Ergodic Two-User Interference Channels: Is Separability Optimal?

Lalitha Sankar*, Xiaohu Shang†, Elza Erkip‡, H. Vincent Poor*
*Dept. of Electrical Engineering, Princeton University, Princeton, NJ 08544. lalitha,eerkip,poor@princeton.edu
†Dept. of Electrical Engineering, Syracuse University, Syracuse, NY 13244. xshang@syr.edu
‡Dept. of Electrical and Computer Engineering, Polytechnic Institute of NYU, Brooklyn, NY 11201. elza@poly.edu

***Abstract*— The optimality of separable encoding and decoding over parallel channels (fading states) for ergodic fading two-user interference channels (IFCs) is studied using a one-sided IFC as a model. For an ergodic fading one-sided IFC with non-fading direct links and a fading cross-channel link, it is shown that separability can be strictly suboptimal except for the cases where all the parallel channels are of the same type, i.e., all of them are either *strong but not very strong* or *very strong* channels. A recent result on the sum-capacities of the classes of ergodic strong and very strong IFCs is used to show that encoding and decoding jointly over all parallel channels is optimal when either the strong or very strong interference conditions hold on average over all channels.**

I. INTRODUCTION

The two-user interference channel (IFC) is a network with two sources (senders or transmitters) and two destinations (receivers) where each source transmits to its intended destination. Determining the capacity region of both the discrete memoryless (d.m.) and the Gaussian IFC remain open problems; however, for certain classes of time-invariant IFCs satisfying specific well-defined constraints the capacity region is known. Specifically, the capacity region is known for the class of strong Gaussian IFCs [1], [2], [3] and for the class of strong d.m. IFCs [4]. Note that the class of strong IFCs includes the sub-class of very strong IFCs. For a special class of one-sided IFCs where only one of the two cross-channel links from the transmitters to the unintended receivers is non-zero, the sum-capacity is known for the cases of both strong and weak interference [5]. Recently, the sum-capacity of a class of noisy or very weak Gaussian IFCs has been determined independently in [6], [7], and [8]. The sum-capacity for a class of mixed IFCs is developed in [7], [9]. Outer bounds for the IFC are developed in [10], [11] while several achievable rate regions for the Gaussian IFC are studied in [12].

Relatively fewer results are known for parallel or fading IFCs. In [13], the authors develop the sum-capacity of a class of parallel Gaussian IFCs where each parallel channel is strong using independent encoding and decoding in each parallel channel. Since the conditions for the strong channel include those for the very strong channel, in [13], the authors do not explicitly distinguish between the very strong channel and a channel that is strong but not very strong. In this paper, we argue that such a distinction is essential and to this end classify them as two mutually exclusive channels referred to as *strong but not very strong* and *very strong*. Using this distinction, we show that the results in [13] are tight only if the parallel channels are all either strong but not very strong or all very strong. In [14], Sung *et al.* present an achievable scheme for a class of one-sided parallel Gaussian IFCs, where in each independent parallel channel, the interference is either viewed as noise or completely decoded depending on whether the cross-channel gain models a weak or strong (includes very strong) one-sided IFC, respectively. Further, they also present the sum-rate maximizing power policies for this achievable scheme with independent encoding and decoding in each parallel channel. While [14] suggests that separability is optimal, i.e., sum-capacity-achieving for the one-sided parallel IFC, here we prove otherwise. More recently in [15] the conditions for the classes of ergodic strong and ergodic very strong IFCs are defined and their sum-capacity is shown to be achieved by the sum-capacity of the compound multiaccess channel that results when both receivers decode messages from both transmitters. Thus, in contrast to the model in [13], [15] studies the case where the channel is strong or very strong when averaged over all fading states. More recently, [16] develops the sum-capacity and optimal power policies for a class of parallel two-sided noisy IFCs under the requirement of independent encoding and decoding in each parallel channel.

In this paper, we study an ergodic fading one-sided IFC with non-fading direct links and present achievable strategies and, where possible, the sum-capacity achieving schemes by dissecting the problem into three mutually exclusive cases. The three cases correspond to channels where the cross-link fading states (parallel channels) are a) all weak, b) a mixture of strong but not very strong and very strong, and c) a mixture of all three, i.e., weak, strong but not very strong, and very strong. Using the sum-capacity results from [15], we show that, unlike the point-to-point, the multiaccess, and the broadcast (without common message) channels, the ergodic fading one-sided IFC does not always simplify to a collection of parallel IFCs with

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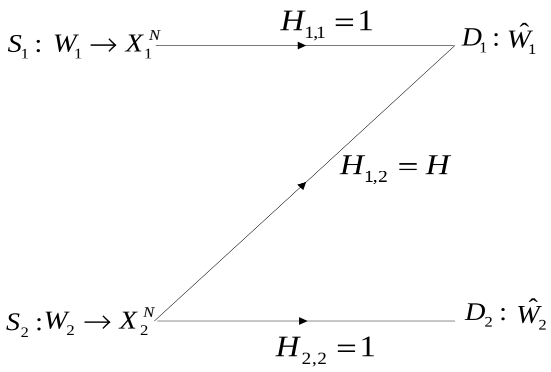

Fig. 1. A one-sided interference channel.

independent encoding and decoding; in fact, with the exception of a few cases we show that encoding and decoding separately over each parallel channel is strictly sub-optimal. Recently, in [17], Cadambe and Jafar demonstrate the inseparability of parallel interference channels using a three-user frequency selective fading IFC as a counter example. The authors use interference alignment schemes to show that separability is not optimal for fading IFCs with three or more users while leaving open the question for the two-user fading IFC. In this paper, we address this question using the ergodic fading one-sided IFC as an illustrative model.

The paper is organized as follows. In Section II, we present the ergodic fading one-sided IFC model. In Section III, we present achievable strategies and where possible sum-capacity results for the three cases described above. We present numerical results in Section IV and conclude in Section V.

II. CHANNEL MODEL

A two-sender (also called two-user) two-receiver Gaussian one-sided IFC consists of two source (user) nodes S_1 and S_2, and two destination nodes D_1 and D_2 as shown in Fig. 1. Source S_k, $k = 1, 2$, uses the channel n times to transmit its messages W_k, distributed uniformly in the set $\{1, 2, \ldots, 2^{B_k}\}$, to its intended receiver, D_k, at a rate $R_k = B_k/n$ bits per channel use. In each use of the channel, S_k transmits the signal X_k while the destinations D_1 and D_2 receive Y_1 and Y_2, respectively, such that

$$Y_1 = X_1 + HX_2 + Z_1 \tag{1}$$

$$Y_2 = X_2 + Z_2 \tag{2}$$

where Z_1 and Z_2 are independent circularly symmetric complex Gaussian noise random variables with zero means and unit variances and the cross-channel gain H is a realization for a given channel use of a stationary and ergodic (not necessarily Gaussian) fading process $\underline{H}$. We assume that the fading H is known instantaneously at all transmitters and receivers. We also assume that over n uses of the channel, the source transmissions are constrained in power according to

$$\sum_{i=1}^{n} |X_{k,i}|^2 \leq n\overline{P}_k, \quad k = 1, 2 \tag{3}$$

where $X_{k,i}$ denotes the transmitted signal from source k in the i^{th} channel use. Since the sources know the fading states of the links on which they transmit, they can allocate their transmitted signal power according to the channel state information. We write $P_k(H)$ to denote the power allocated at the k^{th} transmitter as a function of the channel states H. For an ergodic fading channel, (3) then simplifies to

$$\mathbb{E}\left[P_k(H)\right] \leq \overline{P}_k, \quad k = 1, 2 \tag{4}$$

where the expectation in (4) is over the distribution of H. We write $\underline{P}(H)$ to denote a vector of power allocations with entries $P_k(H)$, for all k, and define $\mathcal{P}$ to be the set of all $\underline{P}(H)$ whose entries satisfy (4).

The capacity region $\mathcal{C}_{\text{IFC}}$ of a two-user IFC is defined as the closure of the set of rate tuples (R_1, R_2) such that the destinations can decode their intended messages with an arbitrarily small positive error probability ϵ. For a one-sided ergodic fading IFC, we seek to maximize the sum-rate R_1+R_2 subject to (4) where $(R_1, R_2) \in \mathcal{C}_{\text{IFC}}$.

In general, the cross-channel fading parameter H in (1) can take any value in the range $[0, \infty)$. To demonstrate the range of H for which separability holds, we consider the following three mutually exclusive cases separately:

1) Strong case: $H \in [1, \infty)$
2) Weak case: $H \in [0, 1)$
3) General (mixed) case: $H \in [0, \infty)$.

Studying the three cases separately, for the strong case we show that separability is optimal only if every parallel channel is of the same type, i.e., either strong but not very strong or very strong. Note that we refer to the first case as strong in keeping with existing literature since it encompasses the two mutually exclusive cases of *strong* interference. For the general case, we present a counter-example to illustrate the strict sub-optimality of separability. Finally, we briefly discuss the weak case. We remark that a more general model for the one-sided fading Gaussian IFC includes fading on all links such that

$$Y_1 = H_{1,1}X_1 + H_{1,2}X_2 + Z_1 \tag{5}$$

$$Y_2 = H_{2,2}X_2 + Z_2 \tag{6}$$

where $H_{k,m}$ for all k, m, are realizations of a jointly stationary and ergodic process. For ease of analysis and illustration, throughout the sequel we focus on the model in (1) and (2). The analysis presented here can be generalized in a straightforward manner to the model in (5) and (6).

For ease of notation, we henceforth omit the functional dependence of $\underline{P}$ on H. We write random variables (e.g. $H_{k,j}$) with uppercase letters and their realizations (e.g. $h_{k,j}$) with the corresponding lowercase letters. We write $\mathcal{K} = \{1, 2\}$ to

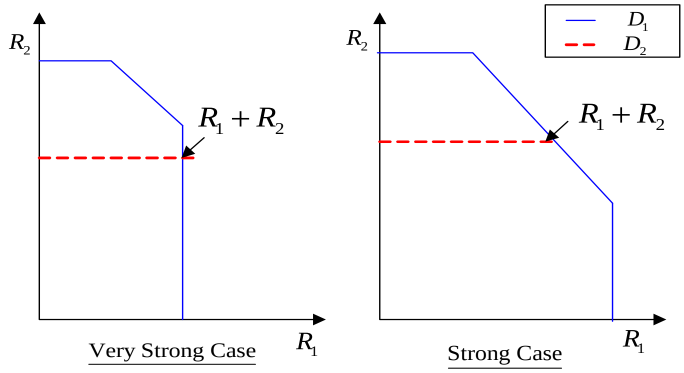

Fig. 2. Intersection of the ergodic rate planes for the two-user very strong and strong one-sided IFCs.

denote the set of transmitters, the notation $C(x) = \log(1+x)$ where the logarithm is to the base 2, and $(x)^+ = \max(x, 0)$. Throughout the sequel, we use the terms fading state and parallel channel interchangeably. Finally, throughout the sequel, we write IFC to denote the general two-sided channel and, in keeping with the common use, we write a strong IFC to denote a channel that is either strong but not very strong or very strong.

III. ERGODIC SUM-CAPACITY

We consider each of the three cases detailed in Section II separately. In each case, we also present the sum-rate optimal power policies at the two transmitters for the achievable scheme considered.

A. *Strong Case:* $H \in [1, \infty)$

We begin by reviewing the known capacity results for both two-sided and one-sided non-fading IFCs. The capacity region of the d.m. and non-fading (fixed $H_{k,m}$ for all k, m denoted by a matrix $\mathbf{H}$) Gaussian (two-sided) IFC under strong interference conditions is known and is given by the intersection of the two multiaccess regions that results from decoding both transmitter messages at both receivers. For a non-fading Gaussian IFC in standard form ($H_{1,1} = H_{2,2} = 1$), the conditions for the strong and very strong interference cases simplify to ($H_{1,2} \geq 1$, $H_{2,1} \geq 1$) and ($H_{1,2} > 1 + P_1$, $H_{2,1} > 1 + P_2$), respectively. Note that the conditions above for a strong IFC subsume those for the very strong. These two cases can be made mutually exclusive by writing the conditions for the strong but not very strong Gaussian IFC as ($1 \leq H_{1,2} \leq 1 + P_2$, $1 \leq H_{2,1} \leq 1 + P_1$). In general, the conditions for the strong but not very strong are

$$\begin{aligned} I(X_2; Y_1|X_1\mathbf{H}) > I(X_2; Y_2|X_1\mathbf{H}) \geq I(X_2; Y_1|\mathbf{H}) \\ I(X_1; Y_2|X_1\mathbf{H}) > I(X_1; Y_1|X_2\mathbf{H}) \geq I(X_1; Y_2|\mathbf{H}) \end{aligned} \tag{7}$$

while those for the very strong are

$$\begin{aligned} I(X_2; Y_1|\mathbf{H}) > I(X_2; Y_2|X_1\mathbf{H}) \\ I(X_2; Y_1|\mathbf{H}) > I(X_1; Y_1|X_2\mathbf{H}). \end{aligned} \tag{8}$$

The conditions in (7) and (8) also apply for the strong but not very strong and very strong d.m. IFC (for a constant $\mathbf{H}$), respectively [4]. For the non-fading one-sided IFC, the conditions in (7) and (8) simplify to just one condition for each type as now the capacity region is simply the intersection of a pentagon and a line as shown by the two possible intersections in Fig 2. The first intersection in the figure corresponds to a very strong one-sided IFC while the second corresponds to a strong but not very strong one-sided IFC. From (7) and (8) we thus obtain the conditions for the strong but not very strong channel as

$$I(X_2; Y_1|X_1H) > I(X_2; Y_2|X_1H) \geq I(X_2; Y_1|H) \tag{9}$$

and that for the very strong channel as

$$I(X_2; Y_1|H) > I(X_2; Y_2|X_1H) = I(X_2; Y_2|H). \tag{10}$$

Finally, we obtain a class of weak one-sided IFCs when the conditions in both (9) and (10) are not satisfied.

For the ergodic IFC, in [15, Theorems 2 and 3] it is shown that, for a given channel statistics, an ergodic strong but not very strong and very strong IFCs result when the sum-capacity optimal power policy $\underline{P}^*$ satisfies the conditions in (7) and (8), respectively. As expected, the sum-capacity in both cases is achieved by decoding both user messages at both receivers. We note that for the ergodic case, the conditioning on the channel state implies that the inequalities in (7) and (8) hold when averaged over all fading states, i.e., not every fading state needs to be of the same type.

Finally, [15, Theorem 2] also shows that for the ergodic very strong IFC, the sum-capacity optimal power policy requires each user to waterfill on its direct link to its intended destination. On the other hand, for the ergodic strong but not very strong IFC, the optimal policy in [15, Theorem 3] involves opportunistically scheduling the users to maximize the smaller of the two sum-capacities achieved at the two receivers when both bounds in (7) are strict inequalities. For those *boundary cases* where one of the two lower bound inequalities in (7) is satisfied with equality, the solution takes an opportunistic non-waterfilling form. These results can be apply directly to the one-sided ergodic fading IFC by setting one of the cross-channel gains to zero.

For the ergodic channel with $H > 1$ for all fading instantiations, in every fading state we obtain either a strong but not very strong or a very strong one-sided IFC. Thus, by definition, the channel is either strong but not very strong or very strong on average. Without loss of generality, we assume L parallel fading states and write h_l to denote the l^{th} channel state and $P^*_{k,l}$ to denote the optimal power policy at the k^{th} user in the l^{th} parallel channel (fading state) such that $\underline{P}^* \in \mathcal{P}$. The sum-capacity from [15, Theorems 2 and 3] for the ergodic one-sided IFC with $H > 1$ is then given as

$$\min \begin{pmatrix} \sum_{l=1}^{L} p_l C\left(P_{1,l}^* + \left|h_l\right|^2 P_{2,l}^*\right), \\ \sum_{l=1}^{L} p_l \left\{C\left(P_{1,l}^*\right) + C\left(P_{2,l}^*\right)\right\} \end{pmatrix} \tag{11}$$

where the first term of the minimum in (11) is the sum-capacity of the ergodic strong one-sided IFC while the second term is the sum-capacity of the ergodic very strong one-sided IFC and $\underline{P}^*$ is the capacity achieving power policy. While the details of the proofs for the two cases can be found in [15, Theorems 2 and 3], we present a simple geometric argument for (11) below. From Fig. 2 and the conditions in (9), for the ergodic strong but not very strong case, the ergodic sum-capacity is limited by the MAC sum-capacity at D_1 if

$$\sum_{l=1}^{L} p_l C\left(P_{2,l}^*\right) \geq \sum_{l=1}^{L} p_l C\left(\frac{\left|h_l\right|^2 P_{2,l}^*}{1+P_{1,l}^*}\right). \tag{12}$$

On the other hand, for the very strong ergodic one-sided IFC, (10) simplifies to requiring

$$\sum_{l=1}^{L} p_l C\left(\frac{\left|h_l\right|^2 P_{2,l}^*}{1+P_{1,l}^*}\right) > \sum_{l=1}^{L} p_l C\left(P_{2,l}^*\right). \tag{13}$$

Combining (12) and (13), we obtain the expression in (11) for the sum-capacity of the two mutually exclusive ergodic strong one-sided IFCs.

Consider the achievable scheme with independent encoding and decoding in each parallel channel as considered in [18]. Since each channel state is either strong but not very strong or very strong, the maximum sum-rate in each parallel channel is a minimum of the bounds for the two cases. The maximum sum-rate is then given as

$$\max_{\underline{P}\in\mathcal{P}} \sum_{l=1}^{L} p_l \min\left(C\left(P_{1,l} + \left|h_l\right|^2 P_{2,l}\right), C\left(P_{1,l}\right) + C\left(P_{2,l}\right)\right). \tag{14}$$

Comparing (11) and (14), we see that separable encoding and decoding is optimal only if all parallel channels are of one type, i.e., all of them are either strong but not very strong or very strong.

B. Weak Case: $H \in [0, 1)$

Next, we consider the case where every fading state is strictly less than 1. In [5, Theorem 1], Costa shows that the sum-capacity of a time-invariant Gaussian one-sided IFC is the same as that of an equivalent degraded Gaussian IFC. Further, this sum-capacity is achieved when the interfered receiver, D_1, ignores the interference from S_2. Under the assumption of independent encoding and decoding in each parallel fading channel, the maximum achievable sum-rate is bounded by

$$\max_{\underline{P}\in\mathcal{P}} \sum_{l=1}^{L} p_l \left[C\left(\frac{P_{1,l}}{1+\left|h_l\right|^2 P_{2,l}}\right) + C\left(P_{2,l}\right)\right] \tag{15}$$

$$s.t. \sum_{l=1}^{L} p_l P_k(H) \leq \overline{P}_k, \quad \text{for all } k. \tag{16}$$

Based on the observations for the two kinds of strong cases above, we conjecture that separability will be optimal for this case. Our conjecture is based on the analogy to the strong case where separability is optimal when all fading states are of the same type. Similarly, for the weak case, since the same strategy of ignoring the interference achieves the sum-capacity for each parallel channel it may also achieve the sum-capacity over all parallel channels.

C. General Case: $H \in [0, \infty)$

We now consider the case where the fading state is arbitrary, i.e., some of the channels can be weak while others can be one of the two mutually exclusive strong types. Thus, for this case the fading channel can be characterized on average as either an ergodic weak or an ergodic strong but not very strong or an ergodic very strong channel.

An achievable scheme for the ergodic one-sided IFC is obtained by assuming independent encoding and decoding for each fading state (see [14]). The resulting sum-rate is given as

$$\max_{\underline{P}\in\mathcal{P}} \sum_{l=1}^{L} C_l\left(h_l\right) \tag{17}$$

where $C_l\left(h_l\right)$ is the sum-capacity achieved in the l^{th} parallel channel (fading state), for all l, and is defined uniquely for the three regimes, weak $(h_l \leq 1)$, strong but not very strong $(1 \leq h_l \leq 1 + P_{2,l})$, and very strong $(h_l \geq 1 + P_{2,l})$, as

$$C_l\left(h_l\right) = \begin{cases} C\left(\frac{P_{1,l}}{1+\left|h_l\right|^2 P_{2,l}}\right) + C\left(P_{2,l}\right) & 0 \leq h_l \leq 1 \\ C\left(P_{1,l} + \left|h_l\right|^2 P_{2,l}\right) & 1 \leq h_l \leq 1 + P_{1,l} \\ C\left(P_{1,l}\right) + C\left(P_{2,l}\right) & h_l \geq 1 + P_{1,l}. \end{cases} \tag{18}$$

We now present an example of a two-state ergodic one-sided IFC for which separability is strictly sub-optimal. Our proof hinges on the fact that the ergodic one-sided IFC can be a strong but not very strong or a very strong channel on average even if this requirement does not hold for every fading state.

Consider a two-state one-sided IFC with states $h_1 < 1$ and $h_2 > 1$ that occur with probabilities $p_1 > 0$ and $p_2 = 1 - p_1$, respectively. The sum-rate achieved when the transmitters and receivers independently encode and decode for each state is obtained from (17) as

$$\max_{\underline{P}\in\mathcal{P}} \begin{bmatrix} p_1 C\left(P_{2,1}\right) + p_1 C\left(\frac{P_{1,1}}{1+\left|h_1\right|^2 P_{2,1}}\right) \\ +p_2 \min\left\{\begin{array}{c} C\left(P_{1,2} + \left|h_2\right|^2 P_{2,2}\right), \\ C\left(P_{1,2}\right) + C\left(P_{2,2}\right) \end{array}\right\} \end{bmatrix} \tag{19}$$

where $P_{k,l}$ is the power allocated by user k to the state l subject to (4). Suppose the sum-capacity optimal power policy for this one-sided IFC satisfies the ergodic very strong condition in (10). Recall that the optimal power policy for the ergodic very strong IFC has each user water-filling over the fading link to its intended receiver. Since these links are non-fading for the model considered here, the optimal policy simplifies to $P_k^*(h_l) = \overline{P}_k$ for $k, l = 1, 2$.

Using the results in [15], since the optimal policy $P_k^*(h_l) = \overline{P}_k$, for all k, l, for the very strong ergodic IFC satisfies (10), and the direct links are non-fading, the sum-capacity for this channel is given as

$$C\left(\overline{P}_1\right) + C\left(\overline{P}_2\right). \tag{20}$$

Comparing (19) with (20), we make the following observations. Let $\underline{P}'$ denote the optimal policy maximizing (19). Since the channel is very strong on average and $h_1 < 1$, the fading state h_2 must be very strong for the optimal allocation, and thus, the maximum achievable sum-rate with independent encoding and decoding is

$$\left[\begin{array}{c} p_1 C\left(P'_{2,1}\right) + p_1 C\left(\frac{P'_{1,1}}{1+|h_1|^2 P'_{2,1}}\right) \\ + p_2 C\left(P'_{1,2}\right) + p_2 C\left(P'_{2,2}\right) \end{array}\right]. \tag{21}$$

Let f denote the weighted sum of capacity expressions in (21). We then have

$$f \leq f|_{|h_1|^2 P'_{2,1}=0} \tag{22}$$

$$\leq C\left(\overline{P}_1\right) + C\left(\overline{P}_2\right) \tag{23}$$

where (23) follows from applying Jensen's inequality and the equality in (22) holds only if either $h_1 = 0$ or if $P'_{2,1} = 0$. For the trivial case of $h_1 = 0$, the channel simplifies to a non-fading very strong channel for which the question of separability is irrelevant. On the other hand, for $P'_{2,1} = 0$, the inequality in (23) is strict. Thus, we see that the sum-rate achieved by independently encoding and decoding over the two fading channels is strictly sub-optimal compared to the rate achieved by jointly encoding and decoding over the two states when the ergodic very strong condition in (10) is satisfied.

One can use similar arguments to show that separability is strictly sub-optimal for an ergodic strong one-sided IFC. Note that an ergodic strong one-sided IFC can result from either a mixture of the two kinds of strong channels or from a mixture of weak, strong but not very strong, and very strong channels such that the optimal policy $\underline{P}^*$ satisfies the conditions for the strong but not very strong case in (10).

Finally, the ergodic weak one-sided IFC by definition results from an appropriate mixture of weak, strong but not very strong, and very strong channels such that conditions in both (9) and (10) are not satisfied. Since the optimal sum-capacity achieving scheme for this channel remains open, it is not known yet whether separability is optimal for this channel. Our observations are summarized in Fig. 3 where we write *uniform* to denote the case where every channel is of the same type and *ergodic* to denote a mixture of different types of parallel IFCs.

Strictly Ergodic	Separability Optimal?	Separability sub-optimal	Separability sub-optimal
Uniform	Separability Optimal?	Separability optimal	Separability optimal
	Weak IFC	Strong but not very strong IFC	V. Strong IFC

Fig. 3. Optimality of separation for ergodic two-user one-sided IFCs.

IV. Illustration of Results

We illustrate the results developed in Section III using a one-sided IFC with two states. We first consider an example of a ergodic very strong one-sided IFC with $h_1 = 1.25$ and $h_2 = 1.75$. In Fig. 4, for different values of the probability, p_1, of the state h_1 such that $\Pr(h_2) = p_2 = 1 - p_1$, we show that the range of average powers $\overline{P}_1 = \overline{P}_2 = \overline{P}$ satisfying the very strong condition in (10) can be a non-zero set. Further, as shown in Fig. 4, as the probability p_2 of the stronger state, h_2, increases, the region of power values for which the very strong condition is satisfied increases. This can be explained using Fig. 2 where we see that satisfying the ergodic very strong condition requires the cross-channel gain to be strong (and frequent) enough that user 2 achieves a larger rate at D_1 even in the presence of interference from user 1 than it does at D_2 in the absence of interference. Thus, as the probability of the relatively stronger state h_2 increases, the set of power values for which the very strong condition is satisfied on average also grows.

In Fig. 5 we plot the power region where the very strong condition in (10) satisfied for the one-sided IFC with $h_1 = 0.5$ and $h_2 = 1.5$, i.e., one of the two states is weak. Here again, we see that as the probability p_1 of the weak state increases, the feasible power region grows smaller until for $p_1 > 0.6$, the region is no longer non-zero.

In Fig. 6, for the two ergodic very strong channels described above, we plot the sum-rate achieved by encoding and decoding the parallel channels separately as a function of the probability, p_1 of the state h_1 where $h_1 < h_2$. Further, Fig. 6 also includes the sum-capacity plots for each of the cases achieved by jointly decoding over channel states such that D_1 decodes the messages from both users. In both cases, the sum-capacity achieving policy requires water-filling over the links from each user to its intended destination, and thus for the model considered, simplifies to $P_1(H) = P_2(H) = \overline{P}$ for $H = h_1, h_2$. On the other hand, the sum-rate maximizing policy for the separable case is obtained by maximizing the expression in (14) when $h_1 > 1$ and that in (19) for $h_1 < 1$ over all feasible power policies.

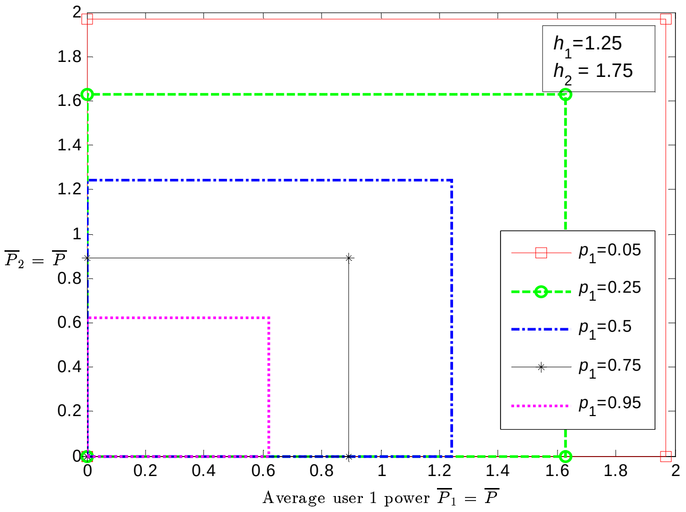

Fig. 4. Feasible power regions for an ergodic very strong one-sided IFC with $(h_1, h_2) = (1.25, 1.75)$.

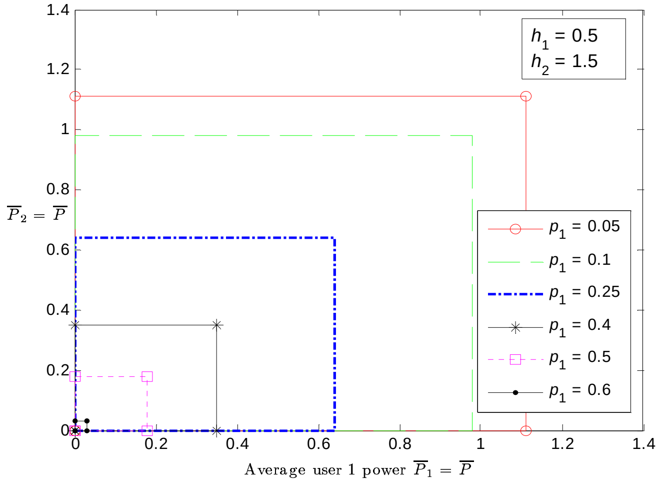

Fig. 5. Feasible power regions for an ergodic very strong one-sided IFC with $(h_1, h_2) = (0.5, 1.5)$.

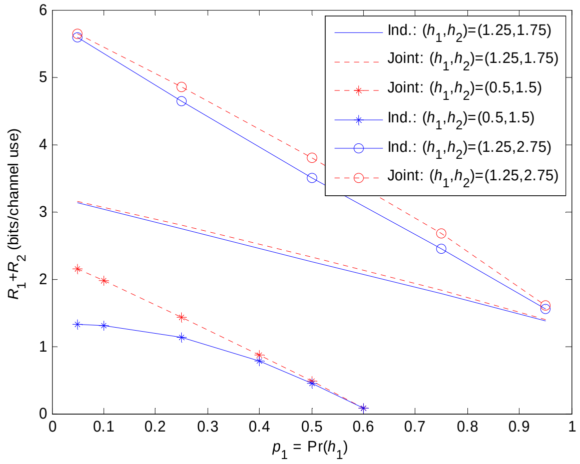

Fig. 6. Plots of the sum-rate achieved by separable coding schemes and the ergodic sum-capacity for three ergodic very strong channels.

In Fig. 6, for each choice of p_1, the average power $\overline{P}_1 = \overline{P}_2 = \overline{P}$ is chosen as the largest value for which the ergodic very strong condition holds. Thus, for example, for $h_1 = 1.25$, $h_2 = 1.75$, and $p_1 = 0.05$, we choose $\overline{P} = 1.97$ (see Fig. 4). In addition to the above two example channels, in Fig. 6 we also consider an ergodic very strong IFC with $h_1 = 1.25$, $h_2 = 2.75$. In contrast to the first example, this channel has a larger difference between the two states, though in both cases each individual parallel channel is strong. The three examples demonstrate how the difference in sum-rates achieved by the optimal joint decoding scheme and the separation-based scheme depends on both the channel states and their statistics. Thus, for the first example with an ergodic very strong channel where the two states $h_1 = 1.25$ and $h_2 = 1.75$ are not significantly different, the gains from joint decoding are minimal. On the other hand, in Fig. 6, we also see that the gains from joint decoding are not negligible when the channel gains are distinctly different as in the third example where $(h_1, h_2) = (1.25, 2.75)$. Note that in both examples the gains are the largest for channel statistics where the two states are more or less equally likely and taper off at the extreme points where one state significantly dominates the other. Finally, Fig. 6 also indicates that the gains from joint decoding are significant in example 2 when the weak channel state occurs relatively infrequently, i.e., for $\Pr(h_1 = 0.5) < 0.25$.

We remark that typical multicarrier or fading channels involve more than two parallel channels. In wireless environments where the range of channel gains are markedly different, joint encoding and decoding may achieve larger gains relative to the separable approach when the parallel channels on average satisfy the strong but not very strong or the very strong interference conditions.

V. CONCLUDING REMARKS

Using a one-sided ergodic interference channel as a model, in this paper we have shown that encoding and decoding independently over parallel channels can be strictly sub-optimal for the two-user ergodic IFC. We have used recent results on the sum-capacity of the class of ergodic strong but not very strong and very strong IFCs to show that an ergodic one-sided IFC is on average either weak, or strong but not very strong, or very strong with the cases where every parallel channel is of the same type being appropriately subsumed into one of these three mutually exclusive cases. We have further shown that for the ergodic strong but not very strong and the ergodic very strong channels, separability is optimal only when the ergodic condition results from the same condition holding true for every parallel channel, i.e., every parallel channel is of the same type.

The suboptimality of separability for the two kinds of ergodic strong channels comes from the fact that for a non-fading IFC, different encoding and decoding approaches are required in the different interference regimes; further, even when the encoding approach is the same as is the case for the two types of strong channels, the sum-capacity requires taking a minimum over the capacity expressions for these two mutually exclusive types. Using this and the fact that for certain classes of ergodic fading IFCs a single coding scheme achieves the ergodic (fading averaged) sum-capacity, we have shown that joint encoding and decoding it is optimal relative to the approach of using the optimal encoding and decoding scheme for each channel independently. Using prior work on the sum-capacity of the two classes of ergodic strong IFCs, we have shown that separability is, in general, sub-optimal for these two classes of fading one-sided IFCs. This observation also holds for the two-sided ergodic strong and very strong IFCs. Finally, a natural extension to this work is to verify whether separability is optimal for the ergodic weak channel including the uniformly weak ergodic IFC.

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