A Critique of White's §2-5, pp. 69-73

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Abstract

F. M. White's *Viscous Fluid Flow* is a good book, but it has sections that deserves criticisms. I am presenting this critique of one brief section to the class, trying to convey the idea that one must always read written words critically.

1 The Particulars

I am unhappy on the following White presentations:

1. The words introducing his eq.(2.31)
2. The words introducing his eq.(2.32)
3. How to get his eq.(2.34) from his eq.(2-32)
4. What happens to eq.(2-34) if $\mathbf{g}(t)$ is time dependent?

To clear the air, all of White's results are correct provided $\mathbf{g}$ is not time dependent. However, if his approach is followed, you can get into big trouble when you consider other kinds of body forces (such as Lorentz Force for a conducting fluid). And this can be serious.

1.1 Item # 1

White said "$W=$ work done on system" but did not *emphasize* that it includes work done by *both* surface forces *and* work done by body forces. In his followup development, indeed he ignored the work done by body forces (see top of p.71)—by introducing potential energy in his E_t. It is my position that this approach is a very bad idea.

1.2 Item # 2

Who said the total energy of the system includes kinetic and potential energy in addition to thermal energy? All the First Law of Thermodynamics ever said was: E_t is a state variable. That's all it said.

Now, what is a state variable? A state variable describes the state of the system. In "pure" thermodynamics, examples of state variables are pressure, density, temperature. These are called thermodynamic state variables. When the system of interest is moving, it is reasonable to say that E_t of a fluid parcel may also depend on additional state (kinematic) variables such as velocity V and position $\mathbf{r}$—they certainly describe the state of the parcel. So we can generalize the classical E_t for non-moving system to $E_t(\ldots; V, \mathbf{r})$ for moving systems. The issue now is how to determine the yet unknown V and $\mathbf{r}$ dependence. As we shall show later, we shall find

$$E_t = \rho(e + \frac{1}{2}V^2) \tag{1}$$

to be the correct V dependence. The $\mathbf{r}$ dependence is trickier.

An editorial blunder here is that the symbol E_t was defined by White twice between eq.(2-31) and eq.(2.32). The first time, he said E_t is the total energy of the system (its dimensional unit should be energy). Then his eq.(2-32) said E_t is energy per unit volume. (Whose thermo books said the net work and heat added to the system equals to the increase of energy <u>per unit volume</u>? Thermodynamics always deal with an agreed-upon *system*!. Check your thermodynamics books.

1.3 Item # 3

If you take the substantial derivative of E_t from eq.(2-32), you do not get his eq.(2-34) since ρ is not a contant.

What happened here?

You see, eq.(2-32) is missing the factor Δv (the volume $dxdydz$ of the chosen fluid particle) on the right hand side. The product $\rho\Delta v$ is a constant—conservation of mass. So we must insist on White's first definition of E_t and add the factor Δv to both eq.(2-32) and eq.(2-34) (and also eq.(2-35) and eq.(2-37); more on the latter equation).

1.4 Item # 4

If there is a body force $\rho\mathbf{f}$ per unit volume already included in the momentum equation, then in the energy equation you are obligated to make a considered judgment on whether $\rho\mathbf{f}\cdot\mathbf{V}$ is the amount of work done per unit volume by this same body force (you got to think physically!).[1] It does NOT matter whether this body force is conservative (expressible in terms of the gradient of a potential function). For example, if you have a Lorentz force on your electrically conducting medium, it has a legitimate role to play in the energy equation. So long as you include the work done by a (conservative or not) body force of interest, you don't have to include any potential energy term in your E_t. In fact, when $\mathbf{g}$ is a constant, you can *derive* the potential energy term in the energy equation without putting it in E_t by fiat. When $\mathbf{g}$ is time dependent, this approach handles everything in strides, while the White approach gives the wrong answer.

White seems to advocate not to pay attention to work done by body forces (he never said so explicitly). Very bad idea.

1.5 Q and W

They should denote heat transferred in and (surface and body) work done on the system (the fluid parcel), and *not* the per unit volume amount. In other words, do *not* divide by $dxdydz$ in going to eq.(2-35) from the equation above.

2 Kinetic Energy

How do we deduce the concept that kinetic energy per unit mass is $\frac{1}{2}V^2$?

Look at §4.3 of my class notes *The Viscous Term and Its Impacts*. It is an equation with the same physical dimensional units as the energy equation, but it is not an energy equation. The left hand side contains the substantial derivative of $\frac{1}{2}V^2$. This equation came from the momentum equation.

If we had not included $\frac{1}{2}V^2$ in the E_t or if we had left it in the form of $E_t(\ldots, V)$, and proceed to get the entropy equation (one of

[1] Ask me in class under what conditions a body force in the momentum equation makes no impact in the energy equation.

your homework problems), we will find the result to be troubling: it is not guaranteed to be Galilean Invariant! In other words, what the equation says changes when you change from one inertia coordinate system to another. The only way to fix this problem is to include an additive $\frac{1}{2}V^2$ (representing kinetic energy per unit mass) along side e (representing thermal plus chemical plus nuclear ... energy per unit mass) in E_t.

3 Parting Remarks

I hope you learn the following things from the above:

- It is always a bad idea to use one symbol to represent several different things in the same exposition. White should have kept E_t to denote total energy of a material system (a fluid parcel), and not change it to total energy per unit volume. He got the right answer because he made two mistakes (he omitted the substantial derivative of ρ) which cancelled each other.
- If you introduced a body force in the momentum equation, you must also account for the work it did in the energy equation. This is not open to debate.
- The check on Galilean Invariants is best done in the entropy equation. The issue is what "kinematic" variables such as V and $\mathbf{r}$ should be included in E_t, and in what form.

In any case, always be critical when you read written words, including those I write. Always.