MAE 533

Final Take-Home Exam
Fall Term, 1995

Due: 4PM, January 27, 1996
(course evaluation form to Etta)

Open Book, open notes.
(Write with a dark pencil. Explain what you are attempting to do with the mess of equations and numbers)

1. We are in the preliminary design stage of a turbo-jet. The working fluid is obviously air, and we shall make the standard perfect gas assumptions (constant $\gamma$ and specific heats, etc.). For this problem, we are interested only in the thrust $F$ of this engine when the aircraft is at rest (with $p_\infty$, $\rho_\infty$ and $T_\infty$).

We are fortunate (again) to have ABET Isentropic Inc. to provide us with an isentropic compressor and an isentropic turbine. We shall take their words for their products, and assume that the entropy of the working fluid indeed does not change as it flows through the compressor or the turbine.

There is an inlet section with a flair. Between station #1 and station #2 is the (isentropic) compressor. Between station #2 and #3 is the combustion chamber, and the area of this section is a constant. The (isentropic) turbine is located between station #3 and #4, the latter is connected smoothly to the exit station where a subsonic (hot) jet is expected to flow out straight and parallel. The compressor is solely driven by the turbine (the connecting mechanical shaft is not shown; of course the engine needs a starter ...) and the frictional losses is negligible. The incoming Mach number is zero (because the aircraft is at rest).
Let the amount of stagnation temperature rise across the compressor be denoted by $\kappa T_\infty$, where $\kappa$ is a design parameter. Let the amount of stagnation temperature rise across the combustion chamber be denoted by $\mu T_\infty$, where $\mu$ is another design parameter (the amount of fuel mass injected per unit time is negligible compared to the air mass flow rate). Both $\kappa$ and $\mu$ are positive. In the analysis, assume the flow is not choked or supersonic anywhere.

(a) Provide some justifications or rationalizations for the estimate that the Mach number in the combustion chamber is small. Even if you can't rigorously justify or rationalize this estimate, assume $M_2$ and $M_3 \ll 1$ in the analysis anyway and take advantage of it to simplify your algebra in the following analysis. (5 points)

(b) Find $M_e$ as a function of $\kappa$ and $\mu$, and whatever else design parameters that you find you need. (20 points)

(c) Assume $A_e = A_2 = A_3$, and provide a quantitative value of $M_3$ (which you had estimated to be small) consistent with the $M_e$ in (b). (5 points)

(d) We want the thrust $F$ of this turbojet engine when the aircraft is at rest. Find $F/A_e$ as a function of $\kappa$ and $\mu$ and whatever other parameters it should depend on. If we want $F$ to be about 1000 lbs, then how big is $A_e$ approximately? (1 square in? two square miles?) The turbine people informed the design team that the turbine inlet temperature should be below 1500 Rankine. Is this an important information? Do you need any additional information to make the estimate? (15 points)

2. For two-dimensional, low speed (incompressible) thin airfoils, we have learned that we can use a vorticity distribution on the mean surface to construct a quantitative (linearized) theory of the effects of angle of attack and camber of a thin airfoil. The exact solution of the resulting Glauert Integral equation can be obtained using Fourier Series.

But in the computer age, we can surely represent the vortex distribution by a large number of segments, each containing a single vortex, and then cook up our own system of algebraic equations for the unknown vortex strength in each of the segments and let a computer solve the problem.
(a) Show how to use a single segment to represent the flat plate at an angle of attack problem and yet get the "correct answer" using paper and pencil. Justify why you would place the vortex at the quarter chord, and explain why you would satisfy the boundary condition at the three-quarter chord. (5 points)

(b) There are two thin 2-dimensional flat plates in a steady, incompressible flow, both at the same geometric (small) angle of attack \( \alpha \). The chord length of both "wings" are identical (unity). Wing #1 is located at \( y=0 \) with its mid-chord at the origin, wing #2 is located at \( y=-1 \) with its leading edge at \( x=0 \). The flow is from left to right with uniform freestream velocity \( U_\infty \). We are interested in the interference effects of the two wings. Estimate the lift and drag forces on each wing by using one vortex segment for each wing. (15 points)

3. You have a constant area tube. At \( t=0 \), the left side of the tube is filled with hot air (\( T_1 \)), and the right side is filled with cold air (\( T_2 \)), and both are at rest. The ratio \( T_1 / T_2 \) is a number of order entity. We are interested in time sufficiently short that thermal diffusion can be neglected. The initial pressure is independent of \( x \).

(a) At \( x=-1 \) ft, there is a piston which begins to move at \( t=0 \) with some small constant velocity, sending a weak wave toward the interface. Find the velocity of the interface after the wave hits—taking whatever advantage that is available because the disturbance is known to be weak. Show that something called the acoustic impedance plays a role in this problem. (20 points)

(b) Alternatively, at \( x=+1 \) ft there is a piston which begins to move at \( t=0 \) with some small constant velocity, sending a weak wave toward the interface. Find the velocity of the interface after the wave hits—taking whatever advantage that is available because the disturbance is known to be weak. (5 points)

(c) How does one do the problem (either one) if the wave is an expansion wave which is not weak? How does one do the problem (either one) if the wave is a compression wave which is not weak? Just discuss and outline the difference in the strategies when the weak disturbance simplification is no longer available (10 points)