MAE 222
Mechanics of Fluids
Princeton University

Assignment # 2

February 11, 1998

Due on Wednesday, 2PM, February 18, 1998

Chapter 3 : Elementary Fluid Mechanics—The Bernoulli's Equation.

§3.1, Newton's Second Law . We all know Newton's Second Law is $\mathbf{F} = m\mathbf{a}$. And you have all heard me say that in fluid mechanics we are interested in applying Newton's Law to *any* glob of fluid of my choice in the flow field—not just to the single apple that fell on Newton's head. And we introduced the fluid mechanics way of doing things in Chapter 2: pick an arbitrary glob of fluid from our flow field, and apply Newton's Law to it. In Chapter 2, the fluid is at rest, so the acceleration vector $\mathbf{a}$ is zero. Remember, hydrostatics is applicable whenever the acceleration vectors of the fluid under consideration is zero (*i.e.* the fluid could be moving). Now in Chapter 3, the fluid acceleration is not zero. The first question is: how do we represent the acceleration vector $\mathbf{a}$ of a tiny (infinitesimal) glob of fluid? §3.1 gives a very rudimentary answer. I will give the more advanced answer given on page 120-121 of YMO, §4.2.1. Acceleration is the *substantial derivative* of velocity, and velocity is the substantial derivative of postion! The answer given by eq.(4.3) on page 120 (or eq.(4.6) on page 121) is the general form of the answer given by eq.(3.1) on page 72—the latter is only true if the flow is *steady*. What is the definition of steady flow? A flow is said to be steady if the flow variables of interest (pressure, velocity, *etc.*) observed at a fixed position of a coordinate system is not a function of time. In other words, when

$$\frac{\partial(.)}{\partial t} = 0,$$

the flow is steady.

§3.2, $\mathbf{F} = m\mathbf{a}$ along a streamline. We will derive the Bernoulli's equation in this section. The Bernoulli's equation eq.(3.4) on page 74 is valid only under the following assumptions: (i) the flow is steady, (ii) only surface force considered is the pressure surface force (no viscous effects included, *i.e.* the flow is *inviscid*), (iii) density is a constant, and (iv) it can be applied only along a streamline. Here $\gamma = \rho g$ and z is the cartesian coordinate pointing up on a flat earth. What does it do for you? Well, if you can determine the Bernoulli's constant on a streamline of interest, then this celebrated equation allows you to find velocity by measuring the pressure and altitude at any point on this streamline. You will use this formula in your labs.

§3.3, $\mathbf{F} = m\mathbf{a}$ Normal to the streamlines. Just scan this. In fact, I don't mind if you skip it. The important point is: in general, the Bernoulli's constants on different streamlines may be different. So: do not assume all inviscid streamlines of a steady flow have the same Bernoulli's constants.

§3.4, 3.5, 3.6, 3.7 Applications! We will talk about them in class.

§3.8, Restrictions ... I have already listed the assumptions underlying the validity of the Bernoulli's equation. But, when can we justifiably claim the viscous surface force is negligible? The answer is: when the flow Reynolds number is large. When can we say the density of air in a flow field is nearly constant? When the flow Mach Number is small. Watch for my exposition in class.

Problems at end of Chapter 3:

- Problem 3.3. Neglect gravity, or consider the streamline under consideration to be at a constant altitude. Now, *pressure gradient* is, in the general case, a vector. Here, along the streamline we are interested in, this vector only has nonzero component in the x-direction. So you need only to fine $\partial p/\partial x$ using eq.(3.4) with s being identified with the coordinate x. Once you find this pressure gradient (in symbolic form), you should be able to integrate it, determining the integration constant at far, far upstream. Now check your answer with the answer obtained by the Bernoulli's equation as instructed.

- Problem 3.17. The front of this wind tunnel is open to the laboratory room where the static pressure of the room air (at rest) is called the atmospheric pressure here. Note: the pressure in the test section is below atomspheric pressure! Use your hydrostatics to work out the manometer reading.
- Problem 3.26. You need the information given in §3.6.2 in YMO for this problem. In addition, you must assume the velocity profile at the two stations of interest are uniform: the axial flow velocity at station #1 is V_1, and that at station #2 is V_2. In English: the flux of mass accross a surface is the product of density, the component of the fluid velocity normal to the surface, and the surface area. Since whatever mass flux comes in from station #1 must come out at station #2, we have

  $$\rho V_1 \frac{\pi D_1^2}{4} = \rho V_2 \frac{\pi D_2^2}{4}$$

  Now you need to apply the Bernoulli's equation for the center streamline. Denote the static pressures at the two stations on the center streamlines by p_1 and p_2. How is p_1 related to the static pressure at the interface between the liquid in the pipe and the manometer fluid? By hydrostatics, of course. How is the static pressure on the left interface related to the static pressure on the right interface? Use hydrostatics! Now relate the static pressure at the right interface to p_2, again use hydrostatics. The answer is given in the back of the book. But I want to see the derivation in details, with English sentences explaining as you go. With the answer, we can use this manometer as a measurement instrument to measure the mass flux in a pipe.
- 3.43. Assume the velocity profile to be uniform in computing Q.
- 3.55. You are told that downstream of the opening, the water depth is a constant and equals to one foot. What is the static pressure distribution with respect to z (the altitude)? It's hydrostatic. So you can show to your satisfaction that the fluid velocity at the constant depth section is a constant. How do you find that constant? Apply Bernoulli's equation to the topmost streamline!!!

Use email or the newsgroup to ask questions.