# ME351B Final Exam
# (Win) 2003-04

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Mechanical Engineering
Stanford University

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This is an open-book, open-notes take-home final exam.

It is due at 2:00PM, Friday, 3/19/2004—to be handed in at Room 500A, Building 500 (Deb Michael's office).

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1. You already have a working code to solve any generic two-dimensional steady laminar boundary layer problem under the constant properties assumption.

   (**50 points**) Modify your code so that it can be used for any perfect compressible gas (with constant specific heats and $\gamma$). That is: I want the boundary layer energy equation fully coupled with the momentum equation, and they are to be solved by marching downstream simultaneously.

   For this final exam, I just want you to run your new code on the Blasius problem with the following specifications:

   **a** : $U_e$ =constant, $M_e = 1.5$, $\gamma = 1.4$, $P_r = 1.0$ (air at standard condition). Assume $\mu$ is proportional to $T$.

   **b** : No-slip condition at the wall.

**c** : Wall is thermally insulated[1] for the first 100 steps (for whatever $\Delta x$ you used to march), then $T_{wall} = T_e$ =constant there after (another 100 steps).

Show a plot of the heat transfer rate, sensibly non-dimensionalized, as a function of $x$, sensibly non-dimensionalized. How did Busemann's energy integral do? How about Crocco's energy integral?

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2. We are interested in a physical system governed by the following (one-dimensional unsteady) PDE:

$$\frac{\partial \phi}{\partial t} = \frac{\sin \phi}{\tau} + \mu \frac{\partial^2 \phi}{\partial x^2} \tag{1}$$

where $\tau$ and $\mu$ are positive constants. The boundary conditions are:

$$x = 0 : \qquad \phi = 0 \tag{2}$$

$$x = L : \qquad \phi = 0. \tag{3}$$

It is obvious that $\phi_{ss} = 0$ is an exact "steady state" solution to this problem.

**a** : (**15 points**) Analyze the stability of this steady state solution $\phi_{ss} = 0$ to tiny disturbances in the initial condition. You have three dimensional parameters, $\tau$, $\mu$ and $L$. When is this $\phi_{ss}$ stable? Use conventional mathematics.

**b** : (**10 points)** You are fully competent with Matlab, and you want to solve this simple unsteady "reactive-diffusive" problem with your personal computer—for some given initial distribution of $\phi(x, t = 0)$. Discuss how you would pick your marching $\Delta t$—particularly when $\tau$ is "small" (compared to what?). Make some comments on the choice between explicit and implicit methods. (**I don't want you to do the problem; I just want you to talk about it intelligently**)

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[1] $\kappa \partial T / \partial y$ at the wall is zero; or the temperature at the first grid point away from the wall is identical to the (unknown) wall temperature.

3. You have a boundary layer problem.

   **a:** (**5 points**) After making the laminar assumption, your code can generate answers for wall friction and heat transfer for the specified boundary conditions (no-slip on walls, etc). Under the laminar assumption, what happens to your (dimensional) wall friction, heat transfer and separation predictions when the characteristic flow velocity is quadrupled?

   **b:** (**10 points**) Someone put a "boundary layer trip" near the front of your boundary layer and the boundary layer is "tripped" into a turbulent boundary layer. You have White's book at your disposal. How do you go about getting the answers for wall friction and heat transfer (which formula and which graphs are useful)? (If you need additional information, say so). According to the formulas, at what Reynolds Number does the laminar and turbulent wall frictions cross? Which formula would you have faith in for Reynolds Number below the crossing value?

   **c:** (**10 points**) Under the turbulent assumption, what happens to your (dimensional) wall friction and heat transfer predictions when the characteristic flow velocity is quadrupled? I don't need precise answers—I am just interested in an intelligent estimate of the differences in the response between laminar and turbulent boundary layers.