MAE 222
Mechanics of Fluids
Princeton University

Assignment # 3

February 18, 1998

Due on Wednesday, 2PM, February 25, 1998

Chapter 4 : Fluid Kinematics. This is a chapter of tools.

§4.1, The velocity field . Velocity **V** is a vector. When we hear someone say: "I know the velocity field of this fluid flow field," we get the idea that if we point to any tiny glob of fluid in the flow field and ask him/her "What is the velocity of this glob?" he/she will draw an arrow on the blackboard, and explain to you the amplitude and direction of this arrow. If you then say: "I want numbers!" and he/she will inquire what coordinates you have in mind, and then give you numbers appropriate only to that coordinate system.

On page 111, the formula for **V**given is for people who prefer Cartesian coordinate: $\widehat{\mathbf{i}}$, $\widehat{\mathbf{j}}$ and $\widehat{\mathbf{k}}$ are *unit vectors* in the x, y, z directions, respectively. What is a unit vector? It is a vector with unit amplitude, and its direction has been explained to you. How do you *explain* direction? Usually by a picture; sometimes by an angle in reference to some other known direction, sometimes by using your index finger.

As we have already explained in class, there are the *Eulerian* and the *Lagrangian* descriptions of a flow field. The description on page 111 is Eulerian. When you think about it, Eulerian in the only sensible thing to do. If that is the case, why do we even mention the Lagrangian description? Because! Because Law of Physics are normally stated in the Lagrangian sense. When Newton says: $\mathbf{F} = m\mathbf{a}$ for his apple, he is applying his second law to the apple as it falls to the ground. When he says the z-velocity component of the apple is w=-gt, we all know that it refers to the apple always, and not to some imaginary point fixed in space where the apple happens to pass through.

Know what a streamline is: it is a line whose tangent at every point is parallel to the fluid velocity vector there. The streakline and the pathline are also defined in this section. The main thing to learn is that they are, in unsteady flow, not the same as the streamline. If you mark the fluid passing a fixed point in space (by a dye or a source of smoke) and then take a snapshot, what you have is a streakline. If you mark one tiny glob of fluid, and let it trace its trajectory for you over time, what you have is a pathline. If the flow is steady, these three kind of lines are the same. Don't spend too much time on Example 4.2 and 4.3. They are merely exercises of solving (micky mouse) ordinary differential equations—once you have understood the concepts.

§4.2, The Acceleration Field. Here comes the substantial derivative (or material derivative, or Lagrangian derivative). We have seen eq.(4.4) before. But this time, you are introduced to the elegant representation of eq.(4.5) using vectors: eq.(4.6). You must agree that eq.(4.6) is prettier than eq.(4.5). We are introduced to the concept of a *gradient*. Given some scalar field (.), the gradient of that scalar is denoted by $\nabla(.)$. For example, given the scalar pressure field $p(\mathbf{x}, t)$, the gradient of pressure is $\nabla p(\mathbf{x}, t)$. If we use Cartesian coordinates, then we have

$$\nabla p \equiv \hat{\mathbf{i}}\frac{\partial p(x,y,z,t)}{\partial x} + \hat{\mathbf{j}}\frac{\partial p(x,y,z,t)}{\partial y} + \hat{\mathbf{k}}\frac{\partial p(x,y,z,t)}{\partial z}. \qquad (1)$$

What happens if we would like to use polar coordinates? We will need to find out how to compute ∇p in polar coordinates (look it up in your old math books). What happens if we would like to use some crazy coordinates such as the streamline coordinates in §4.2.4? In good time, we will get the answer to the last question in this course.

§4.3, Control Volume and System Representations. What is a system? The apple is the system Newton studied as it fell on his head. In general, a system is a collection of matter of fixed identity. In a flow field, a system is a glob of fluid I have arbitrarily identified and have decided to put under surveillance. It moves. What is a control volume? A control volume is an arbitrary volume *fixed in space* that I have decided to put under surveillance. It does not move. (In more sophisticated quarters, some people do talk about moving control volumes. We will stay out of that).

§4.4, Reynolds Transport Theorem This is important stuff! We have done a lot of beating around the bush, just to get ready to deal with this important concept!!! In §4.4, YMO actually derive the Reynolds Transport Theorem twice: a streamtube version (Fig. 4.8), and an arbitrary glob version (Fig.4.9). Personally, I think the arbitrary glob version is much, much easier to read and understand. The punch line of the streamtube version is eq.(4.12); the punch line of the arbitrary glob version is eq.(4.17). There is no question about it: eq.(4.17) is much prettier! What is $\widehat{\mathbf{n}}$? It is our friend, the unit *outward* normal to the element of surface dA. You must understand the physical meaning of each term in eq.(4.17). In fact, you are expected to know this formula by heart.

Problems at end of Chapter 4:

- Problem 4.2. totally straightforward.
- Problem 4.8. to make sure you know how to work out the substantial derivative.
- Problem 4.13. Another substantial derivative problem.
- Problem 4.17. More substantial derivative problem.
- Problem 4.23 Basically, you are asked to find the mass flow rate through this square duct in a steady flow of constant density fluid (water). You are asked to compute it first using the surface $A-B$, next using the surface $C-D$, and last using the wriggly surface $E-F$. Clearly you can do the first two cases, long hand. (use a coordinate on the surface, pick any infinitesimal surface element dA, find the normal component of the fluid velocity, and integrate!) You should not be surprised that these two answers agree. Now, how do you do the last case? YMO did not even give you the equation for the wriggly line. Control Volume comes to the rescue!!! Watch for this in the lectures.