ME 351B – ANSWERS TO HOMEWORK 1

PROBLEM 1 (Cullen Buie)

Zabersky head loss Darcy - Weisbach

From the diagram, the pressure drop is a function of Reynolds number for laminar flows and not a function of the relative roughness. For turbulent flows the pressure drop is a function of the relative roughness, and not the Reynolds number.

The power required is...

$P \propto Qh_L \qquad h_L \equiv f\frac{lV^2}{D2g}$ (head loss) $\qquad$ P - power

$f = \frac{.316}{Re^{1/4}}$ (friction factor) $\qquad V = \frac{Q}{A} = \frac{4Q}{\pi D^2}$

Blasius correlation for $Re \leq 10^5$

$$h_L = f\frac{lV^2}{2Dg} = f\frac{l}{2Dg}\left(\frac{4Q}{\pi D^2}\right)^2 = f\frac{8lQ^2}{g\pi^2 D^5}$$

$$h_L \propto \frac{1}{D^5} \quad \therefore P \propto \frac{1}{D^5}$$

Note that to obtain this 5th power law we ignored the dependence of the friction factor f on D through the relative roughness (we effectively assumed that in rough pipes f is a constant). This is correct to a first approximation, since the -5 power is strong dependence: if D is doubled, the pumping power is reduced by approximately a factor of 32!

PROBLEM 2 (Wendy Ong)

Find diagram of drag co-efficient of a sphere as a funct. of Re no. What happens when transition occurs?

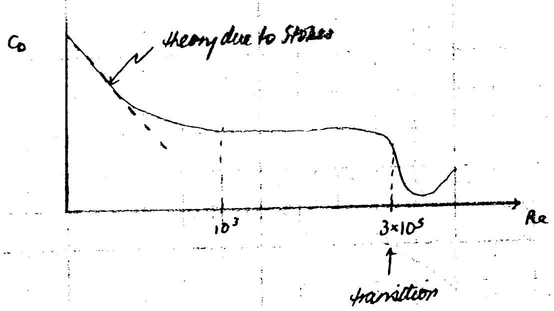
At very low Re no., $Re \leq 1$ there is no flow separation from a sphere. For v. low Re # flows where inertia forces may be neglected, the drag force on a sphere is given by : $F_D = 3\pi\mu V d$

$\therefore \; C_D = \frac{24}{Re}$ (laminar)

This expression agrees w/ experimental data at low Re #, ~~but~~ begins to deviate significantly at $Re > 1.0$.
As the Re # is increased up to abt. 1000, the C_D drops continuously.

In the range $10^3 < Re < 3\times10^5$, the C_D curve is relatively flat.

However, at a critical Re # of $\sim 3\times10^5$, the drag-coeff undergoes a sharp drop. The reason for this is that at Re # $> 3\times10^5$, transition occurs and the boundary layer on the forward portion of the sphere becomes turbulent. The pt. of separation then moves downstream from the sphere mid-section, & the size of the wake is decreased. The net pressure force on the sphere is reduced and the drag co-eff. decreases abruptly.

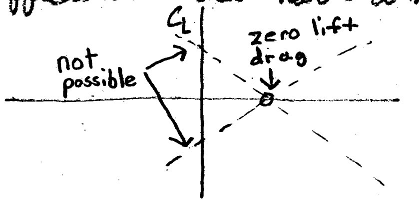


No, Transition in the boundary layer is affected by roughness of the sphere surface + turbulence in the flow stream.

Beyond the sudden drop, the drag coefficient is more or less constant, and is relatively independent of the Reynolds number.

PROBLEM 3 : (Cullen Buie)

If the drag coefficient depended linearly upon the lift, then there would be a lift coefficient that corresponds to a negative drag. Since a negative drag is physically impossible the drag coefficient must have a non-linear relationship with the lift.



The linear dependence (which may be acceptable for very small angles of attack, cf. Taylor expansion) cannot be right as it could predict thrust without an engine (i.e. a negative drag coefficient).

PROBLEM 4 : (Carlo Beretta)

- Water at 20°C, P_{atm} : $\mu = 10^{-3}\ \frac{Ns}{m^2}$

$$\nu = 10^{-6}\ \frac{m^2}{s} \qquad \left[\rho = 1000\ \frac{Kg}{m^3}\right]$$

μ decreases with T and increases with P.

- Air at 20°C, P_{atm} : $\mu \simeq 1{,}812 \cdot 10^{-5}\ \frac{Ns}{m^2}$

$\mu = 0.67 \rho a \lambda$ with class notation $\qquad \nu \simeq 1{,}493 \cdot 10^{-5}\ \frac{m^2}{s} \quad \left[\rho \simeq 1{,}2136\ \frac{Kg}{m^3}\right]$

μ increases with T and P.

These values should be known by heart, at least approximately.

PROBLEM 5 : (Alvin Barlian)

$$Re = \frac{\rho U D}{\mu} = \frac{UD}{\nu}$$

$$1 = \frac{UD}{\nu}$$

$$U = \frac{\nu}{D} = \frac{15.712E^{-6}}{6.25E^{-2}} = \boxed{2.514E^{-4}\ \frac{m}{s}}$$

$D \to$ 6.25 cm.

This value for the velocity is very low: most of the macroscale flows around us occur at high Reynolds number.

PROBLEM 6:

(Alvin Barlian)

The flow at transition to turbulence → $Re \approx 2E^5$

$Re = \frac{UD}{\nu}$ → $D = 7.48$ cm

ν → assuming at $T = 298°K$

$2E^5 = \frac{UD}{\nu}$

$U = \frac{2E^5 \nu}{D} = \frac{(2E^5)(15.712E^{-6})}{7.48E^{-2}} = \boxed{42.01 \frac{m}{s}}$

42 m/s is approximately 94 miles/hr. Does anybody know how fast a baseball pitcher can pitch?

In practice the seams on the baseball will cause transition to occur at much lower Reynolds number (same effect with the dimples on golf balls).

PROBLEM 7:

(Guillaume Blanquart)

. Let's recall from a dimentional analysis ;

$\mu \simeq 0.5 \rho a l$

or $\nu \simeq 0.5 a l$ with a: speed of sound

l: mean free path.

Then the mean free path is around: $l \simeq 7.8 \times 10^{-8}$ m

$l \simeq 78$ nm

$a = 340 \, m.s^{-1}$

80 nm is very small. White's book is 1.3 in thick and has 600 pages. So one sheet is about 5.5x10^-5 m thick, which is more than 1000 times larger.

PROBLEM 8:

(Seongwon Kang)

From Avogadro No.

6.022×10^{23} in 22.4 liters

$= 22.4 \times 10^{-3} \, m^3$

Average volume occupied by a molecule

$$= \frac{22.4 \times 10^{-3}}{6.022 \times 10^{23}} = 3.72 \times 10^{-26} \, m^3$$

Mean distance between air molecules

$$\approx (\text{Average volume occupied by a molecule})^{\frac{1}{3}}$$

$$= 3.4 \times 10^{-9} \, m$$

The mean free path is about an order of magnitude larger than the mean distance between the air molecules. Why could this have been expected for a gas?

PROBLEM 9: (Guillaume Blanquart)

To consider a "universal" graph we should use the following nondimentionnal parameters:

- Air flow rate: $\dot{V} \longrightarrow Re = \frac{\bar{u} D}{\nu}$, with $\dot{V} = \frac{1}{4} \pi D^2 \bar{u}$.
- Pressure drop: $\Delta P \longrightarrow C_p = \frac{\Delta P}{\rho \bar{u}^2}$

Of course, to consider these non dimentionnal parameter we need:

→ ρ: the density
→ D: the diameter of the ~~the~~ incoming pipe.

The air flow rate is the product of the flow velocity times the cross-section A (say, of the inlet pipe). If A is fixed (as it is in the experiments and the computations that generated the data), then the air flow rate is linearly proportional to the velocity (or Reynolds number). If, after getting A from your phone call, you replot the dimensionless pressure drop versus the Reynolds number, you would see that Fig 1-13 would show a pressure drop nearly independent of Re.