MAE 222
Mechanics of Fluids
Final Exam with Answers
January 13, 1994

Closed Book Only, three hours: 1:30PM to 4:30PM

1. Give succinct answers to the following word questions.

(a) Why is dimensional analysis useful to engineers?(10 points)

Ans: It allows engineers to present data, either theoretical or experiemtal, in a form that alllows the most intelligent usage. It also allows engineers, in dealing with a new problem, to recognize how many independent parameters there are in the problem.

(b) What is the physical meaning of vorticity ζ? How do you calculate it if the velocity field **V** is known?(10 points)

Ans: It is twice the averaged angular velocity of a fluid element. It can be computed by taking the curl of the velocity vector.

(c) What are the reasonings that would lead one to conclude that a flow is irrotational? Use complete, English sentences.(10 points)

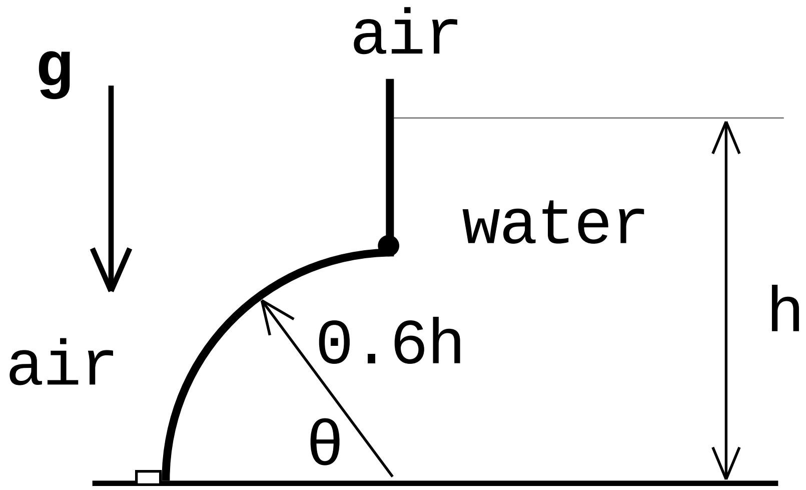
Ans: If the Reynolds number is high and the density is a constant, then the vorticity of a fluid element——which was never in a boundary layer——will remain zero if it was previously zero. Thus for a generic aerodynamics problem where all the upstream fluid elements have zero vorticity, the flow field outside of the boundary layer (and its wake) will have no vorticity.

(d) Why do subsonic airfoils all have sharp trailing edges and round, smooth leading edges? Explain it from the point of view of potential flow theory.(10 points)

Ans: Subsonic (irrotational) aerodynamics problems do not have unique solutions. When there is a sharp trailing edge on a streamlined body, the empirical fact is that the flow always leaves the body smoothly at the sharp trailing edge. Thus all subsonic airfoils are designed with a sharp trailing edge, and we select the solution (among infinite number of mathematically available solutions) which has a smooth flow at the trailing edge. This is called the Kutta Condition. Mathematically, once we respect the Kutta condition, we have no control on the flow around the leading edge. So a well designed subsonic airfoil always has a smooth and rounded leading edge to make sure the flow does not separate there.

Give answers to the following questions (exceptions noted) in terms of symbols and formulas. No numbers. Draw diagram clearly, label your axis, unit normal, and show control volume as appropriate.

2. Given the following two-dimensional hydrostatics problem (width=1 meter):



where the bottom circular arc gate is hinged at the top to a rigid vertical wall, and is held in place by a small stop.

(a) Find the total vertical force acting on the circular arc gate.(5 points)

Ans: Use Archemedes Principle here.

(b) Find the total horizontal force acting on the circular are gate.(10 points)

Ans: Use a control volume consisting of the circular arc gate, the flat bottom, and the vertical side below the hinge. The x-force balance says the horizonal force on the gate equals to the x-force acting on the vertical surface (be careful with signs). Work out the x-force acting on the vertical surface.

(c) Use θ as a coordinate of locations on the circular arc gate. Find the water gage pressure p as a function of θ, h, ρ_{water}, and g.(5 points)

Ans: The hydrostatic pressure of water is a function of y alone. So this question is equivalent to finding the value of y on the circular gate. Let the water pressure at the bottom be given by p(bottom; gage)=ρgh. Then

p(θ; gage; on the circular gate)=p(bottom; gage)-0.6*h*Sin(θ).

(d) Consider a differential segment of the circular arc gate dθ. Find its contribution to the moment about the stop by the water in terms of p(θ), θ, h.(10 points)

Ans: The elemental force d**f** acting on a surface element 0.6*h*dθ by hydrostatic pressure is:

$$d\mathbf{f} = \mathbf{e}_r *0.6*h*p(\theta)*d\theta$$

where the outward unit normal of the circular gate has been denoted by $\mathbf{e}_r$.

The moment arm is the (perpendicular) distance from the line of action of the pressure force (which is radially outward) to the stop.

Using trigonometry, the moment arm=0.6*h*Cos(θ). So the elemental moment, considered positive in the counterclockwise direction about the stop, is

$$d\mathbf{M} = \{0.6*h*Cos(\theta)\}*0.6*h*p(\theta)*d\theta .$$

3. Consider an open channel of rectangular cross-section, with width b(x). The elevation of its bottom surface is h(x). The depth of water is denoted by y(x). Thus, the elevation of the free surface is y+h. The flow is considered steady, frictionless and one-dimensional. The flow velocity is denoted by V(x).

(a) State the continuity equation (between station 1 and station 2) in English, and then write down the equation which expresses that statement.(5 points)

(b) Write down the Bernoulli’s equation for this problem. How is the Bernoulli’s constant normally determined?(5 points)

(c) Show that:

$$Q = \sqrt{g}\ b(x) \left(y_o - h(x) \right)^{3/2} D(F_r)$$

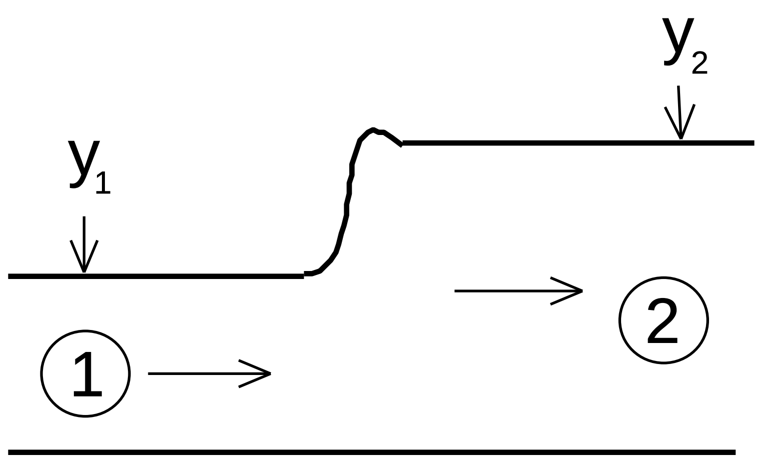
where

$$D(F_r) = \frac{F_r}{\left(1 + \frac{1}{2}F_r^2\right)^{3/2}} .$$

Explain what is Q, what is y_o and what is F_r.(15 points)

Ans: See supplementary notes in Assignment #10.

4. Consider a hydraulic jump observed in a steady frame.



(a) Explain what a hydraulic jump is, and tell all you know about it, including to what happens to the Bernoulli's equation.(10 points)

Ans: If a shallow water open-channel flow is supercritical, the flow at any station has the option of continuing the flow in a continuous manner, or it may suddenly 'jump' to a subcritical flow via a hydraulic jump. A subcritical flow does not have this option.

The Bernoulli's equation does not hold across a hydraulic jump. The Bernoulli's constrant decreases across a hydraulic jump.

(b) State the x-momentum equation in English, using words such as flux, control volumne, etc.(10 points)

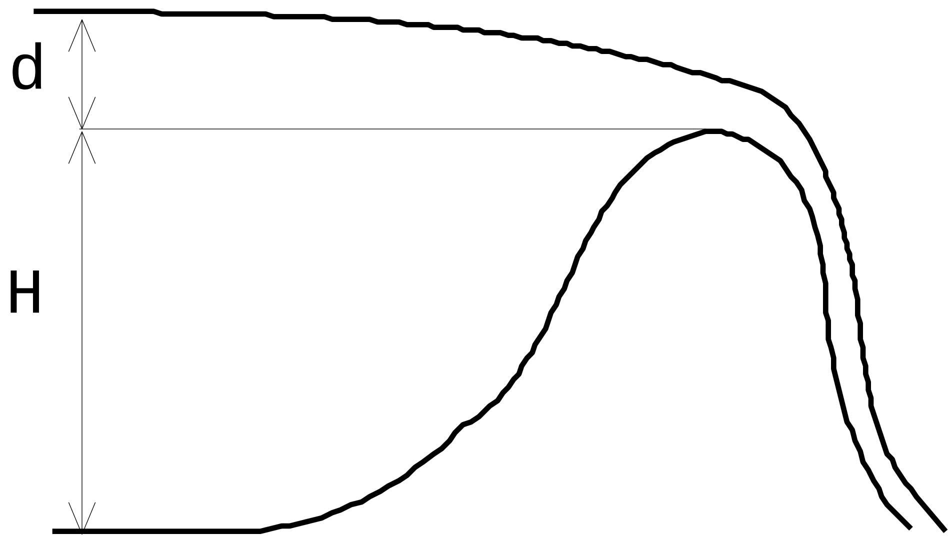
Ans: For a steady flow: the net external force (including pressure force) acting on a control volume fixed in space equals to the net outfluz of momentum from the control volume.

(c) Apply this x-momentum equation to the hydraulic jump. Show your control volume on your diagram.(10 points)

Ans: You need to work out the x-momentum equation only. Work out the (hydrostatic) pressure forces at the upstream and downstream stations. Thats all the external forces we got. Now work out the x-momentum fluxes from the two stations (first get the volume flux, then get the mass flux, then get the x-momentum flux).

5. Water is spilling over a two-dimensional obstacle as shown below. Far, far upstrean, the free surface is d meters above the highest point of the obstacle, and the bottom is H meters below. You may assume that H>>d, and that the Froude number far, far upstream is very, very small.

Show that the volume flow rate (per unit width) is $\sqrt{g}(2d/3)^{3/2}$.(20 points)



Hint: What do you know about the Froude number to the right of the obstacle?

Ans: At the peak of the obstacle, the Froude number is unity. The rest is just like the last problem in the second mid-term.