MAE 222

Mechanics of Fluids

Princeton University

Assignment # 10

April 22, 1998

Due on Wednesday, 2PM, April 29, 1998

The second mid-term is scheduled on on Friday (May 1). It will cover everything except Chapter 10 (unless by popular demand!). The Wednesday class will be a review session.

Chapter 10. Open Channel Flow. This is also called "free surface flows."

§10.1-10.2, General Characteristics, Surface Waves and F_r. In this chapter, attention is mainly confined (except for hydraulic jump) to "shallow water" open channel flows—flows in which the characteristic depth of the water is small in comparison to the characteristic length in the streamwise direction. For shallow water open channel flow problems, the "one-dimensional" assumption can be justified: the streamwise velocity V is a constant across any cross-section of the channel. In other words, $V = V(x,t)$ where x is the streamwise coordinate and t is time.

You learn from these sections that the speed of a shallow water wave is $c = \sqrt{gy}$ where y is the vertical distance from the free surface to the bottom of the channel. You also learn that *Froude number* F_r is defined as the ratio of V to c.

§10.3. Energy Considerations. Skip this section.

§10.4. Uniform Depth Flow. Skip this section.

§10.5. Gradually Varied Flow . Such flows are usually called shallow water flows. We will see that flows with $F_r \le 1$ (subcritical) behaves very differently from supercritical flows (The Froude number is, more or less, the "Mach number" of open channel flows). I will go over this in class.

§10.6. Rapidly Varied Flow . The big deal here is the hydraulic jump. Eq.(10.19) is the major result: the ratio of water depth across a hydraulic jump is a function of the upstream Froude number. The upstream Froude number MUST be larger than unity (supercritical), and the downstream Froude number is always subcritical. The Bernoulli's constant changes value across a hydraulic jump; it always decreases!

Skip §10.6.2, (Sharp-crested Weirs)

§10.6.3, Broad-crested Weirs is interesting stuff! See Smits Notes.

Skip §10.6.4, (Underflow gates).

Problems at end of Chapter 10, page 461:

- Problem 10.2. Totally straightforward.
- Problem 10.4. I will discuss this in class. We want to use the formula $c = \sqrt{gy}$ to provide us with a convincing argument that incoming waves on a sloping beach ought to "break."
- Problem 10.9. The flow is assumed steady and one-dimensional. You have the continuity equation, and you have the Bernoulli's equation for the free surface streamline. Thus you have two equations for the two unknowns, V_2 and y_2.
- Problem 10.14. Watch for this in class. If I forget, remind me.
- Problem 10.46. This is a somewhat tricky problem. You need to change your coordinate system so that it is moving with the moving hydraulic jump. In that coordinate system, you can apply eq.(10.19) (using F_r in that coordinate system).
- Problem 10.51. See Fig. 10.17 on page 457.

Use email or the newsgroup to ask questions.