

# USE OF SURROGATE PULSE-RESPONSE MATRIX FOR THE UDCL

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Abstract: The *universal dynamic control law* (UDCL) advocated by the author can achieve ODE-based control objectives without the need for *detailed* knowledge of the system itself. Thus it is very robust with respect to system uncertainties and unmodeled disturbances. The present paper shows, via an example, how to find a usable “surrogate pulse-response matrix” needed by this control law. Numerical simulations are presented. A brief discussion on design guidelines for sensors and actuators for easy UDCL implementation is also included.

Keywords: Control Theory, Nonlinear Systems, Robust Control, Singular Perturbation Methods.

## 1. INTRODUCTION

Consider a  $N$ -dimensional system governed by the following first order ODEs:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f} + \sum_{i=1}^I \mathbf{b}_i u^i, \quad (1)$$

where  $\mathbf{x}$  is the state (column) vector,  $\mathbf{f}(\mathbf{x}, t)$  and the  $\mathbf{b}_i(\mathbf{x}, t)$ 's are differentiable vector functions of their arguments. The second term on the right hand side represents the “forces” exerted by  $I$  actuators ( $I \leq N$ ). Physically,  $\mathbf{f}$  represents the “open-loop” dynamics of the system, the  $\mathbf{b}_i$ 's represent the *direction* and the  $u^i$ 's the *amplitude* of the control forces. There are  $I$  output variables  $y^1, \dots, y^I$  which are related to  $\mathbf{x}$  by:

$$y^i = \Psi^i(\mathbf{x}, t), \quad i = 1, \dots, I. \quad (2)$$

where the  $\Psi^i(\mathbf{x}, t)$ 's are differentiable functions of  $\mathbf{x}$  and  $t$  but are strictly forbidden to have any dependence on the  $u^i$ 's. The  $y^i$ 's are assumed to be dimensionless and are nominally  $O(1)$  entities.

The ODE system governing the  $y^i(t)$ 's is readily derived:

$$\frac{dy^i}{dt} = g^i + \sum_{i'=1}^I B_{i'}^i u^{i'} \quad (3)$$

where

$$g^i = \frac{\partial \Psi^i}{\partial t} + \mathbf{c}^i \cdot \mathbf{f}, \quad (4a)$$

$$B_{i'}^i = \mathbf{c}^i \cdot \mathbf{b}_{i'}, \quad (4b)$$

$$\mathbf{c}^i \equiv \frac{\partial \Psi^i}{\partial \mathbf{x}}. \quad (4c)$$

The matrix  $B_{i'}^i(t)$  is called the *pulse-response* matrix, since it gives the response of the output variable  $y^i$  to a pulsed input of  $u^{i'}$ . Note that  $B_{i'}^i$  does not depend on  $\mathbf{f}(\mathbf{x}, t)$ . The control goal is to use the actuator forces to make the output

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variables honor approximately the following *ODE-based constraints*:

$$\frac{dy^i}{dt} + \phi^i(y^{i'}, t; \varpi^{i'}) = O(\epsilon), \quad (5)$$

where the  $\phi^i(y^{i'}, t; \varpi^{i'})$ 's are user-specified functions of the arguments (the  $\varpi^{i'}$ 's are parameters to be exploited later), and  $\epsilon$  is the allowable error.

The theory of the Universal Dynamic Control Law (UDCL) advocated by the author (Lam, 1997a,b, 1998, 1999a,b) is applicable to this class of problem when  $N$ ,  $\mathbf{f}(\mathbf{x}, t)$ , the  $\mathbf{b}_i$ 's and the  $\Psi^i(\mathbf{x}, t)$ 's (therefore the  $g^i(\mathbf{x}, t)$ 's) are unknown to the controller—only  $I$  and a “surrogate” of  $B_{i'}^i(t)$  are assumed known—provided good real time measurements of the output variables and their time derivatives are made available to the controller blackbox.

Instead of looking for a static control law which classically gives the  $u^i$ 's as *algebraic* functions of the state vector and time (Ioannou and Sun, 1996; Isidori, 1995; Krstić, Kanellakopoulos and Kokotović, 1995; Marino and Tomei, 1995), the theory of UDCL provides (Lam, 1997a,b, 1998) a system of ODEs for the  $u^i$ 's:

$$\frac{du^i}{dt} = - \sum_{m=1}^M \frac{[\bar{B}_i^m]^+}{\Delta t} \{ \dot{y}_*^m + \phi^m(\mathbf{y}_*, \dots) \}, \quad (6)$$

where  $M = I$  when  $B_{i'}^i$  is nonsingular and  $M > I$  when  $B_{i'}^i$  is either singular or nearly singular,  $\bar{B}_i^m$  is a “surrogate” of  $B_i^m$ , and  $\Delta t$  is a “sufficiently small” positive user-specified time constant to be chosen by the control engineers. The matrix  $[\bar{B}_i^m]^+$  is the *pseudo-inverse* (Golub and Van Loan, 1989) of  $B_i^m$ —it is identical to the conventional inverse when  $M = I$ . Any reasonable initial condition, such as  $u^i = 0$ , may be used.

The condition on the surrogate pulse-response matrix  $\bar{B}_i^m$  is:

$$\sum_{m=1}^M [\bar{B}_{i'}^m]^+ B_{i'}^m = W_{i'}^i \quad (7)$$

where  $W_{i'}^i$  is *any*  $I \times I$  real matrix whose eigenvalues all have  $O(1)$  positive real parts. The ideal choice for  $\bar{B}_i^m$  is obviously  $B_i^m$  so that  $W_{i'}^i$  is simply the unit matrix. The present paper shows, via an example, how to go about finding a usable  $\bar{B}_i^m$ .

## 2. A SIMPLE EXAMPLE

Consider a system with  $I = 2$ . The control objective is to make  $y^1$  and  $y^2$  decay exponentially with (user-chosen) characteristic time  $\tau$  toward (user-chosen)  $\varpi^1$  and  $\varpi^2$  approximately—provided it is possible to do so. In other words, the desired  $\phi$ 's are:

$$\phi^1 = \frac{y^1 - \varpi^1}{\tau}, \quad (8)$$

$$\phi^2 = \frac{y^2 - \varpi^2}{\tau}. \quad (9)$$

Measured real time data of the output variables are marked by asterisks. Thus:

$$y_*^i(t) = y^i(t) + \delta_0^i(t), \quad (10a)$$

$$\dot{y}_*^i(t) = \dot{y}^i(t) + \delta_1^i(t), \quad (10b)$$

where  $\delta_p^i$  represents measurement noise—the subscript  $p$  denotes the order of time derivative involved. It is assumed that all  $\delta_p^i(t)$ 's are “sufficiently” small, but they do not need to have zero mean. In actual implementation, it is strongly preferred that the data for  $\dot{y}_*^i(t)$  be measured directly—numerical differentiation of  $y_*^i(t)$  is frowned upon.

The *sampling time* of the controller blackbox is  $t_s$ , assumed to be very small in comparison to the characteristic time scale  $t_g$  of the physical system. Without loss of generality,  $t_g$  is assumed to be unity.

### 2.1 The Regular Case

For this case, the following information is known about this matrix:

- (1) When  $u^1$  is pulsed,  $y_*^1$  always jumps in the positive direction by  $O(1)$  while the response of  $y_*^2$  is not so predictable.
- (2) When  $u^2$  is pulsed,  $y_*^2$  always jumps “the other way” in a more responsive way, while the response of  $y_*^1$  is not so predictable.

The above information suggests the following non-singular matrix as a surrogate pulse-response matrix for the system under consideration:

$$\bar{B}_{i'}^i(t) = \begin{vmatrix} 1 & 0 \\ 0 & -2 \end{vmatrix}. \quad (11)$$

The UDCL for this regular case is:

$$\frac{du^i}{dt} = - \sum_{i'=1}^2 \frac{[\bar{B}_{i'}^i]^{-1}}{\mu t_s} \{ \dot{y}_*^{i'} + \phi^{i'}(\mathbf{y}_*, \dots) \}. \quad (12)$$

Theoretically, the value of  $\mu t_s$  should be as small as possible. In practice,  $\mu$  should be  $O(1)$  and not smaller—in order to make sure that numerical integration of (12) using  $t_s$  as integration step size will be stable.

## 2.2 The Irregular Case

For this case, the following information is known about this matrix:

- (1) When  $u^1$  is pulsed, neither  $y_*^1$  nor  $y_*^2$  hardly jumps at all.
- (2) When  $u^2$  is pulsed,  $y_*^2$  continues to jump “the other way” as in the regular case, while the response of  $y_*^1$  remains unpredictable.

The above information suggest that  $B_{i'}^i$  is either nearly singular or precisely singular. Since  $y^1$  is not responsive to the actuators, the ODE-based constraint on  $y^1$  is expected to incur some *unavoidable error*. This error is formally introduced as an “extended” output variable,  $y^3$ :

$$y^3 \equiv \frac{dy^1}{dt} + \phi^1(\mathbf{y}, t, \varpi^i). \quad (13)$$

The best one can do is to drive  $y^3$  to zero as quickly as possible. An explicit expression for  $\Psi^3$  can be obtained by eliminating  $dy^1/dt$  using (3). It is of interest to note that unless  $B_1^1$  and  $B_2^1$  are *identically* zero, the resultant  $\Psi^3$  will have a weak dependence on the  $u^i$ 's. This weak dependence has very significant consequences, as we shall see later.

Instead of trying to achieve  $y^3 = O(\epsilon)$ , the UDCL tries to make  $y^3$  decay to zero. A possible choice for  $\phi^3$  is:

$$\phi^3 = \frac{2y^3}{\tau} \quad (14)$$

where the factor 2 is (arbitrarily) inserted to make  $y^3$  decays faster than  $y^1$  (and  $\varpi^3$  has been set to zero).

In order for this new control objective to be achievable, the following information about the extended pulse-response matrix is assumed:

- When  $u^1$  is pulsed,  $y^3$  always jumps by  $O(1)$  in the positive direction, while its response to a  $u^2$  pulse is unpredictable.

Based on this information, the following can be used as a surrogate pulse-response matrix:

$$\bar{B}_{i'}^m = \begin{vmatrix} 0 & 0 \\ 0 & -2 \\ 1 & 0 \end{vmatrix}. \quad (15)$$

The pseudo-inverse of this  $3 \times 2$  matrix is readily found to be :

$$[\bar{B}_{i'}^m]^+ = \begin{vmatrix} 0 & 0 & 1 \\ 0 & -0.5 & 0 \end{vmatrix}. \quad (16)$$

This nonsingular matrix has two finite singular-values (1 and 0.5). The UDCL for this irregular case is:

$$\frac{du^i}{dt} = - \sum_{m=2}^3 \frac{[\bar{B}_i^m]^+}{\mu t_s} \{ \dot{y}_*^m + \phi^m(\mathbf{y}_*, \dots) \}, \quad (17)$$

where again the value of  $\mu$  is to be chosen by the control engineers. However, because  $\Psi^3$  may have a weak dependence on the  $u^i$ 's, the value of  $\mu u$  should be a relatively larger  $O(1)$  number—unless it is known that  $B_{i'}^i$  is precisely singular. Note that in (17) the  $m$  summation omits  $m = 1$  since (8) is replaced by (13) which is the definition of  $y^3$ .

It is assumed that good quality real time measurements  $y_*^3(t)$  and  $\dot{y}_*^3(t)$  are available to the controller blackbox.

## 3. NUMERICAL SIMULATIONS

In order to program numerical simulations, the value of  $N$ , detailed information on  $\mathbf{f}(\mathbf{x}, t)$ , the  $\mathbf{b}_i$ 's and the  $\Psi_i$ 's must be known.<sup>2</sup> Consider a physical system with  $N = 3$  (and  $I = 2$ ). The actual  $\mathbf{b}_i$ 's are given by:

$$\mathbf{b}_1 = [\gamma, 0.2x_3^2, 1]^T, \quad (18a)$$

$$\mathbf{b}_2 = [0.1\gamma x_3, -2, 1]^T, \quad (18b)$$

where  $\gamma$  is a parameter to be exploited later. For the sake of simplicity, the output variables are simply  $y_1 = x_1 = \Psi_1$  and  $y_2 = x_2 = \Psi_2$ . Thus, the actual row vectors  $\mathbf{c}_i$ 's are given by:

$$\mathbf{c}_1 = \frac{\partial \Psi_1}{\partial \mathbf{x}} = [1, 0, 0], \quad (19a)$$

$$\mathbf{c}_2 = \frac{\partial \Psi_2}{\partial \mathbf{x}} = [0, 1, 0]. \quad (19b)$$

The actual  $2 \times 2$  pulse-response matrix is then:

$$B_{i'}^i = \begin{vmatrix} \gamma & 0.1\gamma x_3 \\ 0.2x_3^2 & -2 \end{vmatrix}. \quad (20)$$

It is readily seen that when  $\gamma = O(1) > 0$  and when  $\gamma$  is negligibly small, this pulse-response matrix does have the properties described in the

<sup>2</sup> In this subsection only, superscript of column vectors will appear as a subscript to avoid confusion with exponents.

previous section for the regular and irregular case, respectively.

The following is (arbitrarily) chosen for  $\mathbf{f}(\mathbf{x}, t)$ :

$$f_1 = \sin(x_1 - \alpha) + \exp(-x_2^2) + x_3, \quad (21a)$$

$$f_2 = \arctan(x_1^2 + x_2 + \sin(\alpha x_3) + \alpha), \quad (21b)$$

$$f_3 = -\beta x_3 + \cosh(x_1 + x_2) \cos(x_1 + \alpha x_2), \quad (21c)$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are nominally  $O(1)$  parameters which may be smooth functions of time. It is important to note that  $\partial f_1 / \partial x_3$  is unity—a  $O(1)$  constant.

The initial conditions used were chosen (arbitrarily) to be  $x_1(0) = 1$ ,  $x_2(0) = -3$ ,  $x_3(0) = 1$ . In all cases, the integration timestep used was 0.01, and it is to be interpreted as the  $t_s$  of the controller blackbox. All measurements are assumed noise-free—the issue of measurement noise has been treated in a separate paper (Lam, 1999a).

### 3.1 The Regular Case with $\gamma = 1$

Fig. 1 shows the results for  $\gamma = 1$ ,  $\beta = 1$ , and  $\alpha = -2 \sin(0.1\pi t)$  using  $\mu = 3$ ,  $\varpi_1 = \varpi_2 = 0$ . When other  $O(1)$  choices for  $\alpha$  were used (*e.g.*  $\alpha = 5, 1, 0, -1, -5$ ), the results for  $x_1(t)$  and  $x_2(t)$  were indistinguishable from Fig. 1. The results for  $x_3(t)$  are  $\alpha$ -dependent and are well behaved so long as  $\beta$  is positive.

When  $\beta$  is negative,  $x_3$  diverges—as expected—and the system eventually crashes. This (zero dynamics) instability can easily be arrested by exploiting the  $\varpi^i$ 's—assuming sensor measurement of  $x_3(t)$  is available to the controller. For example, when  $0 \geq \beta \geq -0.57$ , the choice of  $\varpi_1 = -0.55x_3$  and  $\varpi_2 = 0$  was empirically found to stabilize  $x_3$  at the expense of  $x_1$  which is now  $\alpha$ -dependent and no longer decays toward zero. Fig. 2 shows the results of this simulation. Note that the exponential decay of  $x_2$  is strictly enforced in both Fig. 1 and Fig. 2.

### 3.2 The Irregular Case with Small $\gamma$

After introducing the new output variable  $y_3$  by (13), the actual  $3 \times 2$  extended  $B_i^m$ , which now does depend on  $f_1(\mathbf{x}, t)$ , is:

$$B_i^m = \begin{pmatrix} O(\gamma) & O(\gamma) \\ 0.2x_3^2 & -2 \\ 1 + 0.2x_3^2 \frac{\partial f_1}{\partial x_2} & 1 - 2 \frac{\partial f_1}{\partial x_2} \end{pmatrix}. \quad (22)$$

Since both singular values of  $B_i^m$  are now  $O(1)$ , the irregular problem has been “regularized” with  $M = 3$ . This actual matrix (22) should be compared with the surrogate matrix proposed in (15).

Simulations were performed with

$$\gamma = 0.05 \cos(0.1\pi t), \quad (23)$$

a variety of the  $\alpha$ 's,  $\beta$  of either signs, and  $\varpi_1 = \varpi_2 = 0$ . Since the chosen  $\gamma(t)$  is merely small and not precisely zero, the original  $B_i^i$  is nearly singular. As a consequence,  $\Psi_3$  does have a weak dependence on the  $u_i$ 's. This complication introduces a lower bound for  $\Delta t$ . In the simulations,  $\mu = 3$  indeed caused the system to crash, while  $\mu = 7$  was found empirically to work well in all cases. A sample simulation is presented in Fig. 3. It is seen that  $x_1(t)$ —which did not respond to pulsing of the  $u_i$ 's—has a hump before it decays toward zero, in contrast to its previous behavior as shown in Fig. 1. The new output variable  $y_3(t)$  as defined by (13) always honors the ODE-based constraint as specified by (14), and is not shown. Simulations for  $\beta = -1$  were also performed, and the results for  $x_1(t)$  and  $x_2(t)$  look essentially the same as Fig. 3.

## 4. DESIGN GUIDELINES FOR SENSORS AND ACTUATORS

Traditionally, control engineers often begin the design of the controller black box after the choices of sensors and actuators have been made by others. An obviously good idea is to invite the control engineers to play an active role in the whole design process, including the design of sensors and actuators. The following are design guidelines which can greatly facilitate the use of UDCL by the controller blackbox:

- Variables of special importance to the control tasks at hand should be selected as primary output variables and accurate real time measurements must be provided. All needed rate data should be directly measured if at all possible. All unmeasured state variables are assumed to be of secondary interest and, hopefully, are well-behaved without the need of intervention by the controller.
- The actuators should be chosen to yield a full-rank primary impulse-response matrix  $B_i^i$ , such that a (constant or slowly varying) surrogate  $\bar{B}_i^m$  can readily be identified off-line, or determined on-the-fly. Regular prob-

lems are preferred since its pulse-response matrix is independent of  $\mathbf{f}$ .

- If the problem is intrinsically irregular, the number of derived output variables should be as small as possible. Precisely singular  $B_i^i$  is preferred over nearly singular ones.
- The physical unit of the singularvalues of  $B_i^m$  is reciprocal time. The actuators must be powerful enough such that the smallest dimensional singularvalue is  $O(1/t_g)$  where  $t_g$  is the characteristic time of the global system.
- The ODE-based constraints should be chosen such that the associated  $u^i$ 's are expected to be  $O(1)$  and smooth.

It is taken for granted that the sensor measurements (including all the needed time derivatives) are reliable and have good signal-to-noise ratios, and  $t_s$  is sufficiently small in comparison to  $t_g$ .

## 5. CONCLUDING REMARKS

The main merit of the UDCL is that no detailed knowledge of the system is needed. The Achilles' heel is that it requires not only reliable  $y_*^m(t)$ 's, but also  $\dot{y}_*^m(t)$ 's. For example, for mechanical systems measurements of accelerations are needed when the primary output variables are either positions or velocities. This is the price paid for robustness with respect to uncertainties of the system model. Ideally, the needed time derivatives of the primary output variables should be directly measured; numerical differentiation should be avoided if at all possible for obvious reasons. From the point of view of sensor technology, there is no fundamental reason to believe that time derivatives  $\dot{y}^m(t)$ 's are generically more difficult to directly measure than the  $y^m(t)$ 's themselves. Obviously, the UDCL can take advantage of any additional detailed knowledge of the system to reduce its present strong reliance on the  $\dot{y}_*^m(t)$ 's. This issue and the impacts of noise are discussed in more details in a separate paper (Lam, 1999a).

The most crucial information about the system needed by the UDCL is a surrogate of  $B_i^m(t)$ —the precise  $B_i^m(\mathbf{x}, t)$  itself is not necessary. Roughly speaking, the  $i$ -th column of the surrogate matrix  $\bar{B}_i^m$  should mimic the  $M$ -dimensional vector representing the jump of the  $y_*^m$ 's in response to a unit pulse of  $u^i$ . The easiest problems for the UDCL are those for which a constant or nearly constant  $\bar{B}_i^m$ 's is adequate—as was the case in the example presented. Whenever UDCL is adopted,

the task of system identification reduces to the determination of  $\bar{B}_i^m(t)$  or  $\bar{B}_i^m(y_*^m, t)$ . For the  $M = I = 1$  case, only the sign of the scalar  $B_1^1(t)$  needs to be determined. If the elements of the actual  $B_i^m$  has very complicated dependence on  $\mathbf{x}(t)$  and  $t$  and no reliable  $\bar{B}_i^m(y_*^m, t)$  can be found, then the UDCL is not useful.

In essence, the UDCL ignores the unobserved variables—assuming that the zero dynamics of the problem needs no intervention. If intervention is needed, the  $\varpi^i$ 's are available to be exploited. The intelligent use of this degree of freedom is at the disposal of the control engineers—provided additional information and/or real time sensor measurements are available.

The highest order time derivative needed by UDCL for  $y^i(t)$  is commonly known as the “relative degree” of  $y^i$ . In general, the theoretically achievable accuracy of UDCL degrades as the relative degree increases. Generally speaking, the UDCL is most useful for regular problems (*i.e.* all relative degrees are unity) and for irregular problems whose relative degrees are not large. It can be used also in a restricted class of optimal control problems (Lam, 1999c).

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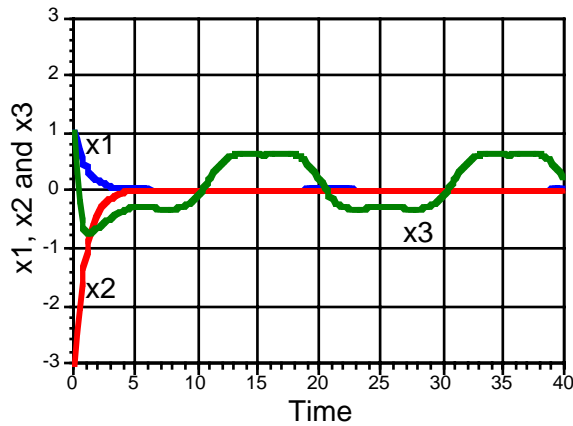


Fig. 1. The regular case ( $\gamma = 1$ ) with  $\beta = 1.0$  ( $x_3$  zero-dynamics is stable without intervention);  $\tau = 1$ .

$\alpha = -\sin(0.1\pi t)$ ,  $\mu = 3$  and  $\varpi_1 = \varpi_2 = 0$ . Measurements are noise-free.

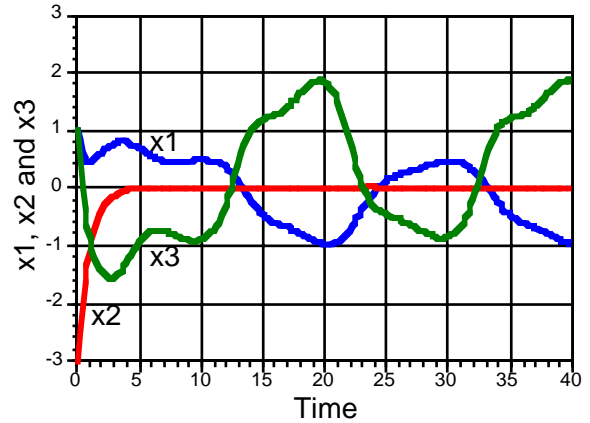


Fig. 2. The regular case ( $\gamma = 1$ ) with  $\beta = -0.57$  (unstable  $x_3$  zero-dynamics stabilized by exploiting  $\varpi_1$ );  $\tau = 1$ .

$\alpha = -\sin(0.1\pi t)$ ,  $\mu = 3$ ,  $\varpi_1 = -0.55x_3$  and  $\varpi_2 = 0$ . Measurements are noise-free.

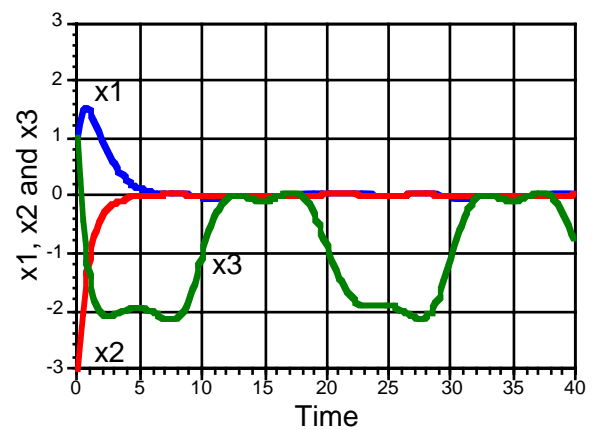


Fig. 3. The irregular case ( $\gamma = 0.05 \cos(0.1\pi t)$ ) ( $N = M$ , no zero-dynamics);  $\tau = 1$ .

$\alpha = -2 \sin(0.1\pi t)$ ,  $\beta = 1.0$ ,  $\mu = 7$  and  $\varpi_2 = \varpi_3 = 0$ . Measurements are noise-free.