MAE 222 Fluid Mechanics
Second Mid-Term

Spring, 1998

ANSWERS

Closed Book

(70 points)

1. You are in charge of a NASA project to provide experimental DRAG data to design a boat to be used on Mars. It is speculated that the sea on Mars is pure gasoline. Your laboratory is, of course, on earth, and you are provided a scaled model and a water tank.

How would you present your experimental data for publication? (10 points)

Answer:

Boats make surface waves, and surface waves depends on the 'gravitational constant' g. I expect the drag coefficient to be primarily dependent on Froude number. Because Reynolds number is expected to be large, its effects are likely to be secondary. So I will plot Drag coefficient against Froude Number.

2. According to the Moody Diagram, the friction factor *f* of a fully developed turbulent flow in a circular pipe is independent of Reynolds number for very large Reynolds number. Using this information, find the ratio of pumping powers (energy per unit time) needed to pump the same amount of fluid through two pipes, the diameter of one being twice the other. (20 points)

Answer:

The volume flow rate Q is $\frac{\pi}{4}UD^2$. So $U=\frac{4Q}{\pi D^2}$.

Pumping power Ppwr=$\Delta p \bullet Q$ (check the dimension! energy/time)

We knon $\Delta p=\frac{f}{2}\rho U^2\frac{L}{D}$, and we are told that *f* is a constant in the range of Reynolds number we will be interested in.

Eliminating U in favor of Q, we obtain Ppwr=constant $\times D^{-5}$.

So a pipe with twice the diameter takes 1/32 amount of pumping power.

3. (a) What is the physical meaning of vorticity? (5 points)

Answer:

It is twice the averaged angular velocity of a fluid element.

(b) How would you justify the claim that the flow about a streamline body of reasonable size is almost everywhere irrotational? (15 points)

Answer:

(a) The Reynolds number must be large so that the inviscid assumption is valid outside the boundary layer,

(b) The density is constant,

(c) The fluid in the flow field was without viscosity once upon a time.

(c) What is so great about irrotational, incompressible flows? (5 points)

4. In an open-channel flow, there is an hydraulic jump: the channel bottom is flat and horizontal, the side walls are parallel, and yet the flow velocities and water heights at stations (1) (upstream) and (2) (downstream) are different. State the momentum equation in English, and then apply it to a control volume enclosing the "discontinuity." I don't want you to "derive" anything; I just want to see the momentum equation properly executed. (15 points)

Answer:

For steady flow: the net external resultant force (vector) acting on a control volume equals the net outflux of momentum (vector).

For the hydraulic jump problem: draw a control volume enclosing the hydraulic jump, and consider the x-momentum equation. The external force acting on the front and rear surfaces are the hydrostatic pressure force which needs to be integrated. There is outflow of x-momentum from the rear surface, and inflow from the front surface. The x-momentum flux is the mass flux times the x-component of the fluid velocity.

Open Book

(30 points)

5. We shall deal with a steady, uniform air flow of velocity U=73 meter/second. The density is ρ=1.23 Kg/m^3, and the kinematic viscosity is $\nu=1.46 \times 10^{-5}$ meter^2/second.

(a) You have a smooth circular cylinder with diameter D=1 cm. Find the drag per unit length of this cylinder using the data on page 406, Fig. 9.15. The drag coefficient plotted on the vertical axis is defined by

$$C_D = \frac{2 \text{ Drag per unit length}}{\rho U^2 D} \qquad \text{(10 points)}$$

Answer:

Compute the Reynolds number, look up the drag coefficient from the chart.

(b) You have a smooth flat plat in the same flow (without the cylinder). If the viscous frictional drag (per unit width) on this flat plate is the same as that of the above cylinder, how long (L, in meters) is the plate, approximately? Use the data presented on page 395, Fig. 9.10. The drag coefficient plotted on the vertical axis is defined by

$$C_{D_f} = \frac{2 \text{ Drag per unit width}}{\rho U^2 L} . \qquad \text{(20 points)}$$

Hint: You need to iterate numerically. For the purpose of the test, you need to perform the iteration only once.

Answer:

Equating the two drags, we have

$$C_D D = C_{D_f} L = \text{a known constant.}$$

And the flat plate drag coefficient depends on $R_{eL} = \frac{\rho U L}{\mu}$. We must iterate.

- Pick a L, and compute the Reynolds number $R_{eL} = \frac{\rho U L}{\mu}$,
- Find the flat plate drag coefficient C_{D_f} from the Chart in Fig. 9.10,
- Compute L from $L = \frac{\text{known constant}}{C_{D_f}}$
- Repeat until convergence.