

# Hints for the Mid-Term

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## Abstract

The Mid-Term will be a closed-book/notes affair. I want to make sure that you know certain things by heart. Here is a list of the things ...

## 1 Things you must know by heart

Dimensional analysis:

1. The physical dimensions of  $\rho, p, \mathbf{V}, \bar{\tau}, \mu, \kappa$ , etc.
2. If I give you an equation in which the dimensions are not consistent, I expect you to notice it.
3. You should know why data presented in non-dimensional form is a good idea.

Vector Calculus:

1. The definitions of gradient, divergence and curl, and the three associated “divergence” theorems.
2. The index notation and the summation convention.
3. These are always in your head:

$$\mathbf{A} \cdot \mathbf{B} = A_i B_i \quad (1)$$

$$\{\mathbf{A} \times \mathbf{B}\}_i = \epsilon_{ijk} A_j B_k \quad (2)$$

$$\{\nabla \phi\}_i = \frac{\partial \phi}{\partial x_i} \quad (3)$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_i}{\partial x_i} \quad (4)$$

$$\{\nabla \times \mathbf{V}\}_i = \epsilon_{ijk} \frac{\partial V_k}{\partial x_j} \quad (5)$$

$$\epsilon_{ijk}\epsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km} \quad (6)$$

4. For curvilinear coordinates, the  $i$ -th component of  $\mathbf{A} \cdot \nabla \mathbf{B}$  is NOT  $\mathbf{A} \cdot \nabla B_i$ , and the  $i$ -th component of  $\nabla^2 \mathbf{A}$  is NOT  $\nabla^2 A_i$ . Know what  $h_1, h_2, h_3$  are, and what they are useful for. I don't expect you to memorize anything.
5. "Proximity rule" applies when there is ambiguity. Remember, index notation is never ambiguous.
6. If  $\mu$  is not a constant, lots of very messy terms needs to be worked out in curvilinear coordinates!

Viscous stress tensor:

1. What was the "thought experiment" performed by Newton which inspired his idea of viscous stress?
2. Given the stress tensor  $\bar{\bar{\tau}}$  and a surface element  $d\sigma$  with a unit normal  $\mathbf{n}$ , what is the surface force vector acting on the surface on the side pointed to by  $\mathbf{n}$ ?
3. What are the assumptions that leads to the Navier-Stokes viscous stress tensor  $\bar{\bar{\epsilon}}$ :<sup>1</sup>

$$\bar{\bar{\epsilon}} = \mu(\nabla \mathbf{V} + [\nabla \mathbf{V}]^T) + \lambda \bar{\bar{\mathbf{I}}} \nabla \cdot \mathbf{V} \quad (7)$$

In particular, why can't a term proportional to  $\nabla \mathbf{V} - [\nabla \mathbf{V}]^T$  be added?

4. What happens if you had "forgotten" to include kinetic energy in your derivation of the energy equation? How would you notice that you have the wrong answer?
5. What happens to the entropy equation when you totally neglect all viscous terms. What does it say to you in English?
6. What happens to the vorticity equation when you totally neglect all viscous terms. What does it say to you in English?
7. Know all conservation laws in terms of English sentences. Then make sure you know how to express the thought mathematically. Conversely, when you see a mathematical equation, learn to read it out loud in English sentences.

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<sup>1</sup>Superscript  $T$  means transposed.

8. You are expected to know the constant property dimensional Navier-Stokes Momentum equation by heart (in other words, that extra viscous term by heart)—in vector form or in index notation for Cartesian coordinates.
9. Should know the reason for defining  $\mu'$  by  $\lambda = -2\mu/3 + \mu'$ .
10. The kinematic viscosity  $\nu$  is  $\mu/\rho$ . You should know by heart the approximate numerical value of  $\nu$  for air and water at standard condition.
11. You should know by heart the definition of Reynold Number.

Low Reynolds Number Flows:

1. You should be able to figure out a formula (except for the constant of proportionality) for the drag of a sphere if someone hinted to you that it is proportional to the first power of velocity. (What happens if you try the same trick for the cylinder drag?)
2. You should know that two dimensional problems (for infinite domain) are not straightforward—remember the hot cylinder problem?
3. Under the Stokes (and Oseen) approximation, the approximate problem is linear. Why is that a big deal?

Prandtl's Boundary Layer Theory:

1. You should know how to explain the boundary layer simplifications using English.
2. What information do you need in order to say you have got a properly formulated boundary layer problem?
3. Under the assumption that the flow is laminar and does not “separate,” does a boundary layer problem need the knowledge of the actual value of the characteristic Reynolds Number?
4. Having seen the messy complications of non-Cartesian coordinates, you should rejoice at the legitimacy of Cartesian coordinates for all boundary layer problems. You are expected to know the two-dimensional laminar boundary layer constant property boundary layer equations by heart.
5. Look at the notes on *Known Boundary Layer Solutions*. Since no homework is assigned for this material, I may ask you to do something quite similar in the closed-book/notes mid-term.

## 2 Remarks

I may add to this set of hints from time to time. If I do, I will put a note on the course website.