of threat. A key difference between the threat model and ours is that the threat model would suggest that non-U.S. NATO spending should have gone down with the decreased threat in the 1970s, which did not happen. Our approach does explain both the increase in spending in the 1970s and the decreased contributions in the post-Cold War period.

on the search costs. Proposition 1 suggests that when the cost of searching for a new ally is sufficiently high, it is the existing alliance that will be activated to pursue joint foreign policy goals. On the other hand, Proposition 2 suggests that when the search cost is sufficiently low, so that r^* is sufficiently large, country 2's offer will lead to a search. Accordingly, low search cost implies that there is a possibility that the alliance breaks down and an *ad hoc* coalition is formed.

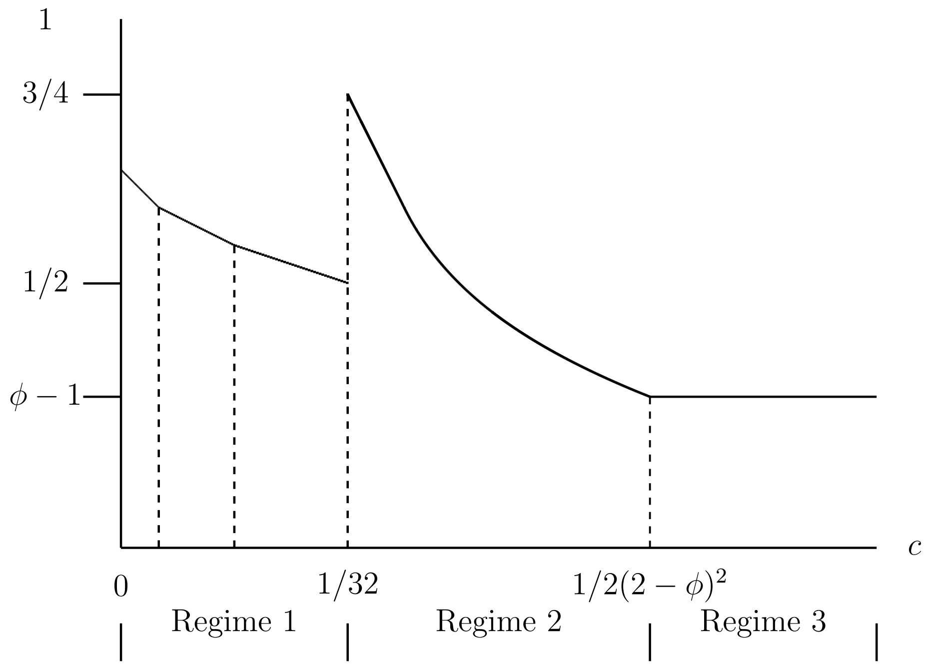


Figure 3: Contribution Regimes for Alliance Bargaining Game.

Regime 1 represents offers described in Proposition 2 that do not deter search. The first dashed vertical line marks the transition from the case where the lower bound of the constraint set is the optimal feasible offer to where the interior solution is the optimal offer. The second dashed line marks the transition from the interior optimal offer to the upper bound on the constraint set. The next dashed line marks the transition from the regime of Proposition 2 to that of Proposition 1. Regime 2 marks the range where the ally contributes r^* and deters search. Regime 3 marks the range where the ally extracts all the surplus and the ability to search has no effect on contributions.

The analysis of the game also leads to three interesting and unexpected results. Our first comparative static shows that the monotonic relationship found in Proposition 1 between the search cost and the optimal offer (r^*) extends to the case in Proposition 2 as well.

Proposition 3. *When $c < 1/32$ and δ is sufficiently large, the equilibrium offer of country 2 is decreasing in country 1's search cost.*

So, like in the first proposition, we can expect small increases in search cost to lead to smaller contributions.

When it comes to the probability of an *ad hoc* coalition forming, however, things are more complicated. Under the conditions of Proposition 1, the equilibrium probability of alliance breakdown is zero. Under Proposition 2, unlike what might be expected when country 1 faces higher search costs, the probability of an *ad hoc* coalition can be decreasing or increasing. When the search cost increases, the offer made in equilibrium ($\hat{r}_1$) goes down and the outside offer that leads country 1 to form an *ad hoc* coalition in the first period also goes down. As a result, the probability of getting an outside offer that leads to alliance break-down increases. However, when this happens, the offer that deters country 1 from searching, r^*, also decreases and the option becomes more attractive to country 2. The probability of alliance break-down, therefore, depends on the net effect of the two countervailing effects. [19]

Third, our model suggests that countries may join alliances in an anarchic environment for the very reason that search is costly as a member of an alliance. Specifically, in our model, when c is sufficiently small, *ex ante* a country can prefer having a constraining alliance and paying the search cost when necessary to the situation where it may search freely. To see the logic of this argument, suppose a country can choose to form an alliance or not. Normalize the search cost without a n alliance to be zero. We then ask: Is it ever the case that a country prefers to form or join an alliance and impose positive search costs on itself? The following proposition shows that answer can be yes.

Proposition 4. *For δ sufficiently large, being in an alliance improves the welfare of the*

[19] Detailed characterization of this result can be found in the appendix.

country that pays the search cost whenever $c < 1/8(1-(\phi-1)^2)^2$.

For both types of equilibrium, even though equilibrium strategies can lead the initiating country to perform costly search, country 1 prefers to be in an alliance than having the freedom to search at no cost. The intuition for this result comes from the fact that the ally's offer insures country 1 against the downside risk of a search that returns a very small outside offer.

Note, however, the existence of outside options is necessary to bring a higher payoff to country 1. Having an alliance in and of itself does not have such a welfare effect. If there were no outside options, country 2 will provide just enough resources to make country 1 indifferent between producing and not producing the collective good. In other words, without an outside option, country 1 receives a net benefit of zero when it takes a joint action, while country 2 receives the maximum net benefit. In this sense, anarchy plays a critical role in creating the incentive to form alliances by presenting outside options. If the alliance were an enforceable contract, as in the economic theory of alliances, country 1 would be better off without an alliance and receive a strictly positive expected payoff from taking a joint action with an *ad hoc* partner. Therefore, alliances of the kind we describe are welfare improving institutions for states in environments where agreements are not enforceable.

Conclusion

In a recent research paper, the Analysis Center for Northrop Grumman considers what trends in the post-Cold War period tell us about military conflicts and strategy in 21st century. One quite dramatic shift in military doctrine has been the change from a focus on collective security to one where countries form coalitions of the willing to achieve foreign policy objectives (Bowie, Haffa and Mullins 2003). Given this trend, a theory of alliances that explains some aspects of both historical alliance behavior and the post-Cold War period's frequent appearance of *ad hoc* coalitions would be valuable.

In this paper, we study burden-sharing in nonbinding alliances by taking countries' outside options seriously and our analysis suggests that costly search for *ad hoc* partners may be an important missing piece in theories of alliances. Our approach allows us to think systematically about how changes in search costs affect burden-sharing and the probability of alliance breakdown. We find that initiators pay a disproportionate share of costs, that there is more free-riding in a tight bipolar system than in a loose bipolar system, and that in multipolar systems allies are unlikely to contribute enough to deter searching. These findings are consistent with historical data on burden-sharing in alliances and better explains the fluctuations in NATO behavior than the joint production model.

Our analysis also suggests answers to the broader questions of why alliances exist and why they are credible. Some argue that alliances exist and are credible because they signal common interests among allies (Morrow 1994). Others argue that the repeated nature of countries' interactions deters a state from reneging on its alliance commitments (Snyder and Diesing 1977). These studies invariably take anarchy as a threat to the maintenance of alliances. Our analysis suggests a very different answer to the questions: anarchy and outside options are what motivate states to form alliances and stick with them. In the classical model without outside options, the country that initiates an allied action will not receive a positive net benefit from joint foreign policy actions because its ally offers just enough of a contribution to get the collective good produced, and keeps all the net benefit to itself. Therefore, if an alliance is an enforceable contract, the initiator is better off by not being a member. Being in an alliance while maintaining outside options, however, improves a country's payoff in situations where the increase in search cost is not too large. In other words, our analysis suggests that countries join alliances because such institutions improve their overall welfare in an anarchic environment.

Appendix

Lemma 1 If in an equilibrium period two is reached and country 1 searched in the first period, then country 2 offers enough to be in the running for the production of the good in the second period. That is, $r_2 \geq \max\{x_1, \phi - 1\}$.

Proof. We first consider the case $x_1 < \phi - 1$. We show that $r_2 \geq \phi - 1$. Suppose not, i.e., $r_2 < \phi - 1$. Then country 2 gets 0 with certainty. Consider its deviation to $r_2 = \phi - 1$. If the game is played such that country 1 searches in the second period, then country 2's expected payoff is $(2 - \phi)(\phi - 1) > 0$. If country 1 does not search in the second period, then country 2's payoff is $2 - \phi > 0$. Clearly, $r_2 = \phi - 1$ is a profitable deviation for country 2. A contradiction.

Now suppose $x_1 \geq \phi - 1$. We show that $r_2 \geq x_1$. Suppose not, i.e., $r_2 < x_1$. Then country 2 gets 0 with certainty. Consider its deviation to $r_2 = x_1$. If the game is played such that country 1 does not search in the second period, country 1 produces the good with country 2, and 2's utility is $1 - x_1 > 0$. If country 1 does search in the second period, by offering $r_2 = x_1$ country 2 gets an expected payoff of $(1 - x_1)x_1 > 0$.
Again, $r_2 = x_1$ is a profitable deviation for country 2. A contradiction.

Finally, if country 1 chooses to produce the good with the *ad hoc* partner when it is indifferent, then country 2 can profitably deviate by offering some slightly larger contribution, leading country 1 to produce the good with country 2. A contradiction. □

The derivation of countries' equilibrium strategies in period 2 after country 1 searched in period 1 (Table 1). We analyze country 1's searching strategy first and then analyze country 2's contributing strategy.

For country 1 the expected payoff from searching in the second period given that country

2 offers r_2 is:

$$\theta(r_2) = -c + \int_0^{r_2} (1 - \phi + r_2)dx + \int_{r_2}^1 (1 - \phi + x)dx. \quad (1)$$

Which can be simplified to

$$\theta(r_2) = -c + 3/2 - \phi + (r_2)^2/2. \quad (2)$$

Note that $\theta(r_2)$ is an increasing function of r_2.

Now let r^* be the level of contribution that makes country 1 indifferent between searching and not searching in the second period. If country 1 accepts r^* and does not search, then its payoff is $1 - \phi + r^*$; if country 1 searches given r^*, then its expected payoff‘ from the search is: $\theta(r^*) = -c + 3/2 - \phi + (r^*)^2/2$.

Because country 1 is indifferent between the two choices, we have

$$1 - \phi + r^* = \theta(r^*). \quad (3)$$

Solving Equation 3, we have $r^* = 1 - \sqrt{2c}$. Since $r^* \in [0, 1]$, $0 < c \leq 1/2$. For each $c \in (0, 1/2]$, r^* is unique and it is decreasing in c. Furthermore, since both $1 - \phi + t$ and $\theta(t)$ are monotonically increasing in $t \in [0, 1]$, with $1 - \phi + t$ increasing at a faster rate, the two functions can only intersect once. Therefore, for all $r_2 < r^*$, the payoff from accepting r_2 is smaller than expected payoff from searching, i.e., $1 - \phi + r_2 < \theta(r_2)$; and for all $r_2 \geq r^*$, the payoff from accepting r_2 is greater than the expected payoff from searching, i.e., $1 - \phi + r_2 \geq \theta(r_2)$. Therefore, in the second period country 1 searches if $r_2 < r^*$, and not search if $r_2 \geq r^*$.

We now turn to country 2's contributing strategy in the second period. Country 2's expected payoff in the second period depends on its level of contribution r_2:

$$u_2(r_2) = \begin{cases} 1 - r_2 & \text{if } r_2 \geq r^* \\ (1 - r_2)r_2 & \text{if } r_2 < r^* \end{cases} \quad (4)$$

That is, if country 2 contributes at least r^*, then country 1 accepts the offer and country 2's payoff is $1 - r_2$. On the other hand, if country 2 contributes an amount that induces country 1 to search, its payoff is a lottery $(1 - r_2)r_2$. In the second scenario, at most it gets 1/4, which is the maximum expected value of the lottery $(1 - r_2)r_2$ achieved at $r_2 = 1/2$. Country 2's best response then depends on the values of r^*, x_1, and 1/2.

If $r^* < 3/4$ (or $1 - r^* > 1/4$) , i.e., if the most country 2 can get from inducing country 1 to search is no more than what it receives from satisfying country 1 outright, then country 2 offers $r_2 = max\{r^*, x_1\}$.

If $r^* \geq 3/4$ (or $1 - r^* \leq 1/4$), i.e., country 2 can be better off by inducing country 1 to search, then the strategies are more involved and we need to define an additional threshold. Let $\tilde{r} > 1/2$ is the solution to the following equality:

$$1 - r^* = (1 - \tilde{r})\tilde{r}. \tag{5}$$

The relationship between r^* and $\tilde{r}$ is shown in Figure 2. Because $u_2(r_2)$ is concave in r_2, $\tilde{r} \leq r^*$. The thresholds, $\tilde{r}$, r^*, and 1/2, mark the regions that the outside offer x_1 may fall and to which country 2 responds differently with its optimal offer.

Now we can characterize countries' equilibrium strategies when $r^* \geq 3/4$.

Case 1: Country 1's first period search goes badly and $x_1 \in [0, 1/2]$. Then, country 2 offers $r_2 = 1/2$ to maximize its expected payoff from inducing country 1 to search, and country 1 searches given the offer.

Case 2: Country 1's first period search goes somewhat better and $x_1 \in (1/2, \tilde{r}]$. Then, country 2 offers $r_2 = x_1$ and country 1 searches. That is, Country 2 matches x_1 so that it is not out of running if the second search goes poorly for country 1.

Case 3: Country 1's first period search goes well and $x_1 \in (\tilde{r}, r^*]$. In this case, country

2 has to outbid the outside offer and deter country 1 from searching in the second period. That is, country 2 offers $r_2 = r^*$, and country 1 accepts.

Case 4: Country 1's outside offer is very good and $x_1 \in (r^*, 1]$. So if the second period is reached, then country 2 again matches the outside offer. That is, $r_2 = x_1$, and country 1 accepts.

The preceding analysis assumes that country 1 searched in the first period. This is the more complicated case. If it did not search in the first period, then country 1's decision problem in the second period is similar to the above, but we need no longer consider the constraint that $r_2 \geq \max\{x_1, \phi - 1\}$. Therefore, if $< 3/4$, then country 2 offers r^* and country 1 accepts; If $r^* \geq 3/4$, then country 2 offers 1/2 and country 1 searches.

The derivation of country 1's equilibrium strategy in period 1 after it searched (Table 2).

First, let r_1^* be country 2's first period offer that gives country 1 the same payoff as offering r^* in the second period. That is, r_1^* is the solution to the equation $1-\phi+r_1^* = \delta(1-\phi+r^*)$.

To characterize country 1's strategy in the first period after a search, we require that the discount factor is sufficiently large, such that $\delta \in [\underline{\delta}, 1]$, where

$$\underline{\delta} = \{\min_{\delta} : \quad 1 - \phi + s < \delta\theta(s), \forall s < \tilde{r}\}.$$

The condition is rather intuitive. First, the condition locates the relative positions of the thresholds that are necessary to characterize the equilibrium strategies: $\tilde{r} < r_1^* \leq r^*$. Second, the condition implies that the discount factor does not change strategic incentives of country 1 in a significant way. Specifically, if an offer from country 2, s, induces country 1 to search in a one-period game (i.e., $1 - \phi + s < \theta(s)$), then by the condition, the same offer leads country 1 to search in a two-period game as well (i.e., $1-\phi+s < \delta\theta(s)$), as long as the offer is sufficiently small, satisfying $s < \tilde{r}$. The condition rules out the possibility that the time preference plays a decisive role in countries' strategic calculations to search, allowing us to focus on the effect of outside options on such decisions. The discount factor still affects the equilibrium contributions.

Now we turn to the derivation of country 1's equilibrium strategy in period 1 after it searched (Table 2). Note that country 1's equilibrium strategy in the first period is influenced by both countries' equilibrium strategies in the second period (Table 1). As in the second period, we need to separately discuss countries' strategies when $r^* < 3/4$ and when $r^* \geq 3/4$. First, suppose that $r^* < 3/4$ (or $1 - r^* > 1/4$) and country 1 has already searched in the first period. Here we have two cases.

Case 1: If country 1's first period search yields $x_1 \in (0, r_1^*]$, then country 1 accepts r_1 if $r_1 \geq r_1^*$; otherwise, it moves the game to the second period, country 2 offers r^*, and country 1 accepts.

Case 2: If country 1's first period search yields $x_1 \in (r_1^*, 1]$, country 1 accepts $max\{x_1, r_1\}$ in the first period, produces the good, and ends the game.

Next, suppose that $r^* \geq 3/4$ (or $1 - r^* \leq 1/4$) and country 1 has already searched in the first period. We have four cases to consider.

Case 1: If $x_1 \in [0, 1/2]$, then country 1 accepts r_1 if $1 - \phi + r_1 \geq \delta\theta(1/2)$; otherwise, it moves the game to the second period, gets a second period offer of 1/2 from country 2, and searches.

Case 2: If $x_1 \in (1/2, \tilde{r}]$, then country 1 accepts r_1 if $1 - \phi + r_1 \geq \delta\theta(x_1)$; otherwise, it moves the game to the second period, country 2 offers x_1, and country 1 searches.

Case 3: If $x_1 \in (\tilde{r}, r_1^*]$, country 1 accepts r_1 if $r_1 \geq r_1^*$; otherwise, it moves to the second period, country 2 offers r^*, and country 1 accepts.

Case 4: If $x_1 \in (r_1^*, 1]$, country 1 accepts $max\{x_1, r_1\}$ in the first period, produces the good, and ends the game.

Lemma 2: For δ sufficiently larger, country 1 accepts a contribution from country 2 in the first period if it is greater than r^*; otherwise, it rejects an offer and searches in the first period.

Proof. As our derivation of the equilibrium strategies assumed $\delta > \underline{\delta}$, assume δ is sufficiently large. Suppose $r_1 \geq r^*$. If country 1 accepts r_1 then its payoff is $1 - \phi + r_1$. If country 1 rejects and moves the game to the second period without search, at most it receives $\delta(1 - \phi + r^*) < 1 - \phi + r_1$. Therefore, country 1 either accepts r_1 immediately or searches and then accepts $\max[x_1, r_1]$. By definition of r^*, we know $1 - \phi + r_1 \geq \theta(r_1)$, so country 1 accepts r_1 immediately.

Now suppose that $r_1 < r^*$. If country 1 accepts r_1, then it gets $1 - \phi + r_1$; if country 1 searches, then its payoff is bounded from below by $\theta(r_1)$. Because $1 - \phi + r_1 < \theta(r_1)$ when $r_1 < r^*$, searching is better than accepting. Where $r_1 < r^*$, however, country 1 must consider both the option of searching and the option of moving directly to the second period. First, suppose $r^* < 3/4$. If country 1 moves to the second period directly, it gets $\delta(1 - \phi + r^*) = 1 - \phi + r_1^*$. If country 1 searches in the first period, its expected utility given r_1 is:

$$\begin{aligned}
\theta(r_1) &\geq -c + \int_0^{r_1^*} \delta(1 - \phi + r^*)dx_1 + \int_{r_1^*}^1 (1 - \phi + x_1)dx_1 \\
&= -c + \int_0^{r_1^*} (1 - \phi + r_1^*)dx_1 + \int_{r_1^*}^1 (1 - \phi + x_1)dx_1 \\
&= \theta(r_1^*)
\end{aligned}$$

Since $r_1^* < r^*$, we have $\theta(r_1) = \theta(r_1^*) > 1 - \phi + r_1^*$. Therefore, the utility of moving to the second period is strictly less than the expected utility from searching and country 1 searches for all $r_1 < r^*$.

Next suppose $r^* \geq 3/4$. If country 1 moves directly to the second period, then country 2 offers 1/2 and country 1 searches. Country 1's payoff from this search is $\delta\theta(1/2)$. If it searches in the first period its utility is bounded from below by:

$$\begin{aligned}\theta(r_1) &> -c + \int_0^{1/2} \delta\theta(1/2)dx_1 + \int_{1/2}^1 (1-\phi+x_1)dx_1 \\ &> -c + \int_0^{1/2} (1-\phi+1/2)dx_1 + \int_{1/2}^1 (1-\phi+x_1)dx_1 \\ &= \theta(1/2)\end{aligned}$$

So, $\theta(r_1) > \theta(1/2) > \delta\theta(1/2)$ for all $r_1 < r^*$ and the utility from moving to the second period directly is strictly less than the expected utility from searching and country 1 searches.

□

Proposition 1: For δ sufficiently large, if $1/2(2-\phi)^2 < c$, then country 2 offers $\phi - 1$ in the first period and country 1 accepts immediately; if $1/32 \leq c < 1/2(2-\phi)^2$, then country 2 offers $r_1 = r^*$ in the first period and country 1 accepts immediately.

Proof. Let $\delta > \underline{\delta}$. If $r^* < \phi - 1$, country 2 will always prefer to accept a contribution that is big enough for the joint production of the public good. As a result, country 2 offers the minimal amount necessary, $\phi - 1$, and takes all the surplus.

Now suppose $\phi - 1 \leq r^* < 3/4$. By lemma 2 country 1 accepts any offers $r_1 > r^*$. So country 2 does not offer anything greater than r^*. Suppose it offers $r_1 < r^*$. Then country 1 searches in the first period for sure. We consider two cases.

Suppose $r_1 < r_1^*$, then with probability $1 - r_1^*$ country 1 produces the good with the *ad hoc* partner. With probability r_1^*, however, country 1 cannot find a sufficiently large outside offer and the game reaches the second period. In the second period, country 2 offers

$r_2 = r^*$ and it gets accepted. Thus, the expected payoff for country 2 making offer $r_1 < r_1^*$ is $\delta(1-r^*)r_1^*$, which is smaller than offering r^* to country 1 outright and getting $1-r^*$.

Now suppose $r_1^* \leq r_1 < r^*$. Then country 1 searches in the first period but the game does not reach the second period. The expected payoff for country 2 is $r_1(1-r_1) \leq 1/4 \leq 1-r^*$, which means that it is better off offering r^* outright. □

Proposition 2. Let $\hat{r}_1$ be the solution of the following constrained maximization problem:

$$\max \omega(r_1) = \int_0^{x(r_1)} (1-r_1)dx_1 + \int_{x(r_1)}^{\tilde{r}} \delta(x_1(1-x_1))dx_1 + \int_{\tilde{r}}^{r_1^*} \delta(1-r^*)dx_1$$
$$s.t. \qquad \delta\theta(1/2) - 1 + \phi \leq r_1 \leq \delta\theta(\tilde{r}) - 1 + \phi.$$

where $x(r_1)$ solves $1-\phi+r_1 = \delta\theta(x_1)$. For δ sufficiently large and $c < 1/32$, country 2 offers $\hat{r}_1$ and country 1 searches in the first period.

Proof. First, note that country 2 never offers anything more than r^*. By offering r^*, country 2 guarantees that the offer gets accepted and the game ends. Now the question is whether there is an offer $r_1 \in [0, r^*)$ that makes country 2 better off than offering r^*. To find such an offer, which induces country 1 to search in the first period, we start by ruling out offers that are strictly dominated by other offers.

Suppose $r_1 \in [r_1^*, r^*)$. Country 2's expected utility from such an offer is $r_1(1-r_1)$ (see Table 2). Since $\tilde{r} < r_1^* \leq r_1$, $r_1(1-r_1) < \tilde{r}(1-\tilde{r})$. We know that $\tilde{r}(1-\tilde{r}) = 1-r^*$, and it follows that $r_1(1-r_1) < 1-r^*$. In other words, the strategy of offering $r_1 \in (r_1^*, r^*)$ is strictly dominated by offering r^* outright.

Suppose $r_1 \in (\delta\theta(\tilde{r}) - 1 + \phi, r_1^*)$. We first confirm for the region that the lower bound $(\delta\theta(\tilde{r})-1+\phi)$ is indeed smaller than the upper bound (r_1^*): since $1-\phi+r_1^* = \delta(1-\phi+r^*) = \delta\theta(r^*)$, $r_1^* = \delta\theta(r^*)-1+\phi > \delta\theta(\tilde{r})-1+\phi$. Next, $r_1 > \delta\theta(\tilde{r})-1+\phi > \delta\theta(x)-1+\phi$, $\forall x \leq \tilde{r}$. So, by Table 2, r_1 will be accepted if $x_1 \leq \tilde{r}$. The total expected utility from offering r_1 is then

$$\int_0^{\tilde{r}} (1 - r_1)dx_1 + \int_{\tilde{r}}^{r_1^*} \delta(1 - r^*)dx_1 \tag{6}$$

Since Equation 6 is a decreasing function of r_1, the maximum is achieved at $r_1 = \delta\theta(\tilde{r}) - 1 + \phi$. Therefore, offering $\delta\theta(\tilde{r}) - 1 + \phi$ strictly dominates any $r_1 \in (\delta\theta(\tilde{r}) - 1 + \phi, r_1^*)$.

Suppose $r_1 \in (0, \delta\theta(1/2) - 1 + \phi)$. Note that $\delta\theta(1/2) - 1 + \phi \in (1/2, \delta\theta(\tilde{r}) - 1 + \phi)$. For any r_1 in this range, it always gets rejected in the first period and the total expected utility from making such an offer is

$$\int_0^{1/2} \delta(1/4)dx_1 + \int_{1/2}^{\tilde{r}} \delta x_1(1 - x_1)dx_1 + \int_{\tilde{r}}^{r_1^*} \delta(1 - r^*)dx_1 \tag{7}$$

If, on the other hand, country 2 just offers $\delta\theta(1/2) - 1 + \phi$, it gets

$$\int_0^{1/2} [2 - \phi - \delta\theta(1/2)]dx_1 + \int_{1/2}^{\tilde{r}} \delta x_1(1 - x_1)dx_1 + \int_{\tilde{r}}^{r_1^*} \delta(1 - r^*)dx_1 \tag{8}$$

The difference between Equations 7 and 8 is the first term, and it is easy to show that $2 - \phi - \delta\theta(1/2) > \delta(1/4)$:

$$2 - \phi - \delta\theta(1/2) > 2 - \phi - \theta(1/2) = c + 3/8 > \delta(1/4)$$

Thus, offering $\delta\theta(1/2) - 1 + \phi$ strictly dominates the strategy of offering any $r_1 \in (0, \delta\theta(1/2) - 1 + \phi)$.

After the above analysis, in terms of country 2's equilibrium strategy in the first period, one of two things must happen: country 2 offers r^*, or country 2 offers some $r_1 \in [\delta\theta(1/2) - 1 + \phi, \delta\theta(\tilde{r}) - 1 + \phi]$ that is better than any other offer in the region. Exactly which one is country 2's best response in the equilibrium depends on its expected utility from the two offers. If $r_1 = r^*$, then country 1 accepts the offer immediately and country 2's utility is $1 - r^*$. If country 2 offers $r_1 \in [\delta\theta(1/2) - 1 + \phi, \delta\theta(\tilde{r}) - 1 + \phi]$, country 1 could have two potential responses: accept after a search, or reject after a search and go to the second period.

For any $r_1 \in [\delta\theta(1/2) - 1 + \phi, \delta\theta(\tilde{r}) - 1 + \phi]$, it is straightforward to see that $\delta\theta(1/2) \leq 1 - \phi + r_1 \leq \delta\theta(\tilde{r})$. By the monotonicity of the function $\theta(\cdot)$, therefore, there is a unique outside offer $x_1 \in [1/2, \tilde{r}]$ that satisfies the equality $1 - \phi + r_1 = \delta\theta(x_1)$. Let $x(r_1)$ denote such an outside offer. Then, country 2's expected utility from offering $r_1 \in [\delta\theta(1/2) - 1 + \phi, \delta\theta(\tilde{r}) - 1 + \phi]$ is $\omega(r_1)$, where

$$\omega(r_1) = \int_0^{x(r_1)} (1 - r_1)dx_1 + \int_{x(r_1)}^{\tilde{r}} \delta(x_1(1 - x_1))dx_1 + \int_{\tilde{r}}^{r_1^*} \delta(1 - r^*)dx_1,$$

Let $\hat{r}_1$ be the solution of the following constrained maximization problem:

$$\begin{aligned} \max \omega(r_1) = & \int_0^{x(r_1)} (1 - r_1)dx_1 + \int_{x(r_1)}^{\tilde{r}} \delta(x_1(1 - x_1))dx_1 + \int_{\tilde{r}}^{r_1^*} \delta(1 - r^*)dx_1 \\ s.t. \quad & \delta\theta(1/2) - 1 + \phi \leq r_1 \leq \delta\theta(\tilde{r}) - 1 + \phi. \end{aligned} \tag{9}$$

The strategy for the rest of the proof of Proposition 2 is as follows. First, we solve for the maximizer of Equation 9 **without the constraint**, $\bar{r}_1$. We show that for all parameter values for c, δ, and ϕ, $\omega(\bar{r}_1) \geq 1 - r^*$. This means that whenever $\bar{r}_1 \in [\delta\theta(1/2) - 1 + \phi, \delta\theta(\tilde{r}) - 1 + \phi]$, $\hat{r}_1 = \bar{r}_1$, and country 2 will offer $\bar{r}_1$ rather than r^*. Next, we show that when $\bar{r}_1 < \delta\theta(1/2) - 1 + \phi$, i.e., when $\bar{r}_1$ is smaller than the lower bound of the feasible range for $\hat{r}_1$, $\hat{r}_1 = \delta\theta(1/2) - 1 + \phi$, and additionally, $\omega(\delta\theta(1/2) - 1 + \phi) \geq 1 - r^*$, which means that country 2 will offer $\delta\theta(1/2) - 1 + \phi$ instead of r^*. Finally, we show that when $\bar{r}_1 > \delta\theta(\tilde{r}) - 1 + \phi$, i.e., when $\bar{r}_1$ is greater than the upper bound of the feasible range for $\hat{r}_1$, $\hat{r}_1 = \delta\theta(\tilde{r}) - 1 + \phi$. Here we show that for $\delta^* = \max\{\underline{\delta}, .8\} \leq 1$, that for every $\delta > \delta^*$ $\omega(\delta\theta(\tilde{r}) - 1 + \phi) > 1 - r^*$. These three steps prove our proposition.

As a first step, we prove $\omega(\bar{r}_1) \geq 1 - r^*$ when $r^* > 3/4$, i.e., $c < 1/32$, where $\bar{r}_1$ is the global maximizer of Equation 9. In other words, we need to prove the following to be true:

$$\int_0^{x(r_1)} (1-r_1)dx_1 + \int_{x(r_1)}^{\tilde{r}} \delta(x_1(1-x_1))dx_1 + \int_{\tilde{r}}^{r_1^*} \delta(1-r^*)dx_1 - (1-r^*) \geq 0 \qquad (10)$$

when $r_1 = \bar{r}_1$.

To find the maximizer $\bar{r}_1$, plug in $x(r_1)$, $\tilde{r}$, r_1^*, and r^*, and then differentiate and set Equation 9 equal to 0. This gives the solution

$$\bar{r}_1 = 1 + \delta - \sqrt{4\delta - 2\delta^2 - 2\delta\phi + 2\delta^2 c + 2\delta^2\phi} \leq 1 \qquad (11)$$

Note that

$$\frac{\partial^2 \omega}{\partial r_1^2} = \frac{\delta(r_1 - 1)}{(\delta\ (2 - 2\,\phi + 2\,r_1 + 2\,\delta\,c - 3\,\delta + 2\,\delta\,\phi))^{3/2}} - \frac{1}{\sqrt{\delta\ (2 - 2\,\phi + 2\,r_1 + 2\,\delta\,c - 3\,\delta + 2\,\delta\,\phi)}},$$

which leads to $\frac{\partial^2 \omega}{\partial r_1^2}|_{\bar{r}_1} \leq 0$. So we know that $\bar{r}_1$ is the unique maximizer.

There are three cases to consider. If $\bar{r}_1 \in [\delta\theta(1/2) - 1 + \phi, \delta\theta(\tilde{r}) - 1 + \phi]$, then $\hat{r}_1 = \bar{r}_1$, and $\omega(\bar{r}_1) \geq 1 - r^*$. Therefore, country 2 will offer $\bar{r}_1$, which will lead to search by country 1. For the case that (i) $\bar{r}_1 < \delta\theta(1/2) - 1 + \phi$ and (ii) $\bar{r}_1 > \delta\theta(\tilde{r}) - 1 + \phi$ we can show that for all $\delta > \delta^* = \max\{\underline{\delta}, .8\}$ the expected utility of offering $\hat{r}_1 = \delta\theta(1/2) - 1 + \phi$ given (i) and $\hat{r}_1 = \delta\theta(\tilde{r}) - 1 + \phi$ given (ii) is greater than offering r^*.[20] This completes the proof of Proposition 2. □

Proposition 3. *When $c < 1/32$ and δ is sufficiently large, the equilibrium offer of country 2 is decreasing in country 1's search cost.*

[20]The proof of these claims are trivial, but tedious. Details are available upon request.

Proof. When $c < 1/32$, we are looking at the equilibrium characterized by Proposition 2. We show that $\hat{r}_1$ is continuous in the parameter space, and it is decreasing in c.

Let $G = (0, 1/32] \times (1, 3/2] \times [\delta^*, 1]$ with generic element $g \in R^3$. Then we can define the constraint set for the optimization problem in Proposition 2 to be

$$\gamma(g) = \{x \in R : \delta\theta(1/2) - 1 + \phi \leq x \leq \delta\theta(\tilde{r}) - 1 + \phi\}. \tag{12}$$

γ is a compact-valued continuous correspondence. Noting that $\omega(r_1) : \gamma(g) \to R$ is continuous, and defining γ^* such that

$$\gamma^*(g) = \{x \in \gamma(g) : x \text{ maximizes } \omega(r_1) \text{ on } \gamma(g)\}, \tag{13}$$

we have by Berge's (1963) Theorem of the Maximum, $\gamma^*(g)$ is upper hemi-continuous on G. Moreover, because γ^* is single-valued in Proposition 2, $\gamma^*(g)$ is a continuous function.[21]

From the characterization in Proposition 2, $\gamma^*(g)$ can be expressed as the following continuous piecewise function:

$$\gamma^*(g) = a_1(\delta\theta(1/2) - 1 + \phi) + a_2(\bar{r}_1) + a_3(\delta\theta(\tilde{r}) - 1 + \phi) \tag{14}$$

with

$$a_1 = \begin{cases} 1 & \text{if } \bar{r}_1 < \delta\theta(1/2) - 1 + \phi \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

$$a_2 = \begin{cases} 1 & \text{if } \delta\theta(1/2) - 1 + \phi \leq \bar{r}_1 \leq \delta\theta(\tilde{r}) - 1 + \phi \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

$$a_3 = \begin{cases} 1 & \text{if } \bar{r}_1 > \delta\theta(\tilde{r}) - 1 + \phi \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

It can be easily shown that $\delta\theta(1/2) - 1 + \phi$, $\bar{r}_1$, and $\delta\theta(\tilde{r}) - 1 + \phi$ are all decreasing functions of c, so $\gamma^*(g)$ is decreasing in c, proving the proposition. □

[21] See Border (1992).

Footnote 18.

Proof. Since in the equilibrium characterized by proposition 1 the probability of an ad hoc coalition is zero, we turn to the case for the equilibrium of proposition 2. From the equilibrium strategies we have

$$\Pr(\text{ad hoc coalition}) = \int_{x(\hat{r}_1)}^{\tilde{r}} \int_{x}^{1} dy dx + \int_{r_1^*}^{1} dx \tag{18}$$

$$= \int_{x(\hat{r}_1)}^{\tilde{r}} (1-x)dx + \int_{r_1^*}^{1} dx. \tag{19}$$

Notice $\hat{r}_1$ and $x(r_1)$ are both directly a functions of c, so to keep this in mind we write $x(\hat{r}_1) = x(\hat{r}_1(c), c)$. By Leibniz's rule,

$$\frac{\partial \Pr}{\partial c} = \frac{\partial \tilde{r}}{\partial c}(1-\tilde{r}) - (1 - x(\hat{r}_1(c), c))\frac{\partial x(\hat{r}_1(c), c)}{\partial c} - \frac{\partial r_1^*}{\partial c}. \tag{20}$$

For a given set of (c, δ, ϕ), $\partial \Pr / \partial c$ can be positive or negative, thus the probability of forming an ad hoc coalition can be increasing or decreasing. If, for example, we consider $c = 1/40$, $\delta = .95$, and $\phi = 1.15$ we get that $\partial \Pr / \partial c = -.765$. If we set $c = 1/75$, $\delta = .98$ and $\phi = 1.4$, then $\partial \Pr / \partial c = 3.47$. □

Proposition 4 For δ sufficiently large, being in an alliance improves the welfare of the country that pays the search cost whenever $c < 1/8(1 - (\phi - 1)^2)^2$.

Proof. We start with the equilibrium characterized by Proposition 1, where $1/32 \leq c$. The condition implies $c > 1/32$. So, when $c > 1/32$ we are in the equilibrium where country 2 makes an offer, r^*, and it is accepted by country 1. The expected payoff of country 1 with an alliance is then $1 - \phi + r^*$, and its expected payoff from searching without an alliance is

$\theta(\phi-1)$ when $c=0$, which equals to $\frac{3}{2}-\phi+\frac{(\phi-1)^2}{2}$. So we need to evaluate the following inequality:

$$W(c,\delta,\phi)=1-\phi+r^*-(\frac{3}{2}-\phi+\frac{(\phi-1)^2}{2})\geq 0$$

If having an alliance is better, then we should have $W(c,\delta,\phi)\geq 0$. We find that for $c\leq \frac{1}{8}[1-(\phi-1)^2]^2$, $W(c,\delta,\phi)\geq 0$, which means that an alliance is better than no alliance, even though the alliance imposes search costs on its members. On the other hand, for $\frac{1}{8}[1-(\phi-1)^2]^2 < c\leq 1/2$, country 1 would prefer to have no alliance and search without cost.

For the remaining case when $0\leq c\leq 1/32$, it involves the equilibrium characterized by Proposition 2. There are three different cases in terms of the contribution that country 2 offers $\delta\theta(1/2)-1+\phi, \bar{r}_1, \delta\theta(\tilde{r})-1+\phi$. Repeating the steps above for each case, the inequality in refeq:welfare is satisfied. This proves the proposition. □

References

Berge, Claude. 1997, [1963]. *Topological Spaces.* New York, NY: Dover Press.

Binmore, Ken. 1985. *Game-Theoretic Models of Bargaining.* Cambridge University Press chapter Bargaining and Coalitions.

Border, Kim C. 1992. *Fixed Point Theorems with Application to Economics and Game Theory.* New York, NY: Cambridge University Pres.

Bowie, Christopher J., Robert P Haffa and Robert E. Mullins. 2003. "Future War: What Trends in America's Post-Cold War Military Conflicts Tell US About Early 21^{st} Century Warfare." *Northrop Grumman: Analysis Center Paper* .

Bruce, Neil. 1990. "Defence Expenditures by Countries in Allied and Adversarial Relationships." *Defence Economics* 1:179–195.

Conybeare, John A. 1994. "The Portfolio Benefits of Free Riding in Military Alliances." *International Studies Quarterly* 38(3):405–419.

Conybeare, John A. and Todd Sandler. 1990. "The Triple Entente and Triple Alliance, 1880–1914: A Collective Goods Approach." *American Political Science Review* 84:1197–1206.

Joll, James. 1992. *The Origins of the First World War.* New York, NY: Longman.

Lee, Ching Chyi. 1994. "Bargaining and Search with Recall: A Two-Period Model with Complete Information." *Operations Research* 42(6):1100–1109.

Morrow, James D. 1994. "Alliances, Credibility, and Peacetime Costs." *Journal of Conflict Resolution* 38(2):270–297.

Morrow, James D. 2000. "Allainces: Why Wrtie Them Down." *Annual Review of Political Science* 3:63–83.

Murdoch, James C. and Todd Sandler. 1982. "A Theoretical and Empirical Analysis of NATO." *Journal of Conflict Resolution* 26(2):237–263.

Murdoch, James C. and Todd Sandler. 1984. "Complementarity, Free-Riding, and the Military Expenditures of NATO Allies." *Journal of Public Economics* 25(1–2):83–101.

Muthoo, Abhinay. 1995. "On the Strategic Role of Outside Options in Bilateral Bargaining." *Operations Research* 43(2):292–297.

Muthoo, Abhinay. 1999. *Bargaining Theory with Applications.* New York, NY: Cambridge University Pres.

Olson, Mancur and Richard Zeckhauser. 1966. "An Economic Theory of Alliances." *The Review of Economics and Statistics* 48(3):266–279.

O'neal, John R. and Mark A. Elrod. 1989. "NATO Burden Sharing and the Forces of Change." *International Studies Quarterly* 33:435–456.

Sandler, Todd. 1977. "Impurity of Defense: An Application to the Economics of Alliances." *Kyklos* 30(3):443–460.

Sandler, Todd and James C. Murdoch. 2000. "On Sharing NATO Defense Burdens in the 1990s and Beyond." *Fiscal Studies* 21(3):297–327.

Sandler, Todd and Keith Hartley. 1995. *The Economics of Defense.* New York, NY: Cambridge University Press.

Sandler, Todd and Keith Hartley. 1999. *The Political Economy of NATO.* New York, NY: Cambridge University Press.

Sandler, Todd and Keith Hartley. 2001. "Economics of Alliances: The Lessons for Collective Action." *Journal of Economic Literature* 39:869–896.

Shaked, Aved. 1994. "Opting Out: Bazaars versus 'High Tech' Markets." *Investigaciones Economicas* 18:421–432.

Smith, Alastair. 1995. "Alliance Formation and War." *International Studies Quarterly* 39(4):405–425.

Smith, Alastair. 1996. "To Intervene or Not to Intervene: A Biased Decision." *The Journal of Conflict Resolution* 40(1):16–40.

Snyder, G.H. and P. Diesing. 1977. *Conflict Among Nations: Bargaining, decision-making, and system structure in international crises.* Princeton, NJ: Princeton University Press.

Snyder, Glenn H. 1984. "The Security Dilemma in Alliance Politics." *World Politics* 36(4):461–495.

Taylor, A.J.P. 1954. *The Struggle For Mastery in Europe: 1848–1918.* New York, NY: Oxford University Press.

Thies, Wallace J. 1987. "Alliances as Collective Goods: A Reappraisal." *Journal of Conflict Resolution* 31(2):298–332.

US Department of Defense. 1999. *Report on Allied Contributions to the Common Defense: A Report to the United States Congress by the Secretary of Defense.* Washington, DC: US DOD.