Throughput Scaling in Wireless Networks with Restricted Mobility

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***Abstract*— We study throughput scaling in an ad-hoc wireless network where the communication domain is divided into overlapping neighborhoods and n mobile nodes are restricted to move within their assigned neighborhood. In our model, when a node is located in a region not shared with any other neighborhood, it transmits to nodes of its own neighborhood only; when it is in an area that overlaps with another neighborhood, it transmits to nodes of the overlapping neighborhood. Communication between source-destination pairs is subject to interference from other nodes. By adopting a deterministic approach, we obtain an achievable throughput which is a function of properties of the node locations and neighborhood dimensions. As special cases of our neighborhood model, the results of Gupta-Kumar [1] and Grossglauser-Tse [2] can be recovered. We then study the case of random placement of nodes with n^α neighborhoods, where $0 \leq \alpha < 1$, and achieve a throughput of $\Omega\left(n^{1-\alpha/2}\right)$. Hence our model captures every order of growth for the throughput, encompassing the results from both [1] and [2] as extreme situations.**

***Index Terms*— Wireless networks, ad hoc networks, limited mobility, multi-hop, throughput, capacity, deterministic, individual sequence, random, scaling.**

I. INTRODUCTION

CURRENT wireless networks utilize a wired infrastructure between base stations. An attractive complement to such traditional networks are all-wireless systems (or wireless ad-hoc networks) where the use of infrastructures can be overcome. In such systems, the nodes communicate over a wireless channel without any centralized control and are thus said to be self-organized. An initial application is in military communications. Recent development of high-performance microprocessors and new sensing materials, combined with several innovations at the physical layer (for example, smart antennas and multiuser detection techniques), have led to considerations for new applications in which the nodes are millimeter sized sensors [3] (biomedical sensors, "smart home", etc.). The special characteristics of an all-wireless network present several unique challenges. First, the lack of centralized control implies a high level of cooperation from the nodes. As individual nodes rely entirely on their own energy source, power saving becomes a key issue. In addition, the transmission power of the nodes must be precisely regulated to allow for communication with desired destinations without generating unnecessarily high levels of interference. [4] gives an overview of the technical issues.

An important aspect of the study of all-wireless systems lies in the analysis of the scaling of such systems with respect to space and to the number of users. Recent progress has been reported in [5], [1], [6], [2], [7], [8], [9], [10], [11], [12]. A model in which the nodes are immobile and sender-receiver pairs are subject to interference from other nodes was studied by Gupta and Kumar [1], who showed that when the traffic pattern and node distribution are random, the total throughput (in bits per second) can grow no faster than $(n/\log n)^{1/2}$, n being the number of nodes. Under the assumption that successful transmission depends only on the signal to noise/interference ratio at the receiver, this limit can be achieved when source-destination pairs utilize intermediate nodes as local relays. As nearest neighbors become closer with increasing n, the number of hops needed to reach the destination increases, imposing a fundamental limit on how the throughput of the entire network scales as a function of n. Using the same model as [1], the authors in [6] identified deterministic properties in the locations of the nodes and combined this deterministic structure with a simple scheduling algorithm to obtain achievability results on throughput for general configurations of immobile nodes. Furthermore, from these deterministic results they were able to recover the results for random node locations and, in particular, the achievability results of [1].

Mobility was introduced into the communication model of [1] by Grossglauser and Tse [2] who considered the case where the trajectories of the mobile nodes are independent from one another, stationary and ergodic with uniform stationary distribution. In this setting, the expected number of feasible successful source-destination pairs is $\Theta(n)$ and the throughput per pair can be kept constant as the number of nodes increases. In [7], it was further shown that the same results remain valid when each node is restricted to move randomly on a randomly and independently chosen great circle on the unit sphere. Other related studies are [13], [14], [15],

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[16], [17], [18], [19].

In this paper we seek to analyze how the throughput scales in situations between the extremes of immobile and fully mobile nodes. We therefore introduce a model of restricted mobility by considering nodes confined to overlapping neighborhoods. The larger the neighborhood size, the less constrained is node mobility. In the limit, the setting is equivalent to the one in [2]. Conversely, by letting the neighborhoods' dimensions go to zero, we obtain a situation approaching the one in [1]. We follow a deterministic approach by capturing essential properties in the location of the nodes with respect to the neighborhoods and then appeal to a deterministic routing algorithm. For arbitrary assignment of nodes to neighborhoods, we obtain a total throughput (in bits/sec) of $\Omega\left(\frac{nc_{min}}{c_{max}(B+2b)^2}\left(\frac{1-b}{2b(B+b)^2}+\frac{2}{B^2}\right)^{-1}\right)$, where $(B+2b)$ is the neighborhood size, B the interior region size, b the overlap region size, and c_{min} and c_{max} respectively the minimum and maximum number of users per neighborhood. This throughput result holds if n, c_{min}, c_{max} go to infinity, and if $c_{min} \leq \left(\frac{B+b}{1-b}\right)^2 n \leq c_{max}$. Besides if $(B+2b) = \Omega(1/n)$ the same result holds for throughput in bit-meters/sec. As special cases of our model, we recover throughputs of $\Omega\left(n\right)$ and $\Omega\left(\sqrt{n}/\sqrt{\log n}\right)$, respectively obtained by [2] and [1]. We further consider the situation where node assignments are i.i.d. and each node belongs equally likely to every neighborhood. With n^α neighborhoods, we obtain $\Omega\left(n^{1-\alpha/2}\right)$ throughput, with $0 \leq \alpha < 1$. Hence, our model covers all possible orders of growth, from that corresponding to immobile nodes, to that achieved when the nodes are allowed to move freely in the entire domain.

The outline of the paper is the following. In Section II, the communication model and the protocol adopted in this study are described. Then the main result of the paper is stated. In Section III, a proof of the main result is presented. In Section IV, we analyze specific cases where the main result applies. Finally, Section V provides some concluding comments.

II. Model Description and Main Throughput Result

The area of communication consists of a square of area 1 m^2 which is divided into N neighborhoods. We assume that the neighborhoods are overlapping as illustrated Figure 1. We denote by "interior region" the part of a neighborhood that does not intersect with any other neighborhood. Conversely the part that does overlap is called the "overlap region". The ad-hoc network is formed by n nodes. Each node is assigned to a given neighborhood. Though the nodes are mobile, their mobility region is restricted to their corresponding neighborhood.

Denote the location of the i^{th} node at time t by $S_i(t)$. For a given neighborhood $\mathcal{N}_k$, we assume that the process $\{S_i(t)|$ node i belongs to neighborhood $\mathcal{N}_k\}$ is stationary and ergodic and that its stationary distribution is uniform on the neighborhood $\mathcal{N}_k$. In addition the nodes belonging to the same neighborhood have i.i.d. trajectories. For expositional ease, we choose the *protocol model* [1] as our transmission

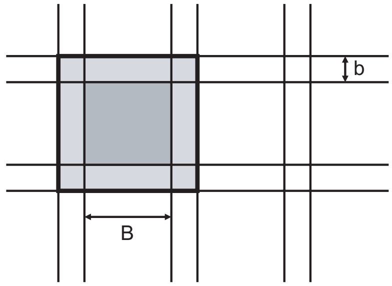

Fig. 1. The area of communication. Each neighborhood is a square with side length $B+b$, the interior region is a square with side length B, and the overlap region has width b.

model. We assume that time is slotted, each slot lasting 1 second. Suppose that at time t node i transmits data to node j at rate R packets/second. Under the protocol model, this transmission is successful if the distance separating both nodes is smaller by a constant factor than the distance between node j and any other node that is simultaneously transmitting, i.e. if $|S_k(t) - S_j(t)| \geq (1+\Delta)|S_i(t) - S_j(t)|$ for every other node k simultaneously transmitting. We recall that Δ is a positive constant representing the guard zone in the protocol model. Throughout our study, we use the same session model as the one in [2]. We suppose that every node is chosen to be a *source node* for one session and a *destination node* for another session. The n source-destination pairs are randomly specified. In addition, we assume that there is a one-to-one correspondence between source and destination nodes and that this specification of source-destination pairs does not change with time. Each source is assumed to have an infinite number of packets to send to its given destination.

For general configurations of node locations, we define the following parameters: c_{min} denotes the minimum number of nodes per neighborhood and c_{max} the maximum number of nodes per neighborhood. The number of neighborhoods being $N = (1-b)^2/(B+b)^2$, c_{min} and c_{max} must satisfy the following:

$$c_{min} \leq \left(\frac{B+b}{1-b}\right)^2 n \leq c_{max}. \quad (1)$$

As in previous work, to cope with both the limitations inherent in the interference caused by simultaneous transmissions and the distance impairment, we adopt local communications and allow relaying of packets. We thus decide that transmissions occur between nearest neighbors only and are subject to the following restrictions:

- When the sender is located in the interior region of its neighborhood, it transmits only to nodes belonging to the same neighborhood.
- When the sender is located in the overlap region of its neighborhood, it transmits only to nodes belonging to the foreign neighborhood that overlaps with its location in the region.

The succession of neighborhoods that a packet traverses from sender to destination is predetermined by a routing algorithm which is presented in Section III-B. A typical scenario is depicted in Figure 2. Consider a source-destination pair, S-R. S first sends a packet to its nearest neighbor within

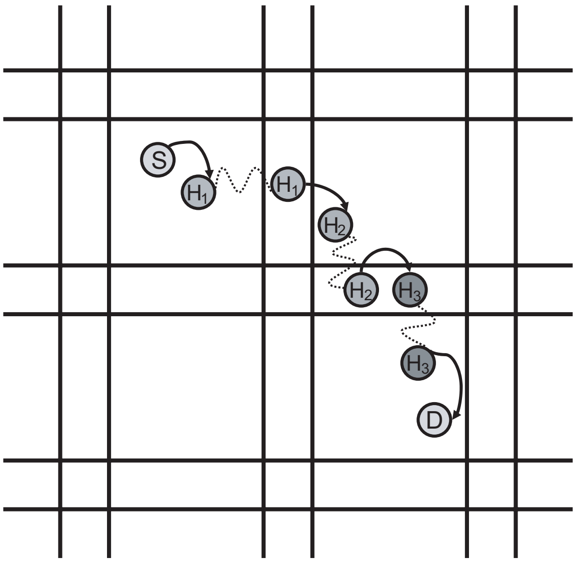

Fig. 2. A typical scenario illustrating the successive hopping between nodes from overlapping neighborhoods according to the route predetermined by the scheduling algorithm.

its neighborhood, say H_1. When H_1 reaches the overlap region with the neighborhood corresponding to the route for S-R's packet, H_1 transmits the packet to its nearest neighbor belonging to the overlapping neighborhood, say H_2. Then H_2 relays the packet to H_3 which is located in the same neighborhood as R. When H_3 encounters R as its nearest neighbor, it delivers to R its destined packet. To allow the storage of the relayed packets, we assume that each node is provided with a buffer of infinite size. At each time t, the scheduling algorithm dictates which nodes are allowed to transmit, which packets they will transmit and to whom.

We now present the main result of this paper. The proof will be given in Section III.

Proposition 2.1: Consider a network with neighborhood size $(B + 2b)$, interior region size B, overlap region size b, minimum and maximum number of users per neighborhood c_{min} and c_{max} respectively. If (1) is satisfied, for c_{min}, c_{max} and n sufficiently large, the total achievable throughput in bit per second is at least

$$\Omega\left(\frac{nc_{min}}{c_{max}(B+2b)^2}\left(\frac{1-b}{2b(B+b)^2P^{over}}+\frac{2}{B^2P^{int}}\right)^{-1}\right), \tag{2}$$

with

$$P^{int} = \left(1+(1+\Delta)^2\frac{\alpha}{1-\alpha}\right)^{-1},$$

$$P^{over} = \left(1+\frac{1}{2}(1+\Delta)^2\frac{\alpha}{1-\alpha}\right)^{-1},$$

and α a positive constant in $(0, 1)$. In addition, the same result holds for an achievable throughput in bit-meters per second if $(B + 2b) = \Omega\left(\frac{1}{n}\right)$.

Noting that P^{int} and $P^{over} \in (0, 1)$, that $P^{over} = \frac{2P^{int}}{1+P^{int}}$ and thus $P^{int} < P^{over} < 2P^{int}$, we obtain an achievable throughput of

$$\Omega\left(\frac{nc_{min}}{c_{max}(B+2b)^2}\left(\frac{1-b}{2b(B+b)^2}+\frac{2}{B^2}\right)^{-1}\right).$$

In the full mobility model [2], the throughput increase brought by mobility results from taking advantage of a form of multiuser diversity. The packet stream between each source-destination pair S-D is split to the other nodes that serve as relays and that have independent time-varying channels to the destination due to their mobility. Using two hops (S-relay and relay-D) each with high throughput, this strategy leads to high overall throughput: at each time, with high probability, there is a node close to S that can serve as relay and there is a relay close to D that can send information to D. In the restricted mobility model considered here, the benefits of multiuser diversity are a function of the number/size of neighborhoods. First, each source splits its packet stream to as many different nodes as possible in its own neighborhood. These relays then spread the packets to as many nodes as possible in the adjacent neighborhoods as determined by the routing algorithm. This relaying procedure is repeated until the destination's neighborhood is reached by the packets, which are finally delivered by the last relays whenever they get close to the destination. As each source has an infinite number of packets to send, in steady state, each node carries packets originating from and destined to every other node belonging to the same neighborhood, say $\mathcal{N}$, as well as source-destination pairs' packets whose routes include neighborhood $\mathcal{N}$. However, the neighborhood parameters control the tradeoff between the number of possible relays in each neighborhood and the number of hops needed between source and destination. As the neighborhood dimension decreases, the number of hops imposed by the routing algorithm increases and there are fewer nodes per neighborhoods, hence the benefits of multiuser diversity are reduced.

III. Proof of the Main Result

The key steps of the proof are as follows. First, we determine in Section III-A the number of feasible simultaneous transmissions for senders in the interior region of a neighborhood (Section III-A.1), and for senders in the overlap region of a neighborhood (Section III-A.2). This is accomplished by determining the asymptotics of the probability of successful transmission between a sender in the interior (or overlap) region of a neighborhood and its nearest candidate receiver in the same (or overlapping) neighborhood, and by determining the number of such senders per neighborhood. Obtaining the asymptotics of the probability of successful transmission involves two main steps: 1) determining the limiting distribution of the distance between a sender in the interior (or overlap) region of a neighborhood and its nearest candidate receiver (see Lemma 3.1 and Lemma 3.3), and the limiting distribution of the distance between this candidate receiver and the nearest simultaneously transmitting node (see Lemma 3.2 and Lemma 3.4); and 2) showing that both limiting distributions are independent.

In Section III-B, we then derive a routing algorithm that makes use of these feasible simultaneous transmissions and arrive at the general throughput result of Proposition 2.1. Following the typical scenario of Section II depicted by Figure 2, each hop from source to destination involves "nearest neighbors" while the succession of neighborhoods traversed by a packet is determined as follows. To prevent certain neighborhoods from becoming "hot spots", i.e. with too much traffic concentration, we obtain the succession of neighborhoods that each packet must follow by establishing a correspondence between an N-neighborhood network and an $N^{1/2}$ by $N^{1/2}$ mesh network, and by using results on routing algorithms for meshes that guarantee minimal queue length and short routing time (see Lemma 3.5).

A. Number of feasible simultaneous transmissions

Recall that there are at least c_{min} nodes in each neighborhood. At each time t, within each neighborhood, randomly designate $n_S = \alpha c_{min}$ nodes as senders and $n_R = (1-\alpha)c_{min}$ nodes as potential receivers, with $\alpha \in (0,1)$ such that $\alpha c_{\min}$ is integer. The remaining nodes are ignored. Note that such a setup is sufficient to permit us to find a lower bound for the throughput.

For simplicity we denote each node and its location the same way. Call $\mathcal{A} = \{A_j\}_{j\in\{1,\dots,N\alpha c_{min}\}}$ the set of all designated senders. Call $\mathcal{B} = \{B_j\}_{j\in\{1,\dots,N(1-\alpha)c_{min}\}}$ the set of all designated potential receivers. Call $\mathcal{N} = \{\mathcal{N}_j\}_{j\in\{1,\dots,N\}}$ the set of all neighborhoods.

The policy is as follows. For each sender node, if the sender is located in the interior region of a given neighborhood, it transmits packets to its nearest neighbor among all the potential receivers belonging to the same neighborhood. If the sender is located in an overlap region of its neighborhood, then it sends packets to its nearest neighbor among the potential receivers belonging to the neighborhood that overlaps its own neighborhood.

1) Senders in the interior region of a neighborhood: Fix a time t. To simplify the notation we will not add a time index. Pick at random a sender located in the interior region of a neighborhood, say A_1. By symmetry we only need to focus on one such sender. Without loss of generality suppose that A_1 belongs to the neighborhood $\mathcal{N}_1$. Denote its interior region by $\mathcal{N}_1^{\text{int}}$. Recall that the candidate receivers are all located in the same neighborhood as the sender A_1. The set of their locations is $\{B_j\}_{j=1,\dots n_R}$. Let $R_j = |A_1 - B_j|$ be the distance between the sender A_1 and the potential receiver B_j, $j = 1,\dots,n_R$. Then the distance between A_1 and its nearest neighbor among the receivers is given by $R = \min_{j\in 1,\dots,n_R} R_j = |A_1 - B_{(1)}|$, $B_{(1)}$ being the location of the nearest potential receiver. We now determine the extreme asymptotic distribution of R as $n_R \to \infty$ or equivalently as $c_{min} \to \infty$. Let

$$L_2(r) = \begin{cases} 1 - \exp\left(-r^2\right) & \text{if } r > 0 \\ 0 & \text{if } r \le 0. \end{cases} \tag{3}$$

The following lemma gives the asymptotic distribution of R. Its proof is deferred to the Appendix.

Lemma 3.1:

$$\frac{\sqrt{\pi(1-\alpha)c_{min}}}{B+2b} R \xrightarrow{\mathcal{D}} R^*,$$

where R^* has the cdf $L_2(r)$ given by (3).

Now, according to the protocol model, the transmission between A_1 and $B_{(1)}$ will be successful if there is no simultaneously transmitting node within a radius of $(1+\Delta)R$ around $B_{(1)}$. Define R'_j as $R'_j = |A_j - B_{(1)}|$, $j = 1,\dots,N\alpha c_{min}$. For the transmission to be successful, we must have

$$R_I = \min_{j\neq 1} R'_j \ge (1+\Delta) R \iff \frac{R}{R_I} \le \frac{1}{1+\Delta}.$$

Note that the I in R_I stands for "interference". We obtain the following lemma whose proof is deferred to the Appendix.

Lemma 3.2:

$$\frac{\sqrt{\pi(\alpha c_{min}-1)}}{B+2b} R_I \xrightarrow{\mathcal{D}} R_I^*,$$

where R_I^* has the cdf $L_2(r)$ given by (3).

We now show that the two limiting distributions of R and R_I are independent. From Lemma 3.2 and (3), we see that the limiting distribution of R_I is independent of $B_{(1)}$. In addition, given $B_{(1)}$, A_1 and R_I are independent. This implies that the limiting distribution of R_I is independent of the pair $\left(A_1, B_{(1)}\right)$. By the Continuous Mapping Theorem, using the fact that R is a continuous function of the pair $\left(A_1, B_{(1)}\right)$, we conclude that the limiting distributions of R and R_I are independent. Thus we get by Lemma 3.1 and Lemma 3.2,

$$\mathbb{P}\big[\text{succ. transmission } A_1 \to B_{(1)}\big] = \mathbb{P}\left[\frac{R}{R_I} \le \frac{1}{1+\Delta}\right] \to \mathbb{P}\left[\frac{R^*}{R_I^*} \le \frac{1}{1+\Delta}\sqrt{\frac{1-\alpha}{\alpha}}\right].$$

Define $F_{X/Y}$ as $F_{X/Y}(z) = \mathbb{P}\left[\frac{X}{Y} \le z\right]$, $z \ge 0$, and P^{int} as

$$P^{int} = F_{X/Y}\left(\frac{1}{1+\Delta}\sqrt{\frac{1-\alpha}{\alpha}}\right),$$

where X and Y are i.i.d. random variables with common distribution L_2 given by (3). After straightforward manipulations we obtain that

$$F_{X/Y}(z) = \frac{1}{1+(1/z)^2}. \tag{4}$$

Hence

$$P^{int} = \left(1 + (1+\Delta)^2 \frac{\alpha}{1-\alpha}\right)^{-1}. \tag{5}$$

Within the whole neighborhood $\mathcal{N}_1$, there are $n_S = \alpha c_{min}$ senders that are attempting to transmit. In steady state, the probability for a sender to be located in $\mathcal{N}_1^{int}$ at time t is

$$p = \frac{B^2}{(B+2b)^2}.$$

Let k be the number of such senders. Then, by Hoeffding's inequality (Theorem 8.1 in [20]), we have

$$\mathbb{P}\left[\left|\frac{k}{n_S} - p\right| \ge \epsilon\right] \le e^{-n_S\epsilon^2}.$$

By the Borel-Cantelli lemma (see [21]), we can thus say that, when $n_S \to \infty$, there are almost surely at least $\alpha c_{min} B^2/(B+2b)^2$ senders in $\mathcal{N}_1^{int}$ that are attempting to transmit. Therefore, the following proposition holds.

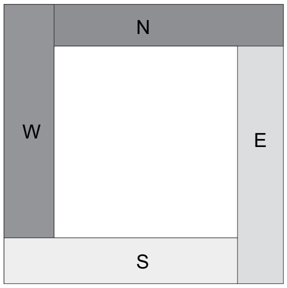

Fig. 3. Subdivision of the overlap region of a neighborhood.

Proposition 3.1: Call n^{int} the number of feasible sender-receiver pairs with the sender located in the interior region of a neighbohood. Then, if (1) holds, for n and c_{min} sufficiently large

$$\mathbb{P}\left\{ n^{int} \geq \alpha c_{min} \frac{B^2}{(B+2b)^2} P^{int} \right\} = 1, \qquad (6)$$

where P^{int} is given by (5).

2) Senders in the overlap region of a neighborhood: For each neighborhood $\mathcal{N}_k$, $k = 1, \ldots, N$, we consider 4 particular subregions of equal area in the overlap region $\mathcal{N}_k^{over}$: $\mathcal{N}_k^{over}(N)$, $\mathcal{N}_k^{over}(S)$, $\mathcal{N}_k^{over}(E)$, and $\mathcal{N}_k^{over}(W)$. These subregions are depicted in Figure 3. By symmetry we only need to focus on one particular subregion, say $\mathcal{N}_1^{over}(E)$. The senders in $\mathcal{N}_1^{over}(E)$ attempt to communicate with their respective nearest neighbors among the receivers located in the neighborhood overlapping with $\mathcal{N}_1^{over}(E)$, say $\mathcal{N}_2$. Pick one of these senders at random, say A_2. Denote by $\{B_j\}_{j=1\ldots n_R}$ the set of the candidate receivers. Let $R_j = |A_2 - B_j|$ be the distance between the sender A_2 and the potential receiver B_j, $j = 1, \ldots, n_R$. Then the distance between A_2 and its nearest neighbor among the receivers is given by $\tilde{R} = \min_{j \in 1,\ldots,n_R} R_j = |A_2 - B_{(2)}|$, $B_{(2)}$ being the location of the nearest potential receiver. We obtain the following lemma whose proof is deferred to the appendix.

Lemma 3.3:

$$\frac{\sqrt{\pi(1-\alpha)c_{min}}}{B+2b} \tilde{R} \xrightarrow{\mathcal{D}} \tilde{R}^*,$$

where $\tilde{R}^*$ has the cdf $L_2(r)$ given by (3).

Once again, according to the protocol model, the transmission between A_2 and $B_{(2)}$ will be successful if no node is simultaneously transmitting within a radius of $(1+\Delta)\,\tilde{R}$ around $B_{(2)}$. In steady state, when $c_{min} \to \infty$, $B_{(2)}$ becomes close enough to A_2 that $B_{(2)} \in \mathcal{N}_1^{over}(E)$ almost surely. Define R'_j as $R'_j = |A_j - B_{(2)}|$, $j = 1, \ldots, N\alpha c_{min}$. For a transmission to be successful, we must have

$$\tilde{R}_I = \min_{j \neq 2} R'_j \geq (1+\Delta)\,\tilde{R} \iff \frac{\tilde{R}}{\tilde{R}_I} \leq \frac{1}{1+\Delta}.$$

We obtain the following lemma, whose proof is deferred to the appendix.

Lemma 3.4:

$$\frac{\sqrt{\pi\,(2\alpha c_{min} - 1)}}{B+2b} \tilde{R}_I \xrightarrow{\mathcal{D}} \tilde{R}_I^*,$$

where I_2^* has the cdf $L_2(r)$ of (3).

As in Section III-A.1, it follows that the limiting distributions of $\tilde{R}$ and $\tilde{R}_I$ are independent and thus,

$$\mathbb{P}[\text{succ. transmission } A_2 \to B_{(2)}] = \mathbb{P}\left[\frac{\tilde{R}}{\tilde{R}_I} \leq \frac{1}{1+\Delta}\right] \to \mathbb{P}\left[\frac{\tilde{R}^*}{\tilde{R}_I^*} \leq \sqrt{2}\frac{1}{1+\Delta}\sqrt{\frac{1-\alpha}{\alpha}}\right].$$

Define P^{over} as

$$P^{over} = \mathbb{P}\left[\frac{X}{Y} \leq \sqrt{2}\frac{1}{1+\Delta}\sqrt{\frac{1-\alpha}{\alpha}}\right],$$

where X and Y are i.i.d. random variables with common distribution L_2 given by (3). As X/Y as c.d.f. given by (4), we obtain

$$P^{over} = \left(1 + \frac{1}{2}(1+\Delta)^2 \frac{\alpha}{1-\alpha}\right)^{-1}. \qquad (7)$$

Within the whole neighborhood $\mathcal{N}_1$, there are $n_S = \alpha c_{min}$ senders that are attempting to transmit. In steady state, the probability for a sender to be located in $\mathcal{N}_1^{over}(E)$ at time t is $p = (B+b)b/(B+2b)^2$. Let k be the number of such senders. By Hoeffding's inequality (Theorem 8.1 in [20]), we have

$$\mathbb{P}\left[\left|\frac{k}{n_S} - p\right| \geq \epsilon\right] \leq e^{-n_S \epsilon^2}.$$

When $n_S \to \infty$, we can thus say that there are almost surely at least $\alpha c_{min}(B+b)b/(B+2b)^2$ senders in $\mathcal{N}_1^{over}(E)$ that are attempting to transmit. Therefore, the following proposition holds.

Proposition 3.2: Call n^{over} the number of feasible sender-receiver pairs with the sender located in a subregion of a neighbohood's overlap region. Then, if (1) holds, for n and c_{min} sufficiently large

$$\mathbb{P}\left\{ n^{over} \geq \alpha c_{min} \frac{(B+b)b}{(B+2b)^2} P^{over} \right\} = 1, \qquad (8)$$

where P^{over} is given by (7).

In the next section we focus on the routing algorithm which determines the successive neighborhoods that a packet has to cross from its source to its destination.

B. Routing Algorithm and Throughput

In our transmission scenario, we consider first the hops that involve senders in the overlap region of their neighborhoods and their nearest candidate receivers in the corresponding overlapping neighborhoods, i.e., from the second hop up to the penultimate one (see Figure 2). As in [6], we appeal to results on routing on meshes. A two-dimensional $l \times l$ mesh is formed by l^2 processing units (PUs) arranged in an $l \times l$ array. Each PU is connected to its (at most) four immediate vertical and horizontal neighbors as depicted in Figure 4. In the *full-port* model, every PU can communicate with all its neighbors (no more than four) simultaneously. Time is slotted and it is assumed that the communication happens between neighboring PUs over slots. The total amount of information that is transmitted during each such communication is exactly

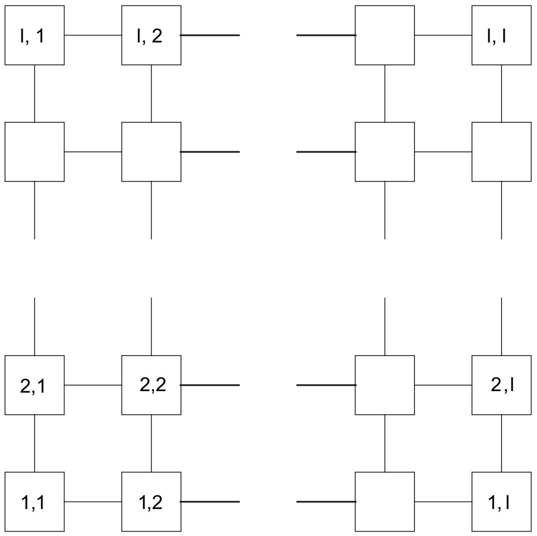

Fig. 4. A mesh network of PUs each connected to its vertical and horizontal neighbors.

one packet. Assume that every PU is the source and destination of exactly k packets. The well studied problem of routing a total of kl^2 packets is called the $k \times k$ *permutation routing problem*. The packets must be routed with minimal queue length requirements as well as with small routing time. The following result has been established:

Lemma 3.5 ([22], [23]): $k \times k$ permutation routing in an $l \times l$ mesh can be performed deterministically in $kl/2 + o(kl)$ steps with maximum queue size at each PU equal to k. Further, every routing algorithm takes at least $kl/2$ steps.

We now apply the $k \times k$ permutation routing algorithm to our problem of routing the packets through the neighborhoods in our ad hoc wireless network by establishing an analogy between PUs and neighborhoods. Recall that we have $N = (1-b)^2/(B+b)^2$ such neighborhoods. Thus we can map the correspondence of PUs and neighborhoods by letting $l = N^{1/2} = (1-b)/(B+b)$. Secondly, let each user have m packets to send. Recall that the maximum number of users per neighborhood is c_{max}. The total number of packets in each neighborhood is therefore no more than mc_{max}. Next, in each neighborhood, we group the packets of the users belonging to the same neighborhood and make them correspond to the k packets of a PU by letting $k = mc_{max}$. We thus have a correspondence between the traffic pattern through the neighborhoods in our wireless network and the mesh network of PUs. According to the *full-port* model, in the mesh of PUs, each PU can transmit and receive up to 4 packets in the same slot. Then, Lemma 3.5 implies that if each neighborhood can transmit and receive up to 4 packets in the same slot then there exists a routing algorithm specifying the succession of neighborhood each packet must follow that requires $k\sqrt{N}/2$ steps and prevents "hot spots". For the details on the actual sequence of PU traversed by a packet, we refer the reader to [22], [23]. In our wireless network, we saw in Section III-A.2 that from a given neighborhood to one of its overlapping (up to 4) neighborhoods, there are at least n^{over} possible simultaneous transmissions, with n^{over} given by (8). Thus each neighborhood can be the site of at least up to $4 \times n^{over}$ possible simultaneous transmissions towards other neighborhoods and at each step of the algorithm we can send n^{over} times more packets. We modify the routing protocol accordingly by allowing an additional factor n^{over} of parallel transmissions. This results in the division of the number of steps by n^{over}.

We now have to consider two extra steps which are the initial step and the final step of the transmission process between a source node and its associated destination. The first one is the communication between the source node and its nearest neighbor and the last one involves the last relay node and the destination node, when the latter is the nearest neighbor of the former. Both steps occur in the interior region of a neighborhood, and we have seen in Section III-A.1 that, in steady state, there can be at least n^{int} possible simultaneous transmissions within the interior region of a neighborhood, with n^{int} given by (6). We should therefore add $2 \times k/n^{int}$ steps. We conclude that, in steady state, the m packets of each user reach their destination in a number of slots equals to

$$\frac{kl}{2n^{over}} + \frac{2k}{n^{int}} = \frac{mc_{max}(B+2b)^2}{\alpha c_{min}} \left(\frac{1-b}{2b(B+b)^2 P^{over}} + \frac{2}{B^2 P^{int}} \right). \quad (9)$$

Before making any statement regarding the throughput of our wireless network, we now have to focus on the sum of the source-destination pairs. This will allow formulating throughput results using bit-meters per second in addition to bits per second. We make the following claim. Its proof is deferred to the Appendix.

Claim 3.1: If $B + 2b = \Omega(1/n)$ the sum of the distances between source-destination pairs is $\Omega(n)$ meters almost surely.

As we have a total of n source-destination pairs, by this claim, the total distance traveled by the mn packets of each source is at least $\Omega(mn)$. By combining this fact with (9), and Propositions 3.1 and 3.2, we are able to state the main throughput result of Proposition 2.1.

IV. Study of Specific Cases

In this section we apply our deterministic results to specific cases.

One neighborhood, no overlap region: Our model allows us to recover the achievability results of [2] (Section C). In Grossglauser and Tse's model, the nodes are allowed to move freely in the whole domain of 1m^2 and the algorithm has at most two phases: the transmission from the source to the nearest relay node, and the transmission from this node to the destination when the destination is near-by. In our model, this corresponds to the situation where $B = 1$, $b = 0$, $c_{min} = c_{max} = n$. Indeed, with $B = 1$, we have made the whole domain a single neighborhood and $c_{min} = c_{max} = n$. Besides, with $b = 0$, we only allow the initial and final steps of our transmission process, hence, in (2), we consider only the second term in the sum. It is straightforward that (1) holds, $B + 2b = \Omega(1/n)$, and $c_{min}, c_{max} \to \infty$ as $n \to \infty$. From Proposition 2.1, we obtain an achievable throughput of $\Omega(n)$ as obtained by [2].

N neighborhoods, no interior region: If there is no interior region, $B = 0$, $N = \left(\frac{1-b}{b}\right)^2$ and the nodes are always located in an overlap region. Therefore, only the first term in the sum in (2) needs to be considered, and a throughput of $\Omega\left(\frac{nc_{min}}{c_{max}}\frac{1}{\sqrt{N}}\right)$ is achievable a.s. provided the conditions of Proposition 2.1 hold. Now suppose that the nodes' initial locations are chosen independently and randomly from a uniform distribution on the whole domain and that each node is assigned to one of the two neighborhoods it lies in equiprobably. Then the node assignments are i.i.d. and the probability that a given node belongs to a given neighborhood equals $1/N$. Set $N = n/(5\log n)$. We now determine c_{min}. The probability that any given neighborhood has at most m nodes is equal to $\sum_{0\le k\le m} C_k^n p^k(1-p)^{n-k}$, with $p = 1/N$. Dudley [24] states the Chernoff-Okamoto inequality

$$\sum_{0\le k\le m} C_k^n p^k(1-p)^{n-k} \le e^{-(np-m)^2/[2np(1-p)]}$$

for $p \le 1/2$ and $m \le np$. From a simple union bound, since we have N neighborhoods, we conclude that the probability that at least one neighborhood has less than m nodes is upper bounded by

$$p_n = Ne^{-(n/N-m)^2/[2n/N(1-1/N)]}. \tag{10}$$

With m equals $5(1-\epsilon)\log n$, where $\epsilon \in (2/\sqrt{5}, 1)$, we arrive at $p_n \le (5n^{\frac{5}{2}\epsilon^2-1}\log n)^{-1}$ and hence $\sum_{n=1}^{\infty} p_n < \infty$. By the Borel-Cantelli lemma (see [21]), we conclude that, almost surely, $c_{min} \ge 5(1-\epsilon)\log n$.
We now determine c_{max}. The number of nodes in a particular neighborhood (denoted by, say Z) is a binomial random variable with parameters(1/N,n). Using a Chernoff bound, we have, for all $m > 0, \theta > 0$,

$$\mathbb{P}[Z > m] \le \frac{\mathbb{E}[\exp(\theta Z)]}{\exp(\theta m)}. \tag{11}$$

Now, with $N = n/(5\log n)$,

$$\mathbb{E}[\exp(\theta Z)] = \left(1 + \left(e^\theta - 1\right)\frac{1}{N}\right)^n \le n^{5(e^\theta-1)}.$$

Choosing $\theta = 1$ and $m = 5e(\log n)^2$ in (11), and using a simple union bound and (11) we have

$$\mathbb{P}\left[c_{max} > 5e(\log n)^2\right] \le \left(5n^{4+5e(\log n-1)}\log n\right)^{-1}.$$

By the Borel-Cantelli lemma (see [21]), we conclude that, almost surely, c_{max} is no more than $5e(\log n)^2$. Since $c_{\min} \to \infty$ and $c_{\max} \to \infty$ as $n \to \infty$, (1) is satisfied. Besides

$$B + 2b = 2b = 2\left(1 + \sqrt{n/(5\log n)}\right)^{-1} = \Omega(1/n).$$

We thus conclude that a throughput $\Omega\left(\sqrt{n}/\sqrt{\log n}\right)$ is achievable almost surely. Note that in that case, $b \to 0$ as $n \to \infty$. Thus the advantages offered by node mobility are quasi non-existent and it is as if the nodes were unable to move. Thus our model gives the same throughput result as the one of Gupta and Kumar [1] for random node locations.

Random node-neighborhood assignment: We now focus further on the situation where nodes are randomly placed in each neighborhood. Fix $N = n^\alpha$, with $0 \le \alpha < 1$ (if $\alpha \ge 1$, c_{min} would not go to infinity). We now determine c_{min}. Applying (10), with m equal to $(1-\epsilon)n^{1-\alpha}$, where $\epsilon \in (0,1)$, we obtain that the probability that at least one neighborhood has less than m nodes is upperbounded by $p_n \le n^\alpha e^{-\frac{1}{2}n^{1-\alpha}\epsilon^2}$ and hence that $\sum_{n=1}^{\infty} p_n < \infty$. Using the Borel-Cantelli lemma (see [21]), we conclude that, almost surely, $c_{min} \ge (1-\epsilon)n^{1-\alpha}$. We now determine c_{max}. With $N = n^\alpha$,

$$\mathbb{E}[\exp(\theta Z)] = \left(1 + \left(e^\theta - 1\right)\frac{1}{N}\right)^n \le e^{\left(e^\theta-1\right)n^{1-\alpha}}.$$

Choosing $\theta = 1$ and $m = e(1+\epsilon)n^{1-\alpha}$ in the Chernoff bound (11) and using a simple union bound we have

$$\mathbb{P}\left[c_{max} > e(1+\epsilon)n^{1-\alpha}\right] \le n^\alpha e^{-(1+e\epsilon)n^{1-\alpha}}.$$

By the Borel-Cantelli lemma (see [21]), we conclude that, almost surely, c_{max} is no more than $e(1+\epsilon)n^{1-\alpha}$. Recall that $N = \left(\frac{1-b}{b+B}\right)^2$. Choosing $b = B = (2n^{\alpha/2}+1)^{-1}$, we indeed have $N = n^\alpha$ and (1) holds. As $B + 2b = \Omega(1/n)$, and $c_{min}, c_{max} \to \infty$ as $n \to \infty$, after straightforward manipulations we obtain that a throughput $\Omega\left(n^{1-\alpha/2}\right)$ is achievable almost surely, with $0 \le \alpha < 1$.

In view of these three specific cases, we conclude that our model covers all possible achievable orders of growth, from that corresponding to immobile nodes to that achieved when the nodes are allowed to move freely in the entire domain.

V. Concluding Comments

In this paper, we studied how throughput scales with the number of nodes in a wireless network, focusing on the scenario in which the nodes are restricted to move within overlapping neighborhoods. Achievability results on throughput for general configurations were derived using a deterministic approach.

Our results depend on properties of the nodes' locations and neighborhood dimensions. For various situations, one can easily verify that these properties hold. In particular, we recovered as a special case the results of [2], when the whole domain is made a single neighborhood in which the nodes are free to move. We also considered i.i.d. uniform node locations in a setting approaching the model of [1] and obtained an achievable throughput $\Omega\left(\sqrt{n}/\sqrt{\log n}\right)$. In addition, results for random assignment of nodes in each neighborhood have been derived using our model. For n^α neighborhoods, $\Omega\left(n^{1-\alpha/2}\right)$ throughput is obtained, $0 \le \alpha < 1$. This result can be interpreted as follows. The network we studied can be assimilated to N "Grossglauser-Tse" subnetworks connected in a "Gupta-Kumar" fashion, since the neighborhoods themselves are immobile. Then, when there are few neighborhoods, their dimensions are large and the advantage offered by the mobility of the nodes dominates the immobile character of the neighborhoods. Hence the throughput results approach the one in [2]. However, as the neighborhood dimensions go to zero, node mobility becomes non-existent and the throughput suffers the same limitations as the model studied in [1].

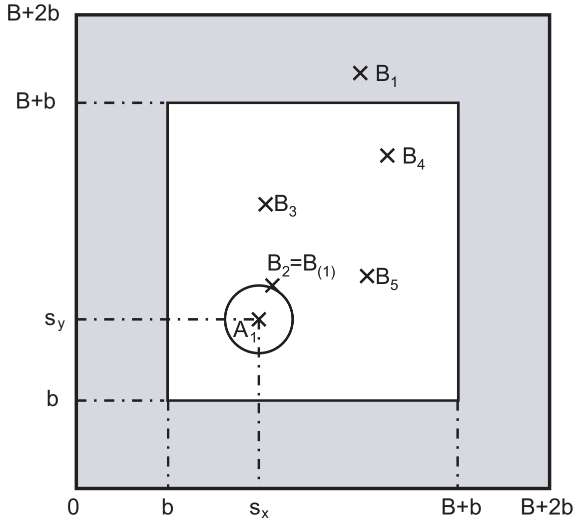

Fig. 5. Case when a sender is in the interior region of its neighborhood.

APPENDIX

Proof of Lemma 3.1 Suppose that at time t, $A_1 = s = (s_x, s_y)$, with s_x and s_y representing the two physical location coordinates. As shown in Figure 5, a disk centered at $A_1 = s$ and of radius

$$r < \min\left(s_x, s_y, B + 2b - s_x, B + 2b - s_y\right)$$

is totally inscribed in the neighborhood $\mathcal{N}_1$. Then, for such r we have

$$\mathbb{P}\left[R_j \leq r | A_1 = s\right] = \frac{\pi r^2}{\left(B + 2b\right)^2}.$$

Also, given $A_1 = s$, the R_j are independent and identically distributed. We use the following theorem.

Theorem 5.1 (Galambos,[25]): Let $X_1, \ldots, X_n$ be n i.i.d. random variables. Define $F(x)$ as $F(x) = \mathbb{P}\left[X_i < x\right]$. Define W_n as $W_n = \min\left(X_1, \ldots, X_n\right)$. Define L_n as

$$L_n(x) = \mathbb{P}\left[W_n < x\right] = 1 - \left(1 - F(x)\right)^n.$$

Define $\alpha(F)$ as $\alpha(F) = \inf\{x : F(x) > 0\}$. Let $\alpha(F)$ be finite. Assume that the distribution function $F^*(x) = F\left(\alpha(F) - 1/x\right)$, $x < 0$ satisfies

$$\lim_{t \to -\infty} \frac{F^*(tx)}{F^*(t)} = x^{-\gamma}\ , \gamma \text{ constant.}$$

Then there exist sequences $c_n > 0$ and $d_n > 0$ such that

$$\lim_{n \to \infty} \mathbb{P}\left[W_n < c_n + d_n x\right] = L_\gamma(x)$$

with

$$L_\gamma(x) = \begin{cases} 1 - \exp\left(-x^\gamma\right) & \text{if } x > 0 \\ 0 & \text{if } x \leq 0. \end{cases}$$

The constants c_n and d_n can be chosen as $c_n = \alpha(F)$ and

$$d_n = \sup\left\{x : F(x) \leq \frac{1}{n}\right\} - \alpha(F).$$

In our case, $F(r) = \mathbb{P}\left[R_j \leq r | A_1 = s\right]$. Then $\alpha(F) = 0$ and thus $c_{n_R} = 0$ and $d_{n_R} = \frac{B+2b}{\sqrt{n_R \pi}}$.
We also have that

$$F^*(r) = \frac{\pi}{\left(B + 2b\right)^2 r^2},$$

$r < 0$, and

$$\lim_{t \to -\infty} \frac{F^*(tr)}{F^*(t)} = r^{-2}.$$

Thus $\lim_{c_{min} \to \infty} \mathbb{P}\left[R < d_{n_R} r | A_1 = s\right] = L_2(r)$, with L_2 given by (3). L_2 is a Rayleigh distribution with variance $1/2$. We observe that the asymptotic distribution of R conditioned on $A_1 = s$ is independent of the location of A_1. We now have, by the Dominated Convergence Theorem (see [21]),

$$\begin{aligned} \forall r > 0,\ & \lim_{c_{min} \to \infty} \mathbb{P}\left[R < d_{n_R} r\right] \\ &= \lim_{c_{min} \to \infty} \int_{s \in \mathcal{N}_1^{int}} \mathbb{P}\left[R < d_{n_R} r | A_1 = s\right] ds \\ &= \int_{s \in \mathcal{N}_1^{int}} \lim_{c_{min} \to \infty} \mathbb{P}\left[R < d_{n_R} r | A_1 = s\right] ds \\ &= L_2(r). \end{aligned}$$

We then obtain the lemma. □

Proof of Lemma 3.2 When $c_{min} \to \infty$, R becomes small enough so that $B_{(1)} \in \mathcal{N}_1^{int}$ a.s. Suppose that at time t, $B_{(1)} = s = (s_x, s_y)$. A circle of radius

$$r < \min\left(s_x - b, s_y - b, B + b - s_x, B + b - s_y\right)$$

is totally inscribed in the interior region of the neighborhood $\mathcal{N}_1$, i.e. $\mathcal{N}_1^{int}$. We thus have

$$\mathbb{P}\left[R'_j \leq r | B_{(1)} = s\right] = \begin{cases} \frac{\pi r^2}{(B+2b)^2} & j = 2, \ldots, n_S \\ 0 & j \geq n_S + 1 \end{cases}$$

For such r,

$$\mathbb{P}\left[\min_{j:2,\ldots,Nn_S} R'_j \leq r | B_{(1)} = s\right] = \mathbb{P}\left[\min_{j:2,\ldots,n_S} R'_j \leq r | B_{(1)} = s\right].$$

In other words we need only to consider the simultaneously transmitting nodes belonging to $\mathcal{N}_1$. By a reasoning similar to the proof of Lemma 3.1, we obtain the lemma. □

Proof of Lemma 3.3 The limiting distribution of $\tilde{R}$ as $c_{min} \to \infty$ is determined by noting that if $A_2 = s = (s_x, s_y)$, a disk centered at $A_2 = s$ of radius

$$r < \min\left(s_x, s_y, B + 2b - s_x, B + 2b - s_y\right)$$

is totally inscribed in the neighborhood $\mathcal{N}_2$ (see Figure 6). Recall that $n_R = (1 - \alpha) c_{min}$. Then, by a reasoning analogous to the one in Section III-A.1, we obtain the lemma. □

Proof of Lemma 3.4 Suppose that at time t, $B_{(2)} = s = (s_x, s_y)$. A circle of radius

$$r < \min\left(s_x, s_y, b - s_x, B + b - s_y\right)$$

is totally inscribed in the region of $\mathcal{N}_1^{over}(E)$. We thus have

$$\mathbb{P}\left[R'_j \leq r | B_{(2)} = s\right] = \begin{cases} \frac{\pi r^2}{(B+2b)^2} & j \neq 2 : A_j \in \mathcal{N}_1 \cup \mathcal{N}_2 \\ 0 & \text{otherwise} \end{cases}$$

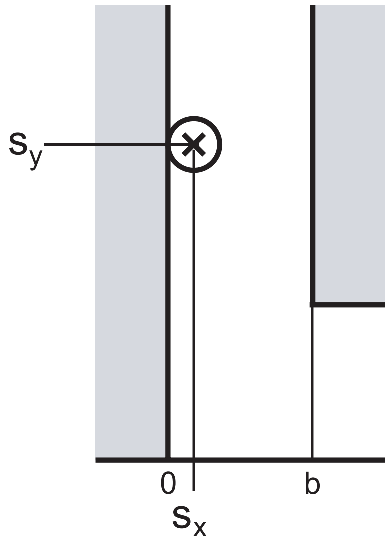

Fig. 6. Case when a sender is in the overlap region of its neighborhood.

For such r,

$$\mathbb{P}\left[\min_{j\neq 2} R'_j \leq r | B_{(2)} = s\right] = \mathbb{P}\left[\min_{j\neq 2: A_j \in \mathcal{N}_1 \cup \mathcal{N}_2} R'_j \leq r | B_{(2)} = s\right].$$

Hence we need only to consider the simultaneously transmitting nodes belonging to $\mathcal{N}_1 \cup \mathcal{N}_2$. There are $2n_R - 1$ such nodes. By a reasoning similar to the proof of Lemma 3.1 we obtain the lemma. □

Proof of Claim 3.1 To prove this claim, as in [6], we need to introduce the following definition.

Definition 5.1 ([26]): Let X and Y be two random variables on $\mathbb{R}$. X is said to be *stochastically larger* than Y, written $X \geq_{st} Y$, if for every $z \in \mathbf{R}$, $\mathbb{P}[X \geq z] \geq \mathbb{P}[Y \geq z]$.

It is then clear that, at each time t, the distance between a source-destination pair is *stochastically larger* if the two nodes belong to different neighborhoods than if they lie in the same neighborhood. Thus, for each time t, the probability of the event that the sum of distances between source-destination pairs is less than z is upper bounded by the probability of the same event conditioned on the fact that, for each source-destination pair, source and destination belong to the same neighborhood. Thus we need only to prove the claim for sources and destinations belonging to the same neighborhood, as we now proceed to do. Recall that the nodes belonging to the same neighborhood have i.i.d. trajectories and that the stationary distribution of their location is uniform on the neighborhood. Within a neighborhood, the distance between any source-destination pair is less than $\sqrt{2}(B + 2b)$. Let (S_i, S_j) be the location of a source-destination pair at time t and let $(x_i, y_i), (x_j, y_j)$ be its corresponding physical coordinates. Denote by $d(S_i, S_j)$ the distance between S_i and S_j. Then we have

$$d^2(S_i, S_j) \leq \sqrt{2}(B + 2b)d(S_i, S_j).$$

The squared distance between any source-destination pair has the same distribution as that of $d^2(S_i, S_j) = (x_i - x_j)^2 + (y_i - y_j)^2$, where, in steady state, x_i, x_j, y_i, y_j are i.i.d. random variables uniformly distributed on $[0, B + 2b]$. Besides, the squared distances between the source-destination pairs are all i.i.d. Call $\mathcal{S}$ the set of source-destination pairs. We thus have, for $\theta > 0, a > 0$,

$$\begin{aligned}\mathbb{P}&\left[\sum_{(S_i,S_j)\in\mathcal{S}} d(S_i, S_j) < \frac{n}{a}\right] \\ &\leq \mathbb{P}\left[\frac{\sum d^2(S_i, S_j)}{\sqrt{2}(B + 2b)} < \frac{n}{a}\right] \\ &\leq \exp\left(\frac{\sqrt{2}(B + 2b)n}{a}\right)\mathbb{E}\big[\exp(-\theta(x_i - x_j)^2)\big]^n, \quad (12)\end{aligned}$$

where x_i and x_j are i.i.d. uniform on [0,B+2b].

Note that the last inequality is obtained by a Chernoff bound and by the fact that the squared distances are i.i.d. Also,

$$\begin{aligned}\mathbb{E}\left[\exp(-\theta(x_i - x_j)^2)\right] &= 2\int_0^1 \int_0^y \exp(-\theta(B + 2b)^2 t^2) dt dy, \\ &= \int_0^1 (1 - t)\exp(-\theta(B + 2b)^2 t^2) dt \\ &< \frac{\sqrt{\pi}}{\sqrt{\theta}(B + 2b)}. \quad (13)\end{aligned}$$

Substituting (13) in (12), with $a = \sqrt{2}(B+2b)\theta$ and $\theta = 2\pi e^2$, we obtain

$$\mathbb{P}\left[\sum d(S_i, S_j) < \frac{n}{2\pi e^2}\right] < \frac{1}{\left(\sqrt{2}(B + 2b)\right)^n}.$$

By the Borel-Cantelli lemma (see [21]), we conclude that, almost surely, the sum of the distances between source-destination pairs grows at least linearly with n provided $(B + 2b) = \Omega(1/n)$. □

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