ME 451C
Mid-Term of
Advanced Fluid Mechanics
Winter Quarter, 2005

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February 8, 2005

> This is an open notes/homework answers mid-term exam. But it is closed-book exam in the sense that you are not supposed to go to books (on your own book shelves or in a library somewhere) on this subject and look at their developments—until you have handed in your mid-term.
>
> There is no time limit except for the tuesday deadline. I expect you to go over your own work yourselves, and make sure you have a perfect paper when you hand it in.
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> Unlike homeworks, students are not supposed to discuss the mid-terms with each other until after next tuesday.

In my presentation of diffusion laws for charged particles in Week #5 Notes, I adopted the phenomenological approach, and I assumed there is no magnetic field **B** worth worrying about.

A Use the "elementary derivation" to derive the diffusion laws. State all your assumptions. Keep both **E** (electric field) and **B** (magnetic field) in the game. We agree that in the presence of **E** and **B**, a charge particle with charge e moving with velocity **V** experiences a *Lorentz force* **F** given by:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{V} \times \mathbf{B}). \tag{1}$$

For an electron, $q = -e$, and for a singly charged positive ion, $q = e$.

B We will ignore Soret and Dufour effects. Compare what you derived with the phenomenological formulas used in class. Identify the conditions under which the phenomenological formulas can be consider a good approximation to your results.

C Show that you can "derive" the Einstein relation (between diffusion and mobility coefficients) attributed to Einstein. Obviously, you need to state the assumptions needed.

D Derive the momentum equation for the mixture (valid for arbitrary reference Debye length).

E We confine our attention to problems for which the reference length L_R is much larger than the reference Debye length. Derive the Ohm's Law under the quasi-neutral approximation, keeping $\mathbf{B}$ in the game.

F When $\mathbf{B}$ is ignored, the old Ohm's Law says the electric current vector $\mathbf{j}$ is parallel to the electric field vector, $\mathbf{E}$. What happens when $\mathbf{B}$ is not zero? In particular, what happens when $\mathbf{B}$ is very, very large?

I expect the mid-term to be handed in on tuesday, 2/15/2005.