

The Solar Wind Theory of Eugene Parker

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Abstract

1 Formulation of the Problem

Eugene Parker published his solar wind theory [1] in 1958. Few people believed him. His theory says a compressible flow in a divergent channel can go from subsonic to supersonic without having a physical throat—provided there is a retarding body force. But he was vindicated and hailed (deservedly) as a space-age hero when his predictions were qualitatively verified.

2 Formulation of the Problem

Assume the atmosphere of the sun to be steady and spherically symmetric. We consider one radial streamtube and denote the distance from the center of the sun by r . The continuity equation is $\dot{m} = 4\pi\rho ur^2$. Taking its logarithmic derivative with respect to r , we have:

$$\frac{1}{\rho} \frac{d\rho}{dr} + \frac{1}{u} \frac{du}{dr} + \frac{2}{r} = 0. \quad (1)$$

The r -momentum equation is:

$$\rho u \frac{du}{dr} = -\frac{dp}{dr} - \frac{\rho G M_o}{r^2}, \quad (\text{no viscous term}) \quad (2)$$

where the second term on the right hand side is the gravitational term with G = the universal gravitation constant and M_o = mass of the sun. Instead of

the energy equation, we shall make the simplifying ad hoc assumption that the temperature T in the whole solar atmosphere is a constant:

$$T = T_o. \quad (3)$$

In addition, we shall assume the perfect gas law $p = \rho RT$. Taking its logarithmic derivative with respect to r , we have:

$$\frac{1}{p} \frac{dp}{dr} = \frac{1}{\rho} \frac{d\rho}{dr}. \quad (4)$$

2.1 Static solution

Let us first find out what the static solution ($u=0$) gives us. Using the equation of state and isothermal assumption in (2), we obtain:

$$\frac{\rho}{\rho_o} = \exp \left[-\frac{2r_*}{r_o} \left(1 - \frac{r_o}{r_*} \right) \right]. \quad (5)$$

where subscript o denote condition on the sun's "surface" and

$$r_* \equiv \frac{GM_o}{2a^2} \quad (6)$$

and " a " is the isothermal speed of sound:

$$a^2 = RT = RT_o \quad (7)$$

For our sun, the value of r_* is 5-10 times the value of r_o . Thus, the value of ρ at $r \rightarrow \infty$ is finite, and its value is much too high to be acceptable.

So we reject the static solution for our sun.

3 The Solar Wind Solution

Manipulating the four equations (1,2,3,4) and with the help of the equation of state itself, we obtain:

$$\frac{1}{u} \frac{du}{dr} = \frac{2a^2(r - r_*)}{r^2(u^2 - a^2)}. \quad (8)$$

Now, we need boundary condition for u . What should u be at the sun's surface?¹

It seems reasonable that u should be subsonic ($u < a$) near the sun's surface. And we know that $r_o < r_*$. So this ODE tells us that u will increase with r . Will it continue to increase with r ?

The answer is immediately clear by inspection. Something happens at $u = a$, where the local flow speed is sonic. Since we don't expect du/dr to blow up there, we ask for a solution that goes sonic at $r = r_*$.

Yes, we are asking a compressible flow in a divergent channel to choke at the fixed location $r = r_*$.

It is easily shown by studying Eq.(8) that once past the choking point at $r = r_*$, there are two solution branches—one supersonic, and one subsonic. Of course, conventional shock waves are available for sudden transition from supersonic to subsonic flows. If the interstellar space has very low pressure, the solar wind will stay on the supersonic branch for a long, long distance. Measurements by satellites showed that the solar wind is indeed supersonic at the earth's orbit, so a “bow wave” is expected by the interactions.

4 Discussion

So why do some heavenly bodies have an atmosphere with an outward flowing wind, and some—like our moon—do not? What would you guess the value of r_* be in comparison to moon's radius? What about the earth?

References

- [1] Parker, E.N., “Dynamics of the interplanetary gas and magnetic fields,” **128**, 664, *Astrophys. J.*, 1958.

¹Note: this ODE can be integrated. But one gets more useful information by staring and analyzing the ODE itself (drawing the “phase portrait”).