Control of Nonlinear Systems with Delays and Hysteresis

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Abstract

The *universal dynamic control law* recently advocated by this author is applied to a class of nonlinear systems with delays and hysteresis. The theory is straightforwardly extended to cover these complications without difficulty. Numerical simulations are presented to demonstrate the performance of the controller.

1 Introduction

A common assumption of most control theories is that the system is either linear or quasi-linear. Usually, the system's open-loop dynamics is assumed to have no time delays and/or hysteresis—conventional wisdom would consider such complications difficult. Recently, Lam [1, 2, 3] proposed a *universal dynamic control law* (UDCL) which is capable of controlling a class of conventional quasi-linear systems even when the detailed mathematical model of the system is unknown or uncertain—provided good quality measurements of the the time derivatives of the output variables (in addition to the output variables themselves) are made available to the controller, and a finite but small error is considered acceptable. This paper demonstrates that the same UDCL, without modification, can deal with all such complications in strides.

2 The Theory

For the sake of simplicity, we confine our attention to single-input, single-output scalar problems. The generic mathematical model of the system is:

$$\frac{dx}{dt} = F(x, t; \dot{x}, \ldots) + u \tag{1}$$

where x, the dependent variable, is $O(1)$, $\dot{x}$ is dx/dt and u is the control input which may be time-delayed. The term $F(x, t; \dot{x}, \ldots)$ represents the open-loop dynamics of the system, and is unknown to the controller except that it is $O(1)$. Note that F is allowed to depend on $\dot{x}$, the time derivative of x, so that (1) is not restricted to be strictly quasi-linear. The unwritten additional parameters in F represent "other" complications. A contrived example for F is:

$$\begin{aligned} F(x, t; \dot{x}, \ldots) = {} & \alpha \arctan(x(t)) \\ & + \beta \cos(x(t - t_o)) \\ & + \gamma H(x(t); sgn(\dot{x}(t))) \\ & + \mu \dot{x}^2(t) \\ & + \sin(0.1\pi t) \end{aligned} \tag{2a}$$

where t_o is a time delay constant, α, β and γ are $O(1)$ constants or smooth functions of time, μ is a small constant or smooth function of time, $sgn(\xi)$ is the sign of ξ, and

$$H(x; sgn(\dot{x})) = \sin(x), \quad \dot{x} \geq 0, \tag{2b}$$
$$= \exp(-x^2), \quad \dot{x} < 0. \tag{2c}$$

The first term on the right hand side of (2a) needs no comments, the second term has a time delay of t_o seconds, the third term emulates hysteresis (at the same time making F's dependence on x to be discontinuous), the fourth term, if present, renders the problem fully nonlinear (the problem is quasi-linear only when $\mu = 0$), and the last term makes sure that (1) is non-autonomous. When F does depend on $\dot{x}$, one can, in principle, solve (1) algebraically for $\dot{x}$ to yield a quasi-linear ODE. If no real roots for $\dot{x}$ can be found, then no solution is possible. If multiple real roots are present, then a choice must be made to select a root. To avoid such issues, we simply assume $(\partial F/\partial \dot{x})_{x,t,\ldots}$ to be "sufficiently small." For the special F given by (2a), this means the parameter μ is assumed to be small.

Let $x_d(t)$ denote the desired trajectory of $x(t)$. We choose to define y, the output variable, to be the error

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between the actual and the desired trajectory:

$$y \equiv x - x_d(t). \quad (3)$$

It is desired that $y(t)$ decays toward zero exponentially with characteristic timescale τ, and remains "small" afterwards. Both $y(t)$ and $\dot{y}(t)$ are assumed accurately measured and are made available to the controller. The control task is to find $u(t)$ to be used in (1) such that the resulting initial-value problem for $y(t)$ is asymptotically stable about $y = 0$ when F (and u) contains the various complications.

2.1 The UDCL

In this section, we assume u to have no time delay. Its time delay effects shall be examined via simulations in the next section.

The UDCL advocated by Lam for this problem is [1, 2]:

$$\frac{du}{dt} = -\frac{1}{\Delta t}\left\{\dot{y}_*(t) + \frac{y_*(t)}{\tau}\right\} \quad (4)$$

where $\dot{y}_*(t)$ and $y_*(t)$ are measured data of the actual $\dot{y}(t)$ and $y(t)$, respectively, and Δt is a "sufficiently small" time constant which is the sole "design parameter" of the controller. It is assumed that $\dot{y}_*(t)$ and $y_*(t)$ are *independently* measured and are of good quality—*i.e.* the inevitable measurement noises are "small" and have zero means. In addition, it is assumed that the system actuator is capable of providing any u demanded by (4).

If the measurements were noise-free, we would have $y_*(t) = y(t)$ and $\dot{y}_*(t) = \dot{y}(t)$. When there are measurement noises, we have:

$$y_*(t) = y(t) + O(\delta), \quad (5)$$
$$\dot{y}_*(t) = \dot{y}(t) + O(\delta), \quad (6)$$

where the noises are represented by the $O(\delta)$ terms. The use of numerical differentiation of the $y_*(t)$ data is definitely *not* recommended; independent measurement of the $\dot{y}_*(t)$ data is *strongly* preferred. In particular, we assume the noise terms are small and have zero mean-values. Using (3) and (1) to eliminate $\dot{y}_*$ in (4), we obtain:

$$\frac{du}{dt} = -\frac{u - u_\infty + \delta}{\Delta t}, \quad (7)$$

where the net effect of measurement noises in both $\dot{y}_*(t)$ and $y_*(t)$ is represented by $\delta(t)$—which is assumed small—and

$$u_\infty(x, t; u) = \frac{dx_d(t)}{dt} - F(x, t, \dot{x}, \ldots) - \frac{x - x_d(t)}{\tau}. \quad (8)$$

Unlike the quasi-linear case, u_∞ now may depend on u because F on the right had side is allowed to depend on $\dot{x}$ which in turn depends on u. Equation (7) is now a quasi-linear ODE for u with a potentially non-linear right hand side, and its stability in the small Δt limit can no longer be guaranteed. However, it is intuitively obvious that if F's dependence on $\dot{x}$ is "sufficiently weak," then (7) should be stable. Whenever (7) is found to be stable when $\Delta t << 1$, the resulting "quasi-steady" solution of (7)—and therefore (4)—is given implicitly by:

$$u = u_\infty(x, t; u) + O(\delta; \Delta t), \quad t >> \Delta t, \quad (9)$$

provided u exists. Substituting (8) and (9) into (1) and eliminating dx/dt in favor of dy/dt using (3), we readily obtain the ODE for y when the system is controlled by (4):

$$\frac{dy}{dt} = -\frac{y}{\tau} + O(\delta; \Delta t). \quad (10)$$

Consequently, y indeed decays exponentially toward zero with timescale τ as expected—except for an $O(\delta; \Delta t)$ error which is inherent to the UDCL approach. The "other" complications in F cause no formal theoretical difficulties at all.

2.2 UDCL vs high gain static control law

The UDCL is theoretically related to a high gain static control law provided $\dot{y}_*(t)$ is indeed the time derivative of $y_*(t)$. This is readily seen by simply integrating (4) with respect to time. Indeed, if numerical differentiation were used to compute $\dot{y}_*(t)$ from $y_*(t)$, the UDCL would inherit all the shortcomings of a high gain static PID control law. Instead of using numerical differentiation, the UDCL *demands* that good quality $\dot{y}_*(t)$ data be independently provided *in addition* to $y_*(t)$ itself—taking advantage of the impressive advances of modern sensor technologies in recent years. Once this demand is granted, the noise term in (7) is small becomes credible, and the theoretical rationale for the UDCL is distinct and self-contained.

3 Numerical Simulations

Equation (1) was numerically integrated concurrently with (4), with F as given in (2a), and y as defined by (3). The needed $\dot{y}_*(t)$ in (4) and $\dot{x}$ in F were obtained by numerical differentiations of the actual (noise-free) x(t) and $y(t)$. Measurement noise was emulated by including a zero mean random number sequence of magnitude ϵ inside the curly bracket of (4), refreshed at every integration time step t_s. For all simulations, the desired trajectory was arbitrarily chosen to be $x_d(t) = \sin(0.2\pi t)$ (which made sure that the hysteresis term was periodically active), a significant noise level $\epsilon = 0.1$ and a very small integration time step $t_s = 0.01$ were used.

3.1 No time delay in u

For this case, Δt was chosen to be $2t_s$. Figs. 1 and 2 show sample simulation results for a set of system and controller parameters identified in the figure captions. Generally speaking, the UDCL works as expected for any set of $O(1)$ system parameters provided $|\mu|$ is sufficiently small. For small $|\mu|$, Fig. 1 is nearly unaffected by the system parameters, while Fig. 2 is clearly affected by them. When all other parameters were held fixed, the system crashed whenever μ ventured beyond some finite upper and lower bound (empirically, the workable range is $|\mu| \leq 0.1$)—as anticipated in §2 previously. Fig. 2 shows that u has a brief initial transient for $t = O(\Delta t)$, moving rapidly from its (arbitrary) initial condition $u(0) = 0$ to its initial quasi-steady value. When $\gamma \neq 0$, the discontinuity of the hysteresis term H is responsible for apparent discontinuities of u as shown in Fig. 2. In addition, the hysteresis term induces small local turmoils in $x(t)$ and $y(t)$ which are barely discernible in Fig. 1. These magnitude of these local turmoils is proportional to the magnitude of $\gamma\Delta t$ used.

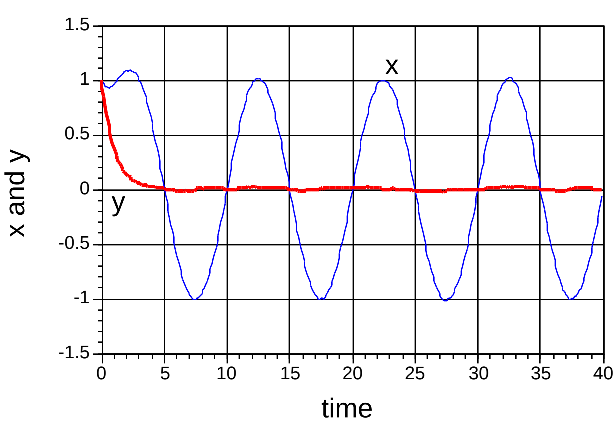

Figure 1: x and y versus t.
$\alpha = 1.0$, $\beta = -1.0$, $\gamma = 0.25$, $\mu = 0.1$
$t_o = 1.0$, $\tau = 1.0$; $\Delta t = 0.02$; $u(0) = 0$

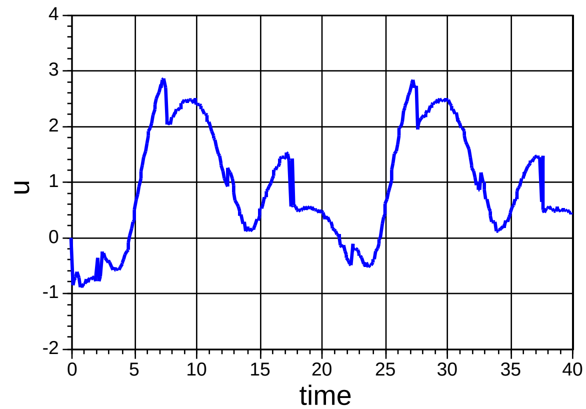

Figure 2: u versus t.
Same parameters as Fig. 1.

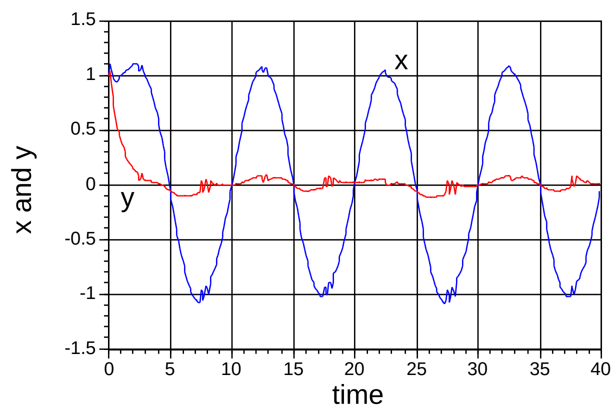

Figure 3: x and y versus t; $t_1 = 0.05$.
Same parameters as Fig. 1 except $\Delta t = 0.1$.

3.2 Time delay in u

It is straightforward to run simulations with the u in (1) replaced by $u(t - t_1)$ where t_1 is a time delay. Empirically, the unmodified UDCL continues to work as expected provided $t_1 < \Delta t$ is comfortably satisfied. Otherwise, the system is (understandably) unstable.

Since Δt is at the disposal of the control engineers, the option of using a somewhat larger Δt to handle short time delay in the actuator is thus available—provided a somewhat degraded performance is acceptable. Fig. 3 shows the performance of the UDCL when $t_1 = 0.05$ using $\Delta t = 10t_s = 0.1$. The local turmoils caused by the hysteresis discontinuity (using the same γ as before) are more clearly visible here.

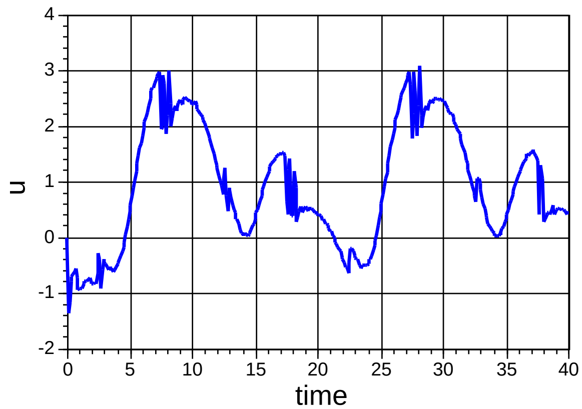

Figure 4: u versus t; $t_1 = 0.05$.
Same parameters as Fig. 3.

4 Discussion

We have demonstrated that the UDCL can be applied, without modification, to nonlinear systems, including time delays and hysteresis in its open-loop dynamics (and short time delay in its actuator)—without the need for a validated mathematical model of the system. The crucial needed knowledge of the system as represented by (1) is that the *pulse-response matrix* is a positive $O(1)$ scalar—*i.e.* a positive unit "pulse" of u will yield a positive $O(1)$ "jump" in the value of x. Note that this information can be directly obtained by the controller on-the-fly by running simple diagnostics.

In our numerous simulation runs, the UDCL had no difficulty at all with the hysteresis term—except for the small local turmoils induced by the discontinuous switchings. Time delay in F caused no degradation of the performance at all, while short time delay in u was handled in strides at the price of some degradation of performance. The ability of the UDCL to handle measurement noise is critically dependent on the availability of good quality $\dot{y}_*(t)$ data—in addition to good quality $y_*(t)$ data. Understandably, the UDCL works well only when the system parameters are $O(1)$.

When the system of interest involves more than one dependent variables, many subtle theoretical issues arise, and they are treated in [1] and [2] in details. For example, the question of how to determine the highest order time derivative of the output variables needed by the UCDL is addressed. The methodology of singular-value decomposition is adopted to provide a unified treatment of a class of so-called "singular" and "nearly singular" problems—the latter are problems whose "relative degrees" are ill-defined. How to exert influence on the "residual" variables of the system is also discussed. How to exploit detailed knowledge of the system (if available) to reduce the need for time derivatives by the UDCL is discussed in a separate paper [3]. In addition, the basic idea of the UDCL has been extended to handle a class of scalar optimal control problems [4].

References

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