

MAE 222
Fluid Mechanics
1st Mid-Term, Spring 1998
(60 monutes, closed book, close notes)
March 13, 1998
Friday

- Assume a flat earth, with constant gravitational acceleration \mathbf{g} .
 - Water density is a constant. Air density is so small that for normal variations of altitudes the air hydrostatic pressure is approximately constant.
 - Look at all the problems, and do the easy ones first.
 - Be brief, but use English whenever appropriate!
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Let the velocity \mathbf{V} of a fluid flow field be represented in Cartesian coordinates, $\mathbf{V}=(u \mathbf{i}, v \mathbf{j}, w \mathbf{k})$ where (u, v, w) are the (x, y, z) components of \mathbf{V} in the directions pointed to by unit vectors $(\mathbf{i}, \mathbf{j}, \mathbf{k})$.

1. What is the physical meaning of substantial derivative $\frac{D}{Dt}$ in English?

If you are given $u=ax+by+\sin(2t)$, $v=1-cx$, $w=ey$, (where a, b, c, e are constants), find the acceleration vector of a fluid particle located at $x=y=z=0$. (20 points)

2. What is the Bernoulli's equation, and what are the four assumptions required for it to be valid? (10 points)

3. Given a circle with radius R situated with its center at the origin of a cartesian coordinate system (x,y) . The inside of the circle is pressurized with air with gage pressure p (outside the circle the gage pressure is zero). Draw diagrams!

- Find the moment arm of the pressure force acting on any 'tiny' surface element dA (your choice of θ) about the top most point of the circle, (5 points) and
- write down the expression (ready for integration) for the contribution by this surface element to the moment (or torque) about this point. (5 points)
- The northeast quadrant of the circle is a door hinged (without friction) at the top most point, and YOU are applying a horizontal force F at the right most point of the door to prevent the door from opening. Find F in terms of an integral. (10 points)

4. Consider a fluid flow problem in steady state. For good and sufficient reason, you have chosen a control volume. Now you want to apply Newton's Law to the system which is identified with the control volume at the moment you chose the control volume.

- State Newton's Law, in English, using words like "external force" and "flux" and "surface of the control volume." (10 points)
- Say the same thing again in mathematics, using \mathbf{n} , the unit outward normal, the density ρ , the velocity \mathbf{V} , and the dot (or inner) product, somewhere in your formula. (10 points)

5. You have a large tank of radius R which is originally fill with water (density is constant and is denoted by ρ) to the height H (at $t=0$), sitting on a weighing scale. For $t>0$, a small round hole of area A is opened on the bottom of the tank, and water streams out smoothly (without interference or splashing or touching the weighing scale). Assume steady state to analyze this problem as the tank drains (why?). Denote the height of the water in the tank by h which, of course, depends on time.

- Find the mass flow rate out of the small hole as a function of h (and whatever else information you need). (10 points)
- Find the reading on the weighing scale as a function of h (and whatever else information you need). (20 points)